

IS SPACE REALLY EXPANDING? A COUNTEREXAMPLE

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Abstract

In all Friedman models, the cosmological redshift is widely interpreted as a consequence of the general-relativistic phenomenon of *expansion of space*. Other commonly believed consequences of this phenomenon are superluminal recession velocities of distant galaxies, and the distance to the particle horizon greater than ct (where t is the age of the Universe), in apparent conflict with special relativity. Here, we study a particular Friedman model: empty universe. This model exhibits both cosmological redshift, superluminal velocities and infinite distance to the horizon. However, we show that the cosmological redshift is there simply a relativistic Doppler shift. Moreover, apparently superluminal velocities and ‘acausal’ distance to the horizon are in fact a direct consequence of special-relativistic phenomenon of time dilation, as well as of the adopted definition of distance in cosmology. There is no conflict with special relativity, whatsoever. In

particular, *inertial* recession velocities are subluminal. Since in the real Universe, sufficiently distant galaxies recede with relativistic velocities, these special-relativistic effects must be at least partly responsible for the cosmological redshift and the aforementioned ‘superluminialities’, commonly attributed to the expansion of space. Let us finish with a question resembling a Buddhism-Zen ‘koan’: in an empty universe, what is expanding?

1 Introduction

What is the physical interpretation of the cosmological redshift? It is well known that, although in general “the redshift cannot be thought of as a *global* Doppler shift, it is correct to think of the effect as an accumulation of the infinitesimal Doppler shifts caused by photons passing between fundamental observers separated by a small distance”. [10] In all Friedman cosmological models, the Universe is isotropic and homogeneous for a privileged set of so-called Fundamental Observers (FOs) co-moving with matter, all measuring the same cosmic time. The cosmological redshift can be thus thought of as a result of relative motions of the FOs along the photon’s trajectory.

An alternative, and widespread, interpretation of the cosmological redshift is that it is a direct consequence of the general-relativistic phenomenon of *expansion of space*. In all homogeneous and isotropic (i.e., Friedman) cosmological models, distances between various galaxies grow by the same factor, called the scale factor, a , which is a function of only the cosmic time, τ : $a = a(\tau)$. The wavelength of the emitted photon, λ , undergoes redshift, z , which is simply related to the values of the scale factor at the time of emission, τ_e , and observation, τ_o ,

$$\frac{\lambda_o}{\lambda_e} \equiv 1 + z = \frac{a(\tau_o)}{a(\tau_e)}. \quad (1)$$

A seemingly natural interpretation of the above equation is that space expands, and the wavelengths of photons grow accordingly in time. But why is then, say, Brooklyn, *not* expanding? Quoting Lineweaver & Davis [6], “In ‘Annie Hall’, the movie character played by the young Woody Allen explains to his doctor and mother why he can’t do his homework. ‘The universe is expanding. . . The universe is everything, and if it’s expanding, someday it will break apart and that would be the end of anything!’. But his mother knows better: ‘You’re here in Brooklyn. Brooklyn is not expanding!’” Certainly. But how can a tiny photon partake in the global expansion of the universe, if something as big as Brooklyn does not?

In his textbook ‘Cosmological Physics’, John Peacock calls the idea of expanding space “perhaps the worst misconception about the big bang”. “Many semi-popular accounts of cosmology contain statements to the effect that ‘space itself is swelling up’ in causing the

galaxies to separate. This seems to imply that all objects are being stretched by some mysterious force: are we to infer that humans who survived for a Hubble time would find themselves to be roughly four meters tall? Certainly not. Apart from anything else, this would be a profoundly anti-relativistic notion, since relativity teaches us that properties of objects in local inertial frames are independent of the global properties of spacetime.” However, expanding space is a deeply rooted myth (e.g., [8, 4, 5]) and such myths die hard. The following properties of the Friedman models are commonly attributed to general-relativistic expansion of space:

1. Cosmological redshift;
2. Superluminal recession velocities of distant galaxies;
3. Distance to the horizon greater than ct , where c is the speed of light and t is the age of the Universe.

Expansion of space is regarded as general-relativistic because the properties 2. and 3. seem to be in sharp conflict with special relativity. In particular, with respect to property 3., it is argued that photons travel locally with the speed of light, but space expands, providing an additional stretching of the distance. With respect to property 2., we can often read that “the velocity in Hubble’s law is a recession velocity caused by the expansion of space, not a motion through space. It is a general-relativistic effect and is not bound by the special-relativistic limit”.^[6]

In this paper, as a counterexample to the idea of expanding space we study the model of an empty universe. It is a rather particular Friedman model, since it is devoid of matter, in a classical sense. However, we will see that it exhibits all properties 1.–3. It is a good counterexample to the idea of expanding space because its spacetime is simply the Minkowskian spacetime of special relativity (SR). The spacelike section of the Minkowskian spacetime is Euclidean (i.e., flat) *static* space. In the empty model, expanding are only fictitious, massless (so non-interacting) FOs, all in constant relative motion. Without them, there would be no expansion at all. After all, how to define motion without any object of reference? This suggests that what really matters is the cosmic substratum (here, massless by assumption) and its relative motions.

One can make a coordinate transformation, transforming the underlying metric of the empty model from its Minkowskian form to the form expressed in the local coordinates of FOs.[10, 7] Then it turns out to be an open Friedman model: its spacelike section is negatively curved space of hyperbolic geometry, evolving in time. There is no absolute space already in SR. However, according to any inertial observer, space is flat and static. We see that in general relativity (GR), even the curvature of space,[13] or its dynamical state (static or evolving) is not an invariant of an arbitrary coordinate transformation. On the other hand, using the latter form of the metric it is straightforward to derive properties 1.–3. of the empty model. On this basis one may still argue that although general-relativistic expansion of space is not absolute, it is a fact in the privileged coordinates of FOs.

To show that even this line of reasoning is incorrect, here we derive properties 1.–3. of the empty model using only special-relativistic concepts. In particular, we do not use the notion of a metric at all. We demonstrate that in this model, the cosmological redshift is a relativistic Doppler shift. Furthermore, we show that the conflict of properties 2. and 3 with SR is only apparent. In fact, they are a direct consequence of special-relativistic phenomenon of time dilation, as well as of the adopted definition of distance in cosmology. In other words, at least in the empty model, properties 1.–3. are in accordance with SR, and are fully explicable as the results of real motions in space. Therefore, at least in this case, the existence of these properties cannot be regarded as an argument for general-relativistic expansion of space. Alternatively, there is at least one Friedman model, in which expansion of space, in detachment from expanding matter, is certainly an illusion.

This paper is organized as follows. In Section 2 we show that in the empty model, the origin of the cosmological redshift is entirely kinematic. In Section 3 we derive the temperature of the cosmic microwave background at redshift z . In Section 4 we derive a formula for the recession velocities of distant objects. In Section 5 we study the distance to the particle horizon. Conclusions are in Section 6.

2 The cosmological redshift

What happens to a photon traveling through an empty universe? A simple (and correct) answer is that nothing. Quoting the lyrics of a song by Grzegorz Turnau, “in fact, nothing happens and nothing occurs till the very end”. [14] The end occurs when the emitted photon is finally absorbed by an observer’s eye, a photographic plate, or a CCD device. This absorption reveals that the frequency of the photon is redshifted. The only possible interpretation of this redshift is a Doppler shift, due to relative motion of the emitter and the absorber.

In an empty universe, these motions can be described entirely by means of the Milne kinematic model. In this model, the cosmic arena of physical events is the pre-existing Minkowski spacetime. In the origin of the coordinate system, O , at time $t = 0$ an ‘explosion’ takes place, sending radially Fundamental Observers (FOs) with constant velocities in the range of speeds $(0, c)$. Let’s place a source of radiation at the origin of the coordinate system. [15] At time t_e the source emits photons, which at time t_o reach a FO moving with velocity v , such that

$$vt_o = c(t_o - t_e). \quad (2)$$

The observer sees them redshifted, due to the Doppler effect. The special-relativistic formula for the Doppler effect is

$$1 + z = \left(\frac{1 + \beta}{1 - \beta} \right)^{1/2}, \quad (3)$$

where z is the photons’ redshift and $\beta \equiv v/c$. From Eq. (2) we have

$$\beta = 1 - p^{-1}, \quad (4)$$

where $p \equiv t_o/t_e$, hence

$$1 + z = (2p - 1)^{1/2}. \quad (5)$$

We emphasize that time t is measured in the inertial frame of the source, i.e., by a set of synchronized clocks (with that at the origin), remaining in rest relative to it. The observer traveling with velocity v relative to the source, carries his own clock which shows his *proper* time, τ . This clock *delays* relative to time t :

$$t = \gamma(v)\tau, \quad (6)$$

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where $\gamma(v) = (1 - \beta^2)^{-1/2}$; hence $t_o = \gamma(v)\tau_o$. On the other hand, the clock at the source measures its own proper time, hence $t_e = \tau_e$. This yields

$$p = \frac{t_o}{t_e} = \gamma(v) \frac{\tau_o}{\tau_e}, \quad (7)$$

The instants of time τ_e and τ_o are measured by the FOs respectively at the point of emission and observation of photons, so they are the instants of *cosmic* time.

From Equation (4) we have $\gamma(v) = [p^{-1}(2 - p^{-1})]^{-1/2}$, or

$$p = [p^{-1}(2 - p^{-1})]^{-1/2} \frac{\tau_o}{\tau_e}. \quad (8)$$

Solving this equation for p we obtain

$$p = \frac{1}{2} \left[1 + \left(\frac{\tau_o}{\tau_e} \right)^2 \right]. \quad (9)$$

Using the above in Equation (5) yields finally

$$1 + z = \frac{\tau_o}{\tau_e}. \quad (10)$$

The scale factor, a , of the Robertson-Walker form of the metric for an empty universe grows linearly with time: $a(\tau) = \tau/\tau_o$. For this form of the metric, the cosmological redshift is in general $1 + z = a(\tau_o)/a(\tau_e)$, so for an empty universe, $1 + z = \tau_o/\tau_e$. Equation (10), derived using only special-relativistic concepts, coincides with this formula. We see thus that in the empty model, the origin of the cosmological redshift is entirely kinematic ([10, 1]; see also [12]).

3 Temperature of the CMB

One can sometimes hear that the temperature of the cosmic microwave background (CMB), measured at redshift z , is a strong observational evidence for expansion of space. For all Friedman models, this temperature is predicted to be $(1 + z)T_o$, where T_o is its present value, as indeed observed.[11] If only matter expanded, this temperature would be expected to be just T_o (Bajtlik, private communication). To see that the last statement is wrong, let us analyze

the redshift of photons in the rest frame of the source. At time t_d , corresponding to the time of decoupling of photons from matter, the source emits photons of temperature T_d towards the observer. At time t_c they reach a molecular cloud. Some of them are absorbed by the cloud, providing a thermal bath for its atoms and molecules, of temperature T_c . The remaining photons are not disturbed and reach the observer at time t_o . What is the value of the temperature T_c ? In the empty model, the redshift of unabsorbed photons, z_{CMB} , is related to the velocity of the observer relative to the source, β_o , by Equation (3). Similarly, Equation (3) also relates the redshift of the cloud relative to the source, z_c , to its velocity, β_c . The redshift of the observer *relative to the cloud*, or the cloud relative to the observer, is

$$1 + z = \left(\frac{1 + \beta'}{1 - \beta'} \right)^{1/2}, \quad (11)$$

where β' is the relative velocity of the observer and the cloud. From the special-relativistic law of addition of velocities,

$$\beta' = \frac{\beta_o - \beta_c}{1 - \beta_o \beta_c}. \quad (12)$$

We thus have

$$(1 + z)^2 = \frac{1 - \beta_o \beta_c + \beta_o - \beta_c}{1 - \beta_o \beta_c - \beta_o + \beta_c} = \frac{(1 + \beta_o)(1 - \beta_c)}{(1 - \beta_o)(1 + \beta_c)} = \frac{(1 + z_{\text{CMB}})^2}{(1 + z_c)^2}, \quad (13)$$

or

$$1 + z_c = \frac{1 + z_{\text{CMB}}}{1 + z}. \quad (14)$$

Since $T_c = T_d/(1 + z_c)$, the above equation yields

$$T_c = \frac{1 + z}{1 + z_{\text{CMB}}} T_d = (1 + z) T_o. \quad (15)$$

In other words, in the empty model the *local* temperature of the CMB photons at a source of redshift z is $(1 + z)T_o$, in agreement with GR. Specifically, Equation (15) immediately follows from Equation (1):

$$1 + z_{\text{CMB}} = \frac{a_o a_c}{a_c a_e} = (1 + z)(1 + z_c), \quad (16)$$

coinciding with Equation (14). Thus the local coordinates of FOs are more convenient for calculations than the Minkowskian coordinates, which we have used. However, we have obtained the same result. This is not surprising since Equation (15) involves three observables: T_c , T_o , and z . Regardless of specific definitions of coordinates in a given coordinate system, their consistent application should lead to the same result in terms of observables.

As a corollary of this section, the temperature of the CMB photons at redshift z is a strong observational evidence for expanding universe, but, at least in the empty model, it is fully consistent with real motions of matter.

4 Superluminal recession velocities

In an empty universe one can use *global* Minkowskian coordinates of distance and time, r and t . We recall that FOs emanate from the origin $r = 0$ at $t = 0$, and travel with constant velocities. The trajectory of a FO is simple in Minkowskian coordinates: $r = v_M t$, where v_M is its Minkowskian velocity. (Analysis in the preceding section involved Minkowskian velocities.) *Any* velocity of any FO is calculated *along its trajectory*. The Minkowskian velocity of a given FO is

$$v = \left. \frac{dr}{dt} \right|_{r=v_M t} = \frac{d(v_M t)}{dt} = v_M. \quad (17)$$

Minkowskian coordinates define what Milne called ‘private space’ of an observer.

However, in cosmology we usually measure distances on the hypersurface of constant *proper* time, τ , of all FOs. Along the line of sight to a distant object we consider a hypothetical series of closely spaced FOs, and as the distance we adopt a sum of all distances measured by them to their nearest neighbours. In Milne’s terminology, this measurement is performed in ‘public space’. Since all FOs are in motion relative to the observer at $r = 0$, the distances and time they measure are subject to special-relativistic phenomena of length-contraction and time-dilation. For a given FO at the point r in the moment t , his clock shows time τ , that is dilated relative to time t . Using Equation (6), $t = \gamma(v_M)\tau$, or

$$t = \gamma(r/t)\tau. \quad (18)$$

We recall that time t is measured in the inertial frame of the FO *at the origin*, i.e., that is measured by a set of synchronized clocks (with that at the origin), remaining in rest relative to it. Similarly, the observer at r measures the distance dl (at τ) that is different from dr measured by the central FO. Specifically,

$$dl|_{\tau} = \frac{dr|_{\tau}}{\gamma(r/t)}. \quad (19)$$

Note that a hypothetical ruler of length dl is stationary not in the frame $O(r, t)$, but in the frame $O'(r', t' = \tau)$, so the above equation may appear at first sight in contradiction with classical special-relativistic length contraction of a moving body. The reason why in this equation, derived from the Lorentz transformation between the two frames, the distance divided by γ is not dl but dr is that these distances are measured not at constant t , but at constant τ . [10] In fact, Equation (19) is the classical length-contraction formula for the frames O and O' with exchanged roles.

From Equation (18) we obtain $t^2 = \tau^2 + r^2/c^2$. Substituting this in Equation (19) yields

$$dl|_{\tau} = \frac{dr|_{\tau}}{\sqrt{1 + r^2/c^2\tau^2}}. \quad (20)$$

Integrating Equation (20) for a constant τ we have

$$l|_{\tau} = \int_0^r \frac{d\tilde{r}|_{\tau}}{\sqrt{1 + \tilde{r}^2/c^2\tau^2}}, \quad (21)$$

hence [7]

$$l = c\tau \sinh^{-1}(r/c\tau). \quad (22)$$

Thus, ‘public-space’ distance, l , is length-contracted compared to ‘private-space’ distance, r . After the proper-time interval $d\tau$, FOs again measure distances to their nearest neighbours, and the cumulative distance is $l(\tau + d\tau)$. The ‘public-space’ velocity of the FO with Minkowskian velocity v_M is thus $v_{\text{rec}} = dl/d\tau$, again calculated

along its trajectory. We have

$$\begin{aligned}
 v_{\text{rec}} &= \frac{d}{d\tau} [c\tau \sinh^{-1}(r/c\tau)] \Big|_{r=v_M t} \\
 &= \frac{d}{d\tau} \left[c\tau \sinh^{-1} \left(\frac{v_M t}{c\tau} \right) \right] \\
 &= \frac{d}{d\tau} [c\tau \sinh^{-1} (\beta_M \gamma_M)], \tag{23}
 \end{aligned}$$

where in the last equality we have used Equation (6); here $\beta_M = v_M/c$, and $\gamma_M = \gamma(v_M)$. Since in an empty universe, v_M of a given FO remains constant, we see that ‘public-space’ distance to this observer (l) grows linearly with ‘public-time’ (τ). Therefore, simply

$$v_{\text{rec}} = c \cdot \sinh^{-1} (\beta_M \gamma_M): \tag{24}$$

also ‘public-space’ velocity of any FO remains constant.

Redshift of photons emitted by a given FO is related by Equation (3) to its *Minkowskian* (i.e., inertial) recession velocity. This yields $\gamma_M = (1+z)/(1+\beta_M)$. After simple algebra, we obtain

$$v_{\text{rec}} = c \cdot \sinh^{-1} \left[\frac{z(1+z/2)}{1+z} \right], \tag{25}$$

or

$$v_{\text{rec}} = c \cdot \ln(1+z). \tag{26}$$

The last step can be easily verified by showing that Equation (26) implies

$$\sinh(v_{\text{rec}}/c) = \frac{z(1+z/2)}{1+z}, \tag{27}$$

thus it reproduces Equation (25).

Solving Equation (3) for v_M we obtain

$$v_M = c \frac{(1+z)^2 - 1}{(1+z)^2 + 1}. \tag{28}$$

Comparing Equation (26) with (28) we see that ‘public-space’ recession velocity of any FO is a different function of redshift than its ‘private-space’ (or inertial) velocity. These velocities are equal only

to second order in redshift ($v \simeq z[1 - z/2 + \mathcal{O}(z^2)]$), because time-dilation and length-contraction factors are unity plus terms which are of second order in β_M , so in z . In almost all Friedman models, objects with sufficiently large redshifts recede from the central observer with superluminal velocities (greater than c). For example, in an Einstein-de Sitter universe ($\Omega_m = 1$ and $\Omega_\Lambda = 0$), the ‘public-space’ recession velocity as a function of redshift is

$$v_{\text{rec}} = 2c \left[1 - (1+z)^{-1/2} \right], \quad (29)$$

hence $v_{\text{rec}} > c$ for $z > 3$. [9] In particular, the velocity of the so-called particle horizon (corresponding to infinite redshift) is $2c$. In an empty universe, ‘public-space’ recession velocities are not only superluminal for sufficiently large redshifts; they are even unbounded. Does it imply violation of special relativity in cosmology? Of course not. Apart from anything else, deriving Equation (26) we have used nothing except special relativity! Constancy of the speed of light, and subluminality of the motion of massive bodies, applies only to *inertial* frames. However, ‘public-space’ distance is a hybrid of distances measured in different inertial frames, all in relative motion. Since the resulting v_{rec} is not measured in any single inertial frame, there is no violation of special relativity. [2]

Specifically, ‘public-space’ distance is measured at constant proper time of FOs. Time-dilation formula tells us that according to the central observer, this measurement is done at the instant of time $t_i = \gamma(v_i)\tau$, where v_i is the Minkowskian velocity of the i -th FO. Since more distant FOs have greater velocities, it is obvious that for two different FOs, $t_i \neq t_j$. Therefore, according to the central observer, different (sub)distances are not measured *simultaneously*. Simultaneity is a crucial condition of special-relativistic measurements of distances to and sizes of bodies in motion. Waiving this condition may have important consequences and indeed, it does have! The problem with the real Universe is that it is filled with matter and expanding, so there are no global inertial frames. Then, measuring distance (along geodesics) on the hypersurface of constant proper time of FOs is something most natural to do. We should, however, bear in mind the ‘costs’ of such a definition of distance. One of them are apparently superluminal recession velocities of distant galaxies.

5 Particle horizon

From the Robertson-Walker (RW) form of the metric for an empty universe it is straightforward to derive the *comoving* radial distance, $x \equiv a(\tau_o)l/a(\tau)$, to a source lying at redshift z ,

$$x = cH_o^{-1} \ln(1 + z). \quad (30)$$

We note in passing that since $v_{\text{rec}} = H_o x$, we have $v_{\text{rec}} = c \ln(1 + z)$, in agreement with Equation (26). By definition, $x = l_o = l(\tau_o)$. Writing $v_{\text{rec}} = dl/d\tau$, from Equation (26) we immediately obtain

$$l = c\tau \ln(1 + z), \quad (31)$$

hence $l_o = c\tau_o \ln(1 + z)$. The central observer observes other FOs receding with constant velocities, so for a given FO, its Minkowskian distance is $r_o = vt_o$, or $v = H_o r_o$, where $H_o = t_o^{-1}$: this is the Hubble law in Minkowskian coordinates. The central observer measures his proper time, so $t_o = \tau_o$. We have thus $\tau_o = t_o = H_o^{-1}$, hence $l_o = cH_o^{-1} \ln(1 + z)$, in agreement with Equation (30).

Equation (1) for the cosmological redshift shows that the limit $z \rightarrow \infty$ corresponds to the limit $\tau_e \rightarrow 0$, so infinitely redshifted photons were emitted just at the Big Bang. The current distance to their source is called the *particle horizon* (at a given instant of time, we cannot see further sources). For example, the present value of the particle horizon in an Einstein-de Sitter universe is $2cH_o^{-1} = 3c\tau_o$, where τ_o is the present age of the universe. This value seems to imply that the horizon recedes with superluminal velocity. Indeed, in Section 4 we have noted that the present value of the ‘public space’ horizon’s velocity in this model is $2c$ (see Eq. 29). We will return to this topic later on.

Returning now to the empty model, from Equation (31), the present value of the particle horizon in an empty universe is

$$\lim_{z \rightarrow \infty} l_o = cH_o^{-1} \lim_{z \rightarrow \infty} \ln(1 + z) = \infty. \quad (32)$$

Therefore, the empty model does not have the particle horizon, or has it at infinity. Why? This is a direct consequence of the special-relativistic phenomenon of time dilation. The present value of the ‘public-space’ distance to any object is measured at the proper time

τ_o . Time-dilation formula tells us that according to the central observer, this measurement is done at the instant of time $t_o = \gamma(v_M)\tau_o$, where v_M is the Minkowskian velocity of the receding object. The limit $z \rightarrow \infty$ implies $v_M \rightarrow c$, hence $t_o \rightarrow \infty$. In other words, in the inertial system of the central observer, a source with $z = \infty$ travels at the speed of light, so it is infinitely time-dilated, so it needs infinite time t to acquire any non-zero (finite) value of its proper time. Travelling with the velocity of light, after infinite time it is infinitely far away, even in terms of the ‘public-space’ distance l .

In the Einstein-de Sitter model, the universe decelerates, so relative to the central observer, any initially ultra-relativistic source slows down. This causes ‘almost’-luminal sources (with $v_M \rightarrow c$, so $z \rightarrow \infty$) to become significantly subluminal. This makes the time-dilation effect finite, or makes the proper time of the FO sitting on the source to flow. Since it takes finite time t_o to acquire the value of the proper time τ_o , the distance travelled is also finite. It is interesting to note that for the currently favoured cosmological model, $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, the radius of the particle horizon is approximately $3.4c\tau_o$, [3] not much greater than the value for the Einstein-de Sitter model ($3c\tau_o$). This is consistent with the fact that the current acceleration of the Universe started fairly recently (in this particular model, at $z \simeq 0.7$).

Another model in which the particle horizon is infinite is the de Sitter model ($\Omega_m = 0$, $\Omega_\Lambda = 1.0$), where $l_o = cH_o^{-1}z$. A de Sitter universe constantly accelerates, so it is not surprising that the divergence of l_o as a function of z is stronger here than in the empty model, since here an ultra-relativistic source becomes even more ultra-relativistic.

6 Conclusions

In this paper, as a counterexample to the idea of expanding space, we have studied the dynamics of the empty model. We have shown that the cosmological redshift is there a result of the real motion of the source, i.e., a Doppler shift. We have verified that the local temperature of the CMB photons at a source of redshift z is a factor of $(1 + z)$ greater than its present value, in agreement with GR. We have shown that the recession velocities of distant galaxies are only apparently superluminal, due to the adopted definition of distance in

cosmology and the effect of special-relativistic time dilation. Alternatively defined, *inertial* velocities are subluminal. The effect of time dilation is also responsible for infinite distance to the particle horizon in this model. Specifically, the distance is infinite because the proper time of a fundamental observer moving with the speed of light does not flow, so it never acquires a non-zero value, necessary to perform the measurement of the distance. (It is always ‘too early’ to send any communication photons.) The particle horizon exists (i.e., the distance to it is finite) for models with a period of initial deceleration, i.e., for which $\Omega_m > 0$.

The empty model shares all properties of the Friedman models, that are commonly considered as an evidence for general-relativistic expansion of space (see Section 1). However, in the empty model these properties are shown to be in agreement with SR and are fully explicable as the effects of real, relativistic motions in space. Therefore, there is at least one Friedman model, in which expansion of space, in detachment from expanding matter, is an illusion. Actually, there is a whole class of such models: with the mean matter density much smaller than the critical density, and vanishing cosmological constant. In these models (at least since some instant of time) expansion is approximately (but with arbitrary accuracy) kinematic, and spacetime is approximately the static Minkowski spacetime. The empty model is an asymptotic state of any open model with $\Omega_\Lambda = 0$. Therefore, in any such universe, during its evolution, expanding space should somehow, mysteriously, disappear. The proponents of expansion of space must be able to describe this process of disappearance. The simplest scenario for disappearing expanding space, that comes to the mind of the author, is that it has never existed. There is neither absolute space, nor expanding space. All that matters is the cosmic substratum and its relative motions. A truly Buddhist enlightenment.

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A Calculation of the redshift in ‘private space’

As mentioned in Section 1, for all Friedman models the cosmological redshift is an accumulation of the infinitesimal Doppler shifts caused by relative motions of closely spaced fundamental observers along photons’ trajectory. In general, this accumulation does not sum up to a *global* Doppler shift, i.e., due to solely the relative motion of the source and the observer. The reason for this is that in a non-empty universe photons also undergo a gravitational shift (e.g., [10]). In section 2 we have shown that in the empty model, the cosmological redshift *is* a global Doppler shift. To do this, we have used the special-relativistic formula for the Doppler effect, Equation (3), and shown that it leads to the correct expression for the redshift. (That is, the same as obtained in this model from general Eq. 1.) In this Appendix we will *derive* this equation, summing up the infinitesimal Doppler shifts of photons passing between neighbouring fundamental observers. Equation (3) involves the *inertial* relative velocity of the emitter and the observer. Therefore, in our calculation we will use *global* Minkowskian coordinates.

At the origin of the coordinate system, let’s place a source of radiation. We will thus perform our calculations in ‘private space’ of the source. (The calculation in ‘private space’ of the observer is similar.) At time t_e the source emits photons, which at time t reach a fundamental observer (FO1) moving with velocity v , such that

$$vt = c(t - t_e). \quad (33)$$

Time t is measured in the global inertial frame of the source, i.e., it is measured by an infinite set of synchronized clocks (with that at the origin), remaining in rest relative to it. In the rest frame of the observer FO1, a neighbouring (the one more distant from the source) fundamental observer (FO2) moves with infinitesimal velocity $\Delta v'$, hence an infinitesimal (so non-relativistic) Doppler shift is

$$\frac{\Delta \nu'}{\nu'} = -\frac{\Delta v'}{c}, \quad (34)$$

where ν' is the photons’ frequency at FO1. The velocity of FO1 relative to the source is $V = r/t$, where r is its distance from the source at time t . Similarly, the velocity of FO2 relative to the source

is $v = (r + \Delta r)/t$. According to the relativistic law of composition of velocities, the velocity of FO2 relative to FO1 is

$$\Delta v' = \frac{v - V}{1 - vV/c^2} = \frac{\Delta v}{1 - v^2/c^2} + \mathcal{O}[(\Delta r)^2], \quad (35)$$

where $\Delta v \equiv v - V = (dv/dt)\Delta t$. From Equation (33), $dv/dt = ct_e/t^2$. Furthermore, $1 - v^2/c^2 = (t_e/t)(2 - t_e/t)$, hence

$$\Delta v' = \frac{c\Delta t}{2t - t_e}, \quad (36)$$

or

$$- \int_{\nu_e}^{\nu_o} \frac{d\nu'}{\nu'} = \int_{t_e}^{t_o} \frac{dt}{2t - t_e}. \quad (37)$$

Integration yields

$$\ln \frac{\nu_e}{\nu_o} = \frac{1}{2} \ln (2t_o/t_e - 1), \quad (38)$$

or, finally,

$$\frac{\nu_e}{\nu_o} = (2t_o/t_e - 1)^{1/2}. \quad (39)$$

Equation (39) exactly coincides with Equation (5), which, in turn, is a direct consequence of Equation (3).

As a corollary, we can calculate an accumulation of the infinitesimal Doppler shifts either in the local coordinates of fundamental observers ('public space'), or in the global Minkowskian coordinates of any selected FO (his 'private space'). The local coordinates of fundamental observers are actually more convenient, because in them, instead of Equation (35), we have simply $\Delta v' = H'\Delta r'$. Whatever is our choice, however, we obtain the same result. Moreover, a calculation in 'private space' is necessary to provide a proper physical interpretation of this result: a global Doppler shift.

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- [15] An analogous calculation with the observer at the origin yields the same result.

Comment by

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Dr Chodorowski's article is interesting, to the point, and well written. It gives Author's comments on the often discussed problem, whether the cosmological expansion may be mimicked by motion in Minkowski spacetime.

Is space really expanding? A counterexample

I do not agree with several Author's interpretations, but I value very much his way of discussing the problem. His arguments are intelligent and based on his solid knowledge of the subject. His explanations are pedagogically very clear.

While I think, that some of Chodorowski's ideas are wrong, they are serious and readers will benefit from knowing them. The Author clearly states that these are his own opinions, and presents different opinions too, with interesting critical comments.

The article is pleasant to read because of the Authors charming (but disciplined) style and his very good command in English.

I strongly recommend this article for publication. It may be published in its present form.