Prog. Theor. Phys. Vol. 60 (1978), Dec.

Irreducibility of Massless Spin 3/2 Field

Takeshi FUKUYAMA

Department of Physics, Osaka University
Toyonaka, Osaka 560

September 13, 1978

In recent years, supergravity and extended supergravity have generated considerable interest, where massless spin 3/2 particle, gravitino, plays an essential role. In this short note we discuss the irreducibility of massless spin 3/2 particle. Several arguments have already been given of this problem,[1] however it seems to us that there is some confusion.

The most general form of the Lagrangian of massless vector spinor field is given by

$$L=-\bar{\psi}_\rho[\gamma\partial\delta_{\rho\sigma}+a(\gamma_\rho\partial_\sigma+\gamma_\sigma\partial_\rho)+b\gamma_\rho\gamma\partial\gamma_\sigma]\psi_\sigma\,, \tag{1}$$

where a and b are real parameters. One can rewrite Lagrangian (1) as

$$L=-\bar{\psi}_\rho[\varepsilon_{\rho\mu\nu\sigma}\gamma_5\gamma_\mu\partial_\nu+(a+1)(\gamma_\rho\partial_\sigma+\gamma_\sigma\partial_\rho)+(b-1)\gamma_\rho\gamma\partial\gamma_\sigma]\psi_\sigma\,. \tag{2}$$

The variation of $\bar{\psi}_\rho$ gives the following Euler equation:

$$\gamma\partial\psi_\rho+a(\gamma_\rho\partial\cdot\psi+\partial_\rho\gamma\cdot\psi)+b\gamma_\rho\gamma\partial\gamma\cdot\psi=0\,. \tag{3}$$

Multiplying Eq. (3) by γ_ρ, one finds that

$$2(1+2a)\partial\cdot\psi=(1-a-4b)\gamma\partial\gamma\cdot\psi\,. \tag{4}$$

Then one obtains

$$\partial\cdot\psi=0\,, \tag{5}$$

if

$$a\neq-1/2 \quad\text{and}\quad b=(1-a)/4\,. \tag{6}$$

Imposing further condition $a=-1$, Lagrangian is reduced to

$$L=-\bar{\psi}_\rho[\varepsilon_{\rho\mu\nu\sigma}\gamma_5\gamma_\mu\partial_\nu-1/2\gamma_\rho\gamma\partial\gamma_\sigma]\psi_\sigma\,. \tag{7}$$

The first term is invariant under the following local spinor gauge transformation:

$$\psi_\mu(x)\to\psi_\mu{}'(x)=\psi_\mu(x)+\partial_\mu u(x)\,. \tag{8}$$

Thus in quantum theory the effective total

Lagrangian L_{eff} defined by

$$L_{\text{eff}} \equiv L_{3/2} + L_{\text{G.F.}} + L_{\text{F.P.}}\,, \tag{9}$$

where

$$L_{3/2} = -\varepsilon_{\rho\mu\nu\sigma}\bar{\psi}_\rho \gamma_5 \gamma_\mu \partial_\nu \psi_\sigma\,, \tag{10}$$

$$L_{\text{G.F.}} = \tfrac{1}{2}\bar{\psi}\cdot\gamma\gamma\partial\gamma\cdot\psi \tag{11}$$

and

$$L_{\text{F.P.}} = i\bar{B}\gamma\partial C\,, \tag{12}$$

is invariant under the B.R.S. transformation[2] expressed below;

$$\psi_\mu(x) \to \psi_\mu{}'(x) = \psi_\mu(x) + \partial_\mu C(x)\,,$$
$$C(x) \to C'(x) = C(x) \tag{13}$$

and

$$\bar{B}(x) \to \bar{B}'(x) = \bar{B}(x) - i\bar{\psi}\cdot\gamma\gamma\overleftarrow{\partial}\,.$$

The eight degrees of freedom of vector spinor field are reduced to six by the condition (5) and furthermore four degrees of freedom can be eliminated by two ghosts. Thus two degrees of freedom of $\pm 3/2$ helicity have been obtained. If $a \neq -1$, one has additive gauge fixing term $L'_{\text{G.F.}}$,

$$L'_{\text{G.F.}} = -(1+a)\,(\bar{\psi}\cdot\gamma\partial\cdot\psi + \bar{\psi}\cdot\overrightarrow{\partial}\gamma\cdot\psi)\,. \tag{14}$$

However this $L'_{\text{G.F.}}$ is inconvenient for the B.R.S. prescription because each term has not such symmetric form about $\bar{\psi}_\rho$ and ψ_σ as $L_{\text{G.F.}} = \frac{1}{2}\bar{\psi}\cdot\gamma\gamma\partial\gamma\cdot\psi$ and one cannot define the corresponding $L_{\text{F.P.}}$. That is, $a = -1$ is inevitable.

In the generally covariant gauge one should modify $L_{\text{G.F.}}$ and the transformation of $\bar{B}$ as

$$L_{\text{G.F.}} = \frac{1}{2\alpha}\bar{\psi}\cdot\gamma\gamma\partial\gamma\cdot\psi\,, \tag{11$'$}$$

$$\bar{B}(x) \to \bar{B}'(x) = \bar{B}(x) - \frac{i}{\alpha}\bar{\psi}\cdot\gamma\gamma\overleftarrow{\partial}\,, \tag{13$'$}$$

where α is an arbitrary real parameter. $L'_{\text{eff}} \equiv L_{3/2} + L'_{\text{G.F.}} + L_{\text{F.P.}}$ is also invariant under the B.R.S. transformations (13) and (13)′. However in this case the constraint (5) is not satisfied except for the special choice of $\alpha = 1$ and the arguments about the unitarity are not so clear. This trouble is a peculiar feature to fermion gauge fields and does not occur in the case of boson gauge fields. We will discuss in detail the uitarity of supergravity in the generally covariant gauge in a subsequent paper.

1) See for instance, G. Sterman, P. Townsend and P. Nieuwenhuigen, Phys. Rev. **D17** (1978), 1501.

2) C. Becchi, A. Rouet and R. Stora, Comm. Math. Phys. **42** (1975), 127.