Is $f_0(1500)$ a scalar glueball?

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Following the discovery of two new scalar mesons $f_0(1370)$ and $f_0(1500)$ at the Low Energy Antiproton Ring at CERN, we argue that the observed properties of this pair are incompatible with them both being $Q\bar{Q}$ mesons. We show instead that $f_0(1500)$ is compatible with the ground state glueball expected around 1500 MeV mixed with the nearby states of the $0^{++}Q\bar{Q}$ nonet. Tests of this hypothesis include the prediction of a further scalar state f_0' (1500–1800) which couples strongly to $K\bar{K}$, $\eta\eta$, and $\eta\eta'$. Signatures for a possible tensor glueball at ~ 2 GeV are also considered.

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I. INTRODUCTION

Glueballs are a missing link of the standard model. Whereas the gluon degrees of freedom expressed in L_{QCD} have been established beyond doubt in high momentum data, their dynamics in the strongly interacting limit epitomized by hadron spectroscopy are quite obscure. This may be about to change, as a family of candidates for gluonic hadrons (glueballs and hybrids) is now emerging [1,2]. In this paper we shall argue that scalar mesons around 1.5 GeV, in particular the detailed phenomenology of $f_0(1500)$ and its partner $f_0(1370)$, suggest that a glueball exists in this region, probably mixed with nearby isoscalar members of the scalar nonet. This hypothesis may be tested in forthcoming experiments.

In advance of the most recent data, theoretical arguments suggested that there may be gluonic activity manifested in the 1.5 GeV mass region. Lattice QCD is the best simulation of theory and predicts the lightest "primitive" (i.e., quenched approximation) glueball to be 0^{++} with mass 1.55 ± 0.05 GeV [3]. Recent lattice computations place the glueball slightly higher in mass at 1.74 ± 0.07 GeV [4] with an optimized value for phenomenology proposed by Teper [5] of 1.57 ± 0.09 GeV. That lattice QCD computations of the scalar glueball mass are now concerned with such fine details represents considerable advance in this field. Whatever the final consensus may be, these results suggest that scalar mesons in the 1.5 GeV region merit special attention. Complementing this has been the growing realization that there are now too many 0^{++} mesons confirmed for them all to be $Q\bar{Q}$ states [1,6,7].

At ~ 1.5 GeV there is a clear 0^{++} signal, $f_0(1500)$, in several experiments [8–15], whose serious consideration for being associated with the primitive glueball is enhanced by the fact that its production is by mechanisms traditionally believed to be those that favor gluonic excitations. Specifically these include [16] the following.

(1) Radiative J/ψ decay: $J/\psi \to \gamma + G$ [17].

(2) Collisions in the central region away from quark beams and target: $pp \to p_f(G)p_s$ [12,15].

(3) Proton-antiproton annihilation where the destruction of quarks creates opportunity for gluons to be manifested. This is the Crystal Barrel [8–11] and E760 [13,14] production mechanism in which detailed decay systematics of $f_0(1500)$ have been studied.

(4) Tantalizing further hints come from the claimed sighting [18] of the $f_0(1500)$ in decays of the hybrid meson candidate [2] $\pi(1800) \to \pi f_0(1500) \to \pi\eta\eta$.

The signals appear to be prominent in decay channels such as $\eta\eta$ and $\eta\eta'$ that are traditionally regarded as glueball signatures. However, such experiments are not totally novel, and some time ago one of us (F.E.C.) addressed the question of why glueballs had remained hidden during 25 years of establishing the Particle Data Group list [7] of $Q\bar{Q}$ states. This was suggested [16] to be due to the experimental concentration on a restricted class of production mechanisms and on final states with charged pions and kaons. The more recent emphasis on neutral final states (involving π^0, η, η') was inspired by the possibility that η and η' are strongly coupled to glue. This dedicated study of neutrals was a new direction pioneered by the GAMS Collaboration at CERN announcing new states decaying to $\eta\eta$ and $\eta\eta'$ [19].

The Crystal Barrel Collaboration at the CERN Low Energy Antiproton Ring (LEAR) has made intensive study of $p\bar{p}$ annihilation into neutral final states involving π^0, η, and η'. They find a clear signal for $f_0(1500)$ in $\pi^0\pi^0$, $\eta\eta$, and $\eta\eta'$. Our present work extends and generalizes the work of Ref. [16] in light of these new data from LEAR. Our purpose is to examine the data on the $f_0(1500)$, compare with predictions for glueballs, and identify the seminal experiments now needed to confirm that gluonic degrees of freedom are being manifested in

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this region. A summary of this work has already been published elsewhere [20].

The structure of the paper is as follows. We first review the experimental data on scalar mesons with special emphasis on states seen in the Crystal Barrel detector at LEAR. We then derive from $SU(3)_f$ the branching ratios for $Q\bar{Q}$ decays into two pseudoscalars, show that this successfully describes the known decay rates in the $2^{++}Q\bar{Q}$ nonet, and then compare our predictions to the observed decay modes of $f_0(1500)$. Previous bubble chamber experiments have not observed a $K\bar{K}$ signal in the 1500 MeV mass region, which, if confirmed, would imply a set of branching ratios that are unnatural for a state belonging to a quarkonium nonet. If a significant signal were to be observed, it would be possible to find a quarkonium mixing angle that reproduces the observed final state abundances; however, the systematics would then imply that $f_0(1500)$ is dominantly $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$. This would have two immediate consequences.

(1) This would leave the $f_0(1370)$ state, which is also seen in $p\bar{p}$ annihilation with decay branching ratios and total width consistent with an $n\bar{n}$ structure [9,22], isolated.

(2) With either the $f_0(1370)$ or $f_0(1500)$ assigned as the $n\bar{n}$ member, the orthogonal quarkonium in the nonet would have to be dominantly $s\bar{s}$, and hence is probably heavier than $f_0(1500)$ and decaying strongly into $K\bar{K}$. Identification of this state is now imperative in order to complete the multiplet and discriminate among hypotheses.

We then show that the decay rates of $f_0(1500)$ are compatible with a glueball state whose mass lies between the $n\bar{n}$ and $s\bar{s}$ scalar quarkonium states and whose nearby presence disturbs the glueball decays in a characteristic flavor dependent manner. In the climax of the paper we show that dynamics inspired by lattice QCD may be consistent with the data, and we consider the implications for glueballs mixing with quarkonia in the 1500 MeV range. We finally show that a reasonable nonet can be constructed with the remaining scalar mesons.

II. SCALAR MESONS IN THE CRYSTAL BARREL

The lowest 0^{++} mesons, namely, the isospin $I = 0 f_0(980)$ and the $I = 1 a_0(980)$, have been assumed to be $K\bar{K}$ molecules [23,24]. This belief is motivated by their strong couplings to $K\bar{K}$—in spite of their masses being at the $K\bar{K}$ threshold—and their small $\gamma\gamma$ partial widths. For $f_0(980), \Gamma_{\gamma\gamma} = 0.56 \pm 0.11$ keV [7]. For $a_0(980)$, one finds with a LEAR measurement of the relative branching fraction for an a_0 decay to $K\bar{K}$ and $\eta\pi$ [25] the partial width $\Gamma_{\gamma\gamma} = 0.33 \pm 0.13$ keV [1]. Thus, the $\gamma\gamma$ partial widths appear to be nearly equal, close to predictions for $K\bar{K}$ molecules (0.6 keV) and much smaller than for $Q\bar{Q}$ states [26].

The nature of these states is likely to be illuminated soon at DAϕNE [24]. If they are not simply $Q\bar{Q}$ then the $0^{++}Q\bar{Q}$ mesons need to be identified. A new $J^{PC}(I^G) = 0^{++}(1^-)$ meson, $a_0(1450) \to \eta\pi$, has been reported by the Crystal Barrel Collaboration at LEAR [25]. This state, with a mass of 1450 ± 40 MeV and a width of 270 ± 40 MeV, appears, together with $a_0(980)$, in the $\eta\pi$ S wave in $\bar{p}p$ annihilation at rest into $\eta\pi^0\pi^0$. We shall show in Sec. VIII that $a_0(1450)$ can be identified with the $I = 1$ member of the ground state $0^{++}Q\bar{Q}$ nonet. In turn, this and the $K^*(1430)$ set the natural energy scale for the scalar nonet.

Recent data in $\bar{p}p$ annihilation at LEAR into $\eta\pi^0\pi^0$ [25], $3\pi^0$ [8,9], and $\eta\eta\pi^0$ [10,22] require an $I = 0$ scalar resonance in the range 1320–1400 MeV, decaying to $\pi^0\pi^0$ and $\eta\eta$. We shall use 1360 MeV as average mass but shall adopt the nomenclature of the Particle Data Group [7] calling this state $f_0(1370)$. Its width varies between 200 and 700 MeV, depending on theoretical assumptions. For example, the $3\pi^0$ data give $\Gamma \sim 700$ MeV [9], decreasing to 300 ± 80 MeV if the 700 MeV broad background structure [27,28] centered at 1000 MeV in the $\pi\pi$ S wave [called $f_0(1300)$ in the latest issue of the Particle Data Group] is introduced in the analysis. This is in good agreement with a coupled channel analysis of $\eta\pi^0\pi^0$, $3\pi^0$, and $\eta\eta\pi^0$, which leads to a mass of 1390 ± 30 MeV and a width of 380 ± 80 MeV [29]. The $\gamma\gamma$ width in this region is also consistent with it containing the ${}^3P_0(Q\bar{Q})$ state [30,31].

There is rather general agreement that the ground state $n\bar{n}$ state is manifested here. The debate is one of detail on the relationship of the $f_0(1370)$ to the broad $f_0(1300)$, in particular as to whether these are two independent states or manifestations of a single state, and to what extent unitarity corrections are important [32]. This issue is peripheral to our main analysis, which will rely only on the generally accepted association of the ground state $n\bar{n}$ as the seed for the phenomenology in the $f_0(1300$–$1370)$ region and that this is distinct from the $f_0(1500)$ state observed by the Crystal Barrel Collaboration decaying to $\pi^0\pi$ [8,9], $\eta\eta$ [10,22], and $\eta\eta'$ [11]. This state was seen in $\bar{p}p$ annihilation into $3\pi^0$, $\eta\eta\pi^0$, and $\eta\eta'\pi^0$, leading to six final state photons. The masses and widths observed in the three decay channels are consistent, giving the average

$$(m, \Gamma) = (1509 \pm 10, 116 \pm 17) \text{ MeV} , \tag{1}$$

while the coupled channel analysis [29] gives 1500 ± 10 MeV and the less precise but compatible width of 154 ± 30 MeV. It is possible that $f_0(1500)$ has also been seen by the Mark III and DM2 Collaborations in $J/\psi \to \gamma + \pi\pi\pi\pi$, hitherto misidentified as 0^{-+} [17] and in $J/\psi \to \gamma\pi\pi$ [33]. A resonance decaying to $\pi^0\pi^0$ and $\eta\eta$ was also reported by E760 in $\bar{p}p$ annihilation at higher energies at the Fermilab accumulator, with masses and widths (1508,103) MeV [13] and (1488,148) MeV [14], respectively. A spin-parity analysis is in progress [34].

The GAMS Collaboration at CERN [19] reports a 0^{++} 180 MeV broad resonance, $f_0(1590)$, decaying to $\eta\eta'$, $\eta\eta$, and $4\pi^0$, also observed in central production [15] and by the VES Collaboration at Serpukhov [35] in $\eta\eta'$: this might be the $f_0(1500)$ state though the status of the $\pi\pi$

branching ratio needs to be clarified.[1] A strong coupling of $f_0(1500)$ to pions would contradict it being primarily an $s\bar{s}$ state, and, as we shall argue later, it is a candidate for a glueball mixed with the $Q\bar{Q}$ nonet, where $f_0(1370)$ is dominantly $n\bar{n}$, and a more massive $s\bar{s}$ remains to be identified. The $f_0(1500)$, clearly established in different decay channels and with detailed information on branching ratios to several channels, will form the fulcrum of our investigation.

There are candidates for this $s\bar{s}$ state, though their existence and/or $s\bar{s}$ assignment remain to be established. A 0^{++} structure, $f_0'(1525)$, with poorly known width (~ 90 MeV) is observed to decay into K_SK_S in K^-p interactions [37]. This state requires confirmation from other experiments. The $\theta(1690)$ (now known as $f_J(1720)$ [7]) is a candidate because of its affinity for $K\bar{K}$ and $\eta\eta$ decays, though its quantum numbers, 0^{++} or 2^{++}, are still controversial.

An $I=0$ scalar with a width of 56 MeV, is observed at 1446 MeV by the WA91 Collaboration at CERN in pp central collisions [12]. It decays to four pions, dominantly through $\rho^0(\pi^+\pi^-)_P$ where the dipion is in a P wave. This may be the same as $f_0(1500)$ produced in the second of the favored glueball mechanisms (Sec. I), with its apparent small width being the result of interference between $f_0(1370)$ and $f_0(1500)$ [38]. In any event, it does not detract from the qualitative observation that there are too many isoscalars observed in various production mechanisms for them all to be explained naturally within a $Q\bar{Q}$ picture. The fact that there does not appear to be such copious activity in the $I=1$ and strange sectors adds weight to the suspicion that glueball excitation is affecting the $I=0$ spectrum.

A substantial part of this paper will examine what the flavor content of two-body decays can reveal about the structure of the initial meson. Based on this analysis, we shall argue that the Crystal Barrel $f_0(1500)$ has decay properties incompatible with a $Q\bar{Q}$ state. In addition, we shall show that a reasonable $Q\bar{Q}$ nonet may be constructed with $a_0(1450)$, $f_0(1370)$, $K_0^*(1430)$, and an $s\bar{s}$ state above 1500 MeV, thus leaving $f_0(980)$ and $f_0(1500)$ as exotic (not simply $Q\bar{Q}$) states.

III. QUARKONIUM DECAY AMPLITUDES

Consider a quarkonium state

$$|Q\bar{Q}\rangle = \cos\alpha|n\bar{n}\rangle - \sin\alpha|s\bar{s}\rangle \ , \tag{2}$$

where

$$n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2} \ . \tag{3}$$

The mixing angle α is related to the usual nonet mixing angle θ [7] by the relation

$$\alpha = 54.7^\circ + \theta \ . \tag{4}$$

For $\theta = 0$ the quarkonium state becomes a pure $\mathrm{SU}(3)_f$ octet, while for $\theta = \pm 90^\circ$ it becomes a pure singlet. Ideal mixing occurs for $\theta = 35.3^\circ$ (-54.7°) for which the quarkonium state becomes pure $s\bar{s}(\bar{n}n)$.

In general, we define

$$\eta = \cos\phi|n\bar{n}\rangle - \sin\phi|s\bar{s}\rangle \tag{5}$$

and

$$\eta' = \sin\phi|n\bar{n}\rangle + \cos\phi|s\bar{s}\rangle \tag{6}$$

with $\phi = 54.7^\circ + \theta_{PS}$, where θ_{PS} is the usual octet-singlet mixing angle in the $\mathrm{SU}(3)_f$ basis where

$$\eta = \cos(\theta_{PS})|\eta_8\rangle - \sin(\theta_{PS})|\eta_1\rangle \ , \tag{7}$$

$$\eta' = \sin(\theta_{PS})|\eta_8\rangle + \cos(\theta_{PS}|\eta_1\rangle \ . \tag{8}$$

The decay of quarkonium into a pair of mesons $Q\bar{Q} \to M(Q\bar{q}_i)M(q_i\bar{Q})$ involves the creation of $q_i\bar{q}_i$ from the vacuum. If the ratio of the matrix elements for the creation of $s\bar{s}$ versus $u\bar{u}$ or $d\bar{d}$ is denoted by[2]

$$\rho \equiv \frac{\langle 0|V|s\bar{s}\rangle}{\langle 0|V|d\bar{d}\rangle} \ , \tag{9}$$

then the decay amplitudes of an isoscalar 0^{++} (or 2^{++}) are proportional to

$$\begin{aligned}
\langle Q\bar{Q}|V|\pi\pi\rangle &= \cos\alpha \ , \\
\langle Q\bar{Q}|V|K\bar{K}\rangle &= \cos\alpha(\rho - \sqrt{2}\tan\alpha)/2 \ , \\
\langle Q\bar{Q}|V|\eta\eta\rangle &= \cos\alpha(1 - \rho\sqrt{2}\tan\alpha)/2 \ , \\
\langle Q\bar{Q}|V|\eta\eta'\rangle &= \cos\alpha(1 + \rho\sqrt{2}\tan\alpha)/2 \ .
\end{aligned} \tag{10}$$

The corresponding decay amplitudes of the isovector are

$$\begin{aligned}
\langle Q\bar{Q}|V|K\bar{K}\rangle &= \rho/2 \ , \\
\langle Q\bar{Q}|V|\pi\eta\rangle &= 1/\sqrt{2} \ , \\
\langle Q\bar{Q}|V|\pi\eta'\rangle &= 1/\sqrt{2} \ ,
\end{aligned} \tag{11}$$

and those for K^* decay

$$\begin{aligned}
\langle Q\bar{Q}|V|K\pi\rangle &= \sqrt{3}/2 \ , \\
\langle Q\bar{Q}|V|K\eta\rangle &= (\sqrt{2}\rho - 1)/\sqrt{8} \ , \\
\langle Q\bar{Q}|V|K\eta'\rangle &= (\sqrt{2}\rho + 1)/\sqrt{8} \ .
\end{aligned} \tag{12}$$

For clarity of presentation we have presented Eqs.

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[1]The $\pi\pi$ branching ratio is the largest deviation but is "not in contradiction" with the $f_0(1590)$ of GAMS and the $f_0(1500)$ of Crystal Barrel being the same state [36].

[2]We shall assume that $\langle 0|V|d\bar{d}\rangle \equiv \langle 0|V|u\bar{u}\rangle$.

(10), (11), and (12) in the approximation where $\eta \equiv (n\bar{n} - s\bar{s})/\sqrt{2}$ and $\eta' \equiv (n\bar{n} + s\bar{s})/\sqrt{2}$, i.e., for a pseudoscalar mixing angle $\theta_{PS} \sim -10°$ ($\phi = 45°$). This is a useful mnemonic; the full expressions for arbitrary η, η' mixing angles θ_{PS} are given in Appendix A and are used in detailed comparisons throughout this paper. Exact $SU(3)_f$ flavor symmetry corresponds to $\rho = 1$; empirically $\rho \geq 0.8$ for well established nonets such as 1^{--} and 2^{++} [21,39].

The partial width into a particular meson pair $M_i M_j$ may be written as

$$\Gamma_{ij} = c_{ij}|M_{ij}|^2 \times |F_{ij}(\vec{q})|^2 \times S_p(\vec{q}) \equiv \gamma_{ij}^2 \times |F_{ij}(\vec{q})|^2 \times S_p(\vec{q}) \, , \tag{13}$$

where $S_p(\vec{q})$ denotes the phase space, $F_{ij}(\vec{q})$ are model-dependent form factors, M_{ij} is the relevant amplitude [Eq. (10), (11), or (12)], and c_{ij} is a weighting factor arising from the sum over the various charge combinations, namely, 4 for $K\bar{K}$, 3 for $\pi\pi$, 2 for $\eta\eta'$, and 1 for $\eta\eta$ for isoscalar decay [Eq. (10)], 4 for $K\bar{K}$, 2 for $\pi\eta$, and 2 for $\pi\eta'$ for isovector decay [Eq. (11)], and 2 for K^* decays [Eq. (12)]. The dependence of $\gamma_{ij}^2 = c_{ij}|M_{ij}|^2$ upon the mixing angle α is shown in Fig. 1(a) for the isoscalar decay in the case of $SU(3)_f$ symmetry, $\rho = 1$. We confront the above with data on the established 2^{++} nonet, determine a probable range of values for ρ, and then compare with $f_0(1500)$ decays.

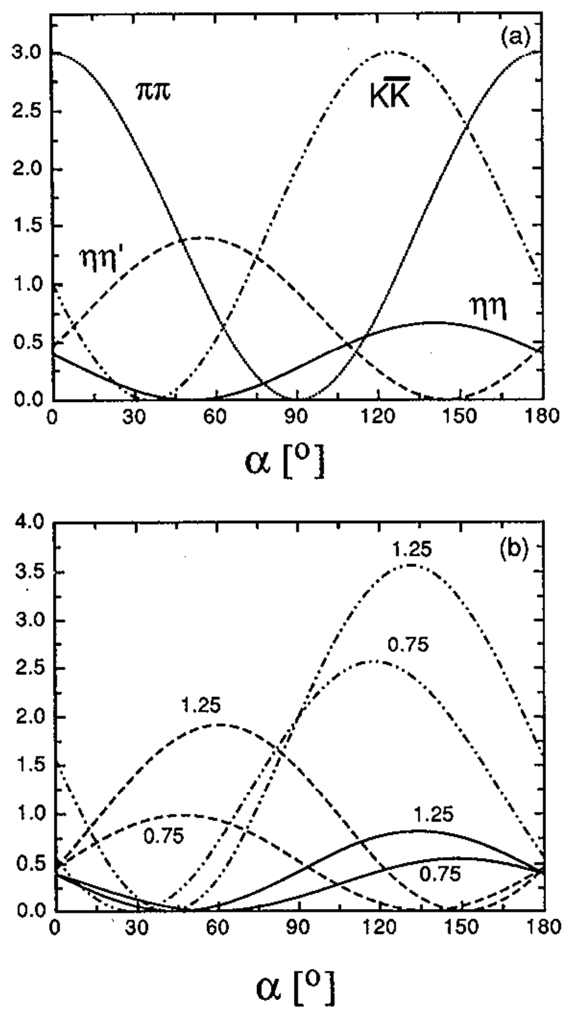

FIG. 1. γ_{ij}^2 as a function of α for $\rho = 1$ (a) and $\rho = 0.75$ or 1.25 (b) for quarkonium decay (up to a common multiplicative factor). Dotted line: $\pi\pi$; dash-dotted line: $K\bar{K}$; dashed line: $\eta\eta'$; solid line: $\eta\eta$.

IV. FLAVOR SYMMETRY IN MESON DECAYS

A. $Q\bar{Q}$ decays: The 2^{++} nonet

Exact $SU(3)_f$ flavor symmetry requires the parameter ρ to be unity. To get a feeling for symmetry breaking in the $Q\bar{Q}$ sector, we have computed some of the expected branching ratios for the tensor mesons $a_2(1320)$, $f_2(1270)$, and $K_2^*(1430)$ decaying to two pseudoscalars and have compared them with data [7]. The decay branching ratio B of a $Q\bar{Q}$ state is proportional to the partial width [Eq. (13)]. We use for the phase space factor $S_p(\vec{q}) = q$ and for the form factor

$$|F_{ij}(\vec{q})|^2 = q^{2l} \exp(-q^2/8\beta^2) \, , \tag{14}$$

where $l = 2$ is the angular momentum in the final state with daughter momenta q and $\beta \simeq 0.4$ GeV/c [39]. The ratios of the various partial widths are rather insensitive to choice among different successful descriptions of meson spectroscopy and dynamics. The detailed sensitivity to form factors is discussed in Appendix B.

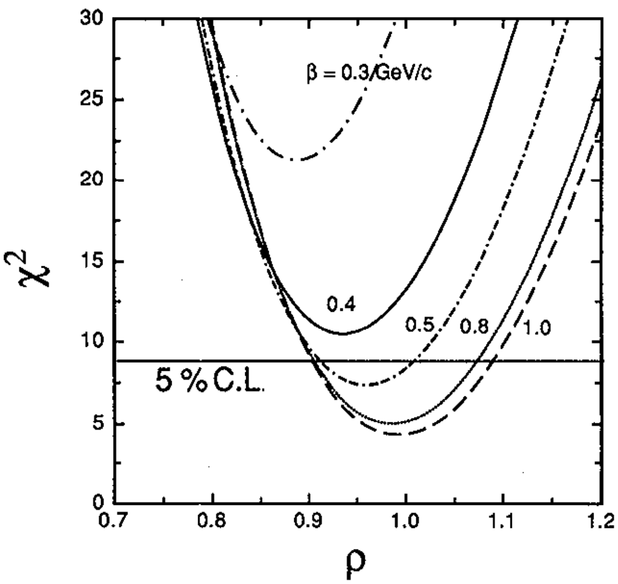

FIG. 2. χ^2 for 2^{++} quarkonium decay as a function of ρ for various values of β. The 5% C.L. limit is shown by the horizontal line.

The pseudoscalar mixing angle θ_{PS} has the empirical value $-(17.3 \pm 1.8)^\circ$ [40] (and hence $\phi = 37.4^\circ$ used in Sec. III). We shall use the full expressions given in Appendix A and the above value of θ_{PS} in all phenomenology. Analogously, for the tensor mixing angle, $\theta = 26^\circ$ [7]. The predictions are fitted to the experimental values [7]

$$\begin{aligned} B(f_2 \to \eta\eta)/B(f_2 \to \pi\pi) &= (5.3 \pm 1.2) \times 10^{-3} , \\ B(f_2 \to K\bar{K})/B(f_2 \to \pi\pi) &= (5.4 \pm 0.6) \times 10^{-2} , \\ B(a_2 \to \eta\pi)/B(a_2 \to K\bar{K}) &= 2.95 \pm 0.54 , \\ B(a_2 \to \eta'\pi)/B(a_2 \to K\bar{K}) &= 0.116 \pm 0.029 . \end{aligned} \tag{15}$$

Figure 2 shows the χ^2 distribution as a function of ρ for various values of β. The distribution does not change significantly for $\beta > 1$ GeV/c. A good fit (with a χ^2 confidence level of more than 5%) is obtained with $\beta =$ 0.5 GeV/c (or larger) for which $\rho = 0.96 \pm 0.04$. This result is consistent with $K_2^*(1430)$ decays, although the experimental errors are large:

$$B(K_2^* \to K\pi)/B(K_2^* \to K\eta) = (3.6^{+7.1}_{-2.3}) \times 10^3$$

[7]. We therefore conclude that flavor symmetry breaking effects cannot be large in this established $Q\bar{Q}$ nonet. Similar conclusions follow for a wide range of $Q\bar{Q}$ decays (at least in the $\beta \to \infty$ limit, where form factors are ignored [21]). Thus it seems reasonable to expect that for $Q\bar{Q}$ scalar decays also, $\rho \approx 1$ and $\beta \sim 0.5$ GeV/c.

We have refrained from using the corresponding ratios for $f_2'(1525)$ decays, since the $\pi\pi$ decay width is (i) poorly known and (ii) very sensitive to the precise tensor nonet mixing angle θ. For the ratio $B(f_2' \to \eta\eta)/B(f_2' \to K\bar{K})$ we find 0.07 for $\rho = 1$ and $\beta = 0.5$ GeV/c, in good agreement with experiment (0.11 ± 0.04) [7] but at variance with the value (0.39 ± 0.05) advocated by the Particle Data Group, which relies on one experiment only.

B. The decay properties of $f_0(1500)$

The branching ratios for $f_0(1500)$ production and decay are [9,11,22]

$$\begin{aligned} B(\bar{p}p \to f_0\pi^0, f_0 \to \pi^0\pi^0) &= (8.1 \pm 2.8) \times 10^{-4} , \\ B(\bar{p}p \to f_0\pi^0, f_0 \to \eta\eta) &= (5.5 \pm 1.3) \times 10^{-4} , \\ B(\bar{p}p \to f_0\pi^0, f_0 \to \eta\eta') &= (1.6 \pm 0.4) \times 10^{-4} . \end{aligned} \tag{16}$$

where the errors do not reflect the statistical significance of the signals but rather uncertainties in the various assumptions made in the fitting procedures. The decay branching ratios are given by Eq. (13) with $S_p(\vec{q}) = q$ and the form factors

$$|F_{ij}(\vec{q})|^2 = \exp(-q^2/8\beta^2) , \tag{17}$$

since $l = 0$ for 0^{++} decays to two pseudoscalars. The $f_0(1500)$ observed in $\eta\eta'$ decay has a mass of 1545 ± 25 MeV and lies just above threshold [11]. We use for q the average decay momentum (194 MeV/c) derived from the damped Breit-Wigner function used in the analysis of Ref. [11]. The uncertainty in the mass (± 25 MeV) is taken into account when computing the error on γ^2. With $\beta = 0.5$ GeV/c (Sec. IV A) we find

$$R_1 = \frac{\gamma^2[f_0(1500) \to \eta\eta]}{\gamma^2[f_0(1500) \to \pi\pi]} = 0.27 \pm 0.11 , \tag{18}$$

$$R_2 = \frac{\gamma^2[f_0(1500) \to \eta\eta']}{\gamma^2[f_0(1500) \to \pi\pi]} = 0.19 \pm 0.08 , \tag{19}$$

where $\pi\pi$ includes $\pi^+\pi^-$. These results are in good agreement with the results of the coupled channel analysis [29]. A signal for scalar decay to $K\bar{K}$ has not yet been observed in $\bar{p}p$ annihilation in the 1500 MeV region. A bubble chamber experiment [41] reports $B(p\bar{p} \to X\pi;\ X \to K\bar{K}) < 3.4 \times 10^{-4}$, which interpreted directly as an intensity leads to a (90% C.L.) upper limit:

$$R_3 = \frac{\gamma^2[f_0(1500) \to K\bar{K}]}{\gamma^2[f_0(1500) \to \pi\pi]} \lesssim 0.1 . \tag{20}$$

However, interference effects among amplitudes could lead to an underestimate for this number. We shall consider the implications of the above R_3 value but shall also give results allowing for larger values.

We find that if in the decay of some state the ratios of partial widths (per charge combination and after phase space and form factor corrections) for $\eta\eta/\pi\pi$ and $\eta\eta/K\bar{K}$ are simultaneously both greater than unity, then this state cannot be a quarkonium decay unless $s\bar{s}$ production is enhanced ($\rho > 1$) (see Fig. 3). The $f_0(1500)$ data on $\eta\eta/K\bar{K}$ satisfy this, but the $\eta\eta/\pi\pi$ is inconclusive; at 1σ the ratio per charge configuration gives 0.81 ± 0.33.

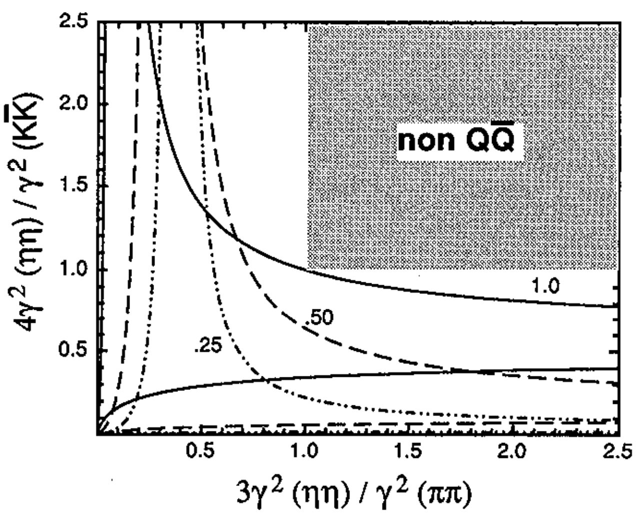

FIG. 3. $\eta\eta/\pi\pi$ vs $\eta\eta/K\bar{K}$ invariant couplings per charge combination for various values of $\rho \leq 1$. The grey region where both ratios are larger than one is not accessible to $Q\bar{Q}$ mesons unless $s\bar{s}$ production is enhanced.

If $f_0(1590)$ and $f_0(1500)$ are the same state, as discussed above (see footnote 1), then if the $\pi\pi$ branching ratio is reduced towards the GAMS limit [15,19] the value of R_1 would rise such that it may be possible to confirm the $f_0(1500)$ as a glueball by this test alone. We cannot overemphasize the importance of a mutually consistent analysis of the data on these experiments, in particular for clarifying the magnitude of the $\eta\eta/\pi\pi$ ratio. If the ratio rises, as for GAMS, it would immediately point towards a glueball; if the ratio remains as in Eq. (18) then the arguments are less direct but there still appears not to be a consistent $Q\bar{Q}$ solution to the flavor dependence of the ratios of partial widths and the magnitudes of the total widths for the $f_0(1500)$–$f_0(1370)$ system.

Figure 1 shows the invariant couplings γ^2 as a function of α for $\rho = 1$ (a) and $\rho = 0.75$ and 1.25 (b), for a pseudoscalar mixing angle $\theta = -17.3$ [40]. The effects of $SU(3)_f$ breaking, $\rho < 1$, in the region where $n\bar{n}$ dominates the Fock state ($0 \leq \alpha \leq 30°$) are interesting [Fig. 1(b)]. We see that the branching ratios for η or η' are little affected (essentially because they are produced via the $\bar{n}n$ component in the η, which is ρ independent) whereas $K\bar{K}$ depends on ρ (due to $s\bar{s}$ creation triggering $K\bar{K}$ production from an $n\bar{n}$ initial state). Thus, we can suppress $K\bar{K}$ by letting $\rho \to 0$ without affecting the $\eta\eta/\pi\pi$ ratio substantially. In this case the measured values for $\eta\eta$ and $\eta\eta'$ and the upper limit for $K\bar{K}$ suggest that $\alpha \sim 0$; hence $f_0(1500)$ is a pure $\bar{n}n$ meson. However, this is still unsatisfactory as the required value of ρ implies a dramatic suppression of $s\bar{s}$ creation to a degree not seen elsewhere in hadron decays.

Figure 4 shows the dependence of $\rho \tan\alpha$ [Eq. (10)] on R_1 and R_2. From the experimental values [Eqs. (18) and (19)] we find consistency for $\rho\tan(\alpha) \sim -0.1$. Hence, either ρ is small or α is small. The former leads to unacceptable violation of $SU(3)_f$, and the latter predicts the ratio $\gamma^2(K\bar{K})/\gamma^2(\pi\pi)$ to be 1/3.

Form factors of the type in Eq. (17) tend to destroy transitions at large q, for example $F_{\eta\eta}(q_\eta)/F_{\pi\pi}(q_\pi) > 1$ (i.e., opposite to naive phase space, which grows with q). Without a form factor, e.g., $|F_{ij}(\vec{q})| \equiv 1$, we obtain $R_1 = 0.31 \pm 0.13$, $R_2 = 0.25 \pm 0.11$, which excludes a common range of $\rho\tan(\alpha)$ (see Fig. 4), and $R_3 < 0.12$. The form factors used in Ref. [44] have a node at $q \sim 0.9$ GeV/c and, hence, lead to an even stronger suppression of the observed $\pi\pi$ intensity, which dramatically reduces R_3, in contradiction with the expected 1/3 for an $\bar{n}n$ state (see also Appendix B).

In Fig. 5 we plot the allowed regions of ρ vs α. The grey area shows the common values of ρ and α that satisfy the Crystal Barrel data, each at 90% C.L., while the black area shows the restricted range allowed by $K\bar{K}$ [Eq. (20)].

If one wishes to force $f_0(1500)$ into a $Q\bar{Q}$ nonet, then independent of form factors and $SU(3)_f$ breaking one is forced to $\alpha \to 0$, whereby $f_0(1500)$ has strong $n\bar{n}$ content. This remains true even were the $K\bar{K}$ branching ratio, currently being remeasured at LEAR, significantly greater than the R_3 value of Eq. (20): the magnitude of the $K\bar{K}/\pi\pi$ ratio is controlled more by flavor symmetry breaking than by the magnitude of the $n\bar{n}$-$s\bar{s}$ mixing angle in the $\alpha \to 0$ region. This immediately implies that the orthogonal isoscalar member will be dominantly $s\bar{s}$, with mass above 1500 MeV and prominent in $K\bar{K}$. The $f_0'(1525)$ if confirmed or the "θ" [$f_J(1720)$], if 0^{++}, could be this state. However, the $f_0(1370)$, seen in both $\pi\pi, \eta\eta$ is then left in isolation.

In the next sections we confront the data on $G = f_0(1500)$ with the extreme hypothesis that it is dominantly a glueball. Its production in the canonical glue

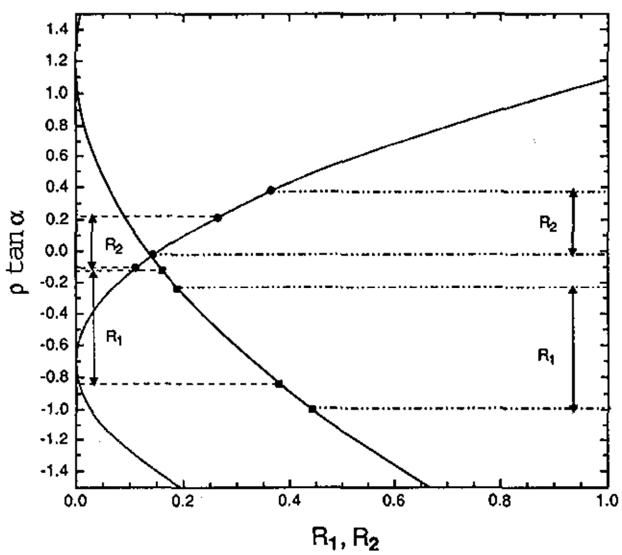

FIG. 4. $\rho\tan(\alpha)$ as a function of $\eta\eta/\pi\pi$ (full curve) and $\eta\eta'/\pi\pi$ (dotted curve). The vertical arrows show the experimental ranges of R_1 and R_2 for $f_0(1500)$ decay, left with form factor, right without form factor.

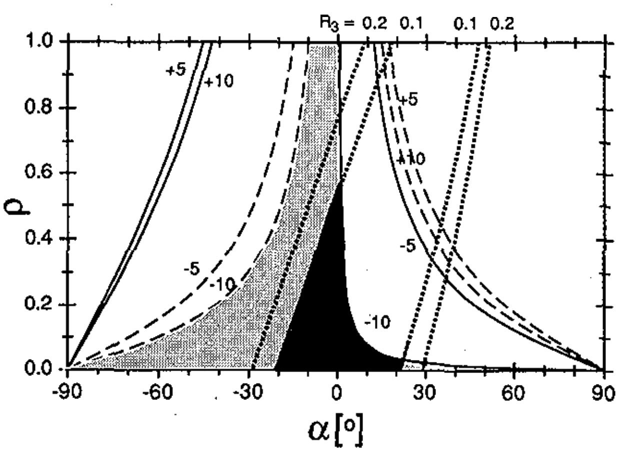

FIG. 5. ρ as a function of α. The full curves show the 5 and 10% C.L. upper (+) and lower (−) limits dependence for the experimental value R_1, the dashed curves for R_2. The dotted curves give the boundaries for the experimental value R_3. The grey region shows the range allowed by the experimental data on $f_0(1500)$ decay to $\pi\pi$, $\eta\eta$, and $\eta\eta'$. The black region includes in addition the bubble chamber upper limit for $K\bar{K}$.

enhanced environments of $J/\psi \to \gamma(G \to 4\pi)$ [17], $pp \to p(G)p$ [12], and $p\bar{p}$ annihilation [8–11,13,14] is consistent with this hypothesis, and recent lattice QCD studies [3,4] predict that a scalar glueball exists in this region of mass. It thus scores well on two of the three glueball figures of merit [16]. We shall now consider the dynamics and phenomenology of glueball decays.

V. PRIMITIVE GLUEBALL DECAYS

The decays of $c\bar{c}$, in particular $\chi_{0,2}$, provide a direct window on G dynamics in the $0^{++}, 2^{++}$ channels insofar as the hadronic decays are triggered by $c\bar{c} \to gg \to Q\bar{Q}Q\bar{Q}$ [Fig. 6(a)]. It is necessary to keep in mind that these are in a different kinematic region to that appropriate to our main analysis but, nonetheless, they offer some insights into the gluon dynamics. Mixing between hard gluons and 0^{++}, 2^{++} $Q\bar{Q}$ states [Fig. 6(c)] is improbable at these energies, as the latter 1–1.5 GeV states will be far off their mass shell. Furthermore, the narrow widths of $\chi_{0,2}$ are consistent with the hypothesis that the 3.5 GeV region is remote from the prominent $0^{+}, 2^{+}$ glueballs, G. Thus we expect that the dominant decay dynamics is triggered by hard gluons directly fragmenting into two independent $Q\bar{Q}$ pairs [Fig. 6(a)] or showering into lower energy gluons [Fig. 6(b)]. We consider the former case now; mixing with $Q\bar{Q}$ [Fig. 6(c)] and $G \to GG$ [Fig. 6(b)] will be discussed in Sec. VI.

The process $G \to QQ\bar{Q}Q$ was discussed in Ref. [16], and the relative amplitudes for the process shown in Fig. 6(a) read

$$\begin{aligned} \langle G|V|\pi\pi\rangle &= 1 , \\ \langle G|V|K\bar{K}\rangle &= R , \\ \langle G|V|\eta\eta\rangle &= (1+R^2)/2 , \\ \langle G|V|\eta\eta'\rangle &= (1-R^2)/2 , \end{aligned} \tag{21}$$

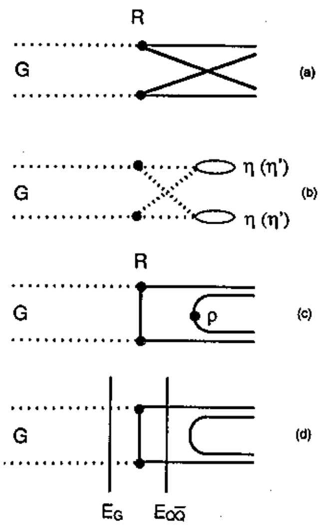

FIG. 6. Contributions to gluonium decay: $QQ\bar{Q}Q$ (a), GG (b), $Q\bar{Q}$ (c), and interpretation as $Q\bar{Q}$ mixing (d) involving the energy denominator E_G-$E_{Q\bar{Q}}$.

with generalizations for arbitrary pseudoscalar mixing angles given in Appendix A and where $R \equiv \langle g|V|s\bar{s}\rangle/\langle g|V|d\bar{d}\rangle$. $SU(3)_f$ symmetry corresponds to $R^2 = 1$. In this case the relative branching ratios (after weighting by the number of charge combinations) for the decays $\chi_{0,2} \to \pi\pi, \eta\eta, \eta\eta', K\bar{K}$ would be in the relative ratios 3:1:0:4. Data for χ_0 are in accord with this, where the branching ratios are (in parts per mil) [7]

$$\begin{aligned} B(\pi^0\pi^0) &= 3.1 \pm 0.6 , \\ \tfrac{1}{2}B(\pi^+\pi^-) &= 3.7 \pm 1.1 , \\ \tfrac{1}{2}B(K^+K^-) &= 3.5 \pm 1.2 , \\ B(\eta\eta) &= 2.5 \pm 1.1 . \end{aligned} \tag{22}$$

No signal has been reported for $\eta\eta'$. Flavor symmetry is manifested in the decays of χ_2 also:

$$\begin{aligned} B(\pi^0\pi^0) &= 1.1 \pm 0.3 , \\ \tfrac{1}{2}B(\pi^+\pi^-) &= 0.95 \pm 0.50 , \\ \tfrac{1}{2}B(K^+K^-) &= 0.75 \pm 0.55 , \\ B(\eta\eta) &= 0.8 \pm 0.5 , \end{aligned} \tag{23}$$

again in parts per mil. The channel $\eta\eta'$ has not been observed either. These results are natural as they involve hard gluons away from the kinematic region where G bound states dominate the dynamics. If glueballs occur at lower energies and mix with nearby $Q\bar{Q}$ states, this will in general lead to a distortion of the branching ratios from the ideal equal weighting values above (a detailed discussion of this follows in Sec. VI A), and also in causing significant mixing between $n\bar{n}$ and $s\bar{s}$ in the quarkonium eigenstates. Conversely, ideal nonets, where the quarkonium eigenstates are $n\bar{n}$ and $s\bar{s}$, are expected to signal those J^{PC} channels where the masses of the prominent glueballs are remote from those of the quarkonia.

An example of this is the 2^{++} sector where the quarkonium members are ideal. Data on glue in the 2^{++} channel, and potential mixing of glue with $n\bar{n}/s\bar{s}$, may be probed by $J/\psi \to \gamma + f_2(1270)/f_2'(1525)$, which measures the $gg \to n\bar{n}/s\bar{s}$ amplitude [Fig. 6(c)] insofar as $J/\psi \to \gamma + gg$ mediates these channels. The branching ratios in parts per mil are

$$\begin{aligned} \tfrac{1}{2}B(J/\psi \to \gamma f_2(1270)) &= 0.69 \pm 0.07 , \\ B(J/\psi \to \gamma f_2(1525)) &= 0.63 \pm 0.1 . \end{aligned} \tag{24}$$

Here again there is no sign of significant symmetry breaking. Furthermore, we note the ideal $n\bar{n}$ and $s\bar{s}$ nature of the 2^{++}, manifested both by the masses and the flavor dependence of the branching ratios, which suggests that G mixing is nugatory in this channel. These data collectively suggest that prominent 2^{++} glueballs are not in the 1.2–1.6 GeV region, which in turn is consistent with lattice calculations, where the mass of the 2^{++} primitive glueball is predicted to be larger than 2 GeV. The sighting of a 2^{++} state in the glueball favored central production, decaying into $\eta\eta$ with no significant $\pi\pi$ [42] could be the first evidence for this state. In view of our earlier remarks on the $\eta\eta/\pi\pi$ and $\eta\eta/K\bar{K}$ ratios being a potentially direct signature for a glueball, we recommend that a detailed search now be made for this state in $\eta\eta$ *and* $K\bar{K}$ (and $\pi\pi$) channels in central production.

The phenomenology of the $J^{PC} = 0^{++}$ sector in the 1.2–1.6 GeV region is rather different from this: the $f_0(1500)$–$f_0(1370)$ system cannot be described within a $Q\bar{Q}$ nonet, nor do the decay branching ratios of the $f_0(1500)$ respect the flavor blindness of glue, [Eqs. (19) and (20)]. We shall now begin to focus on this problem.

It was shown earlier [16] that violation of flavor symmetry ($R^2 \neq 1$) leads to smaller $K\bar{K}/\eta\eta$ and a finite $\eta\eta'$, at least if graph 6(a) dominates glueball decay. This follows immediately from Eq. (21) or from the generalized formulas given in Appendix A. The contributions of graph 6(a) are shown in Fig. 7(a) as a function of R^2.

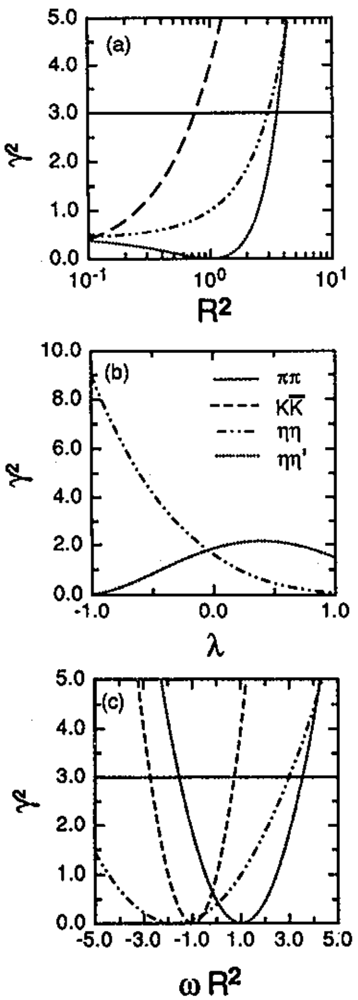

FIG. 7. Predicted decay rates γ_{ij}^2 (up to a common multiplicative constant) (a) as a function of R^2 for $QQ\bar{Q}\bar{Q}$ decay, (b) as a function of λ for GG decay, and (c) as a function of ωR^2 for $Q\bar{Q}$ decays with $\rho = 1$. Solid line: $\pi\pi$; dashed line: $K\bar{K}$; dash-dotted line: $\eta\eta$; dotted line: $\eta\eta'$.

Graph 6(a) becomes compatible with the Crystal Barrel data and the small $K\bar{K}$ ratio if $|R| \sim 0.3$ [1]—a rather strong violation of symmetry, which might be suggestive of significant mixing between G and flavored states. If R_3 were as large as $\frac{1}{3}$ (the value for an $n\bar{n}$ state) an attempt to interpret as gluonium would still require $|R|$ to be as small as 0.5. We now show that mixing of G and $Q\bar{Q}$ is to be expected if the strong coupling picture of QCD, as in the lattice, is a guide to their dynamics.

VI. $Q\bar{Q}$ AND GLUEBALL DECAYS IN STRONG COUPLING QCD

In the strong coupling ($g \to \infty$) lattice formation of QCD, hadrons consist of quarks and flux links, or flux tubes, on the lattice. Primitive $Q\bar{Q}$ mesons consist of a quark and antiquark connected by a tube of colored flux, whereas primitive glueballs consist of a loop of flux Figs 8(a) and 8(b)] [43]. For finite g these eigenstates remain a complete basis set for QCD but are perturbed by two types of interaction [44]: (1) V_1, which creates a Q and a $\bar{Q}$ at neighboring lattice sites, together with an elementary flux tube connecting them, as illustrated in

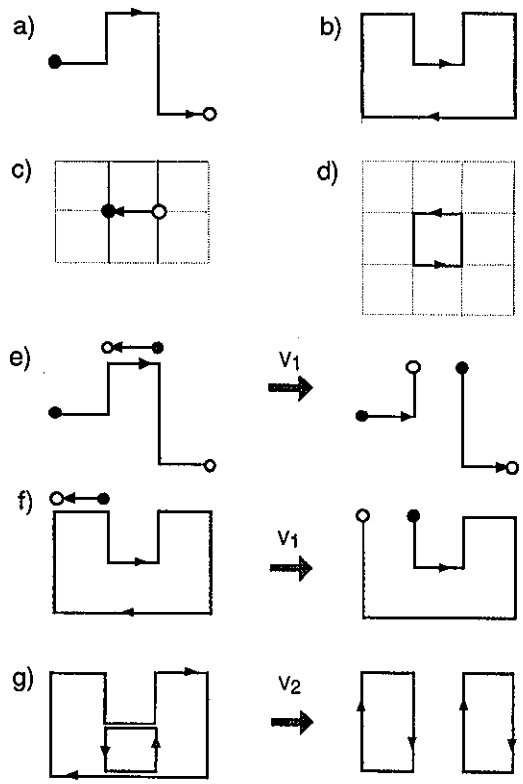

FIG. 8. Glueballs, quarkonia, and perturbations: (a) primitive $Q\bar{Q}$ and (b) primitive glueball G_0 in flux tube simulation lattice QCD; perturbation V_1 (c) and V_2 (d); the effect of V_1 on $Q\bar{Q}$ is shown in (e), and on G is shown in (f); the effect of V_2 on G is shown in (g).

Fig. 8(c); (2) V_2, which creates or destroys a unit of flux around any plaquette (where a plaquette is an elementary square with links on its edges), illustrated in Fig. 8(d).

The perturbation V_1 in leading order causes decays of $Q\bar{Q}$ [Fig. 8(e)] and also induces mixing between the primitive glueball (G_0) and $Q\bar{Q}$ [Fig. 8(f)]. It is perturbation V_2 in leading order that causes glueball decays and leads to a final state consisting of G_0G_0 [Fig. 8(g)]; decays into $Q\bar{Q}$ pairs occur at higher order, by application of the perturbation V_1 twice. This latter sequence effectively causes G_0 mixing with $Q\bar{Q}$ followed by its decay. Application of V_1^2 leads to a $Q^2\bar{Q}^2$ intermediate state, which then turns into color singlet mesons by quark rearrangement [Fig. 6(a)]; application of V_2 would lead to direct coupling to glue in η, η', or $V_2 \times V_1^2$ to their $Q\bar{Q}$ content [Fig. 6(b)].

The absolute magnitudes of these various contributions require commitment to a detailed dynamics and are beyond the scope of this first survey. We concentrate here on their *relative* contributions to the various two-body pseudoscalar meson final states available to 0^{++} meson decays. For $Q\bar{Q} \to Q\bar{q}q\bar{Q}$ decays induced by V_1, the relative branching ratios are given in Eq. (10) where one identifies

$$\rho \equiv \frac{\langle Q\bar{s}s\bar{Q}|V_1|Q\bar{Q}\rangle}{\langle Q\bar{d}d\bar{Q}|V_1|Q\bar{Q}\rangle} . \tag{25}$$

The magnitude of ρ and its dependence on J^{PC} is a challenge for the lattice. We turn now to consider the effect of V_1 on the initial primitive glueball G_0. Here too we allow for possible flavor dependence and define

$$R^2 \equiv \frac{\langle s\bar{s}|V_1|G_0\rangle}{\langle d\bar{d}|V_1|G_0\rangle} . \tag{26}$$

The lattice may eventually guide us on this magnitude and also on the ratio R^2/ρ. In the absence of this information we shall leave R as free parameter and set $\rho = 1$.

A. Glueball-$Q\bar{Q}$ mixing at $O(V_1)$

In this first orientation we shall consider mixing between G_0 (the primitive glueball state) and the quarkonia, $n\bar{n}$ and $s\bar{s}$, at leading order in V_1, but we will ignore that between the two different quarkonia, which is assumed to be higher order perturbation.

The mixed glueball state is then

$$G = |G_0\rangle + \frac{|n\bar{n}\rangle\langle n\bar{n}|V_1|G_0\rangle}{E_{G_0} - E_{n\bar{n}}} + \frac{|s\bar{s}\rangle\langle s\bar{s}|V_1|G_0\rangle}{E_{G_0} - E_{s\bar{s}}} , \tag{27}$$

which may be written as

$$G = |G_0\rangle + \frac{\langle n\bar{n}|V_1|G_0\rangle}{\sqrt{2}(E_{G_0} - E_{n\bar{n}})}\{\sqrt{2}|n\bar{n}\rangle + \omega R^2|s\bar{s}\rangle\} , \tag{28}$$

where

$$\omega \equiv \frac{E_{G_0} - E_{n\bar{n}}}{E_{G_0} - E_{s\bar{s}}} \tag{29}$$

is the ratio of the energy denominators for the $n\bar{n}$ and $s\bar{s}$ intermediate states in old fashioned perturbation theory [Fig. 6(d)].

Denoting the dimensionless mixing parameter by

$$\xi \equiv \frac{\langle d\bar{d}|V_1|G_0\rangle}{E_{G_0} - E_{n\bar{n}}} , \tag{30}$$

the three eigenstates become, to leading order in the perturbation,

$$\begin{aligned} N_G|G\rangle &= |G_0\rangle + \xi\{\sqrt{2}|n\bar{n}\rangle + \omega R^2|s\bar{s}\rangle\} \\ &\equiv |G_0\rangle + \sqrt{2}|Q\bar{Q}\rangle , \\ N_s|\Psi_s\rangle &\equiv |s\bar{s}\rangle - \xi R^2\omega|G_0\rangle , \\ N_n|\Psi_n\rangle &\equiv |n\bar{n}\rangle - \xi\sqrt{2}|G_0\rangle \end{aligned} \tag{31}$$

with the normalizations

$$\begin{aligned} N_G &= \sqrt{1+\xi^2(2+\omega^2R^4)} , \\ N_s &= \sqrt{1+\xi^2\omega^2R^4)} , \\ N_n &= \sqrt{1+2\xi^2} . \end{aligned} \tag{32}$$

Recalling our definition of quarkonium mixing,

$$|Q\bar{Q}\rangle = \cos\alpha|n\bar{n}\rangle - \sin\alpha|s\bar{s}\rangle , \tag{33}$$

we see that G_0 has mixed with an effective quarkonium of mixing angle α, where $\sqrt{2}\tan\alpha = -\omega R^2$ [Eq. (10)]. For example, if $\omega R^2 \equiv 1$, the $SU(3)_f$ flavor symmetry maps a glueball onto quarkonium where $\tan\alpha = -1\sqrt{2}$ and hence $\theta = -90°$, leading to the familiar flavor singlet

$$|Q\bar{Q}\rangle = |u\bar{u} + d\bar{d} + s\bar{s}\rangle/\sqrt{3} . \tag{34}$$

When the glueball is far removed in mass from the $Q\bar{Q}$, $\omega \to 1$ and flavor symmetry ensues; the $\chi_{0,2}$ decay and the 2^{++} analysis of Secs. IV and V are examples of this ideal situation. However, when $\omega \neq 1$, as will tend to be the case when G_0 is in the vicinity of the primitive $Q\bar{Q}$ nonet (the 0^{++} case of interest here), significant distortion from the naive flavor singlet can arise.

If the G_0 component contributed negligibly to the decays, the expectations would be that there is [Eq. (31)] (i) a state $\Psi_n \to \pi\pi, \eta\eta, K\bar{K}$ which is compatible with $f_0(1370)$; (ii) a state $\Psi_s \to K\bar{K}, \eta\eta, \eta\eta'$, but not $\pi\pi$, to be established; (ii) the 1500 MeV state G for which the decay amplitudes relative to $\pi\pi$ are [replacing $\sqrt{2}\tan\alpha$ by $-\omega R^2$ in Eq. (10)]

$$\begin{aligned} \langle G|V|\pi\pi\rangle &= 1 , \\ \langle G|V|K\bar{K}\rangle &= (\rho + \omega R^2)/2 , \\ \langle G|V|\eta\eta\rangle &= (1 + \omega\rho R^2)/2 , \\ \langle G|V|\eta\eta'\rangle &= (1 - \omega\rho R^2)/2 , \end{aligned} \tag{35}$$

with generalization given in Appendix A. The invariant decay couplings γ_{ij}^2 are shown in Fig. 7(c) as a function of ωR^2 for $\rho = 1$. Thus, for example, $SU(3)_f$ may be exact for the glue-quark coupling ($R = 1$), but mass-breaking effects ($\Delta m \equiv m_s - m_d \neq 0$) can cause dramatic effects if $E_G - E_{n\bar{n}}$ or $E_G - E_{s\bar{s}}$ is accidentally small, such that $\omega \to 0$ or ∞, respectively. We now consider an explicit mixing scheme motivated by three mutually consistent phenomenological inputs.

(1) The suppression of $K\bar{K}$ in the $f_0(1500)$ decays suggests a destructive interference between $n\bar{n}$ and $s\bar{s}$ such that $\omega R^2 < 0$ [see Fig. 7(c)]. This arises naturally if the primitive glueball mass is between those of $n\bar{n}$ and the primitive $s\bar{s}$. As the mass of $G_0 \to m_{n\bar{n}}$ or $m_{s\bar{s}}$, the $K\bar{K}$ remains suppressed though non-zero; thus eventual quantification of the $K\bar{K}$ signal will be important.

(2) Lattice QCD suggests that the primitive scalar glueball G_0 lies at or above 1500 MeV, and hence above the $I = 1 Q\bar{Q}$ state $a_0(1450)$ and the (presumed) associated $n\bar{n}$ $f_0(1370)$. Hence $E_{G_0} - En\bar{n} > 0$ in the numerator of ω.

(3) The $\Delta m = m_{s\bar{s}} - m_{n\bar{n}} \approx 200$–$300$ MeV suggests that the primitive $s\bar{s}$ state is in the region 1600–1700 MeV. This is consistent with the requirement from (1) and (2) that $m_{n\bar{n}} < m_{G_0} < m_{s\bar{s}}$.

Higher-order perturbation effects will be required for a complete treatment, in particular including mixing between $n\bar{n}$ and $s\bar{s}$, but that goes beyond this first orientation and will require more data to constrain the analysis. We shall present *a posteriori* evidence supporting this leading-order approximation.

Tests of this scenario and its further development will follow as the predicted states are isolated and the flavor dependence of their branching ratios is measured. In order to compute the decay branching ratios of the physical (mixed) states, we need to incorporate the contributions from the primitive glueball components G_0. We consider this now.

B. $G_0 \to G_0 G_0$ at $O(V_2)$

Here the glueball decays directly into pairs of glueballs or mesons whose Fock states have strong overlap with gg [Fig. 6(b)]. This topology will not feed final states such as $\pi\pi$ nor $K\bar{K}$, since gluons are isoscalar. To the extent that there is significant G coupling to η, η' or to the $\pi\pi$ S wave, $(\pi\pi)_s$ [e.g., $\psi' \to \psi\eta$ and $\psi(\pi\pi)_s$ each have large intrinsic couplings notwithstanding the fact that they are superficially Okubo-Zweig-Iizuka (OZI) violating] one may anticipate $\eta\eta$, $\eta\eta'$, and $(\pi\pi)_s(\pi\pi)_s$ in the decays of scalar glueballs. Analogously for 0^{-+} glueballs one may anticipate $\eta(\pi\pi)_s$ or $\eta'(\pi\pi)_s$ decays.

The manifestation of this mechanism in final states involving the η or η' mesons depends on the unknown overlaps such as $\langle gg|V|q\bar{q}\rangle$ in the pseudoscalars. We consider various possibilities from the literature without prejudice at this stage.

In the limit $m_{u,d} \to 0$ chiral symmetry suggests that the direct coupling of glue to the η or η' occurs dominantly through their $s\bar{s}$ content, thereby favoring the η'. This argument has been applied to the $\eta(1460)$ in Refs. [45–47]:

$$\frac{\langle gg|V|\eta'\rangle}{\langle gg|V|\eta\rangle} = \frac{\langle s\bar{s}|\eta'\rangle}{\langle s\bar{s}|\eta\rangle} = \frac{\cot(\phi) + \sqrt{2}\lambda}{\lambda\sqrt{2}\cot(\phi) - 1}, \tag{36}$$

where

$$\lambda \equiv \frac{\langle gg(0^-)|V|d\bar{d}\rangle}{\langle gg(0^-)|V|s\bar{s}\rangle} \to 0 \tag{37}$$

in the chiral limit, for which the ratio in Eq. (36) is $\sim -4/3$.

The ratio Eq. (36) depends sensitively on the pseudoscalar mixing angle and on a small breaking of chiral symmetry but remains negative in the range $-0.9 < \lambda < 0.5$. Thus we anticipate

$$r_0 \equiv \frac{\langle \eta\eta'|V|G_0\rangle}{\langle \eta\eta|V|G_0\rangle} = \frac{\langle gg|V|\eta'\rangle}{\langle gg|V|\eta\rangle} \sim -\frac{4}{3}. \tag{38}$$

There is some ambiguity as to how this is to be applied quantitatively, since $m_{\eta'} \neq m_\eta$ and the wave functions at the origin $\psi_{\eta'}(0)$ and $\psi_\eta(0)$ are, in general, different. An alternative measure [52] may be the ratio $\Gamma(\psi \to \gamma\eta')/\Gamma(\psi \to \gamma\eta) = 5.0 \pm 0.6$. Dividing out phase-space factors $\sim p_\gamma^3$ we obtain the ratio of matrix elements

$$r_0(J/\psi) = \pm(2.48 \pm 0.15). \tag{39}$$

The solution with the negative sign is compatible with a small breaking of chiral symmetry ($\lambda = 0.18$). This gives similar results to arguments based on the gluon anomaly in the pseudoscalar channel (see Eq. (60) in Ref. [48] and also Ref. [49]).

This enhanced gluonic production of $\eta\eta, \eta\eta'$ does not appear to be dramatic in the $\chi_{0,2}$ decays as the $\eta\eta/\pi\pi$ ratio appears to be "canonical" in the sense of Sec. V. This may be because $\langle gg|V|\eta\rangle$ and $\langle gg|V|\eta'\rangle$ are hidden in the large errors in the present data (in which case isolation of $\chi \to \eta'\eta'$ at a Tau Charm Factory would be especially interesting). Alternatively it may be $\langle gg|V|(\pi\pi)_s\rangle$ that is important, and hence $\chi \to 4\pi$ is the signal. Indeed this channel is the biggest hadronic branching ratio for both χ_0 and χ_2. It would be interesting to compare the $\pi\pi$ spectra in these final states with those in $\eta_c \to \eta'\pi\pi$ and $\eta\pi\pi$, which are dominant modes in the η_c decays and may also be signals for this dynamics. High statistics from a Tau Charm Factory may eventually answer this question. The relative coupling strengths of $\eta\pi\pi$ and $\eta'\pi\pi$ in decays of the glueball candidate $\eta(1420)$ are also relevant here.

Note that for the decays of $J^{PC} = 0^{++}, 2^{++}, \ldots$ states, one will have a sharp test for a glueball if nonperturbative effects favor the direct $\eta\eta$, $\eta\eta'$ decay path over mixing with $Q\bar{Q}$ systems. A state decaying to $\eta\eta$, $\eta\eta'$, and/or $\eta'\eta'$ but not $\pi\pi$ nor $K\bar{K}$ cannot be simply $Q\bar{Q}$, since ρ and α cannot sensibly suppress $\pi\pi$ and $K\bar{K}$ simultaneously (see, e.g., Figs. 1 and 3). GAMS has claimed states at 1590 MeV ($\eta\eta, \eta\eta'$) [15,19], 1740 MeV ($\eta\eta$) [50] and 1910 MeV ($\eta\eta'$) [51] with no strong signal seen in $\pi\pi$ nor $K\bar{K}$. If the existence of any of these enigmatic states X is seen in other experiments such as central production $pp \to p(X)p$ or $\bar{p}p$ annihilation, where $X \to \eta\eta, \eta\eta'$ with no $K\bar{K}$ signal, this will be strong evidence for the presence of G_0 in their wave functions. The possible hints of $f_J(2100)$ in η channels [14] and of $f_2(2170)$ in $\eta\eta$ [42], if confirmed, will put a high premium on searching for or limiting the $K\bar{K}$ branching ratios for these states. A particular realization of these generalities is the model of Ref. [52]. Similar remarks apply to the decay of pseudoscalars $\to \eta(\pi\pi)_s$ in contrast to $\bar{K}(K\pi)_s$. This may be the case for the $\eta(1440)$ and for the η_c, as discussed above.

VII. APPLICATION TO SCALAR MESONS AROUND 1.5 GeV

A. Decays of $f_0(1500)$

We shall now combine these ideas with the other result of Sec. II, namely, that at $O(V_1)$ the G_0 mixes with $Q\bar{Q}$ with amplitude ξ and that the resulting $Q\bar{Q}$ components decay as in Sec. III.

For simplicity of analysis we shall set $\rho = 1$. From Eqs. (31) and (35) we obtain, for $G \equiv f_0(1500)$,

$$\langle K\bar{K}|V_1|G\rangle = \frac{1+\omega R^2}{2}\langle\pi\pi|V_1|G\rangle \ . \tag{40}$$

Eventual quantification of R_3 may be translated into a value of ωR^2 (see Fig. 9). We shall scale all decay amplitudes relative to that for $\langle\pi\pi|V|G\rangle$ and see what this implies for the G_0 decay amplitudes. Thus,

$$r_1 \equiv \frac{\langle\eta\eta|V|G\rangle}{\langle\pi\pi|V|G\rangle} = \frac{\langle\eta\eta|V|G_0\rangle}{N_G\langle\pi\pi|V|G\rangle} + \left(\frac{1+\omega R^2}{2}\right) + \cos 2\phi\left(\frac{1-\omega R^2}{2}\right) = \pm(0.90 \pm 0.20) \tag{41}$$

from R_1 [Eq. (18)] and

$$r_2 \equiv \frac{\langle\eta\eta'|V|G\rangle}{\langle\pi\pi|V|G\rangle} = \frac{\langle\eta\eta'|V|G_0\rangle}{N_G\langle\pi\pi|V|G\rangle} + \sin 2\phi\left(\frac{1-\omega R^2}{2}\right) = \pm(0.53 \pm 0.11) \tag{42}$$

from R_2 [Eq. (19)], from which we predict the ratio

$$r_0 = \frac{\langle\eta\eta'|V|G_0\rangle}{\langle\eta\eta|V|G_0\rangle} = \frac{2r_2 - \sin 2\phi(1-\omega R^2)}{2r_1 - \cos\phi(1-\omega R^2) - 1 - \omega R^2} \ . \tag{43}$$

The ratio r_0 is plotted in Fig. 10 as a function of ωR^2. Note that this ratio is rather sensitive to the precise value of the mixing angle ϕ. The solution with the + sign in Eq. (41) and the − sign in Eq. (42) (which we refer to as the "+−" solution) agrees very well with ωR^2 in the range predicted by the current $K\bar{K}$ suppression (Fig. 9) and with radiative J/ψ decay [Eq. (39)]. Figure 10 also suggests another possible solution ("−−") compatible with the $\psi \to \gamma\eta/\eta'$ ratio.

In the particular limit $\omega R^2 = -1$, the $\eta\eta$ decay mode would be driven dominantly by G_0 decay. The smaller rate for $\eta\eta'$ decay observed by Crystal Barrel would then be due to *destructive* interference between G_0 decay and the admixture of quarkonium in the wave function common to the +− and −− solutions. The amplitudes for G_0 decay [Fig. 6(b)] are given in Appendix A and the invariant couplings shown in Fig. 7(b) as a function of λ.

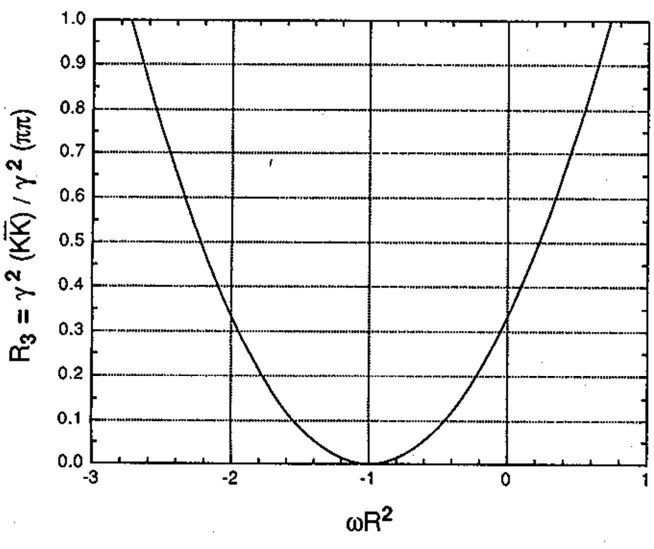

FIG. 9. R_3 vs ωR^2.

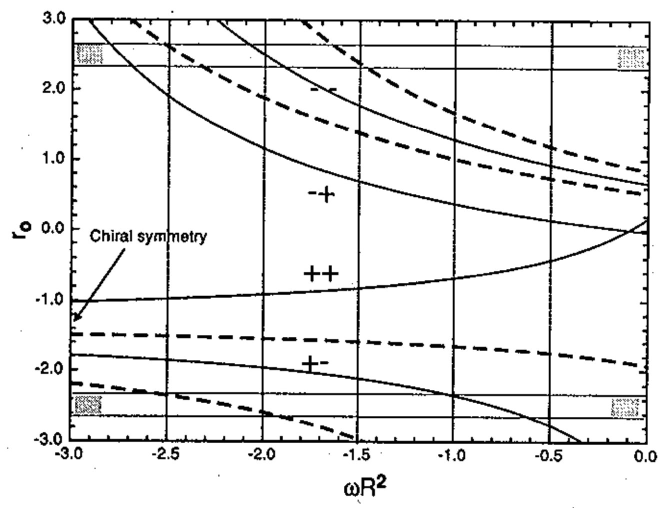

FIG. 10. Predicted ratio r_0 as a function of ωR^2 for $f_0(1500)$ decays. The solid lines show the four possible solutions from the Crystal Barrel results r_1 and r_2 with the corresponding signs. The range allowed by the experimental errors is shown by the dashed lines for the "+−" and "−−" solution. The solutions +− and −− are compatible with ωR^2 in the range allowed by the $K\bar{K}$ suppression and radiative J/ψ decay (area between the parallel lines, indicated by the grey flashes). A small breaking of chiral symmetry favors the +− solution.

B. $f_0(1370)$

The decay amplitudes for $\Psi_n \to \pi\pi$ and $K\bar{K}$ will be those of an $n\bar{n}$ state such that

$$\frac{\gamma^2(\Psi_n \to K\bar{K})}{\gamma^2(\Psi_n \to \pi\pi)} \simeq \frac{1}{3} . \tag{44}$$

Assuming that the lowest state Ψ_n is $f_0(1370)$, the decay to $\eta\eta'$ is kinematically forbidden and so the $\eta\eta$ decay will be the only one immediately sensitive to the predicted G_0 component in the $f_0(1370)$ Fock state. We find, from the decay branching ratios measured by Crystal Barrel [9,22],[3]

$$B(\bar{p}p \to f_0(1370)\pi^0, f_0 \to \pi^0\pi^0) < (2.6 \pm 0.4) \times 10^{-3} ,$$

$$B(\bar{p}p \to f_0(1370)\pi^0, f_0 \to \eta\eta) = (3.5 \pm 0.7) \times 10^{-4} , \tag{45}$$

after phase-space and form-factor corrections,

$$\frac{\gamma^2(\Psi_n \to \eta\eta)}{\gamma^2(\Psi_n \to \pi\pi)} > 0.07 \tag{46}$$

or

$$\frac{\langle \Psi_n|V|\eta\eta\rangle}{\langle \Psi_n|V|\pi\pi\rangle} > 0.46 . \tag{47}$$

On the other hand, Eq. (31) predicts

$$\frac{\langle \Psi_n|V|\eta\eta\rangle}{\langle \Psi_n|V|\pi\pi\rangle} = \cos^2\phi - \sqrt{2}\xi\frac{\langle G_0|V|\eta\eta\rangle}{\langle n\bar{n}|V|\pi\pi\rangle} , \tag{48}$$

which reduces to $\cos^2\phi = 0.63$ for a pure $n\bar{n}$ state. Solving for $\langle G_0|V|\eta\eta\rangle$ and introducing into Eq. (41), one finds, with Eq. (47),

$$\xi^2 = \frac{\cos^2\phi - 0.46}{2r_1 - (1+\omega R^2) - \cos^2\phi(1-\omega R^2)} . \tag{49}$$

The result Eq. (47), then implies that $|\xi|$ must be small and prefers the $+-$ solution rather than the $--$ solution. One finds the 90% C.L. upper limit

$$|\xi| < 0.47 \tag{50}$$

which *a posteriori* justifies the first-order perturbation used in the derivative of Eqs. (27) and (31).

Further data analysis is now needed to quantify the experimental ratio Eq. (46) and compare it in detail with the above. In any event, the branching ratio of this $f_0(1370)$ state is consistent with $n\bar{n}$ dominance and hence further isolates the $f_0(1500)$ as an exceptional state.

[3]The $<$ sign reflects the fact that the measured branching ratio for $f_0(1370)$ decay to $\pi\pi$ also includes some contribution from $f_0(980)$.

C. The $f_0(1370)$-$f_0(1500)$ system

It should by now be clear that it is the combination of the two siblings, $f_0(1370)$ and $f_0(1500)$, rather than either one on its own that reveals the need for degrees of freedom beyond $q\bar{q}$. We now illustrate how the branching ratios and widths of the pair manifest this quantitatively.

In the quark model of Ref. [39] the widths of $^3P_0$ are qualitatively ordered as $\Gamma(\bar{n}n) > \Gamma(s\bar{s}) > \Gamma(a_0) \geq \Gamma(K^*)$. Empirically $\Gamma(a_0) = 270 \pm 40$ MeV, $\Gamma(K^*) = 287 \pm 23$ MeV, which supports this pair to be members of the nonet and leads one to expect for their partners that $\Gamma(\bar{n}n) \sim 700$ MeV and $\Gamma(s\bar{s}) \sim 500$ MeV. In the flux tube model of Ref. [44] (see also Appendix B) after normalizing to the known widths of the 2^{++} nonet one finds typically $\Gamma(K_0^*) > 200$ MeV and $\Gamma(a_0) > 300$ MeV in accord with data, and predict $\Gamma(f_0^{n\bar{n}}) > 500$ MeV. That the $f_0^{n\bar{n}}$ width will be very broad is a rather general conclusion of all standard quark models (see also Ref. [53]); unitarization effects do not alter this conclusion [32].

The $f_0(1500)$ width of 116 ± 17 MeV is clearly out of line with this, being even smaller than the K^* and a_0 widths. The *total* width of $f_0(1370)$ is not yet well determined, 200–700 MeV being possible [9,22] depending on the theoretical model used in the analysis.

These suppressions of widths are natural in the G-$Q\bar{Q}$ mixing scheme as the presence of the G_0 component dilutes the effect of the leading $n\bar{n}$ component:

$$\Gamma(\Psi_n) = \Gamma(n\bar{n})/(1+2\xi^2) > \tfrac{2}{3}\Gamma(n\bar{n}) . \tag{51}$$

The $f_0(1500)$ by contrast is, in our hypothesis, a glueball in leading order and with $n\bar{n}$,$s\bar{s}$ components at $0(\xi)$ in perturbation. The decay amplitudes for $f_0(1500)$ are all at $0(\xi)$, as shown by our analysis above.

Quantitative measures arise if we concentrate on the decays into pseudoscalar pairs. The measurements from Crystal Barrel [9,22] give the ratios of branching ratios for $G = f_0(1500)$ as

$$\frac{\eta\eta}{\pi\pi} = 0.23, \quad \frac{\eta\eta'}{\pi\pi} = 0.07 , \tag{52}$$

which implies that $B(G \to \pi\pi) = 0.7F_2^G$ (where F_2^G is the fraction of two-body decays). With $\Gamma_G = 116$ MeV this implies that $\Gamma(G \to \pi\pi) = 28.2$ MeV per charge mode and hence, after dividing out phase space and form factors (738 MeV/c and 0.755, respectively) we have the reduced dimensionless measure

$$\tilde{\Gamma}(G \to \pi\pi) \equiv \tfrac{1}{3}\gamma^2(G \to \pi\pi) = 0.05F_2^G . \tag{53}$$

We now perform the same manipulations for the $f_0(1370)$. The $\pi\pi$ decay appears to be the dominant two-pseudoscalar decay mode [Eq. (45)]; any error of neglecting $\eta\eta$ in the analysis is likely to be masked anyway by the uncertainty on the total width, which is not known to better than a factor of 3.5 (spanning 200–700 MeV). Dividing out the phase space (671 MeV/c) and form factors (0.798) as before, we form the reduced measure per charge combination

$$\tilde{\Gamma}[f(1370) \to \pi\pi] \equiv \tfrac{1}{3}\gamma^2(G \to \pi\pi) = (0.13 - 0.44)F_2^f \ . \tag{54}$$

Hence, the ratio of measures for the two states is

$$\frac{\tilde{\Gamma}(G \to \pi\pi)}{\tilde{\Gamma}(f \to \pi\pi)} < 0.4\frac{F_2^G}{F_2^f} \ . \tag{55}$$

Assuming that $R = 1$, but allowing ω and ξ to be free, we expect this ratio to be given by

$$\frac{\tilde{\Gamma}(G \to \pi\pi)}{\tilde{\Gamma}(f \to \pi\pi)} = \frac{2\xi^2(1+2\xi^2)}{1+\xi^2(2+\omega^2)} \ . \tag{56}$$

If F_2^f is not small, then independent of F_2^G

$$|\xi| < 0.52 \ , \tag{57}$$

which is consistent with our earlier, independent, estimate in Eq. (50). The sum of the partial widths of the two states for the $\pi\pi$ channels is

$$\Gamma(\Psi_n) + \Gamma(G) \simeq \Gamma(n\bar{n}) \ . \tag{58}$$

Reference [54] reports a strong $\sim$ 300 MeV 4π signal in the 1400 MeV region. It is unlikely that this signal is $f_0(1370)$, since the inelasticity in the $\pi\pi$ S wave would be very large ($\sim$80%). However, given the uncertain dynamics in the $\pi\pi$ sector, one must allow for this possibility, in which case our result, Eq. (57), would break down. On the other hand, a 4π contribution to $f_0(1500)$ decay would decrease the upper limit for $|\xi|$. A more detailed analysis is now warranted to verify if this further qualitative indication is supported and to quantify the resonant contributions F_2^G/F_2^f and their effect on the analysis above.

D. $f_0'(1600)$

The mass of the Ψ_s state, the value of ωR^2 and of $m(G_0)$ are all related in our scheme and currently we can say no more than that the 1520–1850 MeV range is possible for the mass of the $m(\Psi_s)$. A small value of $|\xi|$ would then require that Ψ_s decays essentially like an $s\bar{s}$ state, with couplings

$$\gamma^2(\pi\pi : K\bar{K} : \eta\eta : \eta\eta') = 0 : 4 : \tfrac{1}{2} : 2 \ . \tag{59}$$

The decay of $f_0(1500)$ to $K\bar{K}$ will set the scale. The amount of G_0 mixing depends rather sensitively on the value of ω. However, a general result is that there will be *constructive* interference between G_0 and $s\bar{s}$ for the $\eta\eta'$ channel in either the $+-$ or $--$ solutions

$$\frac{\gamma^2(\Psi_s \to \eta\eta')}{\gamma^2(\Psi_s \to K\bar{K})} > \frac{\gamma^2(s\bar{s} \to \eta\eta')}{\gamma^2(s\bar{s} \to K\bar{K})} = \frac{1}{2} \ . \tag{60}$$

If we take the widths of $K^*(1430)$ and $a_0(1450)$ as a guide to normalize the width of the nonet in the Godfrey-Isgur model [39], we would anticipate $\Gamma(s\bar{s}) \sim 500$ MeV at a mass of 1600 MeV. The Ψ_s width will be suppressed relative to that of a pure $s\bar{s}$ due to the glueball component [reflected in the normalization, Eq. (31)], but the actual branching ratios may be sensitively dependent on dynamics. In the model of Ref. [44] the $K\bar{K}$ and $\eta\eta$ widths are suppressed if $m(\Psi_s) \to 1800$ MeV; see Appendix B. The importance of the $\eta\eta'$ channel appears to be a solid prediction. If the $f_J(1710)$ is confirmed to have a $J = 0$ component in $K\bar{K}$ but not in $\pi\pi$, this could be a viable candidate for a G_0-$s\bar{s}$ mixture, completing the scalar meson system built on the glueball and the quarkonium nonet.

VIII. THE SCALAR $Q\bar{Q}$ MESONS

Based on the analysis of the previous sections, we shall assume that $f_0(1500)$ is mainly glue and shall examine whether a reasonable scalar $Q\bar{Q}$ nonet can be constructed with the other scalar mesons, neglecting first the small glue admixture in the two mainly $Q\bar{Q}$ isoscalars.

As we have seen, there are two many isoscalar 0^{++} mesons to fit in the $Q\bar{Q}$ ground state nonet. A possible classification of the scalar mesons is shown in Table I. The $a_0(980)$ and $f_0(980)$ are interpreted as $K\bar{K}$ molecules, a hypothesis that may be tested at DAΦNE [24]. The $a_0(1450)$, $f_0(1370)$, $f_0'(1600)$, and $K_0^*(1430)$ are the members of the ground state $Q\bar{Q}$ scalar nonet. Note that the masses of the strange and $I = 1$ members are similar; this is also the case for some other nonets, in particular 4^{++} [7].

The Crystal Barrel and Obelix Collaborations at LEAR also report the observation of a 0^{++} state decaying to $\rho^+\rho^-$ and $\rho^0\rho^0$ in $\bar{p}p$ annihilation at rest into $\pi^+\pi^-3\pi^0$ [54] and $\bar{p}n$ annihilation at rest in deuterium into $2\pi^+3\pi^-$ [55] (see also [56]). Given that mass ($\sim$ 1350 MeV) and width ($\sim$ 380 MeV) are compatible with $f_0(1370)$, one might assume that $f_0(1370)$ has been observed here in its $\rho\rho$ decay mode. The large $\rho\rho$ branching ratio points, however, to a large inelasticity in the $\pi\pi$ S wave around 1400 MeV. Thus $f_0(1370)$ may split into two states, a $Q\bar{Q}$ decaying to $\pi\pi$, $K\bar{K}$, and $\eta\eta$ and another state decaying to $\rho\rho$, possibly a molecule [57].

The GAMS meson $f_0(1590)$ [19] decays to $\eta\eta'$ and $\eta\eta$ with a relative branching ratio of 2.7 ± 0.8 [7], which, given the large error, is consistent with Eqs. (18) and

TABLE I. The scalar mesons, their observed decay modes, and their suggested assignment.

Isospin	State	Decays	Nature
1	$a_0(980)$	$\eta\pi, K\bar{K}$	$K\bar{K}$ molecule
0	$f_0(980)$	$\pi\pi, K\bar{K}$	$K\bar{K}$ molecule
1	$a_0(1450)$	$\eta\pi$	$Q\bar{Q}$ 0^{++} nonet
0	$f_0(1370)$	$\pi\pi, \eta\eta, \rho\rho$	$Q\bar{Q}$ 0^{++} nonet
	$[f_0'(\text{“1600”})]$	$[K\bar{K}, \eta\eta, \eta\eta']$	$Q\bar{Q}$ 0^{++} nonet
1/2	$K_0^*(1430)$	$K\pi$	$Q\bar{Q}$ 0^{++} nonet
0	$f_0(1500)$	$\pi\pi, \eta\eta, \eta\eta', 4\pi^0$	glueball

(19) for $f_0(1500)$ (see footnote 1). We shall assume that $f_0(1590)$ is either identical to $f_0(1500)$ or that it is the predicted $s\bar{s}$ state. We are *therefore left with one nonet*, two to three molecules and the supernumerary $f_0(1500)$.

If the $s\bar{s}$ member lies at 1600 MeV then from the linear mass formula

$$\tan^2\theta = \frac{4K_0^* - a_0 - 3f_0'}{3f_0 + a_0 - 4K_0^*}\ , \tag{61}$$

where the symbols denote the particle masses, one obtains for the $Q\bar{Q}$ nonet the singlet-octet mixing angle $(59^{+12}_{-7})°$ (or 121°). This is not far from ideal mixing, $\theta = 35.3°$ (or 125.3°). The relative decay rates of the 0^{++} $Q\bar{Q}$ mesons to two pseudoscalars are given in Appendix A, and hence a further consistency check on this proposed nonet may be applied when the partial decay widths eventually become known. Note that the experimental widths for the known $Q\bar{Q}$ members of the proposed nonet (Table I) are in good agreement with predictions; see Appendix B.

In the quark model, deviation from ideal mixing is due to mixing transitions between $u\bar{u}(d\bar{d})$ and $s\bar{s}$, which lead to a nondiagonal mass matrix in the flavor basis. The mixing energy (nondiagonal elements) is

$$3A = f_0 + f_0' - 2K_0^*\ , \tag{62}$$

which vanishes for ideal mixing. For our 0^{++} nonet one finds 37 MeV. The quark model also predicts the Schwinger sum rule

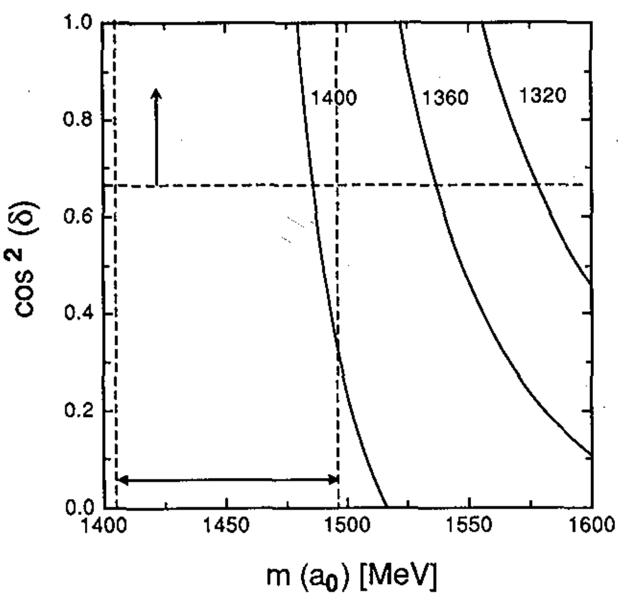

FIG. 11. Fractional contribution of $Q\bar{Q}$ in the $f_0(1370)$ wave function from the Schwinger sum rule as a function of a_0 mass for various $f_0(1370)$ masses, assuming the (mainly) $s\bar{s}$ state to lie at 1600 MeV. The horizontal arrow shows the experimental uncertainty in the a_0 mass and the vertical arrow shows the range allowed for $|\xi| < 0.5$.

$$(f_0 + f_0') \times (4K_0^* - a_0) - 3f_0f_0' - \tfrac{1}{3}(K^* - a_0)^2$$

$$= \frac{8}{3}(a_0 - K^*)^2 \times \cos^2\delta \tag{63}$$

where $\cos^2\delta$ is the fractional $Q\bar{Q}$ in the $f_0(1370)$ or $f_0'(1600)$ wave function [58]. The fraction of glue in $f_0(1370)$ is according to Eq. (31):

$$\sin^2\delta = \frac{2\xi^2}{1+2\xi^2} < 0.33\ . \tag{64}$$

Figure 11 shows how the mixing angle δ in Eq. (64) varies as the function of the a_0 mass for various $f_0(1370)$ masses. A consistent result is indeed obtained when $m(f_0(1370)) \sim 1400$ MeV and $m(f_0(1600)) = 1600$ MeV. The fraction of glue in $f_0(1600)$ depends on ωR^2 [Eq. (31)].

IX. CONCLUSIONS

We have argued that the properties of the $f_0(1500)$-$f_0(1370)$ system are incompatible with them belonging to a quarkonium nonet. We suggest that $f_0(1500)$ is predominantly a glueball mixed with the $n\bar{n}$ and $s\bar{s}$ quarkonia, the $f_0(1370)$ being dominantly $n\bar{n}$.

We have shown that a reasonable scalar nonet can be built with the newly discovered $a_0(1450)$ *setting* the mass scale and its width, together with that of the $K^*(1430)$, setting the scale of the nonet widths, which are within 25% of those expected by earlier quark model calculations. The width of $f_0(1500)$ is dramatically suppressed relative to these and the width of the $f_0(1370)$ may be also somewhat suppressed. These results are in line with these two states being partners in a glueball-$Q\bar{Q}$ mixing scheme. It is this fortunate property that has enabled the $f_0(1500)$ to show up so prominently in several experiments where glueball channels are favored.

Further, supporting these arguments we have shown that there appears to be no dramatic intrinsic violation of flavor symmetry in decays involving gluons in a kinematic region where 0^{++} or 2^{++} glueball bound states are expected to be negligible. The flavor dependence of $f_0(1500)$ decays *suggested that a significant mixing* between G and $Q\bar{Q}$ states is distorting the branching ratios in the 0^{++} sector. We argued that the observed decay branching ratios could be due to the scalar glueball expected in this mass range, mixing with the two nearby $Q\bar{Q}$ isoscalars, one lying below, the other above $f_0(1500)$. The partial decay widths of the lower state, $f_0(1370)$, are consistent with a mainly $(u\bar{u} + d\bar{d})$ state. Our hypothesis also implies that the (mainly) $s\bar{s}$ state lies in the 1600–1700 MeV region.

The quantitative predictions of our analysis depend on the suppression of $f_0(1500)$ decay to $K\bar{K}$. Thus, a detailed study of $p\bar{p} \to \pi K\bar{K}$ can be seminal (i) in confirming the $K\bar{K}$ suppression, (ii) in confirming the $K(1430) \to K\pi$ and $a_0(1450) \to \eta\pi$ and $K\bar{K}$, (iii) in quantifying the signal for $f_0(1370)$ and $f_0(1500)$, and (iv) in isolating the predicted $s\bar{s}$ member of the nonet.

Clarifying the relationship between the Crystal Barrel $f_0(1500)$ and the $f_0(1590)$ of GAMS is important. Particular emphasis should be placed on the strength of the $\pi\pi$ branching ratio and the ratio of branching ratios $\eta\eta/\pi\pi$ in light of its potentially direct significance as a test for glueballs. If the $f_0(1550 \pm 50)$ becomes accepted as a scalar glueball, consistent with the predictions of the lattice, then searches for the 0^{-+} and especially the 2^{++} at mass 2.22 ± 0.13 GeV [5] may become seminal for establishing the lattice as a successful calculational laboratory.

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APPENDIX A: AMPLITUDES FOR ARBITRARY PSEUDOSCALAR MIXING ANGLES

The generalizations of the amplitudes for an arbitrary pseudoscalar mixing angle ϕ are as follows.

Quarkonium decay:

$$\begin{aligned}
\langle Q\bar{Q}|V|\pi\pi\rangle &= \cos\alpha \ , \\
\langle Q\bar{Q}|V|K\bar{K}\rangle &= \cos\alpha(\rho - \sqrt{2}\tan\alpha)/2 \ , \\
\langle Q\bar{Q}|V|\eta\eta\rangle &= \cos\alpha(\cos^2\phi - \rho\sqrt{2}\tan\alpha \sin^2\phi) \ , \\
\langle Q\bar{Q}|V|\eta\eta'\rangle &= \cos\alpha \cos\phi \sin\phi(1 + \rho\sqrt{2}\tan\alpha)
\end{aligned} \tag{A1}$$

for $I = 0$

$$\begin{aligned}
\langle Q\bar{Q}|V|K\bar{K}\rangle &= \rho/2 \ , \\
\langle Q\bar{Q}|V|\pi\eta\rangle &= \cos\phi \ , \\
\langle Q\bar{Q}|V|\eta\eta'\rangle &= \sin\phi \ ,
\end{aligned} \tag{A2}$$

for $I = 1$ and

$$\begin{aligned}
\langle Q\bar{Q}|V|K\pi\rangle &= \sqrt{3}/2 \ , \\
\langle Q\bar{Q}|V|K\eta\rangle &= (\rho \sin\phi - \cos\phi/\sqrt{2})/\sqrt{2} \ , \\
\langle Q\bar{Q}|V|K\eta'\rangle &= (\rho \cos\phi + \sin\phi/\sqrt{2})/\sqrt{2}
\end{aligned} \tag{A3}$$

for $I = 1/2$.

$G \to QQ\bar{Q}\bar{Q}$ [graph 6(a)]:

$$\begin{aligned}
\langle G|V|\pi\pi\rangle &= 1 \ , \\
\langle G|V|K\bar{K}\rangle &= R \ , \\
\langle G|V|\eta\eta\rangle &= \cos^2\phi + R^2 \sin^2\phi \ , \\
\langle G|V|\eta\eta'\rangle &= \cos\phi \sin\phi(1 - R^2) \ .
\end{aligned} \tag{A4}$$

$G \to GG$ [graph 6(b)]:

$$\begin{aligned}
\langle G|V|\pi\pi\rangle &= 0 \ , \\
\langle G|V|K\bar{K}\rangle &= 0 \ , \\
\langle G|V|\eta\eta\rangle &= (\cos\phi\sqrt{2} - \lambda \sin\phi)^2 \ , \\
\langle G|V|\eta\eta'\rangle &= (\cos\phi\sqrt{2} - \lambda \sin\phi)(\sin\phi\sqrt{2} + \lambda \cos\phi) \ ,
\end{aligned} \tag{A5}$$

where

$$\lambda \equiv \frac{\langle G(0^-)|V|d\bar{d}\rangle}{\langle G(0^-)|V|s\bar{s}\rangle} \tag{A6}$$

in the pseudoscalar channel.

$G \to Q\bar{Q}$ [graph 6(c)]:

$$\begin{aligned}
\langle G|V|\pi\pi\rangle &= 1 \ , \\
\langle G|V|K\bar{K}\rangle &= (\rho + \omega R^2)/2 \ , \\
\langle G|V|\eta\eta\rangle &= (\cos^2\phi + \omega\rho R^2 \sin^2\phi) \ , \\
\langle G|V|\eta\eta'\rangle &= \cos\phi \sin\phi(1 - \omega\rho R^2) \ .
\end{aligned} \tag{A7}$$

APPENDIX B: FORM FACTORS

In the main body of the text we used rather simple forms for form factors, Eqs. (14). In order to test sensitivity to these assumptions we consider here the consequences of more structured form factors as arise when the dynamical effects of flux-tube breaking are included. The result is that momentum-dependent multiplicative factors enter additional to those already in Eq. (14); the general structure is discussed in Table II and Appendix B of Ref. [44]. In the approximation where the light hadron wave functions have the same scale parameter [the quantity β in Eq. (14)] the structure of S-wave decay amplitudes (as for ${}^3P_0 \to {}^1S_0 + {}^1S_0$) is

$$S = \left(1 - \frac{2q^2}{9\beta^2}\right) \exp\left(-\frac{q^2}{12\beta^2}\right) \ .$$

The factor q^2/β^2 in parentheses, which was not present before, arises from the coupling of the ${}^3P_0$ of the initial $Q\bar{Q}$ meson with the ${}^3P_0$ of the $q\bar{q}$ pair created by the breaking of the flux-tube and which seed the decay. From detailed fits to meson spectroscopy and decays it is known that $\beta^{-2} \simeq 5$ to 6 GeV^{-2} and so the multiplicative factor $(1 - 2q^2/9\beta^2)$ does not significantly affect our analysis of the $f_0(1500)$.

In this appendix we have followed Ref. [44] in imposing $\beta({}^3P_0) = 0.5$ GeV and have used a more modern value [59] of $\beta = 0.4$ GeV for the ${}^1S_0$ and ${}^3P_2$ states. Ackleh, Barnes, and Swanson [53] have made a similar analysis with $\beta \simeq 0.4$ GeV as a preferred overall value and find similar results. The strategy is to use the known ${}^3P_2$ decays to set the overall scale following the prescriptions in Table II and Appendix B of Ref. [44].

1. $f_0^n(1370)$

Using $\Gamma(f_2 \to \pi\pi) = 157$ MeV yields, in MeV

$$\Gamma(f_0^n \to \pi\pi) = 270 \pm 25 \; ,$$

$$K\bar{K} = 195 \pm 20 \; ,$$

$$\eta\eta = 95 \pm 10 \; ,$$

where the η is assumed to be a 50:50 mixture of $s\bar{s}$ and $n\bar{n}$. The $(1 - 2q^2/9\beta^2)$ factor has suppressed the $\pi\pi$ more markedly than the $K\bar{K}$ and $\eta\eta$, hence leading to a larger $\eta\eta/\pi\pi$ and $K\bar{K}/\pi\pi$ ratio than in the main text. Nonetheless, one sees that the $\Gamma(f_0^n) \gg \Gamma(f_2^n)$ still arises in line with the conclusion that a "narrow" f_0 width is out of line with a ${}^3P_0$ quarkonium state. This conclusion is reinforced by the expectation based on spin counting arguments that $\gamma^2(\rho\rho) > \gamma^2(\pi\pi)$ and, hence, that there should be a non-negligible $\Gamma[f_0^n \to \rho(\pi\pi)_p]$ in addition to the two-body channels.

2. $K_0^*(1430)$

Using $\Gamma(K_2^* \to K\pi) \simeq 50$ MeV or $\Gamma(f_2)$ as above yields consistent similar results; namely, in MeV,

$$\Gamma(K_0^* \to K\pi) = 200 \pm 20 \; ,$$

$$K\eta \simeq 0 \; ,$$

$$K\eta' = 15 \pm 20 \; .$$

The $K\eta'$ has a large coupling for physical $\eta\eta'$ mixing angles, but the width is very sensitive to phase space. We note that the Particle Data Group [7] allow $(7 \pm 10)\%$ "non-$K\pi$" for the K_0^* decay, and we have assigned this to $K\eta'$. These results should be compared with the experimental value $\Gamma(K_0^*) = 287 \pm 23$ MeV.

3. $a_0(1450)$

Using $\Gamma(a_2 \to K\bar{K}) = 5.2 \pm 0.9$ MeV [7] we obtain for the corresponding channel $\Gamma(a_0 \to K\bar{K}) \simeq 110$ MeV. Then with physical $\eta\eta'$ mixing angles we have $\Gamma(\eta\pi) \simeq 90$ MeV, $\Gamma(\eta'\pi) \simeq 80$ MeV. There is considerable uncertainty in these widths, however, since if we were to normalize by $\Gamma(f_2 \to \pi\pi)$ instead of by $\Gamma(a_2 \to K\bar{K})$ we would find $\Gamma(a_0 \to KK : \eta\pi : \eta'\pi) \simeq 200 : 160 : 140$ MeV. Reflecting these uncertainties, we can merely summarize by

$$\Gamma(a_0 \to 0^-0^-)_{\text{theory}} = 390 \pm 110 \text{ MeV} \; ,$$

$$\Gamma(a_0)_{\text{exp}} = 270 \pm 40 \text{ MeV} \; .$$

The general conclusion that $\Gamma(f_0^n) > \Gamma(a_0) \geq \Gamma(K_0^*)$ holds true. Furthermore, we note that these results require that if the $a_0(1450)$ seen by Crystal Barrel is indeed ${}^3P_0(Q\bar{Q})$, then comparable partial widths are expected for all of $K\bar{K}$, $\eta\pi$, and $\eta'\pi$. Quantifying these experimentally will be an important piece of the total strategy in clarifying the nature of these scalar mesons.

4. $f_0^s(1.6\text{–}1.8)$

The $(1 - 2q^2/9\beta^2)$ can have a dramatic effect in the upper part of this range of masses. Using the $\Gamma(f_2^n \to \pi\pi)$ or $\Gamma(f_2^s \to K\bar{K})$ as normalization, the mass dependence of the partial widths of a pure ${}^3P_0(s\bar{s})$ are (in MeV)

	1600	1700	1800
$\Gamma(f_0^s \to K\bar{K})$	270	155	85
$\eta\eta$	45	25	20
$\eta\eta'$	195	170	190

The effect of the momentum node is clearly seen in the KK and $\eta\eta$ channels, whereas the $\eta\eta'$ maintains its strength. There is nothing in this pattern of widths that supports a ${}^3P_0(Q\bar{Q})$ interpretation of $f_0(1500)$. A detailed study of quarkonium decays with similar conclusions is in Ref. [53].

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