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IQC-Synthesis with General Dynamic Multipliers ⋆

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Abstract: In this paper we generalize our previous results on the synthesis of robust controllers. A novel controller/scaling algorithm is proposed that allows for the use of arbitrary real-rational Integral Quadratic Constraint (IQC) multipliers with no poles on the extended imaginary axis. In contrast to the classical μ-synthesis approaches, the techniques completely avoid gridding as well as curve-fitting. Moreover, while the classical approaches are restricted to the use of real/complex time-invariant or arbitrarily fast time-varying parametric uncertainties, the IQC framework can be employed for a much larger class of uncertainties involving nonlinearities and bounds on rates of time-varying parametric uncertainties. The results are illustrated through a numerical example.

Keywords: Robust control, dynamic Integral Quadratic Constraints, IQC-synthesis, μ-synthesis

1. INTRODUCTION

During the last three decades the synthesis of H_∞-controllers has received a lot of attention (Doyle et al. (1989), Gahinet and Apkarian (1994), Iwasaki and Skelton (1994)). Despite the fact that the developed synthesis techniques had a major impact in the control community and have been used for numerous applications, they generally can only be employed in a reliable way if the involved LTI models describe the real system sufficiently well. If, on the other hand, the LTI models are uncertain (i.e. inaccurate), the problem of synthesizing controllers that are optimally robust to these uncertainties is much harder, and, apart from some special cases, no convex solution for the associated optimization problem is known.

Despite the complexity of the robust $\mathscr{H}_\infty$-controller synthesis problem, a number of very useful but non-optimal methods, such as μ- and K_m-synthesis, have been suggested in the literature (see e.g. Safonov et al. (1993), Safonov and Chiang (1993), Helmersson (1995), Young (1994)). Essentially these methods rely on an iteration between the synthesis of an optimal $\mathscr{H}_\infty$-controller, while fixing the involved scalings, and finding scalings by computing an upper-bound of the structured singular value, while fixing the controller. Although the individual steps admit convex solutions and the overall procedure converges, there is no guarantee whatsoever that the resulting controller is globally optimal. Moreover, as another serious limitation of all these methods, they can only be employed, without introducing too much conservatism, for the class of real/complex time-invariant or arbitrarily fast time-varying parametric uncertainties. Therefore, application to a larger class of uncertainties might yield overly conservative results. The methods, nevertheless, are numerically reliable and have been successfully applied in many practical applications.

A well known framework for the analysis of uncertain systems is the IQC approach, which was initially formulated by Megretski and Rantzer (1997). IQCs are very useful in capturing a rich class of uncertainties. One could for example think of repeated static nonlinearities such as saturation (see e.g. Zames and Falb (1968), Chen and Wen (1995), D'Amato et al. (2001)) or smoothly time-varying parametric uncertainties as well as uncertain time-varying time-delays, both with bounds on the rate-of-variation (see e.g. Helmersson (1999), Köroğlu and Scherer (2006), Kao and Rantzer (2007)). Until recently, the IQC framework could also be employed for a limited number of synthesis applications, if the corresponding IQC-multipliers were restricted to be static (see e.g. Packard (1994), Geromel (1999), Scherer (2001), Ferreres and Roos (2005)). Preliminary work on synthesis based on dynamic IQC-multipliers has been done by Scherer and Köse (2008). One of the essential difficulties was the characterization of nominal stability of the closed-loop system. This problem has been resolved in Scherer and Köse (2008), by means of an exact characterization of closed-loop stability in terms of a suitable positivity constraint on the LMI solutions.

The goal of this paper is to generalize our previous results on the synthesis of robust controllers (Veenman and Scherer (2010)) from the use of dynamic DG-scalings to the use of arbitrary real-rational IQC-multipliers with no poles on the extended imaginary axis. Inspired by the results of Scherer and Köse (2008), we formulate an exact characterization of nominal stability in terms of simplified state-space relations with lower state dimensions and LMI certificates for the corresponding Frequency Domain Inequalities (FDI). Analogous to the existing μ- and K_m-synthesis techniques, it will be shown, by employing the insight of Gahinet and Apkarian (1994), Iwasaki and Skelton (1994) and Megretski and Rantzer (1997) that the results are also very useful in a controller/scaling algorithm for the synthesis of robust controllers. The suggested algorithm enables us to perform robust controller synthesis in a systematic fashion for a much larger class of uncertainties if compared to the classical μ and K_m-synthesis approaches.

The paper is organized as follows. After having introduced some preliminaries in Section 2, we formally state the problem in Section 3. Then, in Section 4 and 5, we give a short recap on $\mathscr{H}_\infty$-synthesis and IQC-analysis respectively. The core of the paper is found in Section 6 and forms the basis for the main results in Section 7. We conclude the paper with a numerical example in Section 8 and some final remarks in Section 9.

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2. NOTATION AND PRELIMINARIES

$\mathscr{L}_2$ denotes the space of vector-valued square integrable functions defined on $[0,\infty)$, with the usual inner product given by $\langle .,.\rangle$. $\mathscr{RL}_\infty^{m\times n}$ ($\mathscr{RH}_\infty^{m\times n}$) denotes the space of all real-rational and proper (and stable) matrix functions that have no poles on the extended imaginary axis (in the closed right-half complex-plane). By an operator we mean a map $G:\mathscr{L}_2^a\to\mathscr{L}_2^b$, and for two given linear operators $G=\left(\begin{smallmatrix}G_{11}&G_{12}\\G_{21}&G_{22}\end{smallmatrix}\right)$ and Δ, the Linear Fractional Transformation (LFT) $\Delta\star G$ is defined as $G_{22}+G_{21}\Delta\,(I-G_{11}\Delta)^{-1}G_{12}$, assuming that $(I-G_{11}\Delta)^{-1}$ exists. Realizations of Linear Time Invariant (LTI) systems are denoted by the standard notation $G=\left[\begin{array}{c|c}A&B\\\hline C&D\end{array}\right]:=C\,(sI-A)^{-1}B+D$. We finally use $(\star)^*G_2G_1$ to abbreviate $G_1^*G_2G_1$.

3. PROBLEM FORMULATION

Consider the uncertain plant in Figure 1 where G represents a proper, possibly unstable and weighted LTI system that admits a minimal realization of the form

$$\begin{pmatrix}q\\z\\y\end{pmatrix}=\underbrace{\left[\begin{array}{c|ccc}A&B_{\rm p}&B_{\rm w}&B_{\rm u}\\\hline C_{\rm q}&D_{\rm qp}&D_{\rm qw}&D_{\rm qu}\\C_{\rm z}&D_{\rm zp}&D_{\rm zw}&D_{\rm zu}\\C_{\rm y}&D_{\rm yp}&D_{\rm yw}&0\end{array}\right]}_{G}\begin{pmatrix}p\\w\\u\end{pmatrix},\ A\in\mathbb{R}^{n\times n},\tag{1}$$

where respectively $\mathrm{col}(p,w,u)\in\mathscr{L}_2^{n_{\rm p}+n_{\rm w}+n_{\rm u}}$ and $\mathrm{col}(q,z,y)\in\mathscr{L}_2^{n_{\rm q}+n_{\rm z}+n_{\rm y}}$ denote the collection of uncertainty, exogenous disturbance and control input signals and uncertainty, performance and measurement output signals. The plant is subject to perturbations of the bounded and causal operator Δ, which is allowed to take values from a given star-convex set $\mathbf{\Delta}$ with center zero (i.e. $[0,1]\mathbf{\Delta}\subseteq\mathbf{\Delta}$), describing the uncertainties and/or nonlinearities.

The main goal in robust control is the synthesis of a controller K that dynamically processes the measurement y in order to provide a control input u that robustly stabilizes the system interconnection of Figure 1 while the $\mathscr{L}_2$-gain from w to z is rendered less than γ. Here K is a proper LTI system that admits a minimal realization of the form

$$u=\underbrace{\left[\begin{array}{c|c}A_K&B_{\rm y}\\\hline C_{\rm u}&D_{\rm uy}\end{array}\right]}_{K}y,\ A_K\in\mathbb{R}^{n\times n}.\tag{2}$$

Given the plant G and the set $\mathbf{\Delta}$, the goal of this paper can now be formally stated as follows: Design a robust controller K such that, for all $\Delta\in\mathbf{\Delta}$, the interconnection of Figure 1 is stable and the $\mathscr{L}_2$-gain from w to z is rendered less then $\gamma>0$.

Analogously to the existing μ- and K_m-synthesis techniques and our previous work (Veenman and Scherer (2010)), the main idea of this paper is to develop an algorithm that relies on an iteration between the synthesis of an optimal $\mathscr{H}_\infty$-controller, while fixing the involved IQC-multipliers, and finding IQC-multipliers by performing an IQC-analysis, while fixing the controller. The key problem after each synthesis/analysis iteration step, is to find suitable stable weights for a new augmented plant that can be used for the synthesis of a robust $\mathscr{H}_\infty$-controller in a next iteration step. Although it would be straightforward to proceed if the IQC-multipliers are restricted to be static, it is much more delicate to construct the new augmented plant for the more general case of dynamic IQC-multipliers. The essential difficulty is related to closed-loop stability. Despite the fact that the results of Scherer and Köse (2008) fully apply for general dynamic IQC-multipliers and can be adopted in order to obtain the desired augmented plant, we provide much simpler and easier to implement state-space relations with lower state dimensions by formulating an alternative and constructive proof of (Goh, 1996, Theorem 2).

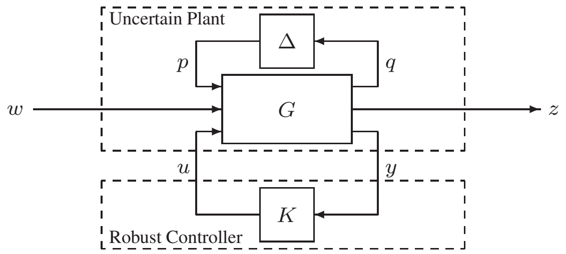

Fig. 1. Setup for Robust synthesis.

4. NOMINAL $\mathscr{H}_\infty$-CONTROLLER SYNTHESIS

In this section we give a brief recap on the well known LMI solution for the synthesis of $\mathscr{H}_\infty$-controllers, as formulated by Gahinet and Apkarian (1994) and Iwasaki and Skelton (1994). For this purpose consider the nominal plant $G_{\rm nom}:=\Delta\star G$, with $\Delta=0$.

Theorem 1. ($\mathscr{H}_\infty$-Synthesis). Suppose that $\Delta=0$. Then there exists a controller K such that the interconnection of Figure 1 is stable and the $\mathscr{L}_2$-gain from w to z is rendered less than $\gamma>0$, if there exist symmetric matrices X and Y for which the following LMIs are feasible:

$$\begin{pmatrix}X&I\\I&Y\end{pmatrix}\succ 0\tag{3}$$

$$T_{\rm y}^T(\star)^T\left(\begin{array}{cc|cc}0&X&0&0\\X&0&0&0\\\hline 0&0&\gamma^{-1}I&0\\0&0&0&-\gamma I\end{array}\right)\left(\begin{array}{cc}I&0\\A&B_{\rm w}\\\hline C_{\rm z}&D_{\rm zw}\\0&I\end{array}\right)T_{\rm y}\prec 0\tag{4}$$

$$T_{\rm u}^T(\star)^T\left(\begin{array}{cc|cc}0&Y&0&0\\Y&0&0&0\\\hline 0&0&\gamma I&0\\0&0&0&-\gamma^{-1}I\end{array}\right)\left(\begin{array}{cc}-A^T&-C_z^T\\I&0\\\hline 0&I\\-B_{\rm w}^T&-D_{\rm zw}^T\end{array}\right)T_{\rm u}\succ 0\tag{5}$$

Here, $T_{\rm y}$ and $T_{\rm u}$ are basis matrices of $\ker\left(C_{\rm y}\ D_{\rm yw}\right)$ and $\ker\left(B_{\rm u}^T\ D_{\rm zu}^T\right)$ respectively.

Once a feasible solution is found, one can, subsequently, determine the controller K along the lines of Gahinet and Apkarian (1994) and Iwasaki and Skelton (1994).

5. ROBUST STABILITY AND PERFORMANCE ANALYSIS WITH DYNAMIC IQCS

Now consider the standard input-output setting for robust stability and performance analysis in Figure 2, where $M:=G\star K\in\mathscr{RH}_\infty^{(n_{\rm q}+n_{\rm z})\times(n_{\rm p}+n_{\rm w})}$ represents the corresponding nominal, stable and weighted closed-loop LTI part of Figure 1 that admits the minimal realization

$$M:=\begin{pmatrix}M_{\rm qp}&M_{\rm qw}\\M_{\rm zp}&M_{\rm zw}\end{pmatrix}=\left[\begin{array}{c|cc}\mathcal{A}&\mathcal{B}_{\rm p}&\mathcal{B}_{\rm w}\\\hline\mathcal{C}_{\rm q}&\mathcal{D}_{\rm qp}&\mathcal{D}_{\rm qw}\\\mathcal{C}_{\rm z}&\mathcal{D}_{\rm zp}&\mathcal{D}_{\rm zw}\end{array}\right].$$

Here the closed-loop realization matrices are defined by

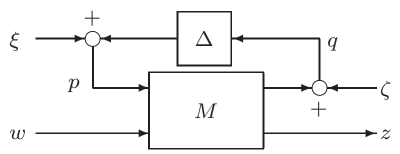

Fig. 2. Standard configuration for robust stability and performance analysis.

$$\left(\begin{array}{c|cc} \mathcal{A} & \mathcal{B}_{\mathrm{p}} & \mathcal{B}_{\mathrm{w}} \\ \hline \mathcal{C}_{\mathrm{q}} & \mathcal{D}_{\mathrm{qp}} & \mathcal{D}_{\mathrm{qw}} \\ \mathcal{C}_{\mathrm{z}} & \mathcal{D}_{\mathrm{zp}} & \mathcal{D}_{\mathrm{zw}} \end{array}\right) = \left(\begin{array}{cc|cc} A & 0 & B_{\mathrm{p}} & B_{\mathrm{w}} \\ 0 & 0 & 0 & 0 \\ \hline C_{\mathrm{q}} & 0 & D_{\mathrm{qp}} & D_{\mathrm{qw}} \\ C_{\mathrm{z}} & 0 & D_{\mathrm{zp}} & D_{\mathrm{zw}} \end{array}\right) + \left(\begin{array}{c|c} 0 & B_{\mathrm{u}} \\ I & 0 \\ \hline 0 & D_{\mathrm{qu}} \\ 0 & D_{\mathrm{zu}} \end{array}\right) \times \left(\begin{array}{c|c} A_K & B_{\mathrm{y}} \\ \hline C_{\mathrm{u}} & D_{\mathrm{uy}} \end{array}\right) \left(\begin{array}{cc|cc} 0 & I & 0 & 0 \\ \hline C_{\mathrm{y}} & 0 & D_{\mathrm{yp}} & D_{\mathrm{yw}} \end{array}\right). \tag{6}$$

M is subject to perturbations by the bounded and causal operator Δ representing the uncertainties and nonlinearities. The feedback interconnection of Figure 2 is well-posed if the operator $I - M_{\mathrm{qp}}\Delta$ has a causal inverse. We say that the system interconnection of Figure 2 is robustly $\mathscr{L}_2$-stable, if for all $\Delta \in \mathbf{\Delta}$, $(I - M_{\mathrm{qp}}\Delta)^{-1}$ defines a bounded and causal operator on $\mathscr{L}_2$. If the system is robustly stable, the outputs q and z have finite-energy, whenever $\Delta \in \mathbf{\Delta}$ and the loop is excited by finite-energy inputs ζ and ξ. Moreover, we say that the system interconnection of Figure 2 has a robust $\mathscr{L}_2$-gain performance of level γ, if it is robustly $\mathscr{L}_2$-stable and if the worst-case $\mathscr{L}_2$-gain from w to z is less than $\gamma > 0$.

Let us introduce the scaling factor $\tau \in [0, 1]$ and recall that an uncertainty/nonlinearity $\Delta \in \tau\mathbf{\Delta}$ is said to satisfy the IQC defined by the multiplier $\Pi \in \mathscr{RL}_\infty^{(n_{\mathrm{q}}+n_{\mathrm{p}})\times(n_{\mathrm{q}}+n_{\mathrm{p}})}$ if the following condition holds true:

$$\left\langle \begin{pmatrix} q \\ \Delta(q) \end{pmatrix}, \Pi \begin{pmatrix} q \\ \Delta(q) \end{pmatrix} \right\rangle \geq 0,\ \forall q \in \mathscr{L}_2^{n_{\mathrm{q}}}. \tag{7}$$

In applications one constructs a whole family of multipliers that are parameterized as $\mathbf{\Pi} = \Psi^* \mathbf{P} \Psi$ with a suitable set of symmetric matrices $\mathbf{P}$ and with a typically tall $\Psi \in \mathscr{RH}_\infty^{n_\psi \times n_{\mathrm{p}}}$ such that the IQC holds for all $\Pi \in \mathbf{\Pi}$ and $\Delta \in \tau\mathbf{\Delta}$. In this paper we assume to have no prior knowledge about the structure of Ψ and $P \in \mathbf{P}$. Hence, conveniently we can partition the columns of Ψ as $(\Psi_1\ \Psi_2)$, compatible with the rows of $\mathrm{col}(M_{\mathrm{qp}}, I)$ and alternatively consider

$$\left\langle \begin{pmatrix} q \\ \Delta(q) \end{pmatrix}, \Psi^* P \Psi \begin{pmatrix} q \\ \Delta(q) \end{pmatrix} \right\rangle \geq 0,\ \forall q \in \mathscr{L}_2^{n_{\mathrm{q}}}, \tag{8}$$

where $\Psi^* P \Psi = (\Psi_1\ \Psi_2)^* P\, (\Psi_1\ \Psi_2)$.

It is well known from Megretski and Rantzer (1997) that stability of the feedback interconnection of Figure 2 can now be characterized as follows.

Theorem 2. (Megretski & Rantzer). Suppose that, for all $\tau \in [0, 1]$ and $\Delta \in \tau\mathbf{\Delta}$, (i) the feedback interconnection of Figure 2 is well-posed and (ii) Δ satisfies (8). Then the feedback interconnection of Figure 2 is stable and the $\mathscr{L}_2$-gain from w to z is less than γ if there exists a $P \in \mathbf{P}$ for which the following FDI is satisfied:

$$(\star)^* \Pi_{\mathrm{e}}(i\omega) \left(\begin{array}{cc} M_{\mathrm{qp}}(i\omega) & M_{\mathrm{qw}}(i\omega) \\ I & 0 \\ \hdashline M_{\mathrm{zp}}(i\omega) & M_{\mathrm{zw}}(i\omega) \\ 0 & I \end{array}\right) \prec 0,\ \forall \omega \in \mathbb{R} \cup \{\infty\}. \tag{9}$$

Here $\Pi_{\mathrm{e}} := \Psi_{\mathrm{e}}^* P_{\mathrm{e}} \Psi_{\mathrm{e}}$ represents the extension of the IQC multiplier $\Pi = \Psi^* P \Psi$ to $\mathscr{L}_2$-performance with $P_{\mathrm{e}} := \mathrm{diag}(P, I, -\gamma^2 I)$ and $\Psi_{\mathrm{e}} := \mathrm{diag}(\Psi, I, I)$.

If all conditions of Theorem 2 are satisfied for Π, they hold as well for $\Pi + \epsilon I$ for some sufficiently small $\epsilon > 0$. We can, hence, assume

$$\Psi_1^* P \Psi_1 \succ 0,\ \forall \omega \in \mathbb{R} \cup \{\infty\}. \tag{10}$$

Let us now suppose that Ψ_1 and $\tau\Psi_2$ respectively admit the minimal realizations

$$\Psi_1 = \left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right],\ \tau\Psi_2 = \left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right], \tag{11}$$

with A_1 and A_2 being stable. We explicitly mention that τ has been incorporated into the realization of Ψ_2. Then, by applying the KYP lemma, the FDIs (9) and (10) are equivalent to the existence of some symmetric matrices $\mathcal{X}$ and $\hat{X}$, for which the following LMIs hold:

$$(\star)^T \left(\begin{array}{c|c|c} 0 & \mathcal{X} & 0 \\ \hline \mathcal{X} & 0 & 0 \\ \hline 0 & 0 & P_{\mathrm{e}} \end{array}\right) \left(\begin{array}{ccc|cc} \multicolumn{3}{c|}{I} & \multicolumn{2}{c}{0} \\ \hline A_1 & 0 & B_1\mathcal{C}_{\mathrm{q}} & B_1\mathcal{D}_{\mathrm{qp}} & B_1\mathcal{D}_{\mathrm{qw}} \\ 0 & A_2 & 0 & B_2 & 0 \\ 0 & 0 & \mathcal{A} & \mathcal{B}_{\mathrm{p}} & \mathcal{B}_{\mathrm{w}} \\ \hline C_1 & C_2 & D_1\mathcal{C}_{\mathrm{q}} & D_1\mathcal{D}_{\mathrm{qp}} + D_2 & D_1\mathcal{D}_{\mathrm{qw}} \\ \hdashline 0 & 0 & \mathcal{C}_{\mathrm{z}} & \mathcal{D}_{\mathrm{zp}} & \mathcal{D}_{\mathrm{zw}} \\ 0 & 0 & 0 & 0 & I \end{array}\right) \prec 0, \tag{12}$$

$$\begin{pmatrix} A_1^T \hat{X} + \hat{X} A_1 + C_1^T P C_1 & \hat{X} B_1 + C_1^T P D_1 \\ B_1^T \hat{X} + D_1^T P C_1 & D_1^T P D_1 \end{pmatrix} \succ 0. \tag{13}$$

If we partition $\mathcal{X}$ as

$$\mathcal{X} = \begin{pmatrix} \mathcal{X}_{11} & \mathcal{X}_{12} & \mathcal{X}_{13} \\ \mathcal{X}_{21} & \mathcal{X}_{22} & \mathcal{X}_{23} \\ \mathcal{X}_{31} & \mathcal{X}_{32} & \mathcal{X}_{33} \end{pmatrix},$$

where $\mathcal{X}_{11}$, $\mathcal{X}_{22}$ and $\mathcal{X}_{33}$ have compatible dimensions with A_1, A_2 and $\mathcal{A}$ respectively, we can define the coupling condition (Scherer and Köse (2008))

$$\begin{pmatrix} \mathcal{X}_{11} - \hat{X} & \mathcal{X}_{13} \\ \mathcal{X}_{31} & \mathcal{X}_{33} \end{pmatrix} \succ 0 \tag{14}$$

and state the following result.

Theorem 3. (IQC-Analysis). The controller K stabilizes the system interconnection of Figure 1 and renders the $\mathscr{L}_2$-gain from w to z less than $\gamma > 0$ for all $\Delta \in \tau\mathbf{\Delta}$ and for all $\tau \in [0, 1]$ if the following statement is true:

$$\exists\ \mathcal{X}, \hat{X}, P \in \mathbf{P} : (12), (13), (14) \text{ hold } \forall \tau \in [0, 1]. \tag{15}$$

Note that the outer factors of the IQC-multipliers Ψ are generally tall and, hence, cannot be inverted. Moreover, in case Ψ is square and invertible, there is no guarantee that Ψ^{-1} is stable. In order to construct suitable stable weights for a new augmented open-loop generalized plant that can be used for the synthesis of a robust $\mathscr{H}_\infty$-controller by employing Theorem 1, we require Ψ to be square, upper triangular, stable and invertible, while the right lower block should have a stable inverse. This is causing the key technical problem of this paper.

6. REFORMULATION OF THE ANALYSIS LMIS

In order to construct the desired weights by using state-space arguments, we will provide an alternative constructive proof of the following result of Goh (1996).

Lemma 4. Suppose that statement (15) holds. Then there exist matrices $\hat{\Psi}_j \in \mathscr{RH}_\infty$, $j = 1, 2, 3$ with $\hat{\Psi}_1^{-*}$, $\hat{\Psi}_2^{-1} \in \mathscr{RH}_\infty$ such that

$$\Psi^* P \Psi = \begin{pmatrix} \hat{\Psi}_1 & \hat{\Psi}_3 \\ 0 & \hat{\Psi}_2 \end{pmatrix}^* \begin{pmatrix} I & 0 \\ 0 & -\gamma^2 I \end{pmatrix} \begin{pmatrix} \hat{\Psi}_1 & \hat{\Psi}_3 \\ 0 & \hat{\Psi}_2 \end{pmatrix} =: \hat{\Psi}^* \hat{P} \hat{\Psi} \tag{16}$$

and $\hat{\Psi}$ admits the controllable realization

$$\hat{\Psi} = \begin{pmatrix} \hat{\Psi}_1 & \hat{\Psi}_3 \\ 0 & \hat{\Psi}_2 \end{pmatrix} = \left[\begin{array}{cc|cc} A_1 & 0 & B_1 & 0 \\ 0 & \hat{A}_2 & 0 & \hat{B}_2 \\ \hline \hat{C}_1 & \hat{C}_3 & \hat{D}_1 & \hat{D}_3 \\ 0 & \hat{C}_2 & 0 & \hat{D}_2 \end{array}\right]. \tag{17}$$

Proof. For notational simplicity we consider the case $\tau = 1$. Since (13) is satisfied there exist some square and non-singular $\hat{D}_1$ such that $D_1^T P D_1 = \hat{D}_1^T \hat{D}_1 \succ 0$. With any such factorization (13) is identical to the ARI

$$A_1^T \hat{X} + \hat{X} A_1 + C_1^T P C_1 - (\star)^T \hat{D}_1^{-1} \hat{D}_1^{-T} \big(B_1^T \hat{X} + D_1^T P C_1\big) \succ 0. \tag{18}$$

Since (11) is minimal and $\hat{D}_1^T \hat{D}_1 \succ 0$, the corresponding ARE has a unique anti-stabilizing solution Z_{11}. With $\hat{C}_1 = \hat{D}_1^{-T}\big(B_1^T Z_{11} + D_1^T P C_1\big)$ we conclude that $A_1 - B_1 \hat{D}_1^{-1} \hat{C}_1$ is anti-stable. Note that Z_{11} has the additional property $Z_{11} \prec \hat{X}$ for all solutions $\hat{X}$ of (18). With $\hat{\Psi}_1 = \left[\begin{array}{c|c} A_1 & B_1 \\ \hline \hat{C}_1 & \hat{D}_1 \end{array}\right]$, we infer that $\Psi_1^* P \Psi_1 = \hat{\Psi}_1^* \hat{\Psi}_1$ where $\hat{\Psi}_1$ is square, invertible and $\hat{\Psi}_1^{-*}$ stable.

It is now possible to define the stable transfer matrix $\hat{\Psi}_3 := \hat{\Psi}_1^{-*} \Psi_1^* P \Psi_2$ which admits the realization

$$\hat{\Psi}_3 = \left[\begin{array}{c|c} \hat{A}_2 & \hat{B}_2 \\ \hline \hat{C}_3 & \hat{D}_3 \end{array}\right] = \left[\begin{array}{cc|c} -A_1^T + \hat{C}_1^T \hat{D}_1^{-T} B_1^T & \hat{P}_1 C_2 & \hat{P}_1 D_2 \\ 0 & A_2 & B_2 \\ \hline -\hat{D}_1^{-T} B_1^T & \hat{P}_2 C_2 & \hat{P}_2 D_2 \end{array}\right], \tag{19}$$

where $\hat{P}_1 = (C_1^T - \hat{C}_1^T \hat{D}_1^{-T} D_1^T) P$ and $\hat{P}_2 = \hat{D}_1^{-T} D_1^T P$. Clearly, since we chose to work with the anti-stabilizing solution Z_{11}, $-A_1^T + \hat{C}_1^T \hat{D}_1^{-T} B_1^T$, and hence $\hat{\Psi}_3$ are stable.

Remark 5. In case (19) has uncontrollable modes one can always perform a simple state-coordinate change in order to arrive at the following controllable realization:

$$\hat{\Psi}_3 = \left[\begin{array}{c|c} \hat{A}_2 & \hat{B}_2 \\ \hline \hat{C}_3 & \hat{D}_3 \end{array}\right] = \left[\begin{array}{cc|c} \star & \star & \star \\ 0 & A_2 & B_2 \\ \hline \star & \hat{P}_2 C_2 & \hat{P}_2 D_2 \end{array}\right]. \tag{20}$$

Now observe that, since statement (15) is satisfied, (9) and (10) hold as well. We can hence infer that

$$\Psi_2^* P \Psi_2 - \Psi_2^* P \Psi_1 (\Psi_1^* P \Psi_1)^{-1} \Psi_1^* P \Psi_2 \prec 0,$$

which can be written as

$$[\star]^* \tilde{P} \left[\begin{array}{c|c} \hat{A}_2 & \hat{B}_2 \\ \hline \tilde{C}_2 & \tilde{D}_2 \end{array}\right] := \begin{pmatrix} \Psi_2 \\ \hat{\Psi}_3 \end{pmatrix}^* \begin{pmatrix} P & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} \Psi_2 \\ \hat{\Psi}_3 \end{pmatrix} \prec 0.$$

By employing the KYP-Lemma, this inequality is equivalent to the existence of some $\tilde{X}$ for which the following LMI holds:

$$\begin{pmatrix} \hat{A}_2^T \tilde{X} + \tilde{X} \hat{A}_2 + \tilde{C}_2^T \tilde{P} \tilde{C}_2 & \tilde{X} \hat{B}_2 + \tilde{C}_2^T \tilde{P} \tilde{D}_2 \\ \hat{B}_2^T \tilde{X} + \tilde{D}_2^T \tilde{P} \tilde{C}_2 & \tilde{D}_2^T \tilde{P} \tilde{D}_2 \end{pmatrix} \prec 0. \tag{21}$$

Here $\tilde{C}_2 = \mathrm{col}(C_2 J, \hat{C}_3)$, $\tilde{D}_2 = \mathrm{col}(D_2, \hat{D}_3)$ and $J = \begin{pmatrix} 0 & I \end{pmatrix}$. In complete analogy to the first part of the proof, we can express $\tilde{D}_2^T \tilde{P} \tilde{D}_2 \prec 0$ as $-\gamma^2 \hat{D}_2^T \hat{D}_2 \prec 0$, where $\hat{D}_2$ is square and invertible. With any such factorization (21) is identical to

$$\hat{A}_2^T \tilde{X} + \tilde{X} \hat{A}_2 + \tilde{C}_2^T \tilde{P} \tilde{C}_2 + \tfrac{1}{\gamma^2} (\star)^T \hat{D}_2^{-1} \hat{D}_2^{-T} \big(\hat{B}_2^T \tilde{X} + \tilde{D}_2^T \tilde{P} \tilde{C}_2\big) \prec 0. \tag{22}$$

Since $(\hat{A}_2, \hat{B}_2)$ is controllable and $\hat{D}_2^T \hat{D}_2 \succ 0$, the corresponding ARE has a unique stabilizing solution Z_{22}, which has the additional property $Z_{22} \prec \tilde{X}$ for all solutions $\tilde{X}$ of (22). With $\hat{C}_2 = \hat{D}_2^{-T}(\hat{B}_2^T Z_{22} + \tilde{D}_2^T \tilde{P} \tilde{C}_2)$ we conclude that $\hat{A}_2 - \hat{B}_2 \hat{D}_2^{-1} \hat{C}_2$ is stable. With $\hat{\Psi}_2 = \left[\begin{array}{c|c} \hat{A}_2 & \hat{B}_2 \\ \hline \hat{C}_2 & \hat{D}_2 \end{array}\right]$, we infer that $\tilde{\Psi}^* \tilde{P} \tilde{\Psi} = \hat{\Psi}_2^* \hat{\Psi}_2$ where $\hat{\Psi}_2$ is square, invertible and $\hat{\Psi}_2^{-1}$ stable. It is finally a matter of direct verification that (16) holds.

Although, the initial 'old' multiplier factorization $\Psi^* P \Psi$ appearing in the FDI (9) can now simply be replaced with the 'new' factorization $\hat{\Psi}^* \hat{P} \hat{\Psi}$, it is far from trivial to see how this can be done by using state-space arguments for the corresponding LMI (12) and its certificate $\mathcal{X}$. We need to find an LMI-certificate for the frequency domain equation

$$\hat{\Psi}^* \hat{P} \hat{\Psi} - \Psi^* P \Psi = 0 \tag{23}$$

and 'merge' the corresponding linear matrix equation with (12). It is now very convenient to see that the constructive proof of Theorem 4 allows us to infer that (23) is equivalent to

$$(\star)^T \left(\begin{array}{cc|cc|cc|c} 0 & 0 & -Z_{11} & \hat{J} & 0 & 0 & 0 \\ 0 & 0 & \hat{J}^T & -Z_{22} & 0 & 0 & 0 \\ \hline -Z_{11} & \hat{J} & 0 & 0 & 0 & 0 & 0 \\ \hat{J}^T & -Z_{22} & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & -P \end{array}\right) \left(\begin{array}{cc|cc} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ \hline A_1 & 0 & B_1 & 0 \\ 0 & \hat{A}_2 & 0 & \hat{B}_2 \\ \hline \hat{C}_1 & \hat{C}_3 & \hat{D}_1 & \hat{D}_3 \\ 0 & \hat{C}_2 & 0 & \hat{D}_2 \\ \hline C_1 & C_2 J & D_1 & D_2 \end{array}\right) = 0, \tag{24}$$

where $\hat{J} = (\, I \; 0 \,)$.

As one of the key technical ingredients of Scherer and Köse (2011), it is now possible to systematically merge the LMIs (12) and (24). Indeed, let $\hat{W}$ and W_{ii}, $i = 1, 2$ respectively satisfy $\hat{J} \hat{A}_2 \hat{J}^T \hat{W} + \hat{W} \hat{J} \hat{A}_2^T \hat{J}^T \prec 0$ and

$$\begin{pmatrix} A_i^T W_{ii} + W_{ii} A_i - C_i^T C_i & W_{ii} B_i - C_i^T D_i \\ B_i^T W_{ii} - D_i^T C_i & -D_i^T D_i \end{pmatrix} \prec 0, \; i = 1, 2.$$

Then there exist $\nu > 0$ and $\mu > 0$ (that can be taken arbitrarily small) such that

$$\hat{\mathcal{X}} = \begin{pmatrix} \mathcal{X}_{11} - Z_{11} + \nu W_{11} & \mathcal{X}_{12} J + \hat{J} & \mathcal{X}_{13} \\ J^T \mathcal{X}_{12}^T + \hat{J}^T & J^T(\mathcal{X}_{22} + \nu W_{22}) J - Z_{22} + \mu \hat{J}^T \hat{W} \hat{J} & J^T \mathcal{X}_{23} \\ \mathcal{X}_{13}^T & \mathcal{X}_{23}^T J & \mathcal{X}_{33} \end{pmatrix}$$

satisfies the LMI

$$(\star)^T \left(\begin{array}{c|c|cc|cc} 0 & \hat{\mathcal{X}} & 0 & 0 & 0 & 0 \\ \hline \hat{\mathcal{X}} & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma^2 I & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{array}\right) \left(\begin{array}{ccc|cc} & I & & & 0 \\ \hline A_1 & 0 & B_1 \mathcal{C}_{\mathrm{q}} & B_1 \mathcal{D}_{\mathrm{qp}} & B_1 \mathcal{D}_{\mathrm{qw}} \\ 0 & \hat{A}_2 & 0 & \hat{B}_2 & 0 \\ 0 & 0 & \mathcal{A} & \mathcal{B}_{\mathrm{p}} & \mathcal{B}_{\mathrm{w}} \\ \hline \hat{C}_1 & \hat{C}_3 & \hat{D}_1 \mathcal{C}_{\mathrm{q}} & \hat{D}_1 \mathcal{D}_{\mathrm{ee}} & \hat{D}_1 \mathcal{D}_{\mathrm{qw}} \\ 0 & \hat{C}_2 & 0 & \hat{D}_2 & 0 \\ \hdashline 0 & 0 & \mathcal{C}_{\mathrm{z}} & \mathcal{D}_{\mathrm{zp}} & \mathcal{D}_{\mathrm{zw}} \\ 0 & 0 & 0 & 0 & I \end{array}\right) \prec 0, \tag{25}$$

where $\mathcal{D}_{\mathrm{ee}} = \mathcal{D}_{\mathrm{dd}} + \hat{D}_1^{-1} \hat{D}_3$. This allows us to arrive at the following reformulation of (12) where the uncertainty channels are re-scaled and combined with the performance channels.

Lemma 6. Statement (15) holds if and only if $\hat{\mathcal{X}} \succ 0$ certifies the LMI

$$\left(\begin{array}{cc} I & 0 \\ \hat{\mathcal{A}} & \hat{\mathcal{B}} \\ \hline \hat{\mathcal{C}} & \hat{\mathcal{D}} \\ 0 & I \end{array}\right)^T \left(\begin{array}{cc|cc} 0 & \hat{\mathcal{X}} & 0 & 0 \\ \hat{\mathcal{X}} & 0 & 0 & 0 \\ \hline 0 & 0 & I & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{array}\right) \left(\begin{array}{cc} I & 0 \\ \hat{\mathcal{A}} & \hat{\mathcal{B}} \\ \hline \hat{\mathcal{C}} & \hat{\mathcal{D}} \\ 0 & I \end{array}\right) \prec 0, \tag{26}$$

where

$$\left[\begin{array}{c|c} \hat{\mathcal{A}} & \hat{\mathcal{B}} \\ \hline \hat{\mathcal{C}} & \hat{\mathcal{D}} \end{array}\right] := \left[\begin{array}{ccc|cc} A_1 & -B_1 \mathcal{D}_{\mathrm{qp}} \hat{D}_2^{-1} \hat{C}_2 & B_1 \mathcal{C}_{\mathrm{q}} & B_1 \mathcal{D}_{\mathrm{qp}} \hat{D}_2^{-1} & B_1 \mathcal{D}_{\mathrm{qw}} \\ 0 & \hat{A}_2 - \hat{B}_2 \hat{D}_2^{-1} \hat{C}_2 & 0 & \hat{B}_2 \hat{D}_2^{-1} & 0 \\ 0 & -\mathcal{B}_{\mathrm{p}} \hat{D}_2^{-1} \hat{C}_2 & \mathcal{A} & \mathcal{B}_{\mathrm{p}} \hat{D}_2^{-1} & \mathcal{B}_{\mathrm{w}} \\ \hline \hat{C}_1 & \hat{C}_3 - \hat{D}_1 \mathcal{D}_{\mathrm{ee}} \hat{D}_2^{-1} \hat{C}_2 & \hat{D}_1 \mathcal{C}_{\mathrm{q}} & \hat{D}_1 \mathcal{D}_{\mathrm{ee}} \hat{D}_2^{-1} & \hat{D}_1 \mathcal{D}_{\mathrm{qw}} \\ 0 & -\mathcal{D}_{\mathrm{zp}} \hat{D}_2^{-1} \hat{C}_2 & \mathcal{C}_{\mathrm{z}} & \mathcal{D}_{\mathrm{zp}} \hat{D}_2^{-1} & \mathcal{D}_{\mathrm{zw}} \end{array}\right]. \tag{27}$$

If (26) is feasible, the closed-loop system (27), from which we can directly extract the desired augmented weighted open-loop generalized plant, is stable.

Proof. Let us combine the uncertainty and performance channels by permuting the 4^{th} and 5^{th} block row/columns of the middle matrix of (25) and, subsequently, eliminate $\hat{C}_2$ and multiply $\hat{D}_2$ by its inverse respectively by performing a simple congruence transformation. This directly yields (26) as well as the closed-loop system (27). Finally, observe that (26) implies $\hat{\mathcal{X}}\hat{\mathcal{A}} + \hat{\mathcal{A}}^T\hat{\mathcal{X}} \prec 0$. Since A_1 and $\hat{A}_2 - \hat{B}_2\hat{D}_2^{-1}\hat{C}_2$ are stable, stability of $\mathcal{A}$ is, hence, equivalent to $\hat{\mathcal{X}} \succ 0$.

7. MAIN RESULTS

Based on the LMI reformulation of Lemma 6 and referring to (6) it is now straightforward to extract the following augmented weighted open-loop generalized plant from the augmented closed-loop system (27) where the uncertainty channels are re-scaled and adjoined to the performance channel:

$$\hat{G}_{\text{nom}} = \left[\begin{array}{ccc|cc:c} A_1 & -B_1D_{\text{qp}}\hat{D}_2^{-1}\hat{C}_2 & B_1C_{\text{q}} & B_1D_{\text{qp}}\hat{D}_2^{-1} & B_1D_{\text{qw}} & B_1D_{\text{qu}} \\ 0 & \hat{A}_2 - \hat{B}_2\hat{D}_2^{-1}\hat{C}_2 & 0 & \hat{B}_2\hat{D}_2^{-1} & 0 & 0 \\ 0 & -B_{\text{p}}\hat{D}_2^{-1}\hat{C}_2 & A & B_{\text{p}}\hat{D}_2^{-1} & B_{\text{w}} & B_{\text{u}} \\ \hline \hat{C}_1 & \hat{C}_3 - \hat{D}_1\hat{D}_{\text{ee}}\hat{C}_2 & \hat{D}_1C_{\text{q}} & \hat{D}_1\hat{D}_{\text{ee}} & \hat{D}_1D_{\text{qw}} & \hat{D}_1D_{\text{qu}} \\ 0 & -D_{\text{zp}}\hat{D}_2^{-1}\hat{C}_2 & C_{\text{z}} & D_{\text{zp}}\hat{D}_2^{-1} & D_{\text{zw}} & D_{\text{zu}} \\ \hdashline 0 & -D_{\text{yp}}\hat{D}_2^{-1}\hat{C}_2 & C_{\text{y}} & D_{\text{yp}}\hat{D}_2^{-1} & D_{\text{yw}} & 0 \end{array}\right]. \tag{28}$$

Here $\hat{D}_{\text{ee}} = (D_{\text{qp}} + \hat{D}_1^{-1}\hat{D}_3)\hat{D}_2^{-1}$. The new augmented plant can be used in Theorem 1 for the synthesis of a new robustified $\mathscr{H}_\infty$-controller.

We now have introduced all the ingredients that allow us to formulate the following novel algorithm for the synthesis of robust controllers. Note that we now incorporate τ as a parameter.

Algorithm 7. (IQC-Synthesis). Initialize the synthesis by respectively performing the following two steps:

- Perform the $\mathscr{H}_\infty$-synthesis of Theorem 1. Feasibility leads to a nominal $\mathscr{H}_\infty$-controller that renders the nominal $\mathscr{L}_2$-gain from w to z less than $\gamma_{\text{nom}} > 0$.
- Construct the weighted closed-loop plant M and find, by bisection, the largest τ for which the analysis LMIs of Theorem 3 are feasible. The maximal τ for which (15) still holds is denoted by $\hat{\tau}$ and the corresponding worst-case $\mathscr{L}_2$-gain from w to z is given by $\hat{\gamma} \geq \gamma_{\text{nom}} > 0$.

Now perform the following iteration:

- Construct the new augmented weighted open-loop generalized plant (28) and perform the $\mathscr{H}_\infty$-synthesis of Theorem 1. Then for all $\Delta \in \hat{\tau}\boldsymbol{\Delta}$ the resulting $\mathscr{H}_\infty$-controller stabilizes the system interconnection of Figure 1 and renders the $\mathscr{L}_2$-gain from w to z less than the new performance level γ_{j-1} with $\hat{\gamma} \geq \gamma_{j-1} \geq \gamma_{\text{nom}} > 0$.
- Construct the weighted closed-loop plant M and apply the IQC analysis of Theorem 3. Then the resulting worst-case $\mathscr{L}_2$-gain γ_j from w to z satisfies $\hat{\gamma} \geq \gamma_{j-1} \geq \gamma_j \geq \gamma_{\text{nom}} > 0$. (If $\hat{\tau} \leq 1$ one can try to increase τ.)
- Repeat the iteration until $|\gamma_{j-1} - \gamma_j| \leq \epsilon$ for some small $\epsilon > 0$.

The iteration defines a sequence of performance levels $\hat{\gamma} \geq \gamma_1 \geq \gamma_2 \geq \gamma_3 \geq \ldots \geq \gamma_{\text{nom}} > 0$ and the resulting $\mathscr{H}_\infty$-controller guarantees stability and renders the $\mathscr{L}_2$-gain from w to z less than $\gamma_j > 0$ for all $\Delta \in \hat{\tau}\boldsymbol{\Delta}$.

Remark 8. We finally remark that the LMI certificate $\hat{\mathcal{X}}$ for the LMI (26) can be used as an initial condition in Theorem 1 in a next iteration step in order to speed up the controller synthesis.

8. NUMERICAL EXAMPLE

The IQC-Synthesis algorithm has been applied to a mixed sensitivity design problem from Skogestad and Postelwaite (2005) extended with an actuator saturation and a measurement delay block, as shown in Figure 3.

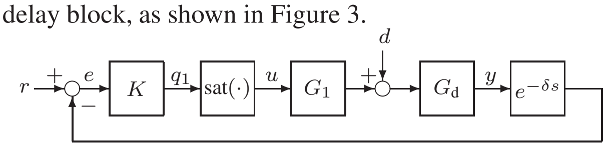

Fig. 3. Block diagram of the disturbance process.

Here, respectively, $G_1 = \frac{400}{(s+20)^2}$, $G_{\text{d}} = \frac{10}{(s+0.1)}$, $\delta \in [0, 0.05]$ and

$$u = \text{sat}(q_1) = \begin{cases} \bar{u}, \ q_1 > \bar{u} \\ u, \ q_1 \in [-\bar{u}, \bar{u}] \\ -\bar{u}, \ q_1 < \bar{u} \end{cases}, \quad \text{with} \quad \bar{u} = 1,$$

represent the plant and the disturbance model, the delay time and the actuator saturation function. The synthesis objective is to track the commanded reference signal r, while rejecting the disturbances appearing at the input d and preventing the control action to exceed the saturation limits.

In order to be able transform the system into the standard LFT form, we introduce the following operator blocks:

$$\Delta_1(q_1) = \begin{cases} q_1 - \bar{u}, \ q_1 > \bar{u} \\ 0, \ q_1 \in [-\bar{u}, \bar{u}] \\ q_1 + \bar{u}, \ q_1 < \bar{u} \end{cases}, \ \Delta_2(s) = \left(e^{-\delta s} - 1\right)\frac{1}{s}.$$

For further details on the parameterization and implementation of the involved IQC-multipliers we refer the reader to Zames and Falb (1968), Chen and Wen (1995) and Jun and Safonov (2000). It is now straightforward to define the weighted closed-loop uncertain system interconnection, shown in Figure 4.

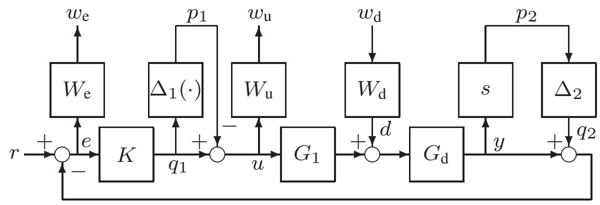

Fig. 4. Weighted closed-loop generalized plant.

The corresponding uncertain weighted open-loop generalized plant G, hence, reads as

$$\begin{pmatrix} q_1 \\ q_2 \\ \hline w_{\text{e}} \\ w_{\text{u}} \\ e \end{pmatrix} = \underbrace{\left(\begin{array}{cc|ccc} 0 & 0 & 0 & 0 & 1 \\ -sG_{\text{d}}G_1 & 0 & sG_{\text{d}}W_{\text{d}} & 0 & sG_{\text{d}}G_1 \\ \hline W_{\text{e}}G_{\text{d}}G_1 & -W_{\text{e}} & -W_{\text{e}}G_{\text{d}}W_{\text{d}} & W_{\text{e}} & -sW_{\text{e}}G_{\text{d}}G_1 \\ -W_{\text{u}} & 0 & 0 & 0 & W_{\text{u}} \\ G_{\text{d}}G_1 & -1 & -G_{\text{d}}W_{\text{d}} & 1 & -G_{\text{d}}G_1 \end{array}\right)}_{G} \begin{pmatrix} p_1 \\ p_2 \\ \hline w_{\text{d}} \\ r \\ q_1 \end{pmatrix},$$

where, respectively, the weights were chosen as

$$W_{\text{p}} = \frac{2(s+18)}{3(s+0.0012)}, \ W_{\text{u}} = \frac{(s+10)}{(s+100)}, \ W_{\text{d}} = \frac{(s+1000)}{100(s+10)}.$$

In Figure 5 the obtained simulation results are shown for two controller implementations (i.e. a nominal and a robust controller design). As can be seen the nominal controller K_{nom} (nom) tracks and rejects the reference and disturbance signal well. However, if the system is simulated, while taking the

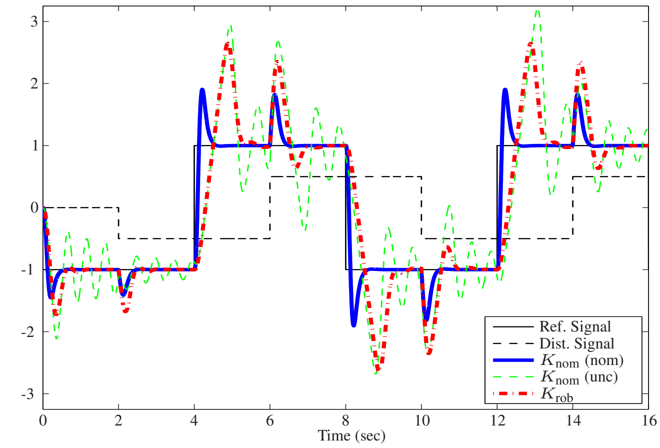

Fig. 5. Time domain simulation results.

actuator saturation and measurement delay into account, the performance degrades drastically (see K_{nom} (unc)). In an attempt to improve robust performance we have considered non-dynamic (static) IQC-multipliers. However, this led to overly conservative results. It is, on the other hand, very satisfactory to see that robust performance can be significantly improved by allowing for dynamics in the IQC-multipliers. Indeed, the robust controller K_{rob}, has been designed by taking into account the saturation constraint as well as the maximal delay time of 0.05 seconds. The improvement of robust performance is also confirmed by the γ-levels that were obtained during the IQC-synthesis iterations, as shown in Figure 6. As can be seen the γ-levels converge and are consistent with the simulation results of Figure 5.

9. CONCLUDING REMARKS

In this paper we have generalized our results on the systematic synthesis of robust controllers to the use of arbitrary real rational IQC-multipliers with no poles on the extended imaginary axis. This allows us to perform robust controller synthesis for a much larger class of uncertainties, involving nonlinearities as well as bounds on rates of time-varying parametric uncertainties, if compared to the classical μ-synthesis approaches. The effectiveness of the synthesis algorithm has been demonstrated through a numerical example. It has been shown that robust performance can be systematically improved in a numerically stable and reliable way.

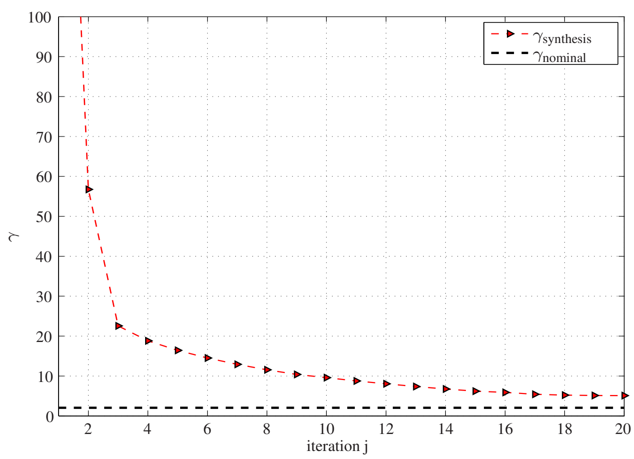

Fig. 6. Obtained γ-levels during the IQC-synthesis iterations.

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