3-28-2017

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IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received January 31, 2017, accepted February 14, 2017, date of publication February 17, 2017, date of current version March 28, 2017.

Digital Object Identifier 10.1109/ACCESS.2017.2671446

IP Flow Mobility in the Industry: From An Economic Perspective

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This work was supported by National Natural Science Foundation of China under Grant 61309031.

ABSTRACT The popularity of social media together with the advancement of mobile Internet applications enabling the uploading of data plays a dominant role in the entire Internet traffic. IP flow mobility (IFOM) is proposed as an effective means to enhance the system capacity by offloading data from the cellular network to WiFi or Femtocells or other complementary networks. Although IFOM has been extensively investigated during the past few years, most of these studies, however, are concerned with IFOM technical issues only; little work regarding the IFOM application has been done from the service providers' perspective. Unlike previous research, in this paper, we address the economic issue involved with the IFOM technology. Specifically, competition among multiple service providers supporting or not supporting IFOM are explored, and a game model for the competition is developed. The Nash equilibrium for the game model is then analyzed. Based on the analysis, an algorithm for Nash equilibrium computation is proposed. Also, numerical experiments are conducted to determine the factors that affect the market share and profit of the service providers. We believe that this research paper will shed light on service providers for the promotion and application of IFOM technology.

INDEX TERMS IP flow mobility (IFOM), service provider, network economics, game theory.

I. INTRODUCTION

The rapid development of smart devices is reshaping the way of people using the Internet ranging from the traditional desktops to mobile terminals, which leads to the exponential increase in wireless data traffic [1], [2]. A recent report released by Ericsson [3] indicates that mobile data traffic will grow 10-fold between 2011 and 2016 (mainly driven by video), and mobile broadband subscriptions having been growing by 60 percent per year and are expected to grow from 900 million in 2011 to almost 5 billion in 2016. In parallel, social medias, such as Facebook, Twitter, and Youtube have significantly prompted users to continuously upload graphics and video files to the servers, resulting in the fact that the amount of uploaded data has exceeded the amount of downloaded data from time to time [4]. Thus, the management of data traffic congestion and quality of service is a challenging issue in the industry that must be resolved.

To address this issue, industries have started developing LTE-advanced networks and standardizing the 5G technology [5], [6]. Although encouraging progress has been made, these efforts are not only expensive but also have limited contributions to mitigating the uploading traffic congestion, especially for the crowded spectrum in 5G cellular networks. Mobile data offloading is proposed as an effective means to deal with this problem. The basic idea of mobile data offloading is to allow cellular data flows to be transmitted through complementary networks such as WiFi and Femtocell. This process can take place anonymously and can be fully and dynamically controlled by the service providers. According to 3GPP Rel. 10 [7], data in LTE networks may be transmitted simultaneously via LTE and the access points of WiFi and Femtocell networks. Such a paradigm is termed IP Flow Mobility (IFOM). The principle of IFOM is illustrated in Fig. 1 where a user device is making a voice call through the base station and uploading data via an access point at the same time, provided that this user needs to complete these two tasks simultaneously and is in the range covered by the access point [5].

In addition to allowing heterogeneous data flows to be transmitted via distinct technologies, IFOM permits the same

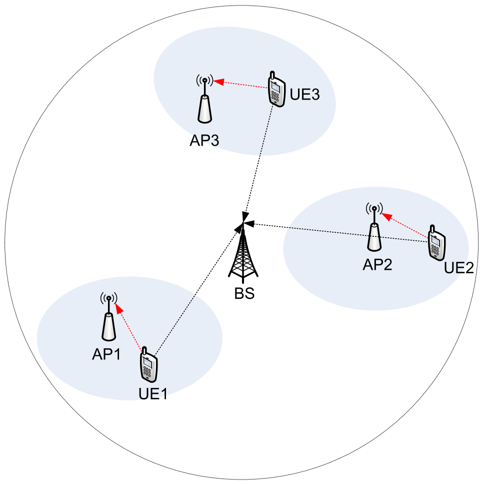


FIGURE 1. An illustration of IFOM technology.

data flow to be split into two (sub-) data flows, and each of which may be transmitted by different wireless networks [8]. Not only service providers and operators can benefit from IFOM in data offloading, but the mobile devices can gain much in terms of power consumption as well. Taking a file transfer as an example, uploading a 10 KB file via an LTE network will roughly consume twice as much energy as uploading the same amount of data via a WiFi network and this ratio would become 2.53 when file size is 10 MB. On the other hand, by Nokia Siemens Networks and ABIs documents, a user would be typically either stationary or in a room when uploading a large volume of data [9]. Based on the fact that over 80% of the mobile data are generated by users in cities or dense areas [9], WiFi and/or Femtocell networks can be set up to intentionally cover these areas. By doing so, IFOM technology can effectively alleviate the burden of the evolved Node B (eNodeB) and improve the quality of service in hot-spot areas.

As IFOM has a great potential to accelerate the development of mobile Internet, it has attracted attentions from both academia and industry during the past few years. While tremendous studies on IFOM have been reported in the research community, most of them, however, are concerned with IFOM technical issues only, little work regarding the IFOM application has been done from an economic perspective. Given the fact that service providers and users are the main players involved in the proliferation of the IFOM technology, how they make their strategy in the case of support or not support IFOM and how the service providers share the market and make profits through IFOM is left untouched. Therefore, it is necessary and interesting to explore the IFOM technology in order to answer the above questions.

Apparently, the number of access points is the key contributing factor to the quality of service since a larger number of access points will generally result in a better quality of experience, provided that they are capable of offloading traffic from eNodeB. In this regard, service providers or network operators are facing a dilemma: if they plan to attract more customers to use their service, they probably are required to offer better services to their customers by installing or renting more infrastructures (access points), this, however, will incur extra costs and investments to the company, and thus they would prefer to use the existing LTE facilities to serve their users as much as possible for saving dollars. On the other hand, if they rely on the existing LTE facilities too much, they will suffer from the inferior quality of service provisioning, and thus would be at risk of losing customers. In addition, the service provider will compete with its peers in the market for its own market share and revenue. So what kind of market strategy and policy, if any, would mostly facilitate the utilization of the IFOM technology and deliver a win-win situation for all competitors? This remains a question that must be answered in the study of the IFOM technology.

In this work, we model the relationships among mobile service providers and users, analyze their marketing strategies, and investigate the contributing factors that impact their behaviors. The contributions of this paper are summarized as follows.

- We investigate the relationships among mobile service providers and users using category theory and game theory. While the category theoretical model presents a macroscopic perspective on such relationships, the game theoretic model provides a microscopic view for the specifics of the competition among service providers and users. The modeling techniques deliver an essential framework for the marketing behaviors of service providers and users to be studied and predicted accurately, which offers valuable guidance for mobile service providers in terms of the application of the IFOM technology.
- We formally prove the existence and the uniqueness of the Nash equilibrium (NE) of the model, and, based on these proofs, propose an iterative algorithm to compute the NE. The algorithm follows the standard Jacobi iterative form for solving systems of linear equations, iteratively updates the data transmission partition ratio of a service provider, and terminates when the difference of such ratios yielded by the last two consecutive iterations is less than a certain small threshold. Moreover, the convergence of the proposed algorithm is explored.
- We conduct thorough numeric experiments to determine the factors that will affect the market share and the profit of service providers with IFOM technology. Several useful insights regarding the strategies of both service providers and users are derived. We believe that this research study will shed light for service providers on the promotion and application of the IFOM technology.

The rest of this paper is organized as follows. Section II briefly reviews the related work. Section III formally establishes the mathematical model for relationships among service providers and users. Section IV presents the NE analysis and the computation of NE is given in Section V. Section VI conducts numerical experiments to investigate various properties regarding the competition. At last, section VII draws the conclusion of this paper.

II. RELATED WORK

Generally, there are evidences that data offloading is playing an increasingly important role in current wireless communication systems. For example, as noted in [10] and [11], WiFi transmissions account for 65% of the total wireless data communications in the U.S. with an average of 55% of power consumption being saved on mobile devices, which is of support for the statement/observation that WiFi data offloading constitutes a substantial portion of wireless communications.

In fact, numerous studies on data offloading have been conducted in the literature. In [12], data offloading in LET networks through WiFi direct D2D technique and the corresponding experiments were investigated. Data offloading in LET-Femtocell networks in terms of maximizing the number of devices and traffic was studied in [13]. In [14], a less-expensive and small-scale data offloading, which uses D2D links and is expected to be both flexible and cost-saving, was suggested by Ishii et al. based on their observation that Femtocell-supported data offloading typically does not work well for large-scale business networks. Al-Kanj *et al.* [15] presented a somehow different data offloading scheme for D2D ad hoc systems where data are classified into different types first and then, according to their types, are transmitted to different (intermediate) mobile devices which will further send the received data to other mobile devices. An optimization technique of data offloading was proposed by Kim *et al.* [16] which can be expected to effectively improve the performance of data offloading. Another data offloading optimization technique, which is based on the download contents as well as the analysis and predication by using Markov chains, was given by Malandrino *et al.* [17]. Yet, a new data offloading mechanism which considers the cluster structure of wireless devices was investigated in [18], and a dynamic programming-supported algorithm for optimizing delayed WiFi offloading was proposed in [19].

It is worth noting that there are some studies on data offloading in the literature that are conducted from some economic perspectives, rather than from technical telecommunication viewpoints. A game theory model was proposed by Iosifidis *et al.* [20] to probe the economic interests between mobile service providers and users in the environment of WiFi or Femtocell data offloading. In [21], a two-stage game was used to simulate the beneficial gains of consumers and mobile network operators, and their study reveals that the close relationship between the packages subscribed by users and their prices may have significant economic impacts on both consumers and service providers. Zhuo *et al.* [22] presented an incentive framework for 3G data offloading where mobile service providers and users mutually negotiate based on the calculation of their own economic benefits that can be obtained from data offloading. The service provider may need to deal with each individual user with an individual and distinct strategy, and as the negotiation evolves, an equilibrium of interests on both sides is expected to reach by reversed auctions. Similar to what proposed in [22], a three-party incentive framework was suggested in [23] where users, mobile network service providers, and WiFi operators all negotiate one another. In particular, mobile service providers will need to compete and bargain with WiFi operators in terms of their own economic interests.

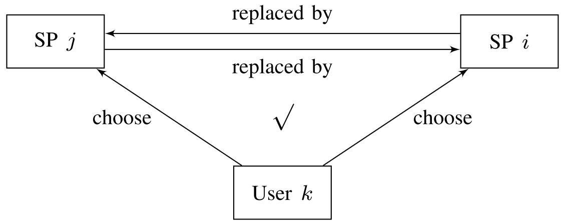
There is a common problem among those economic perspective driven data offloading studies in the literature: most of them assumed (explicitly or implicitly) that there is one mobile network service provider in the market, and all the research work has consequently been performed on the basis of such a (limited) assumption. We, in this paper, tackle the general and more realistic scenario of multiple service providers in the market, present category-theoretical and game-theoretical formulations to model the competitions among multiple mobile service providers and users, investigate the ensuing Nash equilibrium, and conduct extensive experiments to validate the proposed model. Note that this paper is an extension of our previous work [24], and differs from [24] in the sense that is possesses the following new contents: a category theoretic model for formalizing the relationships among mobile service providers and users; a formal proof for the convergence of the NE-computing algorithm; and a set of numerical experiments showing the uniqueness of the NE.

III. SYSTEM MODEL AND GAME FORMULATION

We consider the scenario where there are multiple service providers (SPs) offering mobile Internet services to multiple users in the market. SPs rent or install some access points (APs) within the covered areas of their base stations for data offloading. In the following, we first give a category theoretic formalization of the scenario, and then present a game theoretic model for it. While the category theoretical model offers an upper-level and yet mathematically clear perspective on the relationships among SPs and users, the game theoretic model handles the lower-level specifics and technicalities of the competitions among them.

A. AN OLOG MODEL

Category theory [25] is a branch in mathematics that has recently been recognized to have a wide range of applications in sciences [26]. An Olog [26] diagram describing the relationships among users and service providers is given below.



In this diagram, the bottom rectangle represents an arbitrary user, and top two rectangles represent two arbitrary service providers. An edge from the user to an SP signifies that this user selects (or subscribes) this SP for his mobile communication purposes; an edge from one SP to other signifies the fact that the former can be replaced *by* the latter in terms of the services provided by them. For example, the top edge from SP i to SP j means that the SP i (and its services) can be completely replaced by SP j in the market of mobile communications. The check mark in the triangle denotes that this diagram is *commutative*. That is, the "combination" of any two edges "in the same direction" is the same as another edge that stars with the source of the first edge and ends with the target of the second edge. For instance, the edge from user k to SP i (labeled with "choose") followed by the edge from SP i to SP j (labeled with "replaced by") is the same as the edge from user k to SP j (labeled with "choose"), which can be practically interpreted as "when user k chooses SP i as his service provider and SP i is replaced by SP j, then user k chooses SP j as his service provider."

In fact, the set of all SPs and users in a mobile communication market, connected by the structure indicated in such a diagram, forms a category. Formally,

Proposition 1: For a mobile communication market supported with the IFOM technology, we assume that any SP can be replaced by any other SP in terms of services provided by them (of course, an SP can be replaced by itself), any user can potentially choose any SP as its mobile service provider, and any user does not have any relationship with any other user (except itself, of course). If we use $\mathbb{X}$ to denote the collection of all SPs, all users, and all the relationships among SPs and users in such a market, then $\mathbb{X}$ is a (mathematical) category.

Proof: We conduct the proof by checking the following conditions (as required in the definition of a category).

- Objects. An object in $\mathbb{X}$ is either an SP or a user.
- Morphisms. A morphism in $\mathbb{X}$ is a relationship from one object to another object (including the relationship from an object to itself; see below). Since morphisms are relationships, there is at most one morphism from an object to another object. The domain of a morphism f, $dom(f)$, is the source object of the relationship, and the codomain of a morphism f, $cod(f)$, is the target object of the relationship. We write $f : A \to B$ to signify that f is a morphism with $dom(f) = A$ and $cod(f) = B$.
- Composition of morphisms. The composition of two morphisms f and g with $dom(g) = cod(f)$, $g \circ f$, is defined to the *unique* morphism from $dom(f)$ to $cod(g)$.
- Identity morphisms. For any object A in $\mathbb{X}$,
  - If A is an SP object, then the identity morphism of A, id_A, is the "SP can be replaced by itself" relationship.
  - If A is a user object, then the identity morphism of A, id_A, is simply the "the user is only related to itself" relationship.
- Identity law. For any object A in $\mathbb{X}$, by the way in which morphism composition is defined, we clearly have

$$id_A \circ f = f \quad \text{and } g \circ id_A = g$$

  for any morphism f with $cod(f) = A$ and any morphism g with $dom(g) = A$.
- Associativity law. For any three morphisms $f : A \to B, g : B \to C, h : C \to D$, again, by the definition of the composition of morphisms, we can see that

$$h \circ (g \circ f) = (h \circ g) \circ f.$$

The fact that $\mathbb{X}$ is a category follows immediate from the above observations. ■

B. GAME FORMULATION

Game Theory is a powerful tool concerned with the analysis of strategies for dealing with competitive situations where the outcome of a participant's choice of action depends critically on the choice of actions of other participants. With some successful applications in wireless communications [27], [28], Game Theory is leveraged here to characterize, specify, and compute the relationship between SPs and users and that among SPs themselves.

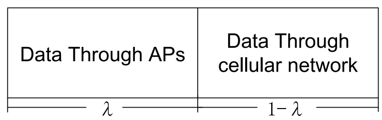


FIGURE 2. Data offloading service partition.

With IFOM support, data uploaded from users is split into two parts: one is transmitted via the base station and the other is transmitted through the APs. Assume that the amount of data transmitted by APs or BS is decided by SP as suggested in Fig. 2, for each total unit amount of data uploaded by users, SP determines that $\lambda (0 < \lambda < 1)$ amount of data transmitting via access points and $(1 - \lambda)$ amount of data transmitting via BS. Note that the value of λ depends on the amount of APs, a larger number of access points will generally result in a wider coverage area and thus a larger λ.

We use p to denote the user's cost for uploading a unit amount of data. μ_{LTE}, μ_{AP} are used to denote the positive outcome with the LTE network when uploading a unit of amount of data and the positive outcome with the APs. The values of μ_{LTE} and μ_{AP} can be the indicators of users' quality of experience with the related services. As APs generally deliver a faster data transmission than LTE networks, we can reasonably assume $\mu_{AP} > \mu_{LTE}$. Let U_{UE} be the users' utility when uploading a unit amount of data. Thus, for SP i's users, we have

$$U_{UE}(\lambda_i) = \mu_{LTE} \cdot (1 - \lambda_i) + \mu_{AP} \cdot \lambda_i - p, \tag{1}$$

where we assume that μ_{LTE} and μ_{AP} are in the same order of magnitude with user's cost p.

The above equation can also be written as

$$U_{UE}(\lambda_i) = \mu_{LTE} + (\mu_{AP} - \mu_{LTE}) \cdot \lambda_i - p. \tag{2}$$

Note that the value of U_{UE} reflects users'preference with respect to choosing SP. To a given SP, the larger the value of U_{UE}, the more preferable it is.

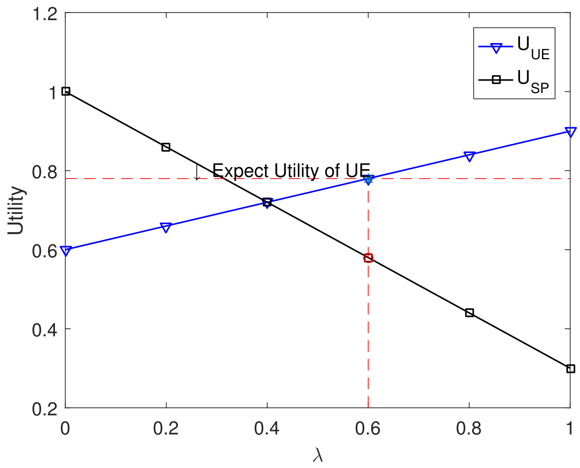


FIGURE 3. Correlations between SP's utility and the user's utility and λ.

Similarly, denote c_{LTE} , c_{AP} as the cost of transmitting a unit amount of data via LTE network (BS) and through APs, respectively. We assume that $c_{AP} > c_{LTE}$ for the reason that enabling data offloading will typically need to rent or set up more APs by an SP. Let U_{SP} be the utility of SPs when a unit amount of data is transmitted. For SP i, we have

$$U_{SP}(\lambda_i) = p - c_{LTE} \cdot (1 - \lambda_i) - c_{AP} \cdot \lambda_i, \tag{3}$$

which can also be written as

$$U_{SP}(\lambda_i) = p - c_{LTE} - (c_{AP} - c_{LTE}) \cdot \lambda_i, \tag{4}$$

where $c_{AP} - c_{LTE} > 0$.

The relationships between SP's utility and λ as well as between user's utility and λ are illustratively depicted in Fig. 3. As this figure shown, a larger λ results in a larger U_{UE}, but a smaller U_{SP}. This indicates that the utility functions are reasonably designed because a larger λ means more data transmission over APs, which, respectively, means a higher cost (thus low utility) for SPs and a higher benefit (utility) for users. Therefore, users would prefer to a larger number of APs so that their utility could be improved toward maximization. On the other hand, SPs would prefer that data be transmitted via the existing LTE network so that the cost for installing new or renting more APs would be reduced, and the utility would be increased. In other words, when an SP makes its strategies, it requires to not only consider the competition between itself and other peer SPs, but also seek a trade-off between the maximization of its own utility and the maximization of the user's utility.

Discount factor is an important parameter in the Game Theory. It roughly describes the patience of players in a game, and is spanned from 0 to 1. In our game model, the discount factor determines the competition capabilities of an SP, that is, a larger discount factor would lead to a more patient of a player.

Let δ_{UE} and δ_{SP} be the discount factor of users and SPs respectively. Considering the profit depends on its services to users and there are multiple SPs in the market competing one another, we can reasonably stipulate that $\delta_{UE} > \delta_{SP}$. In the following section, we will choose an arbitrary SP as the objective to formulate its relationship with users as an infinitely repeated game and investigate the theoretic properties of this game.

IV. NASH EQUILIBRIUM ANALYSIS

A. EQUILIBRIUM BETWEEN SP AND USER

It is practical to assume that a user does not tend to switch his service provider in a short time span when there are multiple SPs competing for the subscribers in the market. Within this short time span, SP i adjusts the value of λ_i, $0 < \lambda_i < 1$ in order to satisfy the user's need. So the relationship between SP i and the user can be formulated under the framework of infinitely repeated games, where users consume the service and experience the service quality provided by the SP i, and the SP i learns about user's satisfaction degree by monitoring the number of users subscribing to its service. Evidently, the model is a repeated game with complete information.

At the initial stage of the game, SP i configures a service partition λ_i^1 to evaluate the user's potential response **(agree or disagree)** to this service partition, and then SP adjusts the service partition in light of the potential response. For example, if the user's utility $U_{UE}^i(\lambda_i^1)$ is lower than the user's expected utility $U_{UE}^i(\lambda_i^0)$, "**disagree**" decision probably will be made by the user and SP i will subsequently set another service partition λ_i^2 to the user such that $U_{UE}^i(\lambda_i^2) \geq U_{UE}^i(\lambda_i^0)$. The game repeats until an agreement on the service partition value set by SP i is achieved.

It is known in Game Theory that if a player's strategy profile cannot yield any better utility with one deviation under any possible circumstances, then this strategy would be the subgame perfect strategy. For dynamic games with complete information, a combination of subgame perfect Nash equilibrium must be the Nash equilibrium of the entire game. Thus, we assume that the game reaches an equilibrium at the t-th ($t \geq 1$) round. As such, ($\lambda_i^t \geq \lambda_i^0$, agree) would be the equilibrium strategy profile for SP i and the user, such that $U_{UE}^i(\lambda_i^t) \geq U_{UE}^i(\lambda_i^0)$ and $U_{SP}^i(\lambda_i^t) \leq U_{SP}^i(\lambda_i^0)$, where λ_i^0 is the expected service partition of the user. Suppose SP i deviates the equilibrium strategy at t-th round of the game by setting $\hat{\lambda}_i^t < \lambda_i^0$. Then in this case the user's response would be "**disagree**", and starting at the $(t+1)$-th round, the SP i's strategy and the user's strategy would come back to ($\lambda_i^{t+1} \geq \lambda_i^0$, **agree**).

As a larger service partition value would lead to a smaller utility for SP i, $\lambda_i^{t+1} = \lambda_i^0$ would be the best choice for SP i. Thus, there exists recurrent strategy profile in the infinitely repeated game between SP i and the user. Based on Nash recurrence theorem and one-deviation property, we propose the following theorem.

Theorem 1: When there is only a single SP in the market (i.e., $n = 1$), the infinitely repeated game between this SP i and the user has no Nash equilibrium. That is, the following *recurrent strategy profile*

$$\{(\lambda_i = \lambda_i^0, agree), (\lambda_i < \lambda_i^0, disagree)\}$$

is not a subgame perfect equilibrium.

Proof: Let $\sum U_{SP}^i$ and $\sum \hat{U}_{SP}^i$ be total utility of SP i when it does not and does deviate at the t-th round in the game. When SP i deviates from λ_i^t to $\hat{\lambda}_i^t$ at the t-th round, we have $\hat{\lambda}_i^t < \lambda_i^0$ and $U_{SP}^i(\hat{\lambda}_i^t) > U_{SP}^i(\lambda_i^t) \geq U_{SP}^i(\lambda_i^0)$. At this point, the user will take the strategy: **disagree**. Starting at the $(t+1)$-th round, the strategies of the SP i and the user will go back to the equilibrium ($\lambda_i = \lambda_i^0$, **agree**). Note that the utility of SP $U_{SP}^i(\lambda_i^{t+1})$ at the $(t+1)$-th round can be regarded as the utility at the t-th round multiplied by the discount factor, that is,

$$U_{SP}^i(\lambda_i^{t+1}) = \delta_{SP} U_{SP}^i(\lambda_i^0).$$

Similarly,

$$U_{SP}^i(\lambda_i^{t+2}) = \delta_{SP}^2 U_{SP}^i(\lambda_i^0)$$

and

$$U_{SP}^i(\lambda_i^{t+k}) = \delta_{SP}^k U_{SP}^i(\lambda_i^0)$$

for any natural number k. When SP i deviates the equilibrium strategy at the t-th round in the game, the user's response would be "**disagree**". Thus, from the $(t+1)$-th round, SP i will keep the equilibrium strategy again. So

$$\begin{aligned}\sum_{l=1}^{\infty} \hat{U}_{SP}^i &= \sum_{l=1}^{t-1} U_{SP}^i + U_{SP}^i(\hat{\lambda}_i^t) + \delta_{SP} U_{SP}^i(\lambda_i^0) \\ &\quad + \delta_{SP}{}^2 U_{SP}^i(\lambda_i^0) + \cdots = \sum_{l=1}^{t-1} U_{SP}^i \\ &\quad + U_{SP}^i(\hat{\lambda}_i^t) + \frac{\delta_{SP}}{1-\delta_{SP}} U_{SP}^i(\lambda_i^0).\end{aligned}$$

When SP i does not deviate, we have

$$\begin{aligned}\sum_{l=1}^{\infty} U_{SP}^i &= \sum_{l=1}^{t-1} U_{SP}^i + U_{SP}^i(\lambda_i^t) + \delta_{SP} U_{SP}^i(\lambda_i^0) \\ &\quad + \delta_{SP}{}^2 U_{SP}^i(\lambda_i^0) + \cdots = \sum_{l=1}^{t-1} U_{SP}^i \\ &\quad + U_{SP}^i(\lambda_i^0) + \frac{\delta_{SP}}{1-\delta_{SP}} U_{SP}^i(\lambda_i^0).\end{aligned}$$

Since $U_{SP}^i(\hat{\lambda}_i^t) > U_{SP}^i(\lambda_i^0)$, we see

$$U_{SP}^i(\hat{\lambda}_i^t) + \frac{\delta_{SP}}{1-\delta_{SP}} U_{SP}^i(\lambda_i^0) > U_{SP}^i(\lambda_i^0) + \frac{\delta_{SP}}{1-\delta_{SP}} U_{SP}^i(\lambda_i^0).$$

That is, $\sum \hat{U}_{SP}^i > \sum U_{SP}^i$. Hence, deviation strategy would be preferred by SP i, since it can always produce a higher utility for SP i. ■

B. THE COMPETITION AMONG SPS

When a user's utility with his/her current SP i is lower than the expected, this user may consider switching to another SP j ($j \neq i$). So there exists a competition among SPs in the market, which will continue until an equilibrium is reached. Considering that during the competition, each SP will strive for its own maximal profit, we therefore model this competition as a non-cooperative game.

Assume that $\lambda_1, \lambda_2, \ldots, \lambda_n$ are used to denote SPs' network service partition value, $R_{SP}^1, R_{SP}^2, \ldots, R_{SP}^n$ are used to denote SPs' market share, and $\pi_1, \pi_2, \ldots, \pi_n$ are used to denote SPs' profit, respectively. Note that the service (or part of it) provided by any SP i may be replaced by another SP $j (i \neq j)$ in the market, we have

$$\begin{cases} R_{SP}^1 = A_1 + a_1\lambda_1 - b_{12}\lambda_2 - b_{13}\lambda_3 - \cdots - b_{1n}\lambda_n \\ R_{SP}^2 = A_2 + a_2\lambda_2 - b_{21}\lambda_1 - b_{23}\lambda_3 - \cdots - b_{2n}\lambda_n \\ \ldots \\ R_{SP}^n = A_n + a_n\lambda_n - b_{n1}\lambda_1 - b_{n2}\lambda_2 - \cdots - b_{n(n-1)}\lambda_{n-1}, \end{cases} \tag{5}$$

where each A_i ($i = 1, \ldots, n$) represents the base market share of SP i. b_{ij} ($0 < b_{ij} \leq 1$, $i, j = 1, \ldots, n$, $i \neq j$) is the replacement coefficient which indicates the extent of the possibility that SP i can be replaced by SP j. A larger value of b_{ij} signifies a higher possibility that SP i can be replaced by SP j. a_i ($0 < a_i < 1$, $i = 1, \ldots, n$) is the service coefficient of SP i, which represents how much of SP i's market share will be affected by the data services, a larger value of a_i indicates that SP i's business depends more on the data services it provides. Thus, the profits of these SPs can be computed as shown in formula (8), as shown at the top of the next page.

The profit maximization of these SPs can be formulated as

$$\begin{cases} \max\limits_{0<\lambda_1<1} [\pi_1(\lambda_1, \lambda_2, \ldots, \lambda_n)] \\ \max\limits_{0<\lambda_2<1} [\pi_2(\lambda_1, \lambda_2, \ldots, \lambda_n)] \\ \ldots \\ \max\limits_{0<\lambda_n<1} [\pi_n(\lambda_1, \lambda_2, \ldots, \lambda_n)]. \end{cases} \tag{6}$$

Toward solving (6), we need

$$\begin{cases} \dfrac{\partial \pi_1(\lambda_1, \lambda_2, \cdots, \lambda_n)}{\partial \lambda_1} = 0 \\ \dfrac{\partial \pi_2(\lambda_1, \lambda_2, \cdots, \lambda_n)}{\partial \lambda_2} = 0 \\ \ldots \\ \dfrac{\partial \pi_n(\lambda_1, \lambda_2, \cdots, \lambda_n)}{\partial \lambda_n} = 0 \\ 0 < \lambda_i < 1, i = 1, \ldots, n, \end{cases} \tag{7}$$

and the solution to (7) would be the Nash equilibrium of the competition of these SPs. Using (8), we can obtain the bonding relationship of the service partition values of these SPs via their best response functions, which are shown in formula (9), as shown at the top of the next page.

$$\begin{cases}\pi_1(\lambda_1,\lambda_2,\cdots,\lambda_n)=R_{SP}^1U_{SP}^1=[A_1+a_1\lambda_1-b_{12}\lambda_2-b_{13}\lambda_3-\cdots-b_{1n}\lambda_n][p-c_{LTE}-(c_{AP}-c_{LTE})\lambda_1]\\ \pi_2(\lambda_1,\lambda_2,\cdots,\lambda_n)=R_{SP}^2U_{SP}^2=[A_2+a_2\lambda_2-b_{21}\lambda_1-b_{23}\lambda_3-\cdots-b_{2n}\lambda_n][p-c_{LTE}-(c_{AP}-c_{LTE})\lambda_2]\\ \cdots\\ \pi_n(\lambda_1,\lambda_2,\cdots,\lambda_n)=R_{SP}^nU_{SP}^n=[A_n+a_n\lambda_n-b_{n1}\lambda_1-b_{n2}\lambda_2-\cdots-b_{n(n-1)}\lambda_{n-1}][p-c_{LTE}-(c_{AP}-c_{LTE})\lambda_n]\end{cases}\tag{8}$$

$$\begin{cases}\lambda_1=f_1(\lambda_2,\lambda_3,\cdots,\lambda_n)=-\dfrac{A_1}{2a_1}+\dfrac{p-c_{LTE}}{2(c_{AP}-c_{LTE})}+\dfrac{b_{12}\lambda_2+b_{13}\lambda_3+\cdots b_{1n}\lambda_n}{2a_1}\\ \lambda_2=f_2(\lambda_1,\lambda_3,\cdots,\lambda_n)=-\dfrac{A_2}{2a_2}+\dfrac{p-c_{LTE}}{2(c_{AP}-c_{LTE})}+\dfrac{b_{21}\lambda_1+b_{23}\lambda_3+\cdots b_{2n}\lambda_n}{2a_2}\\ \cdots\\ \lambda_n=f_n(\lambda_1,\lambda_2,\cdots,\lambda_{n-1})=-\dfrac{A_n}{2a_n}+\dfrac{p-c_{LTE}}{2(c_{AP}-c_{LTE})}+\dfrac{b_{n1}\lambda_1+b_{n2}\lambda_2+\cdots b_{n(n-1)}\lambda_{n-1}}{2a_n}\end{cases}\tag{9}$$

$$Q'=\begin{bmatrix}-1 & \dfrac{b_{12}}{2a_1} & \cdots & \dfrac{b_{1n}}{2a_1} & \dfrac{A_1}{2a_1}-\dfrac{p-c_{LTE}}{2(c_{AP}-c_{LTE})}\\ \dfrac{b_{21}}{2a_2} & -1 & \cdots & \dfrac{b_{2n}}{2a_2} & \dfrac{A_2}{2a_2}-\dfrac{p-c_{LTE}}{2(c_{AP}-c_{LTE})}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \dfrac{b_{n1}}{2a_n} & \cdots & \dfrac{b_{n(n-1)}}{2a_n} & -1 & \dfrac{A_n}{2a_n}-\dfrac{p-c_{LTE}}{2(c_{AP}-c_{LTE})}\end{bmatrix}\tag{10}$$

$$\begin{cases}\pi_1(\lambda_1,\lambda_2)=R_{SP}^1U_{SP}^1=(A_1+a_1\lambda_1-b_{12}\lambda_2)[p-c_{LTE}-(c_{AP}-c_{LTE})\lambda_1]\\ \pi_2(\lambda_1,\lambda_2)=R_{SP}^2U_{SP}^2=(A_2+a_2\lambda_2-b_{21}\lambda_1)[p-c_{LTE}-(c_{AP}-c_{LTE})\lambda_2]\end{cases}\tag{11}$$

We now study the existence and uniqueness of the Nash equilibrium for the model proposed in this paper. A prerequisite for the discussion here is the Glicksberg-Fan fixed point theorem [29], [30], which states that in a finite game, if each player's strategy space is non-empty, compact, and convex, and if each player's payoff function is continuous and quasi-concave over its strategy space, then this game has a Nash equilibrium.

Theorem 2: The game model proposed in Section IV-B has a Nash equilibrium.

Proof: Let N be the set of players, λ_i be play i's strategy, and π_i be player i's payoff function. Then, the following facts can be observed.

- Clearly, the number of mobile network service providers in the market is limited. That is, the set N is finite.
- Note that the strategy space for any player i is $(0, 1)$ since $\lambda_i \in (0, 1)$. Note also that $(0, 1)$ is bounded; thus the strategy space for any player is a non-empty, compact, and convex Euclidean set.
- The fact that $\frac{\partial^2\pi_i}{\partial\lambda_i^2} = -2a_i(c_{AP}-c_{LTE}) < 0$ indicates that π_i is concave (and thus quasi-concave); moreover π_i is obviously continuous.

By the Glicksberg-Fan fixed point mentioned above, the existence of a Nash equilibrium for the proposed model follows immediately. ■

Theorem 3: The Nash equilibrium in Theorem 2 is unique, if $a_i > \frac{1}{2}\sum_{j\neq i}^{n} b_{ij}$ for all $i \in \{1, 2, \ldots, n\}$.

Proof: Acoording to Cachon and Netessine [31], if the response functions of players are contractive, then the Nash equilibrium would be unique. Note that the response functions of players in our game are given by formula (9), so showing the contractiveness of these functions amounts to showing the following Hessian matrix

$$\begin{bmatrix}\dfrac{\partial^2\pi_1}{\partial\lambda_1^2} & \dfrac{\partial^2\pi_1}{\partial\lambda_1\lambda_2} & \cdots & \dfrac{\partial^2\pi_1}{\partial\lambda_1\lambda_n}\\ \dfrac{\partial^2\pi_2}{\partial\lambda_2\lambda_1} & \dfrac{\partial^2\pi_2}{\partial\lambda_2^2} & \cdots & \dfrac{\partial^2\pi_2}{\partial\lambda_2\lambda_n}\\ \vdots & \vdots & \ddots & \vdots\\ \dfrac{\partial^2\pi_n}{\partial\lambda_n\lambda_1} & \dfrac{\partial^2\pi_n}{\partial\lambda_n\lambda_2} & \cdots & \dfrac{\partial^2\pi_n}{\partial\lambda_n^2}\end{bmatrix}$$

is diagonally dominant [32], [33]. For this Hessian matrix to be diagonally dominant, we need

$$\begin{cases}|\dfrac{\partial^2\pi_1}{\partial\lambda_1^2}|>|\dfrac{\partial^2\pi_1}{\partial\lambda_1\lambda_2}|+\cdots+|\dfrac{\partial^2\pi_1}{\partial\lambda_1\lambda_n}|\\ |\dfrac{\partial^2\pi_2}{\partial\lambda_2^2}|>|\dfrac{\partial^2\pi_2}{\partial\lambda_2\lambda_1}|+\cdots+|\dfrac{\partial^2\pi_2}{\partial\lambda_2\lambda_n}|\\ \cdots\\ |\dfrac{\partial^2\pi_n}{\partial\lambda_n^2}|>|\dfrac{\partial^2\pi_n}{\partial\lambda_n\lambda_1}|+\cdots+|\dfrac{\partial^2\pi_n}{\partial\lambda_n\lambda_{n-1}}|.\end{cases}\tag{12}$$

Since

$$\begin{cases}|\dfrac{\partial^2\pi_1}{\partial\lambda_1^2}|=|-2a_1(c_{AP}-c_{LTE})|\\ |\dfrac{\partial^2\pi_2}{\partial\lambda_2^2}|=|-2a_2(c_{AP}-c_{LTE})|\\ \cdots\\ |\dfrac{\partial^2\pi_n}{\partial\lambda_n^2}|=|-2a_n(c_{AP}-c_{LTE})|,\end{cases}$$

and

$$\begin{cases} |\frac{\partial^2\pi_1}{\partial\lambda_1\lambda_2}|+\cdots+|\frac{\partial^2\pi_1}{\partial\lambda_1\lambda_n}|=|(b_{12}+\cdots+b_{1n})(c_{AP}-c_{LTE})| \\ |\frac{\partial^2\pi_2}{\partial\lambda_2\lambda_1}|+\cdots+|\frac{\partial^2\pi_2}{\partial\lambda_2\lambda_n}|=|(b_{21}+\cdots+b_{2n})(c_{AP}-c_{LTE})| \\ \cdots \\ |\frac{\partial^2\pi_n}{\partial\lambda_n\lambda_1}|+\cdots+|\frac{\partial^2\pi_n}{\partial\lambda_n\lambda_{n-1}}|=|(b_{n1}+\cdots+b_{n(n-1)}) \\ \qquad\qquad\qquad\qquad\qquad (c_{AP}-c_{LTE})|, \end{cases}$$

we see that if

$$\begin{cases} a_1 > \dfrac{b_{12}+\cdots+b_{1n}}{2} \\ a_2 > \dfrac{b_{21}+\cdots+b_{2n}}{2} \\ \vdots \\ a_n > \dfrac{b_{n1}+\cdots+b_{n(n-1)}}{2}, \end{cases}$$

then the inequalities in (12) would hold, which consequently implies the contractiveness of the response functions and thus the uniqueness of the Nash equilibrium. ■

Note the coefficient matrix of formula (9) is

$$Q=\begin{bmatrix} -1 & \frac{b_{12}}{2a_1} & \cdots & \frac{b_{1n}}{2a_1} \\ \frac{b_{21}}{2a_2} & -1 & \cdots & \frac{b_{2n}}{2a_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{b_{n1}}{2a_n} & \cdots & \frac{b_{n(n-1)}}{2a_n} & -1 \end{bmatrix}, \tag{13}$$

and its augmented coefficient matrix Q' is displayed in (10), as shown at the top of the previous page.

By the standard theorem regarding the solution existence for non-homogeneous system of equations, we know that (7) has a solution if

$$\begin{cases} r(Q)=r(Q') \\ 0<\lambda_i<1, \quad i=1,\ldots,n \end{cases} \tag{14}$$

where $r(Q)$ and $r(Q')$ are ranks of SPs' coefficient matrix Q and augmented coefficient matrix Q' respectively. Hence, the competing SPs in the market will reach an equilibrium if equation (14) holds. Typically, if $r(Q)\neq r(Q')$ at some point during the course of the SPs' competition, then the values of $r(Q)$ and $r(Q')$ will be eventually equal to each other as the result of the continuous strategy adjustments performed by the SPs and the elimination of some of them by the market.

V. NASH EQUILIBRIUM COMPUTATION

A. AN ITERATIVE ALGORITHM

Following the discussion of the existence and uniqueness of the Nash equilibrium in the previous section, we now delve into the issue of finding the Nash equilibrium. Note that

$$\frac{\partial^2\pi_i}{\partial\lambda_i^2}=-2a_i(c_{AP}-c_{LTE})<0$$

for each player i, which indicates that the profit function π_i of each SP is concave and thus can be maximized. As such, for each SP i, its best response would be service partition value λ^* that maximizes its profit. That is,

$$\begin{cases} \lambda_1^*=\arg\{\max\limits_{0<\lambda_1<1}[\pi_1(\lambda_1,\lambda_2^*,\cdots,\lambda_n^*)]\} \\ \lambda_2^*=\arg\{\max\limits_{0<\lambda_2<1}[\pi_2(\lambda_1^*,\lambda_2,\cdots,\lambda_n^*)]\} \\ \vdots \\ \lambda_n^*=\arg\{\max\limits_{0<\lambda_n<1}[\pi_n(\lambda_1^*,\lambda_2^*,\cdots,\lambda_n)]\}. \end{cases}$$

By using the standard Jacobi iterative method for solving systems of linear equations, these λ_i^* can be computed by the following steps.

1) Initialize each λ_i to a predefined value. That is, $\lambda_i^0=v_i\in(0,1)$.
2) Update each λ_i by formula (9) (λ_i^k is used to represent the outcome of λ_i in its k-th computation).
3) If $|\lambda_i^k-\lambda_i^{k-1}|<\epsilon$ or $\lambda_i^k>1$ (where ϵ is a small predefined threshold), then the computation is terminated, and . Otherwise, repeat the process for the $(k+1)$-th round.
4) For each i, λ_i^* is the Nash-equilibrium λ-value of SP i.

The detailed Jacobi λ_i^* computation algorithm is given in Algorithm 1.

Algorithm 1 Computation of the Nash Equilibrium

```
Input: A_i, a_i, p, c_LTE, c_AP, b_ij, v_i for all
        i, j ∈ {1, 2, ..., n}
Output: λ_i^* for all i ∈ {1, 2, ..., n}.
Initialization λ_i^0 ← v_i, k ← 1, flag_i = 1 for all i
while loop until flag_i = 0 for all i do
    for i = 1 to n do
        if flag_i = 1 then
            λ_i^k ← −A_i/(2a_i) + (p − c_LTE)/(2(c_AP − c_LTE)) + (Σ_{j=1, j≠i}^{n} b_ij λ_j^{k−1})/(2a_i);
            if |λ_i^k − λ_i^{k−1}| < ε or |λ_i^k| ≥ 1 then
                λ_i^* ← λ_i^k, flag_i ← 0;
            end
        else
            λ_i^k ← λ_i^*;
        end
    end
    k ← k + 1
end
```

B. CONVERGENCE OF THE ALGORITHM

We now investigate the convergence issue of the algorithm presented in the previous section. The following theorem gives a sufficient condition for the algorithm to be convergent.

Theorem 4: The Nash equilibrium computation algorithm is convergent if $a_i>\frac{1}{2}\sum_{j\neq i}^{n}b_{ij}$ *for all* $i\in\{1,2,\ldots,n\}$.

Proof: According to [34, p. 47], a sufficient condition for the general multidimensional Jacobi iteration[1]

$$x_i^{(k+1)} = \phi_i(x_1^{(k)}, x_2^{(k)}, \ldots, x_n^{(k)})$$

(where ϕ_i is any function that produces a new value based on old values) to converge is

$$\sum_{j=1}^{n} \left| \frac{\partial \phi_i}{\partial x_j} \right| < 1, \quad \text{for all } i.$$

This condition, when applied to the system of linear equations of the form

$$Ax = b$$

(where $A = (a_{ij})$ is an $n \times n$ matrix, and both x and b are $n \times 1$ column vectors) becomes

$$\sum_{j \neq i}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| < 1, \quad \text{for all } i,$$

or equivalently,

$$|a_{ii}| > \sum_{j \neq i}^{n} |a_{ij}|, \quad \text{for all } i, \tag{15}$$

which simply means that the coefficient matrix A is diagonally dominant. By formula (9), we can see that the linear system of equations that Algorithm 1 attempts to solve is

$$\begin{bmatrix} 2a_1 & -b_{12} & \cdots & -b_{1n} \\ -b_{21} & 2a_2 & \cdots & -b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -b_{n1} & -b_{n2} & \cdots & 2a_n \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} = \begin{bmatrix} -A_1 + \dfrac{a_1(p - c_{LTE})}{c_{AP} - c_{LTE}} \\ -A_2 + \dfrac{a_2(p - c_{LTE})}{c_{AP} - c_{LTE}} \\ \vdots \\ -A_n + \dfrac{a_n(p - c_{LTE})}{c_{AP} - c_{LTE}} \end{bmatrix}.$$

Hence, condition (15) translates to

$$|2a_i| > \sum_{j \neq i}^{n} |-b_{ij}|, \quad \text{for all } i,$$

that is,

$$a_i > \frac{1}{2} \sum_{j \neq i}^{n} b_{ij}, \quad \text{for all } i.$$

■

VI. EXPERIMENTAL RESULTS

Extensive experiments have been conducted toward investigating the following issues: (1) How can SP's strategic decisions affect their market share and profit? (2) Does the number of SPs in the market have any impact on their market share and profit? (3) In what way can SPs' strategies affect the Nash equilibrium? We in this section demonstrate these experimental results. The following parameter setting is used for all experiments in this section. $\mu_{LTE} = 0.6$, $\mu_{AP} = 0.9$, $p = 1.5$, $c_{AP} = 1.2$, $c_{LTE} = 0.5$. Note that the parameters configured here are used for illustrations only, other configurations can be set for the experiments and the same insights can be obtained. The experimental results are statistically collected by averaging 1000 runs, which do not heavily affected by any stochastic factors.

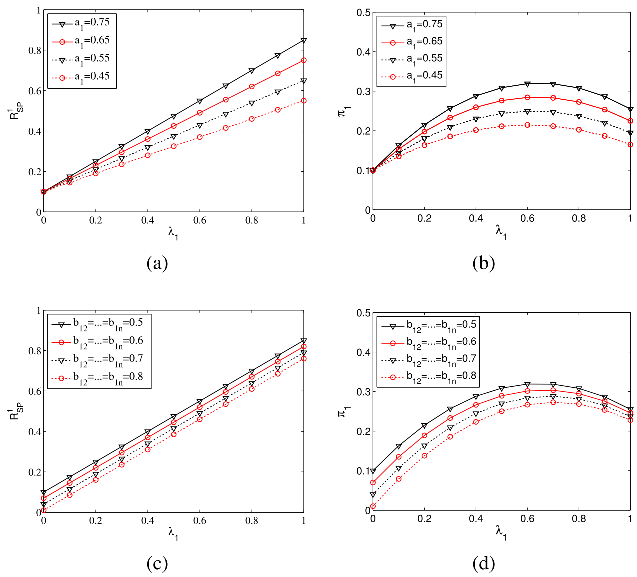


FIGURE 4. Various correlations between SP 1's strategy and its economic gains. (a) Relationships among λ_1, a_1, and R_{SP}^1. (b) Relationships among λ_1, a_1, and π_1. (c) Relationships among λ_1, b_{1i}, and R_{SP}^1. (d) Relationships among λ_1, b_{1i}, and π_1.

A. CORRELATION BETWEEN SPs' BUSINESS STRATEGY AND THEIR ECONOMIC BENEFITS

The business strategy of any SP can be primarily reflected by its network service partition value (λ-value), its service coefficient, and its replacement coefficient. Without loss of generality, we choose SP 1 and study the correlation between its business strategy and its economic benefits.

With $A_1 = 0.25$, $b_{12} = \cdots = b_{1n} = 0.5$ and $\lambda_2 + \lambda_3 + \cdots + \lambda_n = 0.3$, Fig. 4 depicts various such correlations for SP 1. Specifically, Fig. 4(a) shows the correlation among the service partition value λ_1, service coefficient a_1, and market share R_{SP}^1; Fig. 4(b) the service partition value λ_1, service coefficient a_1, and profit π_1; Fig. 4(c) the service partition value λ_1, replacement coefficients b_{1i}, and market share R_{SP}^1; and Fig. 4(d) the service partition value λ_1, replacement coefficients b_{1i}, and market share π_1. We can see clearly from Fig. 4(a) and Fig. 4(b) that for a fixed service partition value, a larger service coefficient would give rise to a larger market share and profit. However, as the service partition value increases, the market share would consistently increase

[1] In [34], Jacobi iterative method is called (basic) Gauss iterative method.

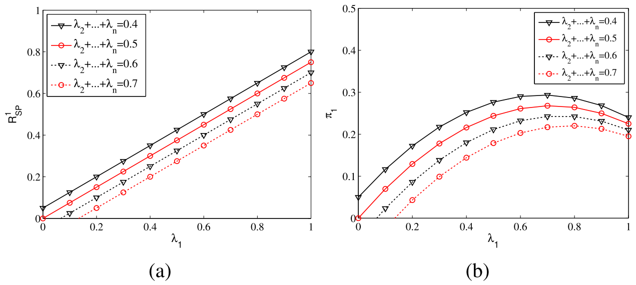


FIGURE 5. Correlations between the number of SPs in the market and the economic benefits of SP 1. (a) Relationships among λ_1, the number of SPs, and R_{SP}^1. (b) Relationships among λ_1, the number of SPs, and π_1.

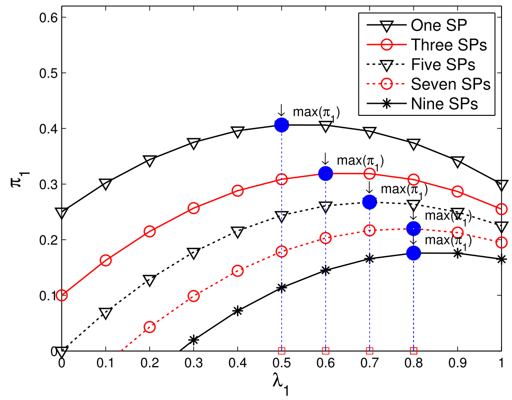


FIGURE 6. A modified redisplay of Fig. 5(b) with specified number of SPs.

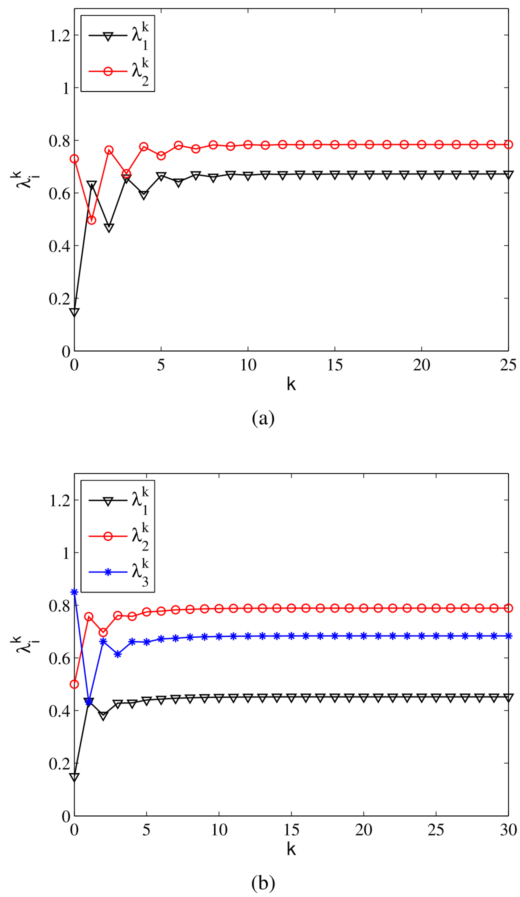


FIGURE 7. The convergence of the λ-value of SPs. (a) Two SPs in the market. (b) Three SPs in the market.

but the profit would first increase and then fall. Fig. 4(c) and Fig. 4(d) exhibit that a higher replacement coefficient would lead to a lower market share and a lower profit as well, when the service partition value is fixed. This result is expected as a higher replacement coefficient means that SP 1 has a higher chance of being replaced by other SPs in the market, and thus has a higher risk of being eliminated from the market. Similar to Fig. 4(a) and Fig. 4(b), Fig. 4(c) and Fig. 4(d) reveal that, as the service partition value increases, the market share would increase all the way up but the profit would increase to a certain point and then start decreasing, which indicates the fact that no SPs can achieve unlimited profits.

B. CORRELATIONS BETWEEN THE NUMBER OF SPs AND THE ECONOMIC BENEFITS OF SP 1

For any SP i,

$$b_{i1}\lambda_1 + \cdots + b_{i(i-1)}\lambda_{i-1} + b_{i(i+1)}\lambda_{i+1} \cdots + b_{in}\lambda_n$$

collectively represent the impact that all other SPs on this SP. In particular, if the replacement coefficients in the above formula are all equal, then $\lambda_1 + \cdots + \lambda_{i-1} + \lambda_{i+1} + \cdots + \lambda_n$ would equivalently represent the number of SPs in the market that will have an economic impact on SP i. Again, without loss of generality, we choose SP 1 and investigate how its business benefits would be influenced by the number of SPs in the market. Fig. 5(a) and Fig. 5(b), respectively, depict the impact of the number of SPs in the market on SP 1's market share and profit with respect to its service partition value λ_1. From these two figures, besides the fact that an increased service partition value would cause an increased market shared and an increased/decreased profit (which is the same as shown in Fig. 4), we can see that an increased number of SPs in the market would lower both the market share and the profit of SP 1, when the service partition value is fixed. This observation matches the real-world competition situation in the market since more number of SPs would intensify the mobile service competition among these SPs, which would most likely render some losses to SP 1. Also, note that Fig. 5(b) reveals that SP 1's service partition must be at least 0.13 in order to have a positive profit when the number of SPs in the market is relatively large ($\lambda_2+\lambda_3+\cdots+\lambda_n = 0.7$).

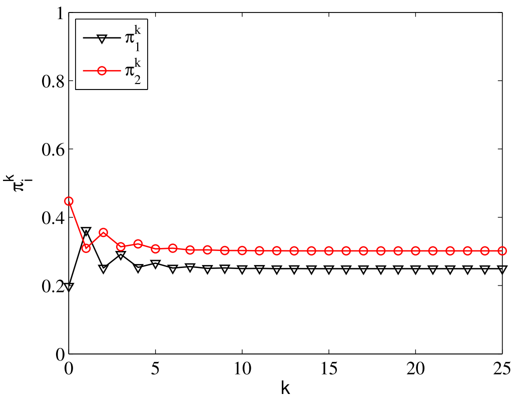
Fig. 6 is a modified redisplay of what is depicted in Fig. 5(b) with clearly specified different number of SPs in the market. Evidently, we see that SP 1 has to keep increasing its service partition value if it would like to continue to make the maximal profit when the number of SPs in the market increases.

C. NASH EQUILIBRIUM ANALYSIS

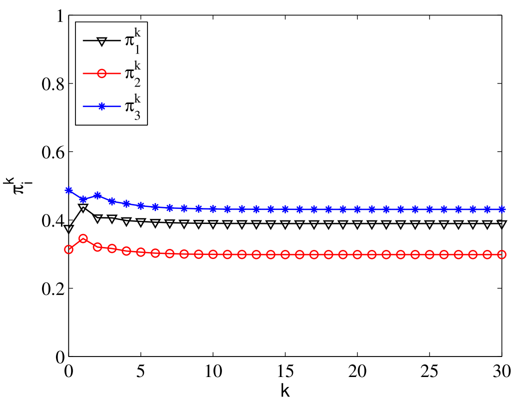
We now delve into the issue of Nash equilibrium.

TABLE 1. Parameter settings for Figs. 7 and 8.

Fig.	p	c_{LTE}	c_{AP}	A_1	a_1	b_{12}	b_{13}	A_2	a_2	b_{21}	b_{23}	λ_1^0	λ_2^0	λ_3^0	A_3	a_3	b_{31}	b_{32}
7(a), 8(a)	1.5	0.5	1.2	0.65	0.55	0.7	n/a	0.45	0.75	0.55	n/a	0.15	0.73	n/a	n/a	n/a	n/a	n/a
7(b),8(b)	1.5	0.5	1.2	0.65	0.55	0.2	0.25	0.45	0.75	0.3	0.35	0.10	0.5	0.85	0.85	0.75	0.4	0.45



(a)



(b)

FIGURE 8. The stabilization of the profit of SPs. (a) Two SPs in the market. (b) Three SPs in the market.

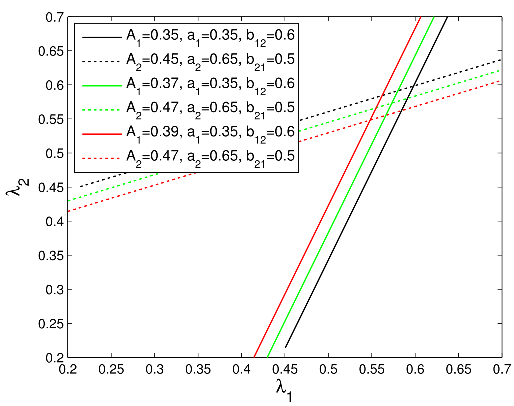


FIGURE 9. The change of the NE point with respect to the variation of A_1, A_2.

Using the data shown in Table 1 (note that the sufficient condition $a_i > \frac{1}{2}\sum_{j\neq i}^{n} b_{ij}$ in Theorems 3 and 4 is satisfied by the data in this table), the uniqueness of the Nash

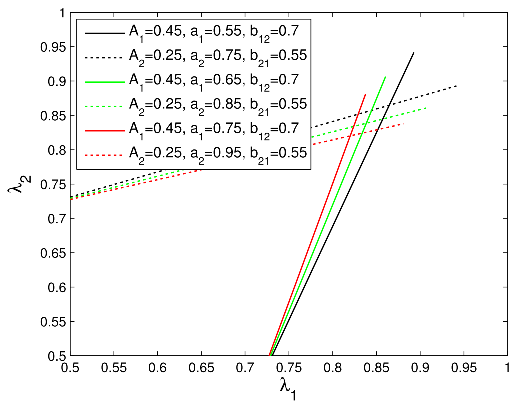


FIGURE 10. The change of the NE point with respect to the variation of a_1, a_2.

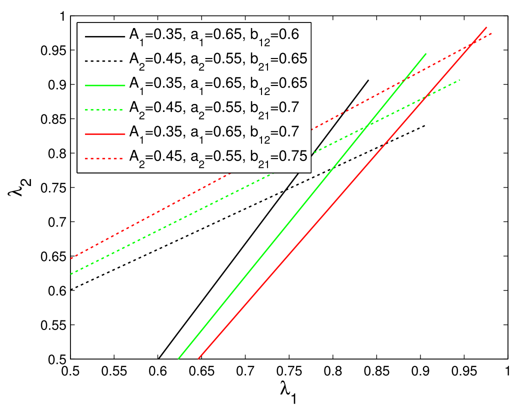


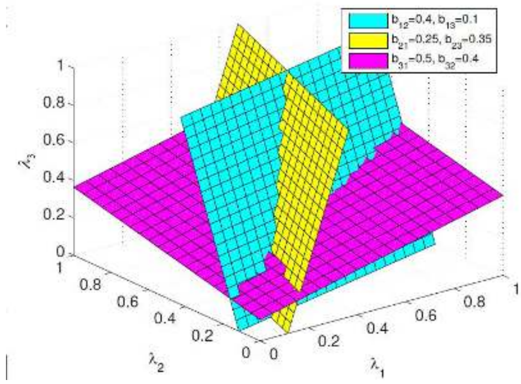
FIGURE 11. The change of the NE point with respect to the variation of b_{12}, b_{21}.

equilibrium (or the convergence of the Nash equilibrium computing algorithm) is demonstrated in Fig. 7, where the two subfigures 7(a) and 7(b) reveal the the cases when there are two and three SPs in the market, respectively. Clearly, the values of λ_i converge to a certain number after a certain amount of iterations, as expected. Similar results, with respect to the profit of SPs rather than their λ-values, are shown in Fig. 8. Again, as expected, the profits of SPs all stabilize at some value after a number of iterations, indicating that an equilibrium state of profits among the SPs has been reached after a number of competitions in the market.

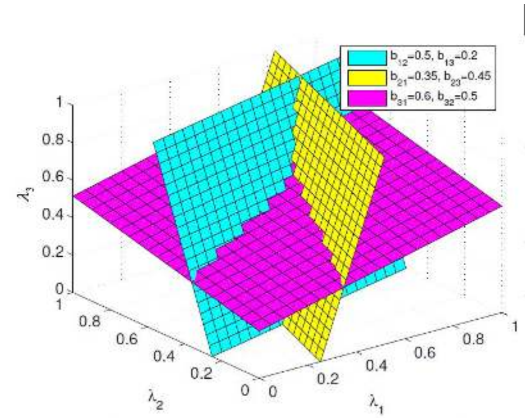
Note that for any player SP i, its NE service partition value λ_i^* primarily depends on its base profit A_i, its service coefficient a_i, its replacement coefficients b_{ij}, and the NE service partition values λ_j^* of other players (SPs). As such, we investigate how the NE point can be affected by these parameters in the rest of this section. For sake of illustrative

TABLE 2. Parameter settings for Figs. 9, 10, and 11.

Fig.	A_1	A_2	a_1	a_2	b_{12}	b_{21}
Fig. 9	varies	varies	0.35	0.65	0.6	0.5
Fig. 10	0.45	0.25	varies	varies	0.7	0.55
Fig. 11	0.35	0.45	0.65	0.55	varies	varies



(a)



(b)

FIGURE 12. The change of the NE point with respect to the variation of b_{ij}.

convenience, we study and demonstrate the case when there are two players SP 1 and SP 2; the general case of multiple players ($n > 2$) can be analyzed in a similar manner. By replacing n with 2 in equation (9), we have

$$\begin{cases} \lambda_1 = f(\lambda_2) = -\dfrac{A_1}{2a_1} + \dfrac{p - c_{LTE}}{2(c_{AP} - c_{LTE})} + \dfrac{b_{12}\lambda_2}{2a_1} \\ \lambda_2 = f(\lambda_1) = -\dfrac{A_2}{2a_2} + \dfrac{p - c_{LTE}}{2(c_{AP} - c_{LTE})} + \dfrac{b_{21}\lambda_1}{2a_2}. \end{cases}$$

Figs. 9, 10, and 11 (in which lines are color-wise paired and the NE point $(\lambda_1^*, \lambda_2^*)$ is the intersection of two same-colored lines) illustrate how the NE point $(\lambda_1^*, \lambda_2^*)$ changes depending on the variation of one of the three parameters (base profit, the service coefficient, replacement coefficient) with the other two being fixed. The settings of these parameters are summarized in Table 2. Evidently, we observe that the NE point $(\lambda_1^*, \lambda_2^*)$ decreases as the values of A_1, A_2 (in Fig. 9) and a_1, a_2 (in Fig. 10) increase, respectively; and increases as the values of b_{12}, b_{21} (in Fig. 11) increase. Analogously, Fig. 12 provides the same observation for the case where there are three players (SPs) in the market.

VII. CONCLUSION

In this paper, we have investigated the relationship between IFOM-supported service provider and user and that among multiple SPs in the mobile communications market. We have formulated those relationships using category theory and game theory. Through the latter one, the competition of SPs are characterized. We have performed the Nash equilibrium (NE) analysis and proposed an iterative algorithm for computing NE. Also, we have examined the correlations between the set of parameters, such as base profits, service coefficients, and replacement coefficients and the set of parameters market shares, profits, and Nash equilibrium. We have found that:

1) SPs with a smaller replacement coefficient have a margin of superiority in the competition. Thus, an SP would consider increasing its service variety and building its business's uniqueness if it is interested in maintaining a strong competitive edge in the market.
2) For an SP, a larger service coefficient indicates a tighter correlation between its service partition value and its market share or profit.
3) An SP's profit increases up to a certain point as its service partition value increases. After that point, the profit would decrease as the service partition value increases. As such, SPs need to make reasonable adjustments to their strategies based on the fluid situation in the market.
4) The NE value of the market increases when the SPs' base profits and service coefficients decrease, and their replacement coefficients increase. This suggests that an SP should focus on improving its service coefficient and base profit, considering that the data-uploading service is typically offered by many SPs and thereby results in a relatively high replacement coefficient.

As our future work, we plan to deepen our current study by investigating the relationships between mobile network operators and complementary network service providers, within the framework of Game Theory .

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