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IS THE FLATTENING OF ELLIPTICAL GALAXIES NECESSARILY DUE TO ROTATION?

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SUMMARY

There is as yet no conclusive observational evidence to indicate at what redshift z_f the first massive galaxies formed. In particular we do not know whether the galaxy formation process was dissipative. If $z_f < 10$ the formation of galaxies is likely to have involved the formation of pancakes of stars. This paper investigates the possibility that elliptical galaxies formed by the violent relaxation of such sheets of stars. A numerical simulation of the relaxation process indicates that the final equilibrium configuration of stars will be markedly flattened whether or not the pancake of stars possessed rotation. The form of the equilibrium configuration which is formed in this way from a pancake appears to be consistent with a Hubble luminosity profile. The model is also consistent with the only published rotation curve of an elliptical galaxy in that it suggests that rotation may not always be dynamically important for such elliptical galaxies.

INTRODUCTION

Perhaps the most important of the unknowns of modern cosmogony is the redshift z_f which saw the formation of the first massive galaxies. Values of z_f as high as 100 and as low as 2 appear to be compatible with present-day cosmogony. Yet z_f is important for several reasons. In the first place the true value radically affects our chances of actually seeing the formation of the first galaxies (Partridge & Peebles 1966). In the second place z_f is intimately connected with the nature of the fluctuations in the expansion of the early Universe from which galaxies later formed and to the present mean density of the Universe. And thirdly the processes which will have been important during the actual formation of galaxies depend very much on z_f. In this paper I wish to examine this last point and to show that we may be able to rule out values of z_f above about 10 on the basis of studies of elliptical galaxies.

THEORIES OF GALAXY FORMATION

Broadly speaking we may divide theories of galaxy formation into two classes. The important factor which distinguishes these two classes from one another is dissipation. Numerical and analytical studies indicate that when a system of mutually gravitating point particles which are initially expanding away from each other in a Hubble flow are dragged back toward one another by their mutual attraction, the system soon settles to a quasi-static configuration (Henon 1964, 1966; Bouvier & Janin 1970). The process of rapid mean field fluctuation by which

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this settling to a quasi-static configuration takes place has been terms ' violent relaxation ' by Lynden-Bell (1966). In any reasonably massive galaxy ($M > 10^8 M_\odot$) the only adjustment of the stars amongst the various orbits which will occur in a Hubble time, takes place in the course of the two or three crossing times after the system ceases expanding and during which the mean gravitational field of the system fluctuates violently. Numerical models of this process indicate that the largest particle density of the final configuration will not be more than a factor ten greater than the lowest particle density which was achieved by the particles during the expansive phase of their motion. Thus suppose that the gas from which the galaxies presumably formed broke up into stars or other tightly bound objects before the recontraction of the protogalactic cloud was well under way. This will cause the protogalaxy to pass through its recollapse phase as an effectively collisionless ensemble of particles, and hence result in the formation of a galaxy whose highest mass density would not be more than about 15 times greater than the lowest density which was achieved by the protogalactic cloud during the expansive phase of its motion.* This latter will have been 5·6 times greater than the critical density at the redshift at which the protogalaxy achieved its maximum expansion (Weinberg 1972). Thus models of this type envisage the formation of galaxies whose peak densities are now,

$$5{\cdot}6 \times 15 \times (1+z_f)^2 \times (1+2q_0 z_f) \times \rho_c \approx 5 \times 10^{-28} \left(\frac{H}{55}\right)^2 (1+z_f)^2 \times (1+2q_0 z_f) \text{ g cm}^{-3}.$$

But in a typical elliptical galaxy a density in excess of 10^{-24} g cm^{-3} may be achieved throughout a region which is several kiloparsecs in diameter. Thus one sees that ordinary galaxies can only have formed by this process if they formed before about $z_f = 15$. For simplicity let us take $z_f = 10$ as the latest redshift at which dissipationless galaxy formation is possible.

If on the other hand the gas of protogalaxies failed to fragment early in the recollapse, it is easy to understand how galaxies may have formed after redshift 10. For under these circumstances much of the kinetic energy which would be acquired by the material of a protogalaxy during the recollapse phase would have been dissipated in one or more gas-on-gas collisions. At some point the gas will have fragmented into stars or other small objects and the protogalaxy will have settled into a quasi-static configuration in much the same way as in the case in which fragmentation occurred soon after the onset of the collapse. The final galaxy will, however, have a higher particle density than would have been the case had the gas fragmented immediately the recollapse had commenced. Indeed the density of the gas at the time when it actually fragmented and the final relaxation process commenced will have been much higher than it was when the Hubble expansion ceased. Thus may galaxies such as those we now see have come into being at redshifts $z_f < 10$.

Clearly the theory of galaxy formation at redshifts $z_f > 10$, in which stars formed as soon as the recollapse commenced, is much to be preferred on grounds of simplicity. The theory involves no complex gas dynamics and can be fully worked out on the basis of Newtonian gravity alone. Indeed rather detailed simula-

* The density enhancement which is produced by the violent relaxation process depends to some extent on the degree of uniformity of the initial configuration. Collapse from a cold precisely spherical configuration produces a density enhancement by more than a factor 15 (Peebles 1969). But such initial conditions seem somewhat artificial.

tions of the recollapse process have been made and the results been compared with observations (Gott 1973, 1975). The situation in regard to theories of galaxy formation at late redshifts is much less satisfactory. The physics involved is intrinsically complex and we have to date only some very rudimentary studies of the processes involved. One line of attack is that of Larson, who supposes that a protogalactic cloud fragments during its collapse into some sort of subclouds whose collisional cross-sections are so small that the collisional relaxation of the system takes place over many crossing-times of the system (Hoyle 1949; Peebles 1968). The alternative approach, and the one which will be adopted here, is to assume that the clouds into which a collapsing protogalaxy fragment have sufficiently high collisional cross-sections that each one is certain to suffer a major collision during the first crossing-time of the system. The picture which has emerged from studies of this model is the following (Sunyaev & Zeldovich 1972; Doroshkevich & Shandarin 1973; Binney, in preparation). A cloud of material containing upwards of $10^{10} M_\odot$ of matter separates out from the general Hubble flow and falls back in on itself. The initial temperature of this material may have been as high as 10^4 K or as low as 0·2 K. But is unlikely to have been high enough that the gas pressure was comparable with self-gravity as a dynamical factor. It has been long known that the recollapse of a cloud of material under the influence of gravity alone will rapidly become highly anisotropic (Lin, Mestel & Shu 1965). Parcels of material from different parts of the cloud will fall along intersecting trajectories. A shock will develop along the caustic surface generated by the points of intersection of these trajectories and this shock will destroy that component of the motion of the matter which falls into the shock which is perpendicular to the shock. Thus at the end of this stage the protogalaxy consists of a sheet of gas, each part of which is moving in towards the centre of the sheet. One of two things may happen at this point. Either the gas of the sheet will fragment into stars or it will not. In the first case the next stage in the evolution of the protogalaxy will consist in the infall and violent relaxation of an ensemble of stars. In the second case one expects the sheet of gas to collapse in on itself forming a second sheet of smaller diameter. Sooner or later the material of one of these sheets will fragment and the stage of violent relaxation will be reached.

What then will be the outcome of this violent relaxation? Will the highly anisotropic initial configuration of the stars be reflected in the final quasi-static stellar distribution? Some rather crude numerical experiments which I shall now describe have led me to believe that the answer to this question is yes. The shape of the final galaxy will reflect the anisotropy of that galaxy's birth.

THE EXPERIMENTS

The aim of the experiments was to follow the violent relaxation of a system of mutually gravitating point objects from an initial configuration which should mimic the result of the fragmentation of a gaseous sheet such as I have described above. The motion of the particles was followed with a slightly modified form of Aarseth's direct algorithm (Aarseth 1972). In this scheme the force on each particle was evaluated from time to time by direct summation of the forces due to each of the other particles. The contribution of each of the latter was taken to be

$$\frac{-r_{ij}}{(|r_{ij}|^2+r_0^2)^{3/2}}. \qquad (1)$$

The motion of each particle was then approximated by a fifth order polynomial in time. Thus no arbitrary restrictions were placed on the degrees of freedom of the gravitational field of the system. It is important that this was so. For to use a technique like that of Gott (1973, 1975), for example, which imposes axial symmetry on the field, is to risk impairing the efficiency of the violent relaxation process. And my aim is to estimate the *maximum* efficiency of the violent relaxation process. The purpose of the cutoff radius r_0, which varied between 3 and 6 per cent of the initial disk radius, was to expedite the calculation. Thus the introduction of r_0 eliminates the sharp accelerations which would otherwise be attendant on close encounters. This enables the time–step length to be increased without loss of accuracy. Soft potentials of this type have been used in several other n-body calculations (Ostriker & Peebles 1973; Henon 1964, 1966; Bouvier & Janin 1970).

In all, four models were run. Three of these had but 100 particles each. Their purpose was to investigate the effects of slight variations in the initial configuration. From these models one can conclude that small variations in the initial configuration do produce but small variations in the end product of the evolution. The noise due to the small numbers of particles involved does, however, make the results of these models less striking than for the 200-particle model. Thus I shall restrict my further remarks to this latter model

The system of 200 particles was evolved for a little under 4 collapse times. A ' collapse time ' is here defined to be the time it actually took the system to collapse from its initial configuration to its minimum configuration. Because in the initial configuration the particle velocities were directed inwards, this time is shorter than the initial free fall time by a factor of about 1·7. It is, however, longer than the free-fall time of the final inner core by a factor of about 8. Angular momentum and momentum were conserved to within about 2·5 per cent per collapse time but energy to within only about 4 per cent collapse time. The initial configuration was the following:

(i) The particles were pseudo-randomly distributed throughout an oblate spheroidal volume. The lengths of the major and minor axes of this volume were in the ratio 10 : 1.

(ii) All particles were initially heading for the centre of the spheroid with velocities which were proportional to radius and of such a size that they would, in the absence of gravity, arrive at the centre just before the elapse of two collapse times.

(iii) In addition the whole collection of particles was initially rotating as a solid body. This rotation was, however, negligible in the case of the 200 particle model; $3{\cdot}4\times10^3$ collapse times being required for one rotation. Thus the system would have had to shrink by a factor of more than 60 in radius before rotation became dynamically important.

(iv) Each particle had an additional isotropic component of velocity chosen at random from the interval $(-u_0, u_0)$, where $u_0 = 1/r_0$ is somewhat higher than the highest orbital velocity possible between two particles in the potential (1). This velocity dispersion was needed to prevent subcondensations appearing during the infall.

In Fig. 1 (a)–(d) I show the appearance of the system at various stages in its evolution. The final configuration clearly shows four zones.

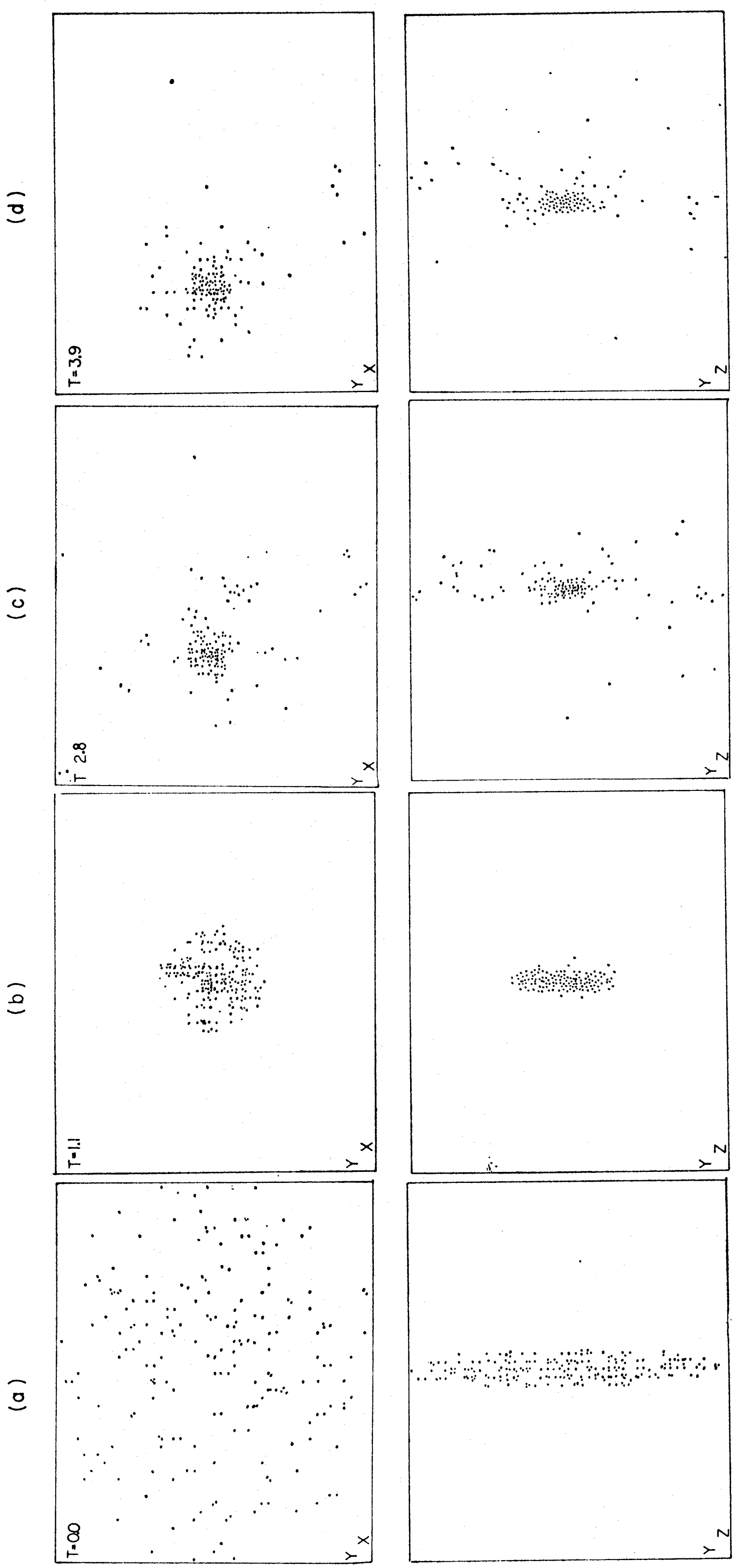

FIG. 1. *Projections onto the YX and YZ planes of the 200-body calculation after (a) 0, (b) 1·1, (c) 2·8, and (d) 3·9 nominal 'collapse times. Each frame represents a square region of space although as reproduced it has sides whose lengths are in the ratio 1·16 : 1.*

(i) A tight inner core of particles forming a spheroid whose major axis is but $\frac{1}{8}$ that of the initial configuration. (ii) A region nearly three times as big as the inner core in which the particle density is as high as in the initial sheet. (iii) A hot region containing a few rather isolated satellites. (iv) The rest of space, which by this time contains the 1 particle in 6 or so which has escaped with positive energy. Per particle these runaways have about as much energy in credit as the others have in debit. Thus the energy which these carry away is not of great importance to the core and halo regions. The inner core in Fig. 1(d) contains about 40 per cent of the total mass and the outer core about another 26 per cent. It emerges very clearly from these figures that after 4 nominal collapse times the system has by no means relaxed to a spherical configuration. Indeed in Fig. 2 I have plotted the ratios of the principal moments of inertia of the bound particles of the system:

$$R_1 = I_y/I_x, \qquad R_2 = I_y/I_z.$$

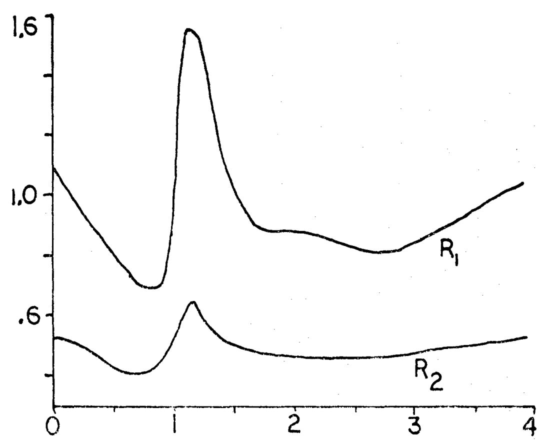

FIG. 2. *The time development of the ratios $R_1 = I_y/I_x$ and $R_2 = I_y/I_z$, where I_x is, for example, the moment of inertia about the x axis.*

One sees in this figure the marked effect of violent relaxation during the interval $t = 0$–$1{\cdot}5$. Thereafter R_2 is fairly constant at about 0·5 until $t = 2{\cdot}8$, when a gradual rise sets in. This leaves R_2 equal to 0·58 at $t = 4$. This increase in R_2 is probably due to the spherizing effects of two-body relaxation. The point which emerged most strongly from this plot and those for the 100-particle experiments was that R_2, which measures the ratio of the extension of the system perpendicular to the initial disk to that in the plane of the disk, is left pretty exactly unchanged by the violent relaxation process.

A glance at Fig. 1(a)–(d) will, however, convince the reader that this exaggerates the true situation. Certainly the core in Fig. 1(d) has an axial ratio which is much less than the initial value of 10 : 1. R_2 remains low because the moments of inertia of the system are dominated by the most distant mass points. An alternative analysis is provided by plots of $\sum_{i=1}^{n(d_i)} \cos(2\theta_i)$ and $\sum^{n(d_i)} \sin(2\theta_i)$. Here θ_i is the angle between the line which joins the most tightly bound particle to the i-th particle and the major axis. In Fig. 3(a) and (b) I have plotted these expressions as functions of the distance d between the most tightly-bound particle and the most distant

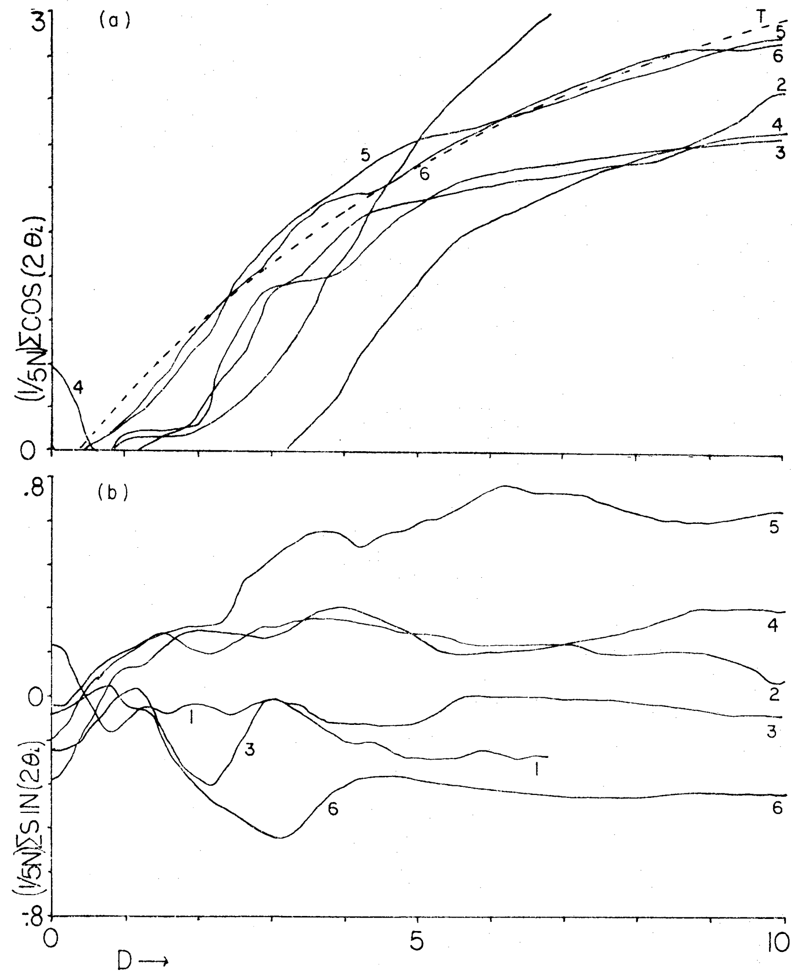

FIG. 3. *The behaviour of the partial sums* $1/(5N) \sum_{d(i)<D} \cos(2\theta_i)$ *and* $1/(5N) \sum_{d(i)<D} \sin(2\theta_i)$. *Here N is the number of particles lying within the sphere* $D < 10$ *and* θ_i *is the angle between the major axis and the line joining the most bound particle to the i-th particle.* $d(i)$ *is the length of this latter line. The curves give these sums for:* $T = 1{\cdot}1$ (1), $T = 1{\cdot}7$ (2), $T = 2{\cdot}2$ (3), $T = 2{\cdot}8$ (4), $T = 3{\cdot}4$ (5) *and* $T = 3{\cdot}9$ (6). *The dashed curve in* (*a*) *gives the partial sum which would be generated by the Hubble-like surface distribution*

$$I(r) = \frac{k}{\left(\sqrt{\left(x^2+\frac{y^2}{1-e^2}\right)}+a_0\right)^2} \qquad \text{for } e = 0{\cdot}945 \text{ and } a_0 = 1.$$

particle included in the sum. The full scale of the abscissae of Fig. 3(a) and (b) correspond to a distance from the most tightly bound particle which is just two fifths of the initial disk radius. In Fig. 3(a) I have also plotted the curve which would be equivalently generated by a natural extension of the Hubble law for spherical galaxies

$$I(r) = \frac{k}{(r+a_0)^2},$$

to elliptical galaxies,

$$I(r) = \frac{k}{(\sqrt{(x^2+y^2/(1-e^2))}+a_0)^2}.$$

Thus in Fig. 6 the marked curve is the value of

$$\int_{x^2+y^2<d^2} \frac{\cos(2\theta)\,dxdy}{(\sqrt{(x^2+y^2/(1-e^2))}+a_0)^2}$$

as a function of d/a_0 with $e = 0{\cdot}945$. The corresponding integral for $\sin(2\theta)$ is of course identically zero. One sees that there is a remarkable similarity between this theoretical curve and that generated by the n-body model at $t=3{\cdot}4$ and $t = 3{\cdot}9$ (curves 5 and 6). Certainly this does not constitute a very severe test of the model, but it does indicate that the outcome of this simple numerical experiment is not incompatible with the one test which can be sensibly applied to a system with as few particles as this. A more thorough investigation of the morphology of models like this one will have to await the completion of more elaborate experiments. This analysis does at any rate confirm the conclusions of Fig. 2: that the absolute magnitude of the asphericity does not change appreciably after the first infall is complete.

CONCLUSIONS FROM THE MODEL

Crude as was the experiment just presented, I think it does establish that if the formation of galaxies did involve gaseous dissipation and consequent flattening into a sheet of stars, there should be significant eccentricities amongst present-day galaxies, independently of whether or not the latter rotate to an important degree.

Let us suppose that the formation of elliptical galaxies was dissipative and that these do possess a dynamically significant amount of angular momentum, acquired perhaps by tidal interaction with protogalactic neighbours as has been suggested by Hoyle (1949) and Peebles (1968). Thuan & Gott (1975) have considered what might be the frequency distribution for the acquisition by this process of angular momentum in various quantities. And they have compared the observed frequency distribution of elliptical galaxies of various eccentricities with that predicted by a combination of the tidal interaction hypothesis and their models of dissipationless protogalactic collapse and relaxation. They find, however, that the theoretical picture predicts more nearly spherical galaxies than actually observed. This discrepancy could be real, or, as they suggest, be caused by noise in the observations. If the discrepancy is real I believe it can be understood in terms of the effect which is the subject of this paper. For suppose the absolute magnitude of the angular momentum which can be acquired by a protogalaxy by tidal interaction with its neighbours is rather less than that anticipated by Thuan & Gott. This would be entirely consistent with our understanding of the tidal interaction process (Oort 1970). And suppose that the first collapse of an elliptical galaxy was in fact dissipational. Then the dissipation caused by the collapse would have introduced a degree of eccentricity which was independent of rotation, but the contraction of the protogalaxy would have increased the dynamical importance of such rotation as the protogalaxy did possess. And whereas the absolute magnitude of the angular momentum which elliptical galaxies would acquire by tidal interaction is highly uncertain, the relative frequency distribution of Thuan & Gott is likely to be realistic. So the likely outcome of such a formation process would be a distribution of ellipticities which at high eccentricity would be that of Thuan & Gott, but at low eccentricities would lie below theirs on account of the presence in all these galaxies of a non-rotational component of ellipticity. Such a distribution of ellipticities would be in more satisfactory agreement with the raw observations than that first predicted by Thuan & Gott. That then is the first, the conservative interpretation of the results of my numerical experiment.

A more radical approach would be to deny that rotation is dynamically important to every elliptical galaxy. Thus one may assume that the eccentricities of elliptical galaxies are due to the anisotropy of their births. And oddly enough it is this more radical view of the matter which would appear at the moment to have the better of the observational data. To date there is only one published rotation curve for an elliptical galaxy. This is due to Bertola & Capaccioli 1975. Bertola & Capaccioli have measured the rotation of NGC 4697 from image tube spectrographs of the Ca II H and K lines. By combining five spectra they obtained a remarkably scatter-free rotation curve which reaches out to 0′·6 from the centre of NGC 4697. This corresponds to 2·58 kpc at an assumed distance of 14·8 Mpc. Their curves show a clear peak in the rotation velocity 15 arcsec from the centre of the galaxy, after which the rotation curve flattens out. This peak rotation velocity is given as 65 km s^{-1}. After correcting for projection effects Bertola & Capaccioli conclude that the true peak rotation velocity is 85 km s^{-1}. Now King & Minkowski (1966) have measured a central velocity dispersion for NGC 4697 of 310 km s^{-1}. Thus the ratio of the rotational to the random kinetic energies of NGC 4697 appears to be only

$$\tau_{\rm rot}/\tau_{\rm ran} = \left(\frac{85}{310}\right)^2 = 0{\cdot}075.$$

Gott has noted that the ratios of the random to the kinetic energies of his models are well approximated by the Maclaurin spheroids of equivalent shape. Yet a Maclaurin spheroid which has $\tau_{\rm rot}/\tau_{\rm ran} = 0{\cdot}075$ would have 10 $(1 = c/a) = 1{\cdot}3$. It would thus be classified E1. This contrasts with NGC 4697 which has been variously classified E6 (de Vaucouleurs & de Vaucouleurs 1964), E5 (Sandage 1961) and E4 (Bertola & Capaccioli 1975). Indeed if NGC 4697 is to be understood in terms of a relaxed model it must have a ratio $\tau_{\rm rot}/\tau_{\rm ran}$ of at least 0·26. If King & Minkowski's velocity dispersion is correct, rotation will be dynamically unimportant in this galaxy unless the peak rotation velocity is at least 150 km s^{-1}, or nearly double that deduced by Bertola & Capaccioli. Actually it is very likely that the King & Minkowski velocity dispersion is high. The question is: is it high by a factor of two? NGC 4697 has an absolute magnitude of $-20{\cdot}5$. Faber & Jackson (1975) obtained dispersions in the range 245–360 km s^{-1} for six ellipticals with absolute magnitudes in the range $M_{\rm B} = -20{\cdot}2$ to $-21{\cdot}7$. So a velocity dispersion as low as 150 km s^{-1} would seem low for a large elliptical like NGC 4697. But such a result is certainty not inconceivable. An accurate redetermination of the velocity dispersion in this galaxy is urgently needed.

There are also a number of theoretical questions here which demand clarification. Can one, for example, generate with this model such good fits to the isophotes of elliptical galaxies as are generated by both Gott (1973, 1975) and Larson (1969) with their rather different models? What will be the effect of starting not from a flat disk of stars but from a more realistically-distorted caustic surface? How will rotation of the initial configuration, perhaps about an axis which is not quite perpendicular to the initial sheet, affect the final configuration? Work is currently in progress on models designed to answer these questions and it is to be hoped that a report will be available in the near future (Aarseth & Binney, in preparation). It also seems reasonable to hope that further rotation curves and more reliable velocity dispersions of flattened ellipticals will become available in the not too distant future.

CONCLUSION

If galaxies formed after redshift 10 their material will probably have fragmented into stars after it had collected together into some sort of sheetlike structure. A simple numerical experiment shows that the violent relaxation consequent on the infall of these stars toward the centre of the sheet will not completely eradicate all memory of the peculiar initial configuration of the stars. The final configuration to which a galaxy which has formed in this way settles will be flattened whether or not the galaxy actually rotates. It may be that the flattening of elliptical galaxies is essentially due to their rotation but that an effect of this nature is responsible for increasing the number of oblate galaxies in the Universe over and above that which one might expect on the basis of rotation alone. It seems also possible that rotation may be dynamically unimportant in many or all elliptical galaxies. If this is indeed the case it may be argued that the fact that most elliptical galaxies are actually flattened is evidence for the existence of a dissipative phase during the galaxy formation process. Which in turn would be indicative that the galaxies formed after redshift 10. At the present time we need both a refinement of the theoretical aspects of violent relaxation from anisotropic initial conditions and more and better data on the ratios of rotational to random kinetic energies in flattened elliptical galaxies.

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