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Is it possible to measure the Lense-Thirring effect on the orbits of the planets in the gravitational field of the Sun?

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Abstract. In this paper we explore a novel approach to try to measure the post-Newtonian $1/c^2$ Lense-Thirring secular effect induced by the gravitomagnetic field of the Sun on planetary orbital motion. Due to the relative smallness of the solar angular momentum J and the large values of the planetary semimajor axes a, the gravitomagnetic precessions, which affect the nodes Ω and the perihelia ω and are proportional to J/a^3, are of the order of 10^{-3} arcsec per century only for, e.g., Mercury. This value lies just at the edge of the present-day observational sensitivity in reconstructing the planetary orbits, although the future mission BepiColombo should allow it to be increased. The major problems come from the main sources of systematic errors. They are the aliasing classical precessions induced by the multipolar expansion of the Sun's gravitational potential and the classical secular N-body precessions which are of the same order of magnitude or much larger than the Lense-Thirring precessions of interest. This definitely rules out the possibility of analyzing only one orbital element of, e.g., Mercury. In order to circumvent these problems, we propose a suitable linear combination of the orbital residuals of the nodes of Mercury, Venus and Mars which is, by construction, independent of such classical secular precessions. A 1-sigma reasonable estimate of the obtainable accuracy yields a 36% error. Since the major role in the proposed combination is played by Mercury's node, it could happen that new, more accurate ephemerides available in the future thanks to the BepiColombo mission will offer an opportunity to improve the present unfavorable situation.

Key words. gravitation – celestial mechanics – solar system: general

1. Introduction

1.1. The Lense-Thirring effect

According to the linearized weak-field and slow-motion approximation of the General Theory of Relativity (GTR), valid throughout the Solar System, the secular gravitomagnetic Lense-Thirring precessions on the longitude of the ascending node Ω and the argument of pericentre ω of the orbit of a test particle freely orbiting around a central mass M with proper angular momentum J are (Lense & Thirring 1918)

$$\dot{\Omega}_{\rm LT} = \frac{2GJ}{c^2a^3(1-e^2)^{\frac{3}{2}}},\ \dot{\omega}_{\rm LT} = -\frac{6GJ\cos i}{c^2a^3(1-e^2)^{\frac{3}{2}}}. \quad (1)$$

In Eq. (1) G is the Newtonian constant of gravitation, c is the speed of light in vacuum, a, e and i are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit to the reference $\{x, y\}$ plane which coincides with the equatorial plane of the central mass. Its spin J is assumed to be directed along the z axis.

1.2. Attempts to measure the Lense-Thirring effect in the solar system

Up to now, there is no direct observational verification possible of this prediction of GTR which can be considered reliable and undisputable.

Indeed, the only attempts performed to detect the Lense-Thirring precessions of Eq. (1) in the Solar System arena are due to I. Ciufolini and coworkers (Ciufolini et al. 1998). They analyzed the orbital data of the existing laser-ranged geodetic LAGEOS and LAGEOS II satellites in the gravitational field of the Earth over an observational time span of a few years. In particular, a linear combination of the residuals of the nodes $\delta\dot{\Omega}_{\rm obs}$ of LAGEOS and LAGEOS II and the perigee $\delta\dot{\omega}_{\rm obs}$ of LAGEOS II (Ciufolini 1996) was adopted. The residuals were suitably built up in order to entirely absorb the Lense-Thirring effect in them by setting it purposely equal to zero in the force models of the equations of motion in the satellite data reduction software. The claimed total accuracy would be of the order of 20–30%, but, according to other scientists, such estimates would be largely optimistic (Ries et al. 2003).

In April 2004 the extraordinarily sophisticated GP-B mission (Everitt et al. 2001) was launched. Its goal is to measure another gravitomagnetic effect in the terrestrial gravitational field, i.e. the precession of the spins of four superconductor gyroscopes (Schiff 1960) carried onboard. The claimed accuracy

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is of the order of 1% or better. The experiment should last one year.

Almost twenty years ago it was proposed to launch a third LAGEOS-like satellite − the LAGEOS III/LARES − and to analyze the time series of the sum of the residuals of the nodes of LAGEOS and LARES (Ciufolini 1986) or some other combinations of residuals of the nodes and the perigees of LARES and both the existing LAGEOS satellites (Iorio et al. 2002). The obtainable accuracy would probably be of the order of 1%. Mainly funding problems have prevented, up to now, implementation of such a relatively easy and cheap mission. Recently, the possibility of measuring the Lense-Thirring precessions of Eq. (1) by means of the relativity-dedicated OPTIS spacecraft, which could be launched in the same orbital configuration of LARES, has been considered (Iorio et al. 2004).

The recently-proposed LATOR (Turyshev et al. 2004) and ASTROD (Ni et al. 2004) missions would be sensitive to the gravitomagnetic part of the bending of light rays in the gravitational field of the Sun.

According to Nordtvedt (2003), the multidecade analysis of the Moon's motion with the Lunar Laser Ranging (LLR) technique strongly supports the existence of the gravitomagnetic force[1] as predicted by GTR, although in an indirect way.

To measure explicitly the Lense-Thirring precessions of Eq. (1) from the analysis of the orbital motion of test masses in the gravitational field of a real rotating astronomical mass like the Earth or the Sun, the main problems come from the aliasing effects induced by a host of classical orbital perturbations of gravitational origin[2] which unavoidably affect the motion of the probes along with GTR. In particular, the even zonal harmonics J_ℓ of the multipolar expansion of the gravitational potential of the central mass induce secular classical precessions which, in many cases, are larger than the gravitomagnetic ones of interest. As we will see, the approach proposed by Ciufolini (1996) and Iorio (Iorio 2002; Iorio & Morea 2004) in the performed or proposed tests with LAGEOS and LAGEOS II consists of suitably designing linear combinations $\sum c_i \delta\dot{\Omega}^i_{\rm obs} + \sum k_j \delta\dot{\omega}^j_{\rm obs}$ that are able to reduce the impact of the even zonal harmonics of the gravitational field of the central mass by cancelling out $N-1$ selected even zonal harmonics[3] where N is the number of orbital elements involved.

1.3. Aim of the paper

In this paper we investigate the possibility of extending the Ciufolini-Iorio approach in order to try to measure the Lense-Thirring effect of Eq. (1) in the gravitational field of the Sun by suitably combining the data from interplanetary ranging to the inner planets of the Solar System. The relevant parameters are listed in Table 1.

What is the current approach in testing post-Newtonian gravity from planetary data analysis followed by, e.g., the Jet Propulsion Laboratory (JPL)? In the interplay between the real data and the equations of motion, which include also the post-Newtonian accelerations expressed in terms of the various PPN parameters (Will 1993), a set of astrodynamical parameters, among which are also γ and β, are simultaneously and straightforwardly fitted and adjusted and a correlation matrix is also determined. This means that the post-Newtonian equations of motion are globally tested as a whole in terms of, among other parameters, γ and β; no attention is paid to any particular feature of the post-Newtonian acceleration. This is similar to the LLR approach outlined before. On the contrary, our aim is isolate one particular piece of the post-Newtonian equations of motion, i.e. the gravitomagnetic acceleration.

2. The gravitomagnetic field of the Sun

In the case of the Sun and the planets, the Lense-Thirring effect is quite small: indeed, for, e.g., the node it is $\leq 10^{-3}$ arcsec per century ($''$ cy^{-1}), as it can be inferred from Tables 1 and 2. The angular momentum of the Sun is relatively small and the Lense-Thirring precessions fall off with the inverse of the third power of the planet's semimajor axis. If we want to consider the detection of Eq. (1) as a genuine test of GTR, it is necessary that the Sun's angular momentum J is known with high accuracy from measurements independent of GTR itself. This is the case: indeed, the helioseismic data from the Global Oscillations Network Group (GONG) and also from the Solar and Heliospheric Observatory (SoHO) satellite yield measurements of J which are accurate to a few percent (Pijpers 2003).

2.1. Sensitivity analysis

According to the results of Table 3 (E.M. Standish 2004, private communication), it should be possible to extract the gravitomagnetic signature from a multi-year analysis of the residuals of the planetary nodal evolution[4]. Standish averaged, among other things, the nodal evolution of some planets over two centuries by using the DE405 ephemerides

[1] According to Nordtvedt (2003), the Earth-Moon range is affected by long-periodic harmonic perturbations of gravitomagnetic origin whose amplitudes are of the order of 5 m and the periods are monthly and semi-monthly. The amplitudes of the lunar motion at both these periods are determined to better than half a centimeter precision in the total orbital fit to the LLR data.

[2] In the case of the LAGEOS-LAGEOS II experiment in the gravitational field of the Earth, the non-gravitational effects also play a very important role, especially in perturbing the perigee of LAGEOS II. In the case of planetary motions the non-conservative forces are, instead, irrelevant.

[3] In general, they are the first low-degree ones J_2, J_4,...

[4] Here we speak about nodal residuals in a, strictly speaking, improper sense. The Keplerian orbital elements are not directly observable: they can only be computed (in the case of the Solar System bodies exhibiting small inclinations the nodes should be obtained by the currently used parameters $\sin i \cos\Omega$ and $\sin i \sin\Omega$). The basic observable quantities are ranges, range-rates and angles. Here we mean the differences between the time series of the node computed from a given observed orbital arc and the time series of the node computed from a propagated orbital arc with the gravitomagnetic force switched off in the force models. The two time series share the same initial conditions. Note that the post-Newtonian equations of motion used at JPL for the computation of the planetary ephemerides (Estabrook 1971) do not include the gravitomagnetic force, so that the Lense-Thirring effect would automatically be absorbed in the time series of $\delta\Omega$.

Table 1. Relevant astronomical and astrophysical parameters used in the text. The value for the Sun's angular momentum J has been obtained from (Pijpers 2003). The planetary data can be retrieved at `http://nssdc.gsfc.nasa.gov/planetary/factsheet/`.

Symbol	Description	Value	Units
G	Newtonian gravitational constant	6.67259×10^{-11}	$m^3\ kg^{-1}\ s^{-2}$
c	speed of light in vacuum	2.99792458×10^{8}	$m\ s^{-1}$
GM	Sun's GM	$1.32712440018 \times 10^{20}$	$m^3\ s^{-2}$
R	Sun's equatorial radius	6.9599×10^{8}	m
J	Sun's proper angular momentum	1.9×10^{41}	$kg\ m^2\ s^{-1}$
AU	astronomical unit	$1.49597870691 \times 10^{11}$	m
$a_{\rm Mer}$	Mercury's semimajor axis	0.38709893	AU
$a_{\rm Ven}$	Venus's semimajor axis	0.72333199	AU
$a_{\rm Ear}$	Earth's semimajor axis	1.00000011	AU
$a_{\rm Mar}$	Mars's semimajor axis	1.52366231	AU
$e_{\rm Mer}$	Mercury's eccentricity	0.20563069	–
$e_{\rm Ven}$	Venus's eccentricity	0.00677323	–
$e_{\rm Ear}$	Earth's eccentricity	0.01671022	–
$e_{\rm Mar}$	Mars's eccentricity	0.09341233	–
$\epsilon_{\rm Mer}$	Mercury's inclination to the ecliptic	7.00487	deg
$\epsilon_{\rm Ven}$	Venus's inclination to the ecliptic	3.39471	deg
$\epsilon_{\rm Ear}$	Earth's inclination to the ecliptic	0.00005	deg
$\epsilon_{\rm Mar}$	Mars's inclination to the ecliptic	1.85061	deg

Table 2. Gravitomagnetic and classical nodal precession coefficients, in $''$ cy^{-1}. The coefficients $\dot\Omega_{.\ell}$ are $\partial\dot\Omega^{J_\ell}_{\rm class}/\partial J_\ell$ and refer to the classical precessions induced by the oblateness of the central mass. The numerical values of Table 1 have been used in Eqs. (1) and (5) (see below). $\dot\Omega_{\rm class}$ are the nominal centennial rates released at `http://ssd.jpl.nasa.gov/elem_planets.html`. They mainly include the N-body secular precessions.

Precessions	Mercury	Venus	Earth	Mars
$\dot\Omega_{\rm LT}$	1.008×10^{-3}	1.44×10^{-4}	5.4×10^{-5}	1.5×10^{-5}
$\dot\Omega_{.2}$	$-1.26878626476 \times 10^{5}$	$-1.3068273031 \times 10^{4}$	$-4.210107706 \times 10^{3}$	$-9.80609460 \times 10^{2}$
$\dot\Omega_{.4}$	5.2774935×10^{1}	1.349709	2.28040×10^{-1}	2.3554×10^{-2}
$\dot\Omega_{\rm class}$	-4.4630×10^{2}	-9.9689×10^{2}	-1.822825×10^{4}	-1.02019×10^{3}

Table 3. Errors in the numerical propagation of the planetary nodal rates averaged over 200 years, in $''$ cy^{-1} (E.M. Standish 2004, private communication).

Mercury	Venus	Earth	Mars
1.82×10^{-4}	6×10^{-6}	2×10^{-6}	1×10^{-6}

(Standish 1998) with and without post-Newtonian accelerations. Standish also included in the force models the solar oblateness with $J_2 = 2 \times 10^{-7}$, so that the so-obtained numerical residuals accounted for the post-Newtonian effects only; the uncertainty in the determined shift for, e.g., Mercury, was $1.82 \times 10^{-4}\,''$ cy^{-1}. The quoted uncertainty of Table 3 does not come from direct observational errors. The uncertainties depend on the fact that in the force models used in the numerical propagation many astrodynamical parameters occur (masses of planets, asteroids, etc.); their numerical values come from multiparameter fits of real data and, consequently, are affected by observational errors. Such numerical tests cannot determine whether GTR is correct or not; they just give an idea of what the obtainable accuracy set up by our knowledge of the Solar System would be if the Einstein theory of gravitation is true. Our knowledge of the orbital motion of Mercury will improve thanks to the future missions Messenger (see `http://messenger.jhuapl.edu/` and `http://discovery.nasa.gov/messenger.html`), which was launched in the summer 2004 and whose encounter with Mercury is scheduled for 2011, and, especially[5], BepiColombo (see `http://sci.esa.int/science-e/www/area/index.cfm?fareaid=30`), which is scheduled for 2010–2012. A complete error analysis for the range and range-rate measurements can be found in Iess & Boscagli (2001). According to them, a two orders of magnitude improvement in the Earth-Mercury range should be possible. According to a more conservative evaluation (Standish 2004, private communication), improvements in Mercury's orbital parameters might amount to one order of magnitude. Indeed, the current accuracy in radar ranging is hundreds of meters (Standish 2002), although the uncertainty in the planetary topography further limits the realistic obtainable precision (Pitjeva 1993); the new data could reach the tens of meters level without the problems related to topography.

On the other hand, there would be severe limitations to the possibility of detecting the Lense-Thirring effect by

[5] While the spacecraft trajectory will be determined from the range-rate data, the planet's orbit will be retrieved from the range data (Milani et al. 2002). In particular, the determination of the planetary centre of mass is important for this goal which can be better reached by a not too elliptical spacecraft orbit. The relatively moderate ellipticity of the planned 400×1500 km polar orbit of BepiColombo, contrary to the much more elliptical path of Messenger, is, then, appropriate.

analyzing the secular evolution of only one orbital element of a given planet due to certain systematic aliasing errors. Indeed, as in the case of the Earth-LAGEOS-LAGEOS II system, we should cope with the multipolar expansion of the central mass, i.e. the Sun in this case. The aliasing secular precessions induced by its quadrupole mass moment J_2 on the planetary nodes and the perihelia would be almost one order of magnitude larger than the Lense-Thirring precessions if we assume $J_2 = 2 \times 10^{-7} \pm 4 \times 10^{-8}$ (Pireaux & Rozelot 2003). There are still many uncertainties about the Sun's oblateness, both from a theoretical modelling point of view and from an observational point of view (Rozelot et al. 2004). Moreover, the perihelia are also affected by another relevant post-Newtonian secular effect, i.e. the gravitoelectric Einstein pericentre advance (Einstein 1915)

$$\dot{\omega}_{\rm GE} = \frac{3nGM}{c^2 a(1-e^2)}. \quad (2)$$

$n = \sqrt{GM/a^3}$ is the Keplerian mean motion of the orbiting particle. This effect was measured for the first time in the Solar System with the interplanetary ranging technique at a 10^{-3} level of relative accuracy (Shapiro et al. 1972, 1976; Shapiro 1990). Recent observations yield a relative accuracy of 2×10^{-4} (Pitjeva 2001). The Einstein precession is almost four orders of magnitude larger than the Lense-Thirring effect, so that its mismodelled part would still be one order of magnitude larger than the Lense-Thirring effect of interest. The classical N-body secular precessions also are to be considered because they are quite large (see Table 2).

3. The linear combination approach

A possible solution could be to extend the Ciufolini-Iorio linear combination approach to the Sun-planets scenario in order to built up some combinations with the nodes of the inner planets which cancel out the impact of the Sun's oblateness and of the N-body precessions.

3.1. A J_2 – (N-body) free combination

We give the expressions for the residuals of the nodes of Mercury, Venus and Mars explicitly in terms of the mismodelled secular precessions induced by the quadrupolar mass moment of the Sun, the secular N-body precessions and of the Lense-Thirring secular precessions, assumed as a totally unmodelled feature. It is accounted for by a scaling parameter $\mu_{\rm LT}$ which is zero in Newtonian mechanics and 1 in GTR[6]

$$\begin{cases} \delta\dot{\Omega}^{\rm Mercury}_{\rm obs} = \dot{\Omega}^{\rm Mercury}_{.2}\delta J_2 + \dot{\Omega}^{\rm Mercury}_{\rm class} \\ \qquad\qquad + \dot{\Omega}^{\rm Mercury}_{\rm LT}\mu_{\rm LT} + \Delta^{\rm Mercury}, \\ \delta\dot{\Omega}^{\rm Venus}_{\rm obs} = \dot{\Omega}^{\rm Venus}_{.2}\delta J_2 + \dot{\Omega}^{\rm Venus}_{\rm class} + \dot{\Omega}^{\rm Venus}_{\rm LT}\mu_{\rm LT} + \Delta^{\rm Venus}, \\ \delta\dot{\Omega}^{\rm Mars}_{\rm obs} = \dot{\Omega}^{\rm Mars}_{.2}\delta J_2 + \dot{\Omega}^{\rm Mars}_{\rm class} + \dot{\Omega}^{\rm Mars}_{\rm LT}\mu_{\rm LT} + \Delta^{\rm Mars}. \end{cases} \quad (3)$$

[6] It can be shown that it can be expressed in terms of the PPN parameter γ.

The coefficients $\dot{\Omega}_{.\ell}$ are defined as

$$\dot{\Omega}_{.\ell} = \frac{\partial \dot{\Omega}^{J_\ell}_{\rm class}}{\partial J_\ell}, \quad (4)$$

where $\dot{\Omega}^{J_\ell}_{\rm class}$ represent the classical secular precessions induced by the oblateness of the central mass. The coefficients $\dot{\Omega}_{.\ell}$ have been explicitly worked out from $\ell = 2$ to $\ell = 20$ (Iorio 2003); they are functions of the semimajor axis a, the inclination[7] i and the eccentricity e of the considered planet: $\dot{\Omega}_{.\ell} = \dot{\Omega}_{.\ell}(a, e, i; GM)$. For the first two even zonal harmonics we have

$$\begin{cases} \dot{\Omega}_{.2} = -\frac{3}{2} n \left(\frac{R}{a}\right)^2 \frac{\cos i}{(1-e^2)^2}, \\ \dot{\Omega}_{.4} = \dot{\Omega}_{.2}\left[\frac{5}{8}\left(\frac{R}{a}\right)^2 \frac{\left(1+\frac{3}{2}e^2\right)}{(1-e^2)^2}\left(7\sin^2 i - 4\right)\right]; \end{cases} \quad (5)$$

R is the Sun's equatorial radius. The quantities Δ in Eq. (3) refer to the other unmodelled or mismodelled effects which affect the temporal evolution of the nodes of the considered planets. In the present case they would mainly be represented by the precessions induced by the octupolar mass moment of the Sun (Rozelot et al. 2004). From the results of Table 2 and from the evaluations of Rozelot et al. (2004) according to which the possible magnitude of J_4 would span the range 10^{-7}–10^{-9}, the secular precession induced by the octupolar mass moment of the Sun is negligible with respect the Lense-Thirring rates. These facts lead us to design a three-node combination which cancels out the effects of J_2 and of the classical N-body precessions while it is affected by the residual effect of J_4.

Indeed, if we solve Eq. (3) for the Lense-Thirring parameter $\mu_{\rm LT}$ it is possible to obtain

$$\begin{cases} \delta\dot{\Omega}^{\rm Mercury}_{\rm obs} + k_1\delta\dot{\Omega}^{\rm Venus}_{\rm obs} + k_2\delta\dot{\Omega}^{\rm Mars}_{\rm obs} \sim X_{\rm LT}\mu_{\rm LT}, \\ k_1 = \frac{\dot{\Omega}^{\rm Mars}_{.2}\dot{\Omega}^{\rm Mercury}_{\rm class} - \dot{\Omega}^{\rm Mercury}_{.2}\dot{\Omega}^{\rm Mars}_{\rm class}}{\dot{\Omega}^{\rm Venus}_{.2}\dot{\Omega}^{\rm Mars}_{\rm class} - \dot{\Omega}^{\rm Mars}_{.2}\dot{\Omega}^{\rm Venus}_{\rm class}} = -1.0441702 \times 10^1, \\ k_2 = \frac{\dot{\Omega}^{\rm Mercury}_{.2}\dot{\Omega}^{\rm Venus}_{\rm class} - \dot{\Omega}^{\rm Venus}_{.2}\dot{\Omega}^{\rm Mercury}_{\rm class}}{\dot{\Omega}^{\rm Venus}_{.2}\dot{\Omega}^{\rm Mars}_{\rm class} - \dot{\Omega}^{\rm Mars}_{.2}\dot{\Omega}^{\rm Venus}_{\rm class}} = 9.765758, \\ X_{\rm LT} = \dot{\Omega}^{\rm Mercury}_{\rm LT} + k_1\dot{\Omega}^{\rm Venus}_{\rm LT} + k_2\dot{\Omega}^{\rm Mars}_{\rm LT} \\ \qquad = -3.51 \times 10^{-4}\ ''\ {\rm cy}^{-1}, \end{cases} \quad (6)$$

where the numerical values of the coefficients k_1 and k_2 and the slope $X_{\rm LT}$ of the gravitomagnetic trend come from the values of Table 2. We construct the time series of the residuals of the nodes of Mercury, Venus and Mars by using real observational data and the full dynamical models in which GTR is purposely set equal to zero, e.g. by using a very large value of c. We expect that, over a multidecade observational time span, the combined residuals will fully show the GTR signature in terms of a linear trend. The measured slope, divided by $X_{\rm LT}$, yields $\mu_{\rm LT}$

[7] As pointed out in Milani et al. (2002), the angle i refers to the inclination between the planet's orbital plane and the fixed reference plane of the celestial reference frame; it is not the angle ϵ between the planet's orbital plane and the ecliptic. It turns out that $i \sim \epsilon/2$. For Mercury $\epsilon = 7.00487°$.

which should be equal to one if GTR is correct and if the bias from the residual N-body effect was sufficiently small. The systematic error affecting Eq. (6) is negligible because it would be due only to the higher degree multipole mass moments of the Sun. The obtainable 1-sigma observational error, according to the results of Table 3, would amount to 36%.

4. Conclusions

In this paper we have explored the possibility of measuring the post-Newtonian Lense-Thirring effect induced by the solar gravitomagnetic field on the motion of some of the Solar System planets. The magnitude of the gravitomagnetic precessions is very small, 10^{-3} $''$ cy^{-1} for Mercury. The main systematic errors which would mask the relativistic effect of interest would be the quite larger secular precessions induced by the post-Newtonian gravitoelectric part of the Sun's gravitational field, by the Sun's oblateness and by the N-body interactions. By using a suitably designed linear combination of the orbital residuals of the nodes of Mercury, Venus and Mars it would be possible to cancel out the corrupting impact of the first solar even zonal harmonic plus the N-body classical secular precessions. Moreover, the proposed measurement would not be aliased by the post-Newtonian gravitoelectric field because it affects only the perihelia and the mean anomalies. The obtainable observable accuracy should be 36% (1-sigma) for the proposed $J_2 - (N\text{-body})$ free combination. It would be a somewhat modest result for a reliable test of GTR. However, we note that should new, more accurate ephemerides for Mercury be available as a by-product of the Messenger and, especialy, BepiColombo missions, the error's evaluation presented here could become more favorable.

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