IRROTATIONAL FLOW ABOUT SHIP FORMS

by

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Sponsored by
Office of Naval Research
Contract Nonr - 1611(07)

IIHR Report No. 123

Iowa Institute of Hydraulic Research
The University of Iowa
Iowa City, Iowa

December 1969

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Introduction

A method of computing the potential flow about ship forms would, in spite of the neglect of viscous effects, be valuable in the preliminary design of a ship, or in investigating means of improving the performance of an existing ship. If an efficient procedure for performing such calculations were available, one could determine, without recourse to model tests, whether the streamlines along the forebody are such that bilge vortices would be generated and what would be the effect on these streamlines of various modifications of the bow. If separation at the stern is not severe and the bow not too blunt near the free surface, useful results could be obtained for the wavemaking of a ship form on the assumptions of irrotational flow and the linearized free-surface condition, but with the exact boundary condition on the hull. The last condition is significant since it would enable the effects of local modifications of form, especially at the bow, to be studied.

A well-known method of calculating the potential flow about a three-dimensional form is that of Hess and Smith [1]*. In common with the Hess-Smith approach, the method to be described herein determines a distribution of sources on the surface of a given body by solving the basic integral equation of potential theory for such a Neumann problem [2]. The methods differ in the treatment of the singularity of the kernel of the integral equation, the selection of an iteration formula for solving the integral equation, the quadrature formula used to reduce the integral equation to a set of linear equations, and the procedure employed to calculate the velocity distribution along the hull once the source distribution has been determined.

Although the procedure to be described has been available as a computer program for several years, and early results with a body of revolution (a spheroid) and a three-dimensional form (an ellipsoid) showed very good agreement with the exact solutions, publication has been delayed because anomalous results were obtained when the method was applied to a

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*Numbers in brackets indicate references.

mathematical form with parabolic lines and sections, having sharp edges at the bow, stern and keel. Presently an attempt is being made to apply the method to a Series-60 model [3]; but since this form is serving as a vehicle for development of a procedure for fitting a mathematical equation to an arbitrary ship form, the hull coordinates and direction cosines of the normals to the hull, required as input in the potential flow program, are not yet available. Consequently it has been decided to present the method and the computer program without further delay, since others may be more efficient in obtaining the required input for their ship forms.

Statement of Problem

Our problem is to develop means of computing the flow about a ship form, including free-surface effects. We shall assume the fluid to be incompressible and inviscid, and the flow irrotational. We shall suppose that a ship form has been prescribed, and that its draft and trim are known. We shall assume that the surface disturbance is sufficiently small that the boundary condition on the free surface may be linearized. The boundary condition on the hull will be satisfied exactly.

An obvious criticism of these assumptions is that, in neglecting the effects of viscosity and sinkage and trim, and employing the approximate linearized form of the free-surface boundary condition, in comparison with the elegant, classical, thin-ship theory, only one of several equally important corrections will have been made. Our view is that the solution of the present problem can be made the basis for incorporating additional corrections by iterative techniques. For example, the resulting pressure distribution can be used to calculate the equilibrium trim and sinkage of the ship, which can then be applied to obtain a second approximation for the flow about it.

Let $f(x, y, z) = 0$ be the equation of the hull surface S, with x in the direction of the stream U, and z positive upwards, with the origin in the undisturbed level of the free surface. Denote the velocity potential by

$$\Phi = Ux + \phi \tag{1}$$

where ϕ is the disturbance potential which, as well as Φ, satisfies the

Laplace equation

$$\nabla^2\phi = 0 \qquad (2)$$

The boundary condition on S is then

$$\frac{\partial\phi}{\partial N} = - U \frac{\partial x}{\partial N} \qquad (3)$$

where N denotes distance in the direction of the outward normal to S. The free-surface boundary condition will be taken in the linearized form

$$\frac{\partial^2\phi}{\partial x^2} + k_0 \frac{\partial\phi}{\partial z} = 0; \quad k_0 = \frac{g}{U^2}, \quad z = 0 \qquad (4)$$

For a source of unit strength at the point $P(\xi, \eta, \zeta)$ in the same uniform stream, the velocity potential Φ_S which satisfies (2) and boundary condition (4) may be written in the form

$$\Phi_S = Ux - \frac{1}{R} - \frac{1}{R'} + H(P, Q) \qquad (5)$$

where Q is a point $Q(x, y, z)$ below the free surface, R is the distance from P to Q, R' is the distance from $P'(\xi, \eta, -\zeta)$, the mirror image of P in the free surface, to Q,

$$R = [(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2]^{\frac{1}{2}}, \quad R' = [(x - \xi)^2 + (y - \eta)^2 + (z + \zeta)^2]^{\frac{1}{2}} \qquad (6)$$

and $H(P, Q)$ is regular harmonic in the lower half space $z < 0$ and given by

$$H(P, Q) = \frac{1}{\pi} ⨍_0^\infty \int_0^{2\pi} \frac{e^{k(z + \zeta) + ik[(x - \xi)\cos\theta + (y - \eta)\sin\theta]}}{k - k_0 \sec^2\theta} k\,dk\,d\theta$$

$$- 4k_0 \int_0^{\pi/2} e^{k_0(z + \zeta)\sec^2\theta} \sin[k_0(x - \xi)\sec\theta] \cos[k_0(y - \eta)\tan\theta\sec\theta] \sec^2\theta \cdot d\theta \qquad (7)$$

where $⨍$ denotes the "Cauchy principal part". The velocity potential of (5) and (6) also satisfies the "radiation condition", that waves are propagated downstream from the source [4].

Now let M(P) denote the strength of a source distribution on S. The velocity potential of this source distribution which satisfies the free-surface condition (4) and the radiation condition, by (5), is given by

$$\Phi(Q) = Ux - \int_{S+S'} \frac{M(P)}{R}\, dS_P + \int_S M(P)\, H(P, Q)\, dS_P \qquad (7)$$

where S' denotes the mirror image of S in the plane z = 0. Taking into account the discontinuity in the normal derivative of the potential at a surface distribution, the boundary condition on the hull surface S then yields

$$M(Q) = -\frac{U}{2\pi}\frac{\partial x}{\partial N_Q} + \frac{1}{2\pi}\int_{S+S'} M(P)\,\frac{\partial}{\partial N_Q}\left(\frac{1}{R}\right) dS_P - \frac{1}{2\pi}\int_S M(P)\,\frac{\partial H(P, Q)}{\partial N_Q}\, dS_P \qquad (8)$$

a Fredholm integral equation of the second kind. The development of a procedure for solving this integral equation numerically for a given ship form is our principal objective.

An essential difficulty in the numerical solution of (8) is that both of the integrands are singular. A means of removing these singularities will be described and justified in the following two sections.

Treatment of Double-Model Integral

At points of S where the normal is continuous, we have by Gauss's flux theorem

$$-\int_{S+S'} \frac{\partial}{\partial N_P}\left(\frac{1}{R}\right) dS_P = 2\pi \qquad (9)$$

This enables us to write

$$\frac{1}{2\pi}\int_{S+S'} M(P)\,\frac{\partial}{\partial N_Q}\left(\frac{1}{R}\right) dS_P = \frac{1}{2\pi}\int_{S+S'}\left[M(P)\,\frac{\partial}{\partial N_Q}\left(\frac{1}{R}\right) - M(Q)\,\frac{\partial}{\partial N_P}\left(\frac{1}{R}\right)\right] dS_P - M(Q) \qquad (10)$$

We shall now show that the singularity of the left member of (10) when P coincides with Q is not present in the right member.

The direction cosines at the point Q of S are given by

$$\ell_Q = \left(\frac{\partial n}{\partial N}\right)_Q = \frac{f_x}{D_Q}, \quad M_Q = \frac{f_y}{D_Q}, \quad n_Q = \frac{f_z}{D_Q} \qquad (11)$$

where f_x, f_y, f_z denote partial derivatives of f(x, y, z) with respect to x, y, z, and

$$D_Q = [f_x^{\,2} + f_y^{\,2} + f_z^{\,2}]^{\frac{1}{2}}$$

Then we have

$$\left.\begin{aligned} \frac{\partial}{\partial N_Q}\left(\frac{1}{R}\right) &= -\frac{1}{R^3}\left[(x-\xi)\ell_Q + (y-\eta)m_Q + (z-\zeta)n_Q\right] \\ &= -\frac{1}{R^3 D_Q}\left[(x-\xi)f_x + (y-\eta)f_y + (z-\zeta)f_z\right] \end{aligned}\right\} \qquad (12)$$

and

$$M(P)\frac{\partial}{\partial N_Q}\left(\frac{1}{R}\right) - M(Q)\frac{\partial}{\partial N_P}\left(\frac{1}{R}\right) = -\frac{1}{R^3}\left\{\frac{M(P)}{D_Q}\left[(x-\xi)f_x + (y-\eta)f_y + (z-\zeta)f_z\right] + \frac{M(Q)}{D_P}\left[(x-\xi)f_\xi + (y-\eta)f_\eta + (z-\zeta)f_\zeta\right]\right\} \qquad (13)$$

When P is near Q, we can write the Taylor expansion

$$f(\xi, \eta, \zeta) = f(x, y, z) - (x-\xi)f_x - (y-\eta)f_y - (z-\zeta)f_z + \tfrac{1}{2}[(x-\xi)^2 f_{xx} + (y-\eta)^2 f_{yy} + (z-\zeta)^2 f_{zz} + 2(y-\eta)(z-\zeta)f_{yz} + \cdots] + \cdots$$

Since

$$f(x, y, z) = f(\xi, \eta, \zeta) = 0$$

we obtain

$$(x-\xi)f_x + (y-\eta)f_y + (z-\zeta)f_z = \tfrac{1}{2}[(x-\xi)^2 f_{xx} + \cdots] + \text{3rd order terms} \qquad (14)$$

Similarly we have

$$(\xi-x)f_\xi + (\eta-y)f_\eta + (\zeta-z)f_\zeta = \tfrac{1}{2}[(x-\xi)\ f_{\xi\xi} + \cdots] + \text{3rd order terms}$$

or

$$(x-\xi)f_\xi + (y-\eta)f_\eta + (z-\zeta)f_\zeta = -\tfrac{1}{2}[(x-\xi)^2 f_{xx} + \cdots] + \text{3rd order terms} \qquad (15)$$

Also we have

$$M(P) = M(Q) + (\xi-x)M_x + (\eta-y)M_y + (\zeta-z)M_z + \cdots \qquad (16)$$

and

$$\frac{1}{D_P} = \frac{1}{D_Q} + (\xi - x)\frac{\partial}{\partial x}\left(\frac{1}{D_Q}\right) + \cdots \qquad (17)$$

Substituting the results in (14), (15), (16) and (17) into (13), we observe that terms of the first and second order in $(x - \xi)$, $(y - \eta)$, $(z - \zeta)$ cancel, leaving antisymmetric terms of the third order in the numerator. Since the denominator R^3 is also of the third order, the ratio is indeterminate as R approaches zero. The integral of this ratio over a small area symmetric about Q, however, is zero. For this reason we propose to set the integrand of the right member of (10) equal to zero when P coincides with Q.

Treatment of Wave Integral

As in the prior case, let us consider the integral of the transposed kernel, $\partial H(Q, P)/\partial N_P$. For points Q below the free surface $(z < 0)$, $H(Q, P)$ is a regular harmonic function of $P(\xi, \eta, \zeta)$ for $\zeta \leq 0$. Hence, by Gauss's flux theorem, the flux of H(Q. P) through the closed surface, consisting of S and the surface S_0 of the plane $z = 0$ capping S, is zero; i.e.

$$\int_S \frac{\partial H(Q, P)}{\partial N_P} dS_P = - \int_{S_0} \left. \frac{\partial H(Q, P)}{\partial \zeta} \right|_{\zeta=0} dS_P \tag{18}$$

Then we can write the wave integral in (8) in the form

$$\int_S M(P) \frac{\partial H(P, Q)}{\partial N_Q} dS_P = \int_S \left[M(P) \frac{\partial H(P, Q)}{\partial N_Q} - M(Q) \frac{\partial H(Q, P)}{\partial N_P} \right] dS_P - M(Q) \int_{S_0} \left. \frac{\partial H(Q, P)}{\partial \zeta} \right|_{\zeta=0} dS_P \tag{19}$$

From (6) we have

$$\frac{\partial H(P, Q)}{\partial N_Q} = \frac{1}{\pi} \int_0^\infty \int_0^{2\pi} \frac{e^{k(z + \zeta) + ik[(x - \xi) \cos \theta + (y - \eta) \sin \theta]}}{k - k_0 \sec^2\theta} k^2 \cdot [n_Q + i(\ell_Q \cos \theta + m_Q \sin \theta)] \, d\theta dk$$

$$+ 4k_0^2 \int_0^{\pi/2} \sec^3\theta \cdot e^{k_0(z + \zeta) \sec^2\theta} \Big\{ \ell_Q \cos [k_0(x - \xi) \sec \theta] \cdot \cos [k_0(y - \eta) \tan \theta \sec \theta]$$

$$- m_Q \tan\theta \sin[k_0(x-\xi)\sec\theta] \sin[k_0(y-\eta)\tan\theta\sec\theta]$$

$$+ n_Q \sec\theta \sin[k_0(x-\xi)\sec\theta] \cos[k_0(y-\eta)\tan\theta\sec\theta]\Big\} d\theta \tag{20}$$

By comparison of the double integral in (20) with that in the relation

$$2\frac{\partial}{\partial N_Q}[(z+\zeta)^2 + (x-\xi)^2 + (y-\eta)^2]^{-\frac{1}{2}} =$$

$$\frac{1}{\pi}\int_0^\infty\int_0^{2\pi} e^{k(z+\zeta) + ik[(x-\xi)\cos\theta + (y-\eta)\sin\theta]} \cdot k[n_Q + i(\ell_Q\cos\theta + n_Q\sin\theta)]\, dk d\theta$$

the integrands of which are asymptotically equal for very large values of k, we see that the former integral is singular only at the free surface $z = \zeta = 0$ when P coincides with Q. As in the previous section, this singularity is removed from the first integral on the right of (19) by subtracting the transpose from the original integrand, although the problem of treating the last integral of (19), which is also singular, remains.

Next let us consider the second integral of (20). With the substitution $\lambda = \tan\theta$, the integral becomes

$$\int_0^\infty \sqrt{1+\lambda^2}\, e^{k_0(z+\zeta)(1+\lambda^2)}\Big\{\ell_Q \cos[k_0(x-\xi)\sqrt{1+\lambda^2}] \cos[k_0(y-\eta)\lambda\sqrt{1+\lambda^2}]$$

$$- m_Q \lambda \sin[k_0(x-\xi)\sqrt{1+\lambda^2}] \sin[k_0(y-\eta)\lambda\sqrt{1+\lambda^2}$$

$$+ n_Q \sqrt{1+\lambda^2} \sin[k_0(x-\xi)\sqrt{1+\lambda^2}] \cos[k_0(y-\eta)\lambda\sqrt{1+\lambda^2}]\Big\} d\lambda \tag{21}$$

Convergence problems arise only when $z = \zeta = 0$. For this case let us expand the integrand of (21) in powers of $1/\lambda$. We have, with

$$\alpha = k_0(x-\xi), \quad \beta = k_0(y-\eta)$$

$$\cos(\alpha\sqrt{1+\lambda^2}) = \cos[\alpha(\lambda + \frac{1}{2\lambda} - \frac{1}{8\lambda^3} + \cdots)]$$

$$= \cos\alpha\lambda \cos[\alpha(\frac{1}{2\lambda} - \frac{1}{8\lambda^3} + \cdots)]$$

$$- \sin\alpha\lambda \sin[\alpha(\frac{1}{2\lambda} - \cdots)]$$

Hence, for very large values of λ, we have

$$\cos\,(\alpha\sqrt{1+\lambda^2}) \approx (1 - \frac{\alpha^2}{4\lambda^2})\cos\alpha\lambda - \frac{\alpha}{2\lambda}\sin\alpha\lambda \tag{22a}$$

Similarly

$$\sin\,(\alpha\sqrt{1+\lambda^2}) \approx (1 - \frac{\alpha^2}{4\lambda^2})\sin\alpha\lambda + \frac{\alpha}{2\lambda}\cos\alpha\lambda \tag{22b}$$

$$\cos\,(\beta\lambda\sqrt{1+\lambda^2}) \approx \cos\,[\beta(\lambda^2+\tfrac{1}{2})] + \frac{\beta}{8\lambda^2}\sin\,[\beta(\lambda^2+\tfrac{1}{2})] \tag{22c}$$

$$\sin\,(\beta\lambda\sqrt{1+\lambda^2}) \approx \sin\,[\beta(\lambda^2+\tfrac{1}{2})] + \frac{\beta}{8\lambda^2}\cos\,[\beta(\lambda^2+\tfrac{1}{2})] \tag{22d}$$

The the terms of the integrand of (21) become asymptotically

$$\sqrt{1+\lambda^2}\cos\,[\alpha\sqrt{1+\lambda^2}]\cos\,[\beta\lambda\sqrt{1+\lambda^2}] \approx \lambda\cos\alpha\lambda\cos\,[\beta(\lambda^2+\tfrac{1}{2})] - \frac{\alpha}{2}\sin\alpha\lambda\cos\,[\beta(\lambda^2+\tfrac{1}{2})] \tag{23a}$$

$$\lambda\sqrt{1+\lambda^2}\sin\,[\alpha\sqrt{1+\lambda^2}]\sin\,[\beta\lambda\sqrt{1+\lambda^2}] \approx \lambda^2 - \frac{\alpha^2}{4} + \tfrac{1}{2})\sin\alpha\lambda\sin\,[\beta(\lambda^2+\tfrac{1}{2})] + \frac{\alpha\lambda}{2}\cos\alpha\lambda\sin\,[\beta(\lambda^2+\tfrac{1}{2})] - \frac{\beta}{8}\sin\alpha\lambda\cos\,[\beta(\lambda^2+\tfrac{1}{2})] \tag{23b}$$

$$(1+\lambda^2)\sin\,[\alpha\sqrt{1+\lambda^2}]\cos\,[\beta\lambda\sqrt{1+\lambda^2}] \approx (\lambda^2 - \frac{\alpha^2}{4} + 1)\cos\alpha\lambda\sin\,[\beta(\lambda^2+\tfrac{1}{2})] + \frac{\alpha\lambda}{2}\cos\alpha\lambda\cos\,[\beta(\lambda^2+\tfrac{1}{2})] + \frac{\beta}{8}\sin\alpha\lambda\sin\,[\beta(\lambda^2+\tfrac{1}{2})] \tag{23c}$$

Considering $\sin\,[\beta(\lambda^2+\tfrac{1}{2})]$ and $\cos\,[\beta(\lambda^2+\tfrac{1}{2})]$ in the forms

$$\sin\,[\beta(\lambda^2+\tfrac{1}{2})] = \sin\,(\beta\lambda^2)\cos\frac{\beta}{2} + \cos\,(\beta\lambda^2)\sin\frac{\beta}{2}$$

$$\cos\,[\beta(\lambda^2+\tfrac{1}{2})] = \cos\,(\beta\lambda^2)\cos\frac{\beta}{2} - \sin\,(\beta\lambda^2)\sin\frac{\beta}{2}$$

it is seen that, with $z = \zeta = 0$, the asymptotic form of the integrand of (21) is a linear combination of the terms

$$\sin\alpha\lambda\,\frac{\sin}{\cos}\,(\beta\lambda^2),\quad \lambda\cos\alpha\lambda\,\frac{\sin}{\cos}\,(\beta\lambda^2),\quad \lambda^2\sin\alpha\lambda\,\frac{\sin}{\cos}\,(\beta\lambda^2) \tag{24}$$

But from the table of definite integrals by Gröbner and Hofreiter [5], we have

$$\int_0^\infty \sin\alpha\lambda\,\frac{\sin}{\cos}\,(\beta\lambda^2)\,d\lambda = 0,\quad \beta \neq 0 \tag{25}$$

Since the derivatives of the first member of (24) with respect to α and β yield the second and third members, one is tempted to conclude from these derivations of (25) that the infinite integrals of the second and third members are also zero. However, since the integrals resulting from these differentiations are not uniformly convergent, this conclusion may not be valid, as is also shown by the integration by parts

$$\int_0^L \sin \alpha\lambda \sin (\beta\lambda^2)\, d\lambda = -\frac{1}{\alpha} \cos \alpha L \sin \beta L^2 \Big|_0^L + \frac{2\beta}{\alpha} \int_0^L \lambda \cos \alpha\lambda \cos (\beta\lambda^2)\, d\lambda \qquad (26)$$

Although the limit of the left member as $L \to \infty$ is zero, the oscillation of the first term on the right between $\pm 1/\alpha$ indicates that the last integral is indeterminate. The mean value of the last integral would, however, be zero in the limit.

If z and ζ were not zero, the asymptotic forms occurring in (24) would have been multiplied by $e^{k_0(z + \zeta)\lambda^2}$. With this factor the integral of the first member of (24) and its derivatives would be uniformly convergent, and consequently the derivative of the integral would be equal to the integral of the derivative. For example we would have

$$\frac{\partial}{\partial\alpha} \int_0^\infty e^{k_0(z + \zeta)\lambda^2} \sin \alpha\lambda \sin (\beta\lambda^2)\, d\lambda = \int_0^\infty e^{k_0(z + \zeta)\lambda^2} \lambda \cos \alpha\lambda \sin (\beta\lambda^2)\, d\lambda$$

with a determinate value for the right member, no matter how close $z + \zeta$ is to zero. Again integrating by parts, we have

$$\int_0^L e^{k_0(z + \zeta)\lambda^2} \sin \alpha\lambda \sin (\beta\lambda^2)\, d\lambda = -\frac{1}{\alpha} e^{k_0(z + \zeta)L^2} \cos \alpha L \sin \beta L^2 \Big|_0^L$$

$$+ \frac{2}{\alpha} \int_0^L [k_0(z + \zeta) \sin (\beta\lambda^2) + \beta \cos (\beta\lambda^2)]\, \lambda e^{k_0(z + \zeta)\lambda^2} \cos \alpha\lambda d\lambda \qquad (27)$$

Now the oscillation of the first member on the right with increasing values of L is damped by the exponential factor and yields in the limit

$$\int_0^\infty e^{k_0(z + \zeta)\lambda^2} \sin \alpha\lambda \sin (\beta\lambda^2)\, d\lambda = \frac{2}{\alpha} \int_0^\infty [k_0(z + \zeta) \sin (\beta\lambda^2) + \beta \cos (\beta\lambda^2)] \cdot$$

$$\lambda e^{k_0(z + \zeta)\lambda^2} \cos \alpha\lambda d\lambda \qquad (28)$$

which, when $z + \zeta$ is very small, becomes

$$\int_0^\infty e^{k_0(z + \zeta)\lambda^2} \sin \alpha\lambda \sin (\beta\lambda^2)\, d\lambda \simeq \frac{2\beta}{\alpha} \int_0^\infty e^{k_0(z + \zeta)\lambda^2} \lambda \cos \alpha\lambda \cos (\beta\lambda^2)\, d\lambda \tag{29}$$

Hence, since by (25) the limit of the integral on the left of (29), as $z + \zeta \to 0$, is zero, we see that

$$\lim_{z + \zeta \to 0} \int_0^\infty e^{k_0(z + \zeta)\lambda^2} \lambda \cos \alpha\lambda \,\frac{\sin}{\cos}\, (\beta\lambda^2)\, d\lambda$$

$$= \text{M.V.O.} \int_0^L \lambda \cos \alpha\lambda \,\frac{\sin}{\cos}\, (\beta\lambda^2)\, d\lambda = 0 \tag{30}$$

where M.V.O. denotes the "mean value of the oscillation" for large values of L. Clearly the foregoing result applies to either $\sin (\beta\lambda^2)$ or $\cos (\beta\lambda^2)$, as is indicated in (30). Similarly, by integrating the last integrals in (26) and (27) again by parts, we can show that

$$\lim_{z = \zeta \to 0} \int_0^\infty e^{k(z + \zeta)\lambda^2} \lambda^2 \sin \alpha\lambda \,\frac{\sin}{\cos}\, (\beta\lambda^2)\, d\lambda$$

$$= \text{M.V.O.} \int_0^L \lambda^2 \sin \alpha\lambda \,\frac{\sin}{\cos}\, (\beta\lambda^2)\, d\lambda = 0 \tag{31}$$

In the above analysis it was assumed in (25) that $\beta = k_0(y - \eta) \neq 0$. If $\beta = 0$, $\alpha \neq 0$, the terms of (24) become

$$\sin \alpha\lambda, \quad \lambda \cos \alpha\lambda, \quad \lambda^2 \sin \alpha\lambda$$

and we can show by integration by parts that

$$\lim_{z + \zeta \to 0} \int_0^\infty e^{k_0(z + \zeta)\lambda^2} \sin \alpha\lambda d\lambda = \lim_{z + \zeta \to 0} \int_0^\infty \lambda e^{k_0(z + \zeta)\lambda^2} \cos \alpha\lambda d\lambda$$

$$= \lim_{z + \zeta \to 0} \int_0^\infty \lambda^2 e^{k_0(z + \zeta)\lambda^2} \sin \alpha\lambda d\lambda = 0 \tag{32}$$

If $\alpha = 0$ also, then we have

$$\int_0^\infty e^{k_0(z + \zeta)\lambda^2} \lambda d\lambda = \frac{1}{2k_0 |z + \zeta|}$$

which indicates that the second of the limits in (32) does not exist.

Our conclusion is that the integral in (21) is determinate except when P and Q coincide and are at the free surface. When P and Q are at the free surface, but not coincident, the integral must be determined as the limiting value as $z + \zeta \to 0$ through negative values. Finally, the singularity when P and Q coincide is removed from the first integral on the right of (19) by subtracting the transpose from the original integrand, although, as for the double integral of (20), the last integral of (19) remains to be treated.

Let us now consider the last integral in (19),

$$\int_{S_0} \frac{\partial H(Q, P)}{\partial \zeta}\bigg|_{\zeta=0} dS_P = \frac{1}{\pi} \int_{S_0} \int\!\!\!\!\!-_{0}^{\infty} \int_0^{2\pi} \frac{e^{kz - ik[(x - \xi)\cos\theta + (y - \eta)\sin\theta]}}{k - k_0 \sec^2\theta} \cdot k^2 d\theta dk dS_P$$

$$-4k_0^2 \int_{S_0} \int_0^{\pi/2} \sec^4\theta \cdot e^{k_0 z \sec^2\theta} \sin[k_0(x - \xi)\sec\theta] \cos[k_0(y - \eta)\tan\theta\sec\theta] \, d\theta dS_P \qquad (33)$$

Let $\eta = \pm\,\eta(x)$ be the equation of the hull waterplane at $\zeta = 0$. Take the origin at the midship section and let ℓ denote the half length of the ship. Then, interchanging the order of integration in (33) with the integration over S_0 taken first, we are led to consider the integral,

$$F(k, \theta) = \int_{-\ell}^{\ell} \int_{-\eta(x)}^{\eta(x)} e^{ik(\xi\cos\theta + \eta\sin\theta)} d\eta d\xi \qquad (34)$$

This becomes

$$F(k, \theta) = \frac{2\csc\theta}{k} \int_{-\ell}^{\ell} e^{ik\,\xi\cos\theta} \sin[k\,\eta\,(\xi)\sin\theta]\, d\xi \qquad (35)$$

or, integrating by parts and noting that $\eta(\ell) = \eta(-\ell) = 0$, we obtain

$$F(k, \theta) = \frac{2i\sec\theta}{k} \int_{-\ell}^{\ell} e^{ik\,\xi\cos\theta} \eta'(\xi) \cos[k\,\eta\,(\xi)\sin\theta]\, d\xi \qquad (36)$$

where $\eta'(\xi)$ denotes the derivative of η with respect to ξ.

Along the parallel middle body of a ship form, $\eta'(\xi) = 0$, and near the bow and stern, $\eta(\xi)$ is very small. This suggests the approximation

$$F(k, \theta) \doteq \frac{2i \sec \theta}{k} \int_{-\ell}^{\ell} e^{ik \xi \cos \theta} \eta'(\xi) \, [1 - \tfrac{1}{2}k^2 \eta(\xi)^2 \sin^2\theta] \, d\xi \qquad (37)$$

Additional terms may be taken in the expansion of cos $[k \eta (\xi) \sin \theta]$ if required for greater accuracy. If the parallel middle body extends over the range $a \leq \xi \leq b$, (37) may be written

$$F(k, \theta) = \frac{2i \sec \theta}{k} \left\{ \int_{-\ell}^{a} + \int_{b}^{\ell} \right\} e^{ik \xi \cos \theta} \cdot$$

$$[\eta'(\xi) - \tfrac{1}{2}k^2 \eta'(\xi) \eta(\xi)^2 \sin^2\theta] \, d\xi$$

or, introducing $\mu = \frac{\xi - a}{\ell + a}$ in the first integral, and $\mu = \frac{\xi - b}{\ell - b}$ in the second,

$$F(k, \theta) = \frac{2i \sec \theta}{k} \left\{ e^{ika \cos \theta} \int_{-1}^{0} e^{ik(\ell + a)\mu \cos \theta} \cdot \right.$$

$$[\eta'(\xi) - \tfrac{1}{2}k^2 \eta'(\xi) \eta(\xi)^2 \sin^2\theta] \, d\mu$$

$$+ e^{ikb \cos \theta} \int_{0}^{1} e^{ik(\ell - b)\mu \cos \theta} [\eta'(\xi) - \tfrac{1}{2}k^2 \eta'(\xi) \eta(\xi)^2 \sin^2\theta] \, d\mu \qquad (38)$$

One can now fit either Fourier series or polynomials in μ to the functions $\eta'(\xi)$ and $\eta'(\xi) \eta(\xi)^2$, the choice depending upon the particular form. If $\eta'(\xi)$ becomes infinite at the bow and stern, as it will if the radii of curvature are not zero at the extremities, a suitable fit which can satisfy this condition can be obtained from a polynomial for $\eta(\xi)^2$ of the form

$$\eta(\xi)^2 = (1 - \mu^2) \, p(\mu) \qquad (39)$$

where $p(\mu)$ is a polynomial such that $p(\pm 1) \neq 0$. For then

$$\eta'(\xi) = [\tfrac{1}{2} (1 - \mu^2) \, p'(\mu) - \mu p(\mu)] \, \frac{d\mu/d\xi}{\eta(\xi)}$$

is seen to become infinite at $\mu = \pm 1$. In this way $F(k, \theta)$ can be expressed as a series of functions of k and θ, each of which is regular even as z approaches zero.

Although not evident from the form of the last integral in (33), by returning to the complex exponential form from which its integrand was derived one finds that the function $F(k, \theta)$ of (34), with $k = k_0 \sec^2\theta$, applies to this integral as well. This will not be developed here in detail, nor will the analysis of the wave kernel be carried any further, since the application to a particular case, on which, it has been seen, the nature of the subsequent analysis would depend, has not yet been performed.

Convergence of Iteration Formulas

Hereafter we shall consider only the case where the boundary condition on the plane $z = 0$ is

$$\frac{\partial \Phi}{\partial z} = 0, \quad z = 0 \tag{40}$$

i.e., the case of "zero" Froude number. The integral equation (8) then reduces to

$$M(Q) = F(Q) + \int_{S+S'} M(P)\, K(P, Q)\, dS_P \tag{41}$$

where

$$K(P, Q) = \frac{1}{2\pi} \frac{\partial}{\partial N_Q} \left(\frac{1}{R}\right), \quad F(Q) = -\frac{U}{2\pi} \ell_Q \tag{42}$$

Equation (9) shows that the homogeneous integral equation

$$f(Q) = \lambda \int_{S+S'} K(Q, P)\, f(P)\, dS_P$$

has the eigenfunction $f(P) = 1$ when $\lambda = -1$. Thus $\lambda = -1$ is an eigenvalue of the kernel $K(Q. P)$, and hence also of its transpose $K(P, Q)$.

Consider the inhomogeneous integral equation

$$M(Q) = F(Q) + \lambda \int_{S+S'} M(P)\, K(P, Q)\, dS_P \tag{43}$$

which reduces to (41) when $\lambda = 1$. The theory of this integral equation states that $M(Q)$, considered as a function of the complex variable λ, is regular in the unit circle about $\lambda = 0$, and has a simple pole at $\lambda = -1$. Writing

$$M(Q) = F(Q) + \lambda F_1(Q) + \lambda^2 F_2(Q) + \cdots, \quad |\lambda| < 1 \tag{44}$$

and substituting (44) into (43), yields the relation

$$F_{n+1}(Q) = \int_{S+S'} F_n(P)\, K(P, Q)\, dS_P \tag{45}$$

Put

$$M_n(Q) = F(Q) + \lambda F_1(Q) + \cdots + \lambda^n F_n(Q) \tag{46}$$

Then, by (45), we obtain the iteration formula

$$M_{n+1}(Q) = F(Q) + \lambda \int_{S+S'} M_n(P)\, K(P, Q)\, dS_P \tag{47}$$

According to (44), however, the sequence of functions $M_{n+1}(Q)$ defined by (47) may not converge when $\lambda = 1$.

We can eliminate the pole at $\lambda = -1$ by considering

$$(1 + \lambda)\, M(Q) = F(Q) + \lambda(F + F_1) + \lambda^2(F_1 + F_2) + \cdots, \quad |\lambda| < |\lambda_2| \tag{48}$$

where λ_2 denotes the next eigenvalue of $K(P, Q)$, arranged in the order of increasing absolute magnitude. Defining

$$M_n' = \frac{1}{1 + \lambda}\,[F(Q) + \lambda(F + F_1) + \cdots + \lambda^n(F_{n-1} + F_n)] \tag{49}$$

then, by (45),

$$M_n'(Q) = F(Q) + \lambda \int_{S+S'} M_n'(P)\, K(P, Q)\, dS_P \tag{50}$$

Comparison with (47) shows that the sequences M_n and M_n' are obtained from the identical iteration formula, but these sequences differ because of the change in the initial approximations,

$$M_0(Q) = F(Q), \quad M_0'(Q) = \frac{F(Q)}{1 + \lambda} \tag{51}$$

Thus, when $\lambda = 1$, $M_0'(Q) = \frac{1}{2} F(Q)$. Alternatively, if we observe from (44) and (49) that

$$M_n' = \frac{1}{1 + \lambda}\,(M_n + \lambda M_{n-1}) \tag{52}$$

we obtain when $\lambda = 1$

$$M_n' = \tfrac{1}{2}\,(M_n + M_{n-1})$$

i.e., the arithmetic means of successive pairs of members of the sequence $M_n(Q)$ form a convergent sequence.

Let us consider the modification of the iteration formula (50),

$$2M''_{n+1}(Q) = M''_n(Q) + F(Q) + \lambda \int_{S+S'} M''_n(Q)\, K(P, Q)\, dS_P \qquad (53)$$

with $M''_0 = M'_0$ given by (51). We have, by (50),

$$M''_1 = \tfrac{1}{2}\, [M'_0 + F + \lambda \int M'_0\, KdS] = \tfrac{1}{2}\, (M'_0 + M'_1)$$

$$M''_2 = \tfrac{1}{2}\, [\tfrac{1}{2}\, (M'_0 + M'_1) + F + \lambda \int \tfrac{1}{2}\, (M'_0 + M'_1)\, KdS] = \tfrac{1}{4}\, (M'_0 + 2M'_1 + M'_2)$$

We can now show by mathematical induction that

$$M''_n = \frac{1}{2^n}\, [M'_0 + \binom{n}{1} M'_1 + \binom{n}{2} M'_2 + \cdots + \binom{n}{n} M'_n] \qquad (54)$$

For if (54) is valid, then by (53)

$$2M''_{n+1} = \frac{1}{2^n} \left\{ [M'_0 + \binom{n}{1} M_1 + \cdots + M'_n] + 2^n F + \lambda \int [M'_0 + \binom{n}{1} M'_1 + \cdots + M'_n]\, KdS \right\}$$

But

$$2^n F \equiv (1 + 1)^n F = [1 + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}] F$$

Then, by (50),

$$2^n F + \lambda \int [M'_0 + \binom{n}{1} M'_1 + \cdots + M'_n] KdS = M'_1 + \binom{n}{1} M'_2 + \binom{n}{2} M'_3 + \cdots + M'_{n+1}$$

Thus we have

$$2M''_{n+1} = \frac{1}{2^n} \left\{ M'_0 + [\binom{n}{0} + \binom{n}{1}] M'_1 + [\binom{n}{1} + \binom{n}{2}] M'_2 + \cdots + [\binom{n}{n-1} + \binom{n}{n}]\, M'_n + M'_{n+1} \right\}$$

or since

$$\binom{n}{n} + \binom{n}{n+1} = \binom{n+1}{n+1}$$

then

$$M''_{n+1} = \frac{1}{2^{n+1}}\, [M'_0 + \binom{n+1}{1} M'_1 + \binom{n+1}{2} M'_2 + \cdots + M'_{n+1}]$$

as we wished to show to complete the proof by induction.

In order to investigate the convergence of the sequence $\left\{M''_n\right\}$, let us take N sufficiently large so that, for $r > N$, $|M'_r - M| < \varepsilon/2$, where M is the limit of the sequence $\left\{M'_n\right\}$. From (54) we have

$$M''_n - M = \frac{1}{2^n} [(M'_0 - M) + \binom{n}{1}(M'_1 - M) + \cdots + \binom{n}{N}(M'_N - M) + \cdots + \binom{n}{n}(M'_n - M)]$$

Then

$$|M''_n - M| < \frac{1}{2^n} [|M'_0 - M| + \binom{n}{1}|M'_1 - M| + \cdots + |M'_N - M|\binom{n}{N}]$$
$$+ \frac{\varepsilon}{2^{n+1}} [\binom{n}{N+1} + \binom{n}{N+2} + \cdots + \binom{n}{n}]$$

But

$$\frac{1}{2^n} [\binom{n}{N+1} + \binom{n}{N+2} + \cdots + \binom{n}{n}] < \frac{1}{2^n} [1 + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}] = 1$$

Then, if ρ is an upper bound of $|M'_i - M|$, $i = 0, 1, 2, \cdots N$, and $n << 2N$, we have

$$|M''_n - M| < \frac{\rho}{2^n} [1 + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{N}] + \frac{\varepsilon}{2} < \frac{\rho}{2^n} (N + 1)\binom{n}{N} + \frac{\varepsilon}{2}$$
$$< \frac{\rho(N + 1)}{N!} \frac{n^N}{2^n} + \frac{\varepsilon}{2} < \varepsilon$$

by taking n sufficiently large. Hence the sequence $\left\{M''_n\right\}$ also converges to M.

The alternative iteration formulas for M' or M" arise when relation (10) is applied to remove the singularity of the kernel in the integral equation (41). We obtain

$$M(Q) = F(Q) - M(Q) + \int_{S+S'} [M(P)\, K(P, Q) - M(Q)\, K(Q, P)]\, dS_P \qquad (55)$$

and the iteration formulas

$$M_{n+1} = F - M_n + \int [M_n(P)\, K(P, Q) - M_n(Q)\, K(Q, P)]\, dS_P \qquad (56)$$

or

$$2M_{n+1} = F + \int [M_n(P)\ K(P,\ q) - M_n(Q)\ K(Q,\ P)]\ dS_P \qquad (57)$$

By (9), the first of these is seen to be of the form (50), the second of the form (53). Both begin with the same initial approximation which, by (51), is given by

$$M_0(Q) = \tfrac{1}{2}\ F(Q) = -\frac{U}{4\pi}\ \ell_Q \qquad (58)$$

Although there is no a-priori basis for preferring one iteration formula over the other, comparison of the numerical results with the known exact solution for the case of an ellipsoid has shown that the sequence given by (57) converged much more rapidly than that obtained from (56).

At points where the normal to S is not continuous, the integral equation (8), and of course the above iteration formulae, are not valid. At such points we can either set $M(Q) = 0$, as can be justified, or round sharp edges with small, nonzero curvature and continue to use the iteration formula (57).

Distribution of Velocity Potential on S

Once the source distribution M has been found, the velocity potential Φ can be computed from (7). For points Q on S we again encounter a singularity when P coincides with Q. This singularity may be removed as follows:

Let $N(P)$ be a source distribution on $S + S'$ which makes the surface an equipotential of potential ϕ_0. This distribution satisfies the homogeneous integral equation

$$\int_{S+S'} N(P)\ K(P,\ Q)\ dS_P = -\ N(Q) \qquad (59)$$

with the same kernel as in (42). In fact, $N(P)$ is the eigenfunction of $K(P,\ Q)$ associated with the eigenvalue $\lambda = -\ 1$. This equation can be solved by means of the iteration formula

$$N_{n+1}(Q) = -\int_{S+S'} N_n(P)\ K(P,\ Q)\ dS_P$$

which, by applying (9), may be written in the singularity-free form

$$N_{n+1}(Q) = N_n(Q) - \int_{S+S'} [N_n(P)\, K(P, Q) - N_n(Q)\, K(Q, P)]\, dS_P \tag{60}$$

Since the matrices occurring in (60) have already been obtained for the numerical evaluation of M(Q) from (57), the corresponding values of N(Q) can be obtained from (60) with little additional computer time. Since the potential is constant in the interior of an equipotential surface, its value may conveniently be computed at the origin as

$$\phi_0 = - \int_{S+S'} \frac{N(P)}{[x^2 + y^2 + z^2]^{\frac{1}{2}}}\, dS_P \tag{61}$$

We can now apply the solution of this Dirichlet problem to eliminate the singularity from the expression for the velocity potential (7), by writing

$$\Phi(Q) = Ux - \int_{S+S'} \frac{1}{R} [M(P) - N(P) \frac{M(Q)}{N(Q)}]\, dS_P + \phi_0 \frac{M(Q)}{N(Q)}$$

$$+ \int_S M(P)\, H(P, Q)\, dS_P \tag{62}$$

Here also we can justify setting the first integrand of (62) equal to zero when P coincides with Q, by the same argument as was used in equation (10).

Application to a Double Ship Model — Zero Froude Number

Since the x-y and x-z planes are planes of symmetry, it is necessary to determine the source distribution over only one-fourth of the hull surface of the double model. Let us consider only points Q for $y, z, \leqq 0$. Denote by S_1, S_2, S_3, S_4 the parts of S + S' for which $y, z > 0$; $y < 0, z > 0$; $y < 0, z < 0$; $y > 0, z < 0$, respectively. Put

$$R_1 = [(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2]^{\frac{1}{2}}$$

$$R_2 = [(x - \xi)^2 + (y + \eta)^2 + (z - \zeta)^2]^{\frac{1}{2}}$$

$$R_3 = [(x - \xi)^2 + (y + \eta)^2 + (z + \zeta)^2]^{\frac{1}{2}}$$

$$R_4 = [(x - \xi)^2 + (y - \eta)^2 + (z + \zeta)^2]^{\frac{1}{2}}$$

the distances from $Q(x, y, z)$ to the congruent points $P \equiv P_1(\xi, \eta, \zeta)$, $P_2(\xi, -\eta, \zeta)$, $P_3(\xi, -\eta, -\zeta)$, $P_4(\ , \ \ , - \ )$. At congruent points we have

$$M(P) = M(P_2) = M(P_3) = M(P_4)$$

and, denoting the direction cosines at P_i by ℓ_i, m_i, n_i, $i = 1, 2, 3, 4$, we obtain the following relations:

$$\ell_p \equiv \ell_1 = \ell_2 = \ell_3 = \ell_4$$

$$m_p \equiv m_1 = - m_2 = - m_3 = m_4$$

$$n_p \equiv n_1 = n_2 = - n_3 = - n_4$$

If the values of the integrand of (57) at congruent points P are collected, the resulting integral would extend only over S_1 and is found to be of the form

$$M_{n+1}(Q) = F(Q) + \frac{1}{4\pi} \int_{S_1} [M_n(P)\, J(P, Q) - M_n(Q)\, J(Q, P)]\, dS_p \qquad (63)$$

where

$$J(P, Q) = [(\xi - x)\ell_Q + (\eta - y)m_Q + (\zeta - z)n_Q]\left(\frac{1}{R_1^3} + \frac{1}{R_2^3} + \frac{1}{R_3^3} + \frac{1}{R_4^3}\right)$$

$$- 2\left(\frac{m_Q\eta}{R_2^3} + \frac{m_Q\eta + n_Q\zeta}{R_3^3} + \frac{n_Q\zeta}{R_4^3}\right); \quad P \neq Q \qquad (64)$$

and, since the integrand of (63) vanishes when P coincides with Q, we may set

$$J(Q, Q) = 0 \qquad (65)$$

Similarly the integrations over S + S' in connection with the Dirichlet problem in (60), (61) and (62) can be expressed in terms of integrals over S_1. Thus (60) becomes

$$N_{n+1}(Q) = N_n(Q) - \int_{S_1} [N_n(P)\, J(P, Q) - N_n(Q)\, J(Q, P)]\, dS_P \qquad (66)$$

in which J(P, Q) and J(Q, Q) are given in (64) and (65), (61) becomes

$$\phi_0 = - 4 \int_{S_1} \frac{N(P)}{[x^2 + y^2 + z^2]^{\frac{1}{2}}}\, dS_P \qquad (67)$$

and (62), without the wave integral, assumes the form

$$\Phi(Q) = Ux - \int_{S_1} L(P, Q) \left[M(P) - N(P) \frac{M(Q)}{N(Q)}\right] dS_P + \phi_0 \frac{M(Q)}{N(Q)} \tag{68}$$

where

$$\left.\begin{aligned} L(P, Q) &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\,; \quad P \neq Q \\ \text{and} \qquad L(Q, Q) &= 0 \end{aligned}\right\} \tag{69}$$

Application to an Ellipsoid

With the equation of the ellipsoid in the alternative forms

$$\left.\begin{aligned} &\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \\ \text{or} \qquad &y = \frac{b}{a}\sqrt{a^2 - x^2}\cos\theta, \; z = \frac{c}{a}\sqrt{a^2 - x^2}\sin\theta \end{aligned}\right\} \tag{70}$$

the direction cosines are

$$\ell_Q = \frac{A(Q)x}{a^2}, \quad m_Q = \frac{A(Q)y}{b^2}, \quad n_Q = \frac{A(Q)z}{c^2}$$

where

$$A(Q) = \left[\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}\right]^{-\frac{1}{2}}$$

Also we have

$$(\xi - x)\ell_Q + (\eta - y)m_Q + (\zeta - z)n_Q = -A(Q)\left[\frac{(\xi - x)^2}{a^2} + \frac{(\eta - y)^2}{b^2} + \frac{(\zeta - z)^2}{c^2}\right] \tag{71}$$

The right member of (71) is preferable to the first for numerical computations, especially when P is near Q, since all terms on the right are then of second order of smallness, and a loss of numerical accuracy would be expected if the same result were obtained from the sum of the first-order terms on the left. In terms of x and θ as the independent variables, the element of area dS_P becomes

$$dS_P = \left[\frac{b^2c^2x^2}{a^4} + \left(1 - \frac{x^2}{a^2}\right)(b^2\sin^2\theta + c^2\cos^2\theta)\right]^{\frac{1}{2}} d\theta dx$$

with θ varying from 0 to 2π and x from -a to a.

Exact solutions of the foregoing integral equations for the ellipsoid are expressible in terms of the Lamé ellipsoidal coordinates (ρ, μ, ν), which are related to the rectangular coordinates (x, y, z) by [6]

$$x = \frac{\rho\mu\nu}{hk}, \quad y^2 = \frac{(\rho^2 - k^2)(k^2 - \mu^2)(k^2 - \nu^2)}{k^2(k^2 - h^2)}, \quad z^2 = \frac{(\rho^2 - h^2)(\mu^2 - h^2)(h^2 - \nu^2)}{h\,(k^2 - h^2)} \tag{72}$$

When $\rho = a$, $k = \sqrt{a^2 - b^2}$, $h = \sqrt{a^2 - c^2}$, (72) is equivalent to the equation of the ellipsoid (70). Here

$$k^2 \le \rho^2 < \infty\ , \quad h^2 \le \mu^2 \le k^2\ , \quad 0 \le \nu^2 \le h^2$$

Solutions of the present problem can be expressed in terms of the Lamé functions of the first and second kind, E_n^m and F_m^n. For the source distribution we find

$$M(Q) = \frac{3U\mu\nu}{4\pi ahk\, \dot{F}_1^1(a)\sqrt{(a^2 - \mu^2)(a^2 - \nu^2)}} = -\frac{U}{4\pi}\left[1 - \frac{F_1^1(a)}{\dot{F}_1^1(a)}\right]\ell_Q \tag{73}$$

and for the velocity potential,

$$\Phi = \left[1 - \frac{F_1^1(a)}{\dot{F}_1^1(a)}\right] Ux \tag{74}$$

Here $\dot{F}_1^1$ denotes the derivative of F_1^1 with respect to its argument, and

$$F_1^1(a) = 3a\int_a^\infty \frac{d\rho}{\rho^2\sqrt{(\rho^2 - h^2)(\rho^2 - k^2)}} = \frac{3a}{kh^2}\left[F(\phi, \lambda) - E(\phi, \lambda)\right] \tag{75}$$

$$\phi = \arccos\frac{b}{a}\ , \quad \lambda = \frac{h}{k}$$

where $F(\phi, \lambda)$, $E(\phi, \lambda)$ are the Legendre incomplete elliptic integrals of modulus λ and amplitude ϕ of first and second kinds.

Exact solutions of the equipotential problem are also of interest. If the total strength of the distribution over the ellipsoid is G, the source strength N(Q) is given by

$$N(Q) = \frac{GA(Q)}{4\pi abc} \tag{76}$$

On and within the ellipsoid the potential has the constant value

$$\phi_0 = \frac{G}{2}\int_0^\infty \frac{ds}{[(a^2+s)(b^2+s)(c^2+s)]^{\frac{1}{2}}} = \frac{G}{k} F(\phi, \lambda) \qquad (77)$$

Calculations have been performed for the case a = 1, b = 0.25, c = 0.50, for which

$$1 - \frac{F_1^1(a)}{F_1^{1}(a)} = 1.12659$$

Then

$$M(Q) = -\ 1.12659\ \frac{U}{4\pi}\ xA(Q)$$

$$\Phi = 1.12659\ Ux$$

$$N(Q) = \frac{2}{\pi}\ GA(Q)$$

$$\phi_0 = 1.76984\ G$$

FORTRAN Program

In the FORTRAN program given in the Appendix, the Gauss 16-point quadrature formula was selected for purposes of illustration. Because of symmetry about the x-z and x-y planes, with this quadrature formula there are 16 i's but only 8 j's, so that the matrices $J_{ij,\ kl}$ and $L_{ij,\ kl}$ contain 128 x 128 = 16384 elements. An advantage of the Gauss formula is that it gives finer intervals at the bow and stern than amidships, as is required by the rapid changes in form of the transverse sections at a ship's extremities. Nevertheless, for a particular form, it may be desired to use a quadrature formula of the Simpson-rule type, with a fine mesh near the bow and stern and a coarser one over the remainder of the hull.

The input data of the FORTRAN program are the coordinates (x_i, y_{ij}, z_{ij}), the direction cosines l_{ij}, m_{ij}, n_{ij}, and the weighting factors of the selected quadrature formula, A_k and A_l. The various do-loops in the program perform the following operations:

Loop 2 computes the first approximations for the iteration formulas (63) and (66), $(M_0)_{ij}$ = SDI (I, J) and $(N_0)_{ij}$ = SDO (I, J). Here is also computed the product of the area element E_{ij} = E (I, J) by the weighting factors, $A_iA_jE_{ij}$ = F (I, J).

Loop 9 computes the kernel matrix of the integral equations (63) and (66), $J_{ij,\ kl}$ which, multiplied by the weighting factors, is denoted by C (IJ, KL). The program yields simultaneously the transpose of the matrix with its corresponding weighting factors, and then computes directly the diagonal terms of the matrix. Also computed in this loop is the matrix $L_{ij,\ kl}$ = P1 (IJ, KL), including the weighting factors and the element of area, used in computing ϕ_i from (68).

Loops 22-23 compute successive approximations to the source distribution M_n = SDI from the iteration formula (63), for 10 iterations.

Loops 25-26 compute successive approximations to the source distribution for the equipotential problem, N_n = SDO, from the iteration formula (66), for 20 iterations.

Loop 28 computes ϕ_0 = PHO from (67).

Loop 35 computes ϕ_{ij} = PHI (I, J) from the expression for the velocity potential in (68).

Results for an Ellipsoid

The program was tested by applying it to an ellipsoid with axes a, b, c in the proportions

$$a : b : c = 4 : 1 : 2$$

Input data are given in Table 1, in which θ is the parametric angle of the equation of the ellipsoid, (70). The values of x and θ in the table correspond to the abscissas of the Gauss 16-point quadrature formula.

Values of the source distribution, M(Q), computed from the exact expression as well as from the discretized iteration formula with 10 iterations, are given in Table 2. The results are seen to agree to within four significant figures for θ near zero, but to only three significant figures for θ near $\pi/2$. Repetition of the calculations employing double-precision arithmetic showed essentially the same results, indicating that the errors are due principally to the discretization of the equations, rather than round-off errors by the computer.

Table 3 gives the results for the source distribution N(Q) when the hull is treated as an equipotential surface. Since the total source strength on the hull was not normalized in the iterations, an unknown constant factor is present in the values computed from the iteration formula. The ratios of the exact values to those computed with twenty iterations show consistency to five significant figures. The exact value of the constant potential on the hull, $\phi_{OE} = -1.76977$, and the value of the potential at the origin computed from the source distribution of the twentieth iteration, $\phi_{OC} = 4.07197$, are in essentially the same ratio as that given in the Table, $\phi_{OE}/\phi_{OC} = 0.434621$.

Finally, the distribution of the velocity potential over the hull is given in Table 4. The exact values are seen to agree with those computed from the previously calculated source distributions, M(Q) and N(Q), to within at least five significant figures. This is remarkable in view of the aforementioned result that the values of the source distribution M(Q) from the tenth iteration have considerably larger errors. A reexamination of the values of M(Q) in Table 2 shows that the computed values tend to be larger than the exact ones at small values of θ, and smaller than the exact ones at the large values of θ, a possible explanation for the unexpectedly good agreement in the values of the velocity potential.

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TABLE I

INPUT DATA - COORDINATES AND DIRECTION COSINES

OF A 4 : 1 : 2 ELLIPSOID

x = 0.755404

θ	y	z	l	m	n
0.149245	0.161994	0.048716	0.279081	0.957566	0.071991
0.442341	0.148048	0.140244	0.296377	0.929365	0.220094
0.719450	0.123216	0.215898	0.331168	0.864286	0.378599
0.970557	0.092529	0.270360	0.380958	0.746614	0.545380
1.186584	0.061402	0.303743	0.435264	0.566082	0.700069
1.359729	0.034320	0.320359	0.476405	0.346307	0.808152
1.483732	0.014244	0.326388	0.495196	0.149402	0.855839
1.554146	0.002727	0.327584	0.499237	0.028841	0.865985

x = 0.865631

θ	y	z	l	m	n
0.149245	0.123773	0.037224	0.399553	0.914131	0.068726
0.442341	0.113123	0.107160	0.421931	0.882226	0.208931
0.719450	0.094149	0.164968	0.465774	0.810548	0.355059
0.970557	0.070701	0.206581	0.525660	0.686941	0.501791
1.186584	0.046917	0.232090	0.586995	0.509045	0.629532
1.359729	0.026224	0.244785	0.630642	0.305678	0.713338
1.483732	0.010884	0.249393	0.649769	0.130718	0.748808
1.554146	0.002084	0.250306	0.653818	0.025186	0.756233

x = 0.944575

θ	y	z	l	m	n
0.149245	0.081162	0.024407	0.587148	0.807201	0.060687
0.442341	0.074174	0.070265	0.612317	0.769333	0.182195
0.719450	0.061733	0.108169	0.658909	0.689016	0.301822
0.970557	0.046359	0.135455	0.716912	0.562963	0.411229
1.186584	0.030764	0.152180	0.769949	0.401221	0.496186
1.359729	0.017195	0.160505	0.804046	0.234186	0.546504
1.483732	0.007137	0.163526	0.818094	0.098896	0.566518
1.554146	0.001367	0.164125	0.821000	0.019004	0.570613

x = 0.989401

θ	y	z	l	m	n
0.149245	0.035899	0.010796	0.864215	0.501707	0.037719
0.442341	0.032808	0.031079	0.877978	0.465817	0.110316
0.719450	0.027306	0.047845	0.900792	0.397762	0.174239
0.970557	0.020505	0.059914	0.925044	0.306740	0.224065
1.186584	0.013607	0.067311	0.943869	0.207696	0.256855
1.359729	0.007605	0.070993	0.954546	0.117401	0.273970
1.483732	0.003157	0.072330	0.958656	0.048936	0.280329
1.554146	0.000604	0.072595	0.959486	0.009378	0.281599

TABLE I, Continued

x = 0.095012

θ	y	z	l	m	n
0.149245	0.246102	0.074009	0.024054	0.996897	0.074948
0.442341	0.224910	0.213060	0.025683	0.972764	0.230372
0.719450	0.187191	0.327995	0.029045	0.915587	0.401071
0.970557	0.140571	0.410734	0.034092	0.807036	0.589518
1.186584	0.093283	0.461450	0.039995	0.628266	0.776970
1.359729	0.052139	0.486692	0.044815	0.393482	0.918240
1.483732	0.021640	0.495853	0.047138	0.171776	0.984008
1.554146	0.004144	0.497669	0.047648	0.033248	0.998311

x = 0.281604

θ	y	z	l	m	n
0.149245	0.237216	0.071337	0.073784	0.994468	0.074766
0.442341	0.216794	0.205367	0.078753	0.970062	0.229732
0.719450	0.180432	0.316152	0.088994	0.912339	0.399648
0.970557	0.135495	0.395903	0.104319	0.803100	0.586642
1.186584	0.089915	0.444788	0.122155	0.624060	0.771769
1.359729	0.050256	0.469118	0.136646	0.390183	0.910542
1.483732	0.020859	0.477948	0.143600	0.170185	0.974893
1.554146	0.003994	0.479699	0.145126	0.032933	0.988865

x = 0.458017

θ	y	z	l	m	n
0.149245	0.219765	0.066089	0.128809	0.988879	0.074345
0.442341	0.200846	0.190259	0.137376	0.963859	0.228263
0.719450	0.167159	0.292894	0.154966	0.904908	0.396393
0.970557	0.125528	0.366778	0.181103	0.794152	0.580107
1.186584	0.083300	0.412067	0.211201	0.614585	0.760052
1.359729	0.046559	0.434608	0.235364	0.382812	0.893341
1.483732	0.019324	0.442788	0.246862	0.166645	0.954615
1.554146	0.003700	0.444410	0.249376	0.032234	0.967870

x = 0.617876

θ	y	z	l	m	n
0.149245	0.194384	0.058456	0.194329	0.978176	0.073541
0.442341	0.177649	0.168285	0.206949	0.952019	0.225459
0.719450	0.147853	0.259066	0.232674	0.890834	0.390228
0.970557	0.111030	0.324417	0.270396	0.777425	0.567888
1.186584	0.073680	0.364476	0.312994	0.597176	0.738522
1.359729	0.041182	0.384413	0.346471	0.369481	0.862231
1.483732	0.017092	0.391649	0.362157	0.160293	0.918231
1.554146	0.003273	0.393083	0.365566	0.030982	0.930270

TABLE 2

SOURCE DISTRIBUTION ON ELLIPSOID, M(P)

Exact

-0.002157	-0.006615	-0.011548	-0.017422	-0.025020	-0.035820	-0.052639	-0.077478
-0.002303	-0.007060	-0.012316	-0.018553	-0.026571	-0.037827	-0.054895	-0.078712
-0.002604	-0.007978	-0.013893	-0.020860	-0.029690	-0.041757	-0.059072	-0.080757
-0.003056	-0.009352	-0.016236	-0.024241	-0.034153	-0.047126	-0.064272	-0.082931
-0.003586	-0.010951	-0.018935	-0.028060	-0.039022	-0.052625	-0.069027	-0.084619
-0.004018	-0.012250	-0.021101	-0.031062	-0.042710	-0.056538	-0.072084	-0.085576
-0.004226	-0.012874	-0.022131	-0.032468	-0.044395	-0.058253	-0.073343	-0.085945
-0.004272	-0.013011	-0.022357	-0.032774	-0.044757	-0.058616	-0.073604	-0.086019

Computed - 10th Interaction

-0.002157	-0.006615	-0.011548	-0.017422	-0.025021	-0.035822	-0.052640	-0.077497
-0.002303	-0.007061	-0.012317	-0.018555	-0.026573	-0.037830	-0.054902	-0.078720
-0.002605	-0.007981	-0.013897	-0.020866	-0.029699	-0.041771	-0.059094	-0.080748
-0.003058	-0.009356	-0.016243	-0.024251	-0.034166	-0.047141	-0.064278	-0.082904
-0.003585	-0.010950	-0.018932	-0.028056	-0.039014	-0.052608	-0.068994	-0.084577
-0.004014	-0.012238	-0.021079	-0.031030	-0.042666	-0.056480	-0.072016	-0.085525
-0.004219	-0.012853	-0.022096	-0.032417	-0.044328	-0.058172	-0.073258	-0.085889
-0.004264	-0.012988	-0.022318	-0.032718	-0.044685	-0.058529	-0.073515	-0.085963

TABLE 3

EQUIPOTENTIAL SOURCE DISTRIBUTION, N(P)

Exact

0.161174	0.166804	0.179037	0.200225	0.235196	0.293847	0.395723	0.556070
0.172087	0.178038	0.190946	0.213227	0.249772	0.310305	0.412686	0.564926
0.194614	0.201188	0.215395	0.239733	0.279094	0.342549	0.444088	0.579606
0.228432	0.235833	0.251724	0.278598	0.321054	0.386591	0.483181	0.595210
0.267979	0.276157	0.293559	0.322490	0.366821	0.431700	0.518927	0.607323
0.300277	0.308914	0.327145	0.356982	0.401492	0.463800	0.541907	0.614193
0.315839	0.324636	0.343125	0.373143	0.417328	0.477866	0.551375	0.616837
0.319261	0.328087	0.346620	0.376656	0.420734	0.480844	0.553333	0.617372

Computed - 20th Iteration

0.370837	0.383791	0.411939	0.460688	0.541154	0.676101	0.910506	1.279444
0.395946	0.409638	0.439338	0.490604	0.574691	0.713969	0.949536	1.299819
0.447777	0.462904	0.495592	0.551591	0.642155	0.788157	1.021787	1.333596
0.525587	0.542617	0.579180	0.641015	0.738699	0.889493	1.111734	1.369499
0.616579	0.635395	0.675437	0.742002	0.844004	0.993282	1.193980	1.397369
0.690892	0.710765	0.752712	0.821363	0.923777	1.067139	1.246855	1.413176
0.726698	0.746938	0.789480	0.858549	0.960213	1.099505	1.268639	1.419261
0.734570	0.754878	0.797521	0.866630	0.968050	1.106355	1.273145	1.420490

Ratio of Exact to Computed Values

0.434622	0.434622	0.434621	0.434621	0.434620	0.434620	0.434619	0.434619
0.434622	0.434622	0.434621	0.434621	0.434620	0.434620	0.434619	0.434619
0.434622	0.434622	0.434621	0.434621	0.434620	0.434620	0.434619	0.434619
0.434622	0.434622	0.434622	0.434621	0.434620	0.434620	0.434619	0.434619
0.434622	0.434622	0.434622	0.434621	0.434620	0.434620	0.434619	0.434619
0.434622	0.434622	0.434622	0.434621	0.434620	0.434620	0.434619	0.434619
0.434622	0.434622	0.434622	0.434621	0.434620	0.434620	0.434619	0.434619
0.434622	0.434622	0.434622	0.434621	0.434620	0.434620	0.434619	0.434619

TABLE 4

VELOCITY POTENTIAL ON SURFACE OF ELLIPSOID

Exact

0.107040	0.317252	0.515997	0.696093	0.851031	0.975211	1.064149	1.114649

Computed

0.107038	0.317247	0.515989	0.696082	0.851017	0.975195	1.064130	1.114623
0.107038	0.317247	0.515989	0.696082	0.851017	0.975195	1.064129	1.114628
0.107038	0.317246	0.515988	0.696081	0.851016	0.975193	1.064127	1.114636
0.107038	0.317246	0.515988	0.696081	0.851016	0.975195	1.064134	1.114643
0.107039	0.317248	0.515991	0.696086	0.851023	0.975204	1.064146	1.114650
0.107040	0.317251	0.515997	0.696093	0.851033	0.975215	1.064156	1.114654
0.107041	0.317254	0.516001	0.696098	0.851038	0.975221	1.064161	1.114655
0.107041	0.317254	0.516002	0.696100	0.851040	0.975223	1.064162	1.114656

APPENDIX

FORTRAN IV(G) Program for IBM 360/

Program for Computing Source Distribution and Velocity Potential on a Ship Surface

```
      DIMENSION X(16),Y(16,8),Z(16,8),EL(16,8),EM(16,8),EN(16,8),A(16),
     1CAS(128),SD(128),SDA(128),SD3(128),SDE(16,8),SDO(16,8),SD1(16,8)
     2PHI(16,8),C(128,128),P1(128,128),F(16,8),E(16,8)
      P = 12.56637061
      READ(5,1) (X(I),I=1,16)
    1 FORMAT(8F10.8)
      READ(5,1) (A(I),I=1,16)
      READ(5,1) (( Y(I,J),I=1,8 ),J=1,8)
      READ(5,1) (( Y(I,J),I=9,16),J=1,8)
      READ(5,1) (( Z(I,J),I=1,8 ),J=1,8)
      READ(5,1) (( Z(I,J),I=9,16),J=1,8)
      READ(5,1) ((EL(I,J),I=1,8 ),J=1,8)
      READ(5,1) ((EL(I,J),I=9,16),J=1,8)
      READ(5,1) ((EM(I,J),I=1,8 ),J=1,8)
      READ(5,1) ((EM(I,J),I=9,16),J=1,8)
      READ(5,1) ((EN(I,J),I=1,8 ),J=1,8)
      READ(5,1) ((EN(I,J),I=9,16),J=1,8)
      DO 2 I = 1,16
      X2 = X(I) * X(I)
      DO 2 J = 1,8
      J1 = J + 8
      Y2 = Y(I,J) * Y(I,J)
      Z2 = Z(I,J) * Z(I,J)
      E(I,J) = ..........
      F(I,J) = E(I,J) * A(I) * A(J1)
      SD1(I,J) = - EL(I,J)/ P
    2 SDO(I,J) = SQRT(X2 + Y2 + Z2)
      KL = 0
      DO 9 K = 1,16
      DO 9 L = 1,8
      KL = KL + 1
      SD3(KL) = SDO(K,L)
      CAS(KL) = SD1(K,L)
      SD(KL) = SD1(K,L)
      IJ = 0
      SUM = 0.0
      DO 8 I = 1,16
      DO 8 J = 1,8
      IJ = IJ + 1
      XO = X(K) - X(I)
      YO = Y(K,L) - Y(I,J)
      Y1 = Y(K,L) + Y(I,J)
      ZO = Z(K,L) - Z(I,J)
      Z1 = Z(K,L) + Z(I,J)
      XO2 = XO * XO
      YO2 = YO * YO
      Y12 = Y1 * Y1
      ZO2 = ZO * ZO
      Z12 = Z1 * Z1
```

APPENDIX, Continued

```
      R11 = SQRT(XO2 + YO2 + ZO2)
      R22 = SQRT(XO2 + Y12 + ZO2)
      R33 = SQRT(XO2 + Y12 + Z12)
      R44 = SQRT(XO2 + YO2 + Z12)
      R1 = R11 * R11 * R11
      R2 = R22 * R22 * R22
      R3 = R33 * R33 * R33
      R4 = R44 * R44 * R44
      V2 = (1.0/R2) + (1.0/R3)
      V3 = (1.0/R4) + (1.0/R3)
      W2 = EM(I,J) * Y(K,L) * V2
      W3 = EN(I,J) * Z(K,L) * V3
      PIJ = XO * EL(I,J) + YO * EM(I,J) + ZO * EN(I,J)
      IF(K.EQ.I.AND.L.EQ.J) GO TO 10
      V11 = (1.0/R11) +(1.0/R22) +(1.0/R33) +(1.0/R44)
      V1 = (1.0/R1) + (1.0/R2) + ( 1.0/R3) + (1.0 / R4)
      P1(IJ,KL) = V11
      BA = PIJ * V1 - 2.0 * (W2 + W3)
      BE = BA * F(I,J)
      GO TO 11
   10 BA = 0.00000000
      BE = 0.00000000
      P1(IJ,KL) = BE
   11 C(IJ,KL) = BA * F(K,L)
    8 SUM = SUM + BE
    9 C(KL,KL) = C(KL,KL) - SUM
      LA = 1
   13 DO 22 IJ = 1,128
      SUM = 0.0
      DO 21 KL = 1,128
   21 SUM = SUM + SD(KL) * C(IJ,KL)
   22 SDA(IJ) = 0.125 * SUM + CAS(IJ)
      IJ = 0
      DO 23 I = 1,16
      DO 23 J = 1,8
      IJ = IJ + 1
      SD(IJ) = SDA(IJ)
   23 SDE(I,J) = SDA(IJ)
      LA = LA + 1
      IF(LA.LE.10) GO TO 13
      WRITE(6,102)((SDE(I,J),I=1,8),J=1,8)
  102 FORMAT(40H SOURCE DISTRIBUTION AFTER 10 ITERATIONS/(8F10.6))
      WRITE(6,110)((SDE(I,J),I=9,16),J=1,8)
  110 FORMAT(1H///((8F10.6))
      LA = 1
   27 DO 25 IJ = 1,128
      SUM = 0.0
      DO 24 KL = 1,128
   24 SUM = SUM + SD3(KL) * C(IJ,KL)
   25 SD(IJ) = SD3(IJ) - 0.25 * SUM
      IJ = 0
      DO 26 I = 1,16
      DO 26 J = 1,8
      IJ = IJ + 1
      SD3(IJ) = SD(IJ)
```

APPENDIX, Continued

```
 26 SDO(I,J) = SD(IJ)
    LA = LA + 1
    IF(LA.LE.20) GO TO 27
    WRITE(6,104)((SDO(I,J),I= 1,8 ),J=1,8)
104 FORMAT(40H DIRICHLET PROBLEM AFTER 20 ITERATIONS  /(8F10.6))
    PHO = 0.0
    DO 28 I = 1,16
    DO 28 J = 1,8
    VV= SQRT(X(I)*X(I) + Y(I,J)*Y(I,J) + Z(I,J)*Z(I,J))
 28 PHO = PHO - 4.0*SDO(I,J) * F(I,J) * P / VV
    IJ = 0
    DO 35 I = 1,16
    DO 35 J = 1,8
    IJ = IJ + 1
    COEF = SDE(I,J) / SDO(I,J)
    KL = 0

    SUM = 0.0
    DO 33 K = 1,16
    DO 33 L = 1,8
    KL = KL + 1
 33 SUM = SUM + P1(IJ,KL)*(SDA(KL)-SDB(KL)*COEF) * F(K,L) * P
 35 PHI(I,J) = X(I) - SUM + PHO * COEF
    WRITE(6,107) (X(I),I=1,8)
107 FORMAT(1H ///(8F10.6))
    WRITE(6,110)((PHI(I,J),I=1,8),J=1,8)
    WRITE(6,107) (X(I),I=9,16)
    WRITE(6,110)((PHI(I,J),I=9,16),J=1,8)
    CALL EXIT
    END
```

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DOCUMENT CONTROL DATA - R & D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) IOWA INSTITUTE OF HYDRAULIC RESEARCH	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

Irrotational Flow About Ship Forms

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Interim Report

5. AUTHOR(S) (First name, middle initial, last name)

Louis Landweber and Matilde Macagno

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1969	33	7

8a. CONTRACT OR GRANT NO. Nonr-1611(07) b. PROJECT NO. Task 062-183 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) IIHR Report No. 123 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
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13. ABSTRACT

A method of computing the potential flow about a ship model in terms of a source distribution on the hull surface is described. After a formulation and discussion of the flow with wavemaking, the zero-Froude number, double-ship-model case is treated in detail. The Fredholm singular integral equation of the second kind for the source distribution is solved numerically by removing the singularity, replacing the integral by a quadrature formula, and solving the resulting high-order set of linear equations by an iteration formula for which convergence is proved. The corresponding velocity potential on the hull surface, the evaluation of which from the source distribution also requires the calculation of a singular integral, is obtained by first solving the equipotential problem for the hull form (which employs the same kernel as the original integral equation). The solution of this Dirichlet problem is then used to remove the singularity from the velocity-potential integral, and then the latter is computed by means of a quadrature formula. The method is applied to an ellipsoid, results from which are compared with the exact solution. A FORTRAN program is included as an Appendix.

DD FORM 1473 1 NOV 65 (PAGE 1)
S/N 0101-807-6811

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Ship Form						
Irrotational Flow						
Three Dimensional Flow						
Source Distributions						
Integral Equations						
Wave Making						

DD FORM 1 NOV 65 1473 (BACK)
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