Is the price system or rationing more effective in getting a commodity to those who need it most?

Martin L. Weitzman

Professor of Economics
Massachusetts Institute of Technology

Using a simple formal model, the present paper analyzes under what conditions the price system or crude rationing is more effective in matching up the limited supply of a deficit commodity with those users who need it most. The answer depends in a well-defined way on the distribution of needs and income. Other things being equal, the price system has greater comparative effectiveness in sorting out the deficit commodity and in getting it to those who need it most when wants are more widely dispersed or when the society is relatively egalitarian in its income distribution. Conversely, rationing is more effective as needs for the deficit commodity are more uniform or as there is greater income inequality.

1. Introduction

■ The question of whether it would be better in various circumstances to use quotas or market clearing prices to allocate resources is a debate of long standing. From time to time it has flared up as a policy issue of genuine importance. While each specific debate in this series is about a particular issue, and therefore has its own special features, it does seem to me that the same general themes reappear again and again. The purpose of the present paper is to single out and clarify one of the common strands.

A favorite argument for relying on the market to allocate a particular good or service concerns what might be called its built-in selectivity. The price system, it has been said, is really quite a sophisticated mechanism for matching up a scarce commodity with those who need it most. And this is done automatically, simply by giving consumers a chance to express their preferences in the market place. By contrast, rationing is seen as a crude allocation device which cannot effectively take account of individual differences. Any rationing scheme typically ends up over-delivering goods to some people who do not really want them so much, at the same time that it will be withholding from others with a genuine need for more.

The rejoinder is that using rationing, not the price mechanism, is in fact the better way of ensuring that true needs are met. If a market clearing price is used, this may mean only that it will be driven up until those with more money end up with more of the deficit commodity. How can it honestly be said that such a system selects and fulfills real needs when awards are being made as much on the basis of income as anything else? One fair way to make sure that everyone has an equal chance to satisfy his wants would be to give more or less the same share to each consumer, independent of his budget size.

Both of these arguments are right, or at least each contains a strong element of truth. With the aid of a very simple model, I hope to indicate how the two effects just described interact in determining which allocation system is actually more effective for meeting real needs.

2. A way of formulating the problem

■ At the outset it is probably best to admit that my formulation of the problem is not quite consistent with standard economic theory. In this paper I am interested in the question of how well an allocation mechanism matches up a commodity with those people who have the greatest need for it. Treating only one-dimensional equity in the distribution of a single commodity, to the exclusion of all other considerations, is a violation of consumers' sovereignty and the usual welfare economics. Nevertheless, society sometimes seems to act as if one-dimensional equity is a valid principle, at least for some commodities. As an example, it seems perfectly meaningful to be concerned about the effectiveness of the mental health profession in reaching people with counseling needs. Similar issues arise repeatedly in other service professions—medicine, education, basic food and shelter, legal aid, etc. While the judgment about what sort of commodity qualifies for this kind of special treatment is often arbitrary, the issue itself seems real enough. My point of departure is the notion that there is a class of commodities whose just distribution to those having the greatest need for them is viewed by society as a desirable end in itself. The current paper analyzes whether rationing or the price system is a better way of allocating a good of this sort.

In what follows, the abstraction is going to be on a heroic scale. Because the primary goal of this paper is to capture sharply the interplay of issues discussed in the introduction, a high premium will be placed on the use of analytically convenient functional forms which do not at the same time grossly violate reality. I am well aware that an especially simple case is being treated and that is precisely my purpose. Many essential features of the model would probably remain under more general representations of the economic environment. But since the basic message would then tend to get diluted, such an approach is not taken.[1]

Suppose the population under consideration consists of n consumers. Any person in this population is endowed with a particular set of needs and a certain level of income. I am purposely using the loaded

[1] The point of this exercise is to use the simplest consistent model of consumer behavior along with the simplest explicit notion of social welfare to obtain an easily interpretable formula comparing the market allocation of a commodity with equal rationing. Certainly it is possible to imagine more general formulations. For example, higher than second-order terms could be included in the utility function (2). Or, the loss function (13) might be made something other than quadratic. Such cases would be much more difficult to handle analytically. With luck, I think one ought to be able to prove results analogous to those of the present paper, because the distinction between the distribution of tastes and the distribution of income in explaining the relative performance of the price system and rationing is basic. But these more general results might not be worth the extra effort involved in obtaining them, because they would undoubtedly be of a very messy form, as opposed to the relatively crisp formula which can be derived under the simplifying assumptions of the present paper.

word "need" instead of "taste" or "preference" to bring home the point that, for whatever reason, we are dealing with a commodity whose just distribution is considered a worthy end in itself. Let ε be a variable which quantifies needs for the deficit commodity. ε is intended to be some measure of how much a consumer "needs," "wants," or "enjoys" the deficit commodity. It is operationally measured by how much the consumer purchases relative to what other consumers *would* be buying *if* they belonged to his same income category. Levels of income (or of wealth) will be indirectly quantified by λ, a variable meant to represent the marginal utility of an extra dollar. For ease of exposition, it is assumed that the marginal value of an extra dollar is identical for all consumers having the same income. To the extent that this is not true, it will alter only the interpretation of certain results.

Each consumer endowed with traits (ε,λ) is postulated to demand

$$x(p;\varepsilon,\lambda) = A - B\lambda p + \varepsilon \tag{1}$$

units of the deficit commodity when it is offered for sale at price p. The above form might be defended as a first-order approximation to a demand curve, if incomes and needs did not vary significantly. At any rate, choosing a demand function of this type will considerably simplify the analysis.

Different needs or tastes are parameterized in expression (1) by various values of ε. A higher value of ε denotes a more intense desire for x and is represented by a rightward shift in the underlying linear demand curve. Note that the demand schedule is written as a function of λp, the *real* price of the commodity. It seems to me that an allowable partial equilibrium assumption in the present context is that people with the same needs or preferences but different incomes should have the same demand for x when it is expressed as a function of a price normalized to measure the opportunity loss of income foregone. In effect, I am assuming that the consumer (ε,λ) picks $x(p;\varepsilon,\lambda)$ to maximize

$$U(x;\varepsilon) - \lambda px,$$

where $U(\cdot)$ is a quadratic utility function of the form

$$U(x;\varepsilon) = C + \frac{(A + \varepsilon)x}{B} - \frac{x^2}{2B}. \tag{2}$$

Let $f(\varepsilon)$ denote the number of consumers of need type ε and let $g(\lambda)$ be the number with marginal utility of income equal to λ. Naturally

$$\sum_{\epsilon} f(\varepsilon) = \sum_{\lambda} g(\lambda) = n.$$

As a norm of sorts, and to make the distinction between their roles especially clear, it will be assumed that tastes and income are independently distributed.[2] In other words, the number of consumers possessing the trait combination (ε,λ) is

$$h(\varepsilon,\lambda) = \frac{f(\varepsilon)\, g(\lambda)}{n}. \tag{3}$$

[2] It should not be difficult for the reader to carry through the analysis when ϵ and λ have a nonzero correlation. Equation (18) or (19) will simply end up with a covariance term.

Purely for notational convenience and without loss of generality, ε and λ are normalized in (1) so that their average values are respectively zero and one:

$$E[\varepsilon] \equiv \sum_{\varepsilon} \frac{\varepsilon f(\varepsilon)}{n} = 0 \tag{4}$$

$$E[\lambda] \equiv \sum_{\lambda} \frac{\lambda g(\lambda)}{n} = 1. \tag{5}$$

The variance of ε,

$$V[\varepsilon] \equiv E[(\varepsilon - E[\varepsilon])^2] = \sum_{\varepsilon} \frac{\varepsilon^2 f(\varepsilon)}{n}, \tag{6}$$

can be interpreted from equation (1) as the mean square deviation in the demand for x *when income is held constant* (given our assumption of a unique relationship between the marginal utility of an extra dollar and income). This is by contrast with the mean square deviation in the overall demand for x (*without* controlling for income). The latter variance will be denoted

$$\sigma^2(p) \equiv E[(x(p;\varepsilon,\lambda) - E[x(p;\varepsilon,\lambda)])^2], \tag{7}$$

where the symbol $E[\cdot]$, as usual, indicates the average per capita value of the variable in question. From (1), (3), (4), (5),

$$E[x(p;\varepsilon,\lambda)] \equiv \sum_{\varepsilon} \sum_{\lambda} \frac{x(p;\varepsilon,\lambda)\, h(\varepsilon,\lambda)}{n} = A - Bp. \tag{8}$$

Using (8), (7) becomes

$$\sigma^2(p) = \sum_{\varepsilon} \sum_{\lambda} \frac{(Bp(\lambda - 1) + \varepsilon)^2\, h(\varepsilon,\lambda)}{n}.$$

By (3), (4), and (5), this reduces to

$$\sigma^2(p) = B^2 p^2 V[\lambda] + V[\varepsilon], \tag{9}$$

where

$$V[\lambda] \equiv E[(\lambda - E[\lambda])^2] = \sum_{\lambda} \frac{(\lambda - 1)^2\, g(\lambda)}{n}. \tag{10}$$

3. Alternative allocation mechanisms

■ Suppose there is a fixed supply $\bar{X}$ of the deficit commodity which is available to be allocated among the n consumers.[3] The average x available per capita is

$$\bar{x} \equiv \frac{\bar{X}}{n}.$$

A distribution plan or allocation mechanism which gives $\chi(\varepsilon,\lambda)$ to each person of type (ε,λ) is *feasible* if

$$E[\chi] = \bar{x}, \tag{11}$$

[3] This is also an abstraction, since the supply may be variable and might itself depend on the allocation system, e.g., through the price. Tobin (1970) stresses that the elasticity of supply can be an important determinant of whether to use a price or quota system.

where

$$E[\chi] = \sum_{\varepsilon} \sum_{\lambda} \frac{\chi(\varepsilon,\lambda)\ h(\varepsilon,\lambda)}{n}.$$

Note that without loss of generality we are excluding trivial inefficiency from our definition of feasibility by only considering feasible plans of the equality rather than inequality form in (11).

The *ideal* feasible allocation of x to a person of type (ε,λ) is

$$\chi^*(\varepsilon,\lambda) = \bar{x} + \varepsilon. \tag{12}$$

Remember that we are dealing with a commodity whose just distribution based purely on need (and abstracting away from income) is considered a socially desirable end in itself. Under these circumstances, (12) is a reasonable definition of an ideal distribution, because it is exactly what the consumer with traits (ε,λ) would be purchasing if all incomes were the same and the price were set to clear the market.

The *loss* of effectiveness of a feasible plan $\{\chi(\varepsilon,\lambda)\}$ in meeting true needs for the deficit commodity is defined as its mean square deviation from the ideal plan:

$$L(\{\chi(\varepsilon,\lambda)\}) = \sum_{\varepsilon} \sum_{\lambda} \frac{(\chi(\varepsilon,\lambda) - \chi^*(\varepsilon,\lambda))^2\ h(\varepsilon,\lambda)}{n}. \tag{13}$$

Equation (13) is the standard quadratic loss function, chosen (as usual) because it yields simple results.

The ideal distribution plan $\{\chi^*(\varepsilon,\lambda)\}$ is not in general attainable, because the government typically lacks the information, authority, or inclination to identify people of type (ε,λ) for all ε and λ. Even when approximate grouping distinctions can be made, the same problem of not being able to demarcate types accurately will crop up again *within* each group.

Our point of departure is a collection of consumers with similar enough observable characteristics to make further subdivision of types too expensive or downright infeasible. In this context only those allocation mechanisms can be employed which do not depend on the ability of anyone to screen out types. Furthermore, we limit ourselves to *simple* distribution rules.[4]

One simple rule to follow in the absence of identifiable type-features is to give[5] every consumer the same ration $\bar{x}$. The loss of rationing in meeting true needs for the deficit commodity is

$$L(\{\bar{x}\}) = \sum_{\varepsilon} \sum_{\lambda} \frac{[\bar{x} - \chi^*(\varepsilon,\lambda)]^2\ h(\varepsilon,\lambda)}{n}. \tag{14}$$

The price or market system gives a consumer with traits (ε,λ) that allocation

$$\chi(\varepsilon,\lambda) = x(\hat{p};\varepsilon,\lambda),$$

[4] More complicated rules than we shall consider are certainly possible, for example, an entire schedule of prices as a function of the quantity purchased, or even just a two-tiered version. Naturally an optimal schedule cannot help but be better than a single price or quantity. But a schedule is hard to institute, and lacks the important quality of simplicity, which the two special cases provide.

[5] This is what might be called a *pure* rationing system. We are abstracting away from prices' doing any of the allocating by in effect assuming that the price charged is so low that it deters no one from purchasing his allotted quota.

which he demands by equation (1). In the above expression $\hat{p}$ is the competitive price of x which satisfies the market clearing condition

$$E[x(\hat{p};\varepsilon,\lambda)] = \bar{x}.$$

Substituting from (8), the above expression becomes

$$A - B\hat{p} = \bar{x}. \tag{15}$$

The loss of the price system in meeting real needs for the deficit commodity is

$$L(\{x(\hat{p};\varepsilon,\lambda)\}) = \sum_{\varepsilon}\sum_{\lambda} \frac{(x(\hat{p};\varepsilon,\lambda) - \chi^*(\varepsilon,\lambda))^2\, h(\varepsilon,\lambda)}{n}. \tag{16}$$

The above expression points up how incomplete, limited, and partial our treatment of welfare is. In the price allocation system, and this should never be lost from view, revenues of $\hat{p}\bar{X}$ are collected from the consumers by someone or other. A full treatment would take account of where those funds are coming from and going to, and who is benefiting from them. But since anything is possible here, we merely evade the more general issue after warning that what happens to collected revenues might just be the most relevant consideration of all.[6] Instead, we continue to concentrate on the narrower technical question of how well the deficit commodity in and of itself alone is being allocated, abstracting away from all else that might be happening.

4. Rationing vs. the price system

■ The comparative effectiveness[7] of the price system over rationing in meeting true needs for the deficit commodity is defined as

$$\delta \equiv L(\{\bar{x}\}) - L(\{x(\hat{p};\varepsilon,\lambda)\}). \tag{17}$$

From (12), (3), (4), and (6), expression (14) becomes

$$L(\{\bar{x}\}) = V[\varepsilon]. \tag{18}$$

Plugging in (1) and (12) into (16), it becomes

$$L(\{x(\hat{p};\varepsilon,\lambda)\}) = E[(A - B\lambda\hat{p} + \varepsilon - \bar{x} - \varepsilon)^2].$$

Using (15), (3), (5), and (10), the above expression reduces to

$$L(\{x(\hat{p};\varepsilon,\lambda)\}) = B^2\hat{p}^2V[\lambda]. \tag{19}$$

Thus, (17) becomes

$$\delta = V[\varepsilon] - B^2\hat{p}^2V[\lambda]. \tag{20}$$

An equivalent but perhaps more useful (because it is somewhat more operational) expression for δ is obtained by substituting from (9) into (20) to obtain

$$\delta = 2V[\varepsilon] - \sigma^2. \tag{21}$$

Here $\sigma^2 \equiv \sigma^2(\hat{p})$ is the mean square deviation in demand for x at its market clearing price $\hat{p}$.

---

[6] As an example, it certainly matters whether the funds taxed by the government are used to relieve flood victims or end up in the bank account of an oil billionaire. Diamond and Mirrlees (1971) develop a general equilibrium taxation framework which sheds light on the nature of the general problem.

[7] This criterion is analogous to the one employed in Weitzman (1974).

Expression (20) or (21) is the fundamental result of this paper. The next section is devoted to examining it in some detail.

5. Analyzing the coefficient of comparative effectiveness

■ Starting with an examination of (20), from (1), $V[\varepsilon]$ is interpretable as the variance of the demand for the deficit commodity when the marginal utility of income is held constant. Thus, it is a measure of the heterogeneity of tastes. The larger is $V[\varepsilon]$, the more widely dispersed are "true needs" for the deficit commodity.

Now, $B^2\hat{p}^2V[\lambda]$ is the variance of the marginal utility of income expressed in terms of the deficit commodity as numeraire. The smaller is $V[\lambda]$, the more uniform is the distribution of income.

Expression (20) is essentially the difference of two terms—a "taste distribution effect" and an "income distribution effect." Other things being equal, the price system has greater comparative effectiveness in sorting out the deficit commodity and in supplying it to those who need it most when wants are more widely dispersed or when the society is relatively egalitarian in its income distribution. Conversely, rationing is more effective as needs for the deficit commodity are more uniform or as there is greater income inequality.[8]

Expression (21) is really the same as (20), except that in (21) the variables can be operationally interpreted, whereas (20) is defined in terms of variances of marginal utilities. From (21), δ is a difference of two observable mean square deviations—twice the variance of demand among a subpopulation *controlled* for income (see (6)) minus the *uncontrolled* variance of demand (7).

Thus, if the mean square deviation in demand by the entire consumer population is not much larger than it is by fixed income subgroup, the price mechanism has greater effectiveness in screening out the deficit commodity and in funneling more of it to those who need it relatively more. Conversely, the greater the dependence of demand on income (as measured by (21)), the larger the comparative effectiveness of a quota system, because it essentially prevents those with larger incomes from monopolizing consumption of the commodity in question. Naturally if both variances σ^2 and $V[\epsilon]$ are small, it does not make much difference which system is used.

6. Concluding remarks

■ It might be appropriate to end this paper by commenting on its relation to some more standard approaches. Economists sometimes maintain or imply that the market system is a superior mechanism for distributing resources. After all, the argument goes, consider any other allocation. There is always some corresponding way of combining the price system with a specific lump-sum transfer arrangement which will make everyone better off (or at least no worse).[9]

That is true enough in principle, but not typically very useful for policy prescriptions, because the necessary compensation is practically never paid. When dispensation *is* made, the point deserves em-

[8] Note that a *ceteris paribus* increase in the market clearing price makes rationing relatively more effective, because the income distribution effect takes on added importance.

[9] Given the usual general equilibrium assumptions. See, for example, Arrow and Hahn (1971).

phasis. For example, by the standard argument of consumer sovereignty, it certainly makes everyone better off if ration tickets can be sold (and bought) than if resale is not allowed. The "white market" is superior to strict rationing, because in redistributing property rights, the losers are being adequately compensated or "bought off." However, this does not necessarily mean that pure direct wants for the deficit commodity are better served in the sense that those who "need" it most actually end up receiving more.

Besides, why stop here? Surely the *status quo* income distribution is nonoptimal. An even better system from an overall social welfare position might be to give *all* the rationing tickets to the infirm (who perhaps do not even consume the commodity in question) or to Bangladesh, or so forth. There is no end to what could be done on the income distribution side.

This is one reason why I have preferred to leave income considerations in abeyance and to concentrate in this paper on inquiring about the pure distributive effectiveness of an allocation system in getting the deficit commodity to those who need it most. The other and more substantive reason is that for many situations such a formulation is probably the most appropriate way of posing the question in the first place.

There is a class of commodities whose just distribution is sometimes viewed as a desirable end in itself, independent of how society may be allocating its other resources. While it is always somewhat arbitrary where the line should be drawn, such "natural right goods" as basic food and shelter, security, legal aid, military service, medical assistance, education, justice, or even many others are frequently deemed to be sufficiently vital in some sense to give them a special status. The principal of limited dimensional equity in the distribution of a commodity is an open violation of consumer sovereignty. Yet society does not seem to mind, at least sometimes. In cases when this is so, our model may have some relevant things to say about whether market prices or quotas are better allocation instruments.

References

ARROW, K. J. AND HAHN, F. *General Competitive Analysis*. San Francisco: Holden-Day, 1971.

DIAMOND, P. A. AND MIRRLEES, J. A. "Optimal Taxation and Public Production." *The American Economic Review*, Vol. 61, No. 1 (March 1971), pp. 8–27 and Vol. 61, No. 3 (June 1971), pp. 261–278.

SAMUELSON, P. A. *Foundations of Economic Analysis*. Cambridge: Harvard Univ. Press, 1947.

TOBIN, J. "On Limiting the Domain of Inequality." *Journal of Law and Economics* (October 1970).

WEITZMAN, M. "Prices vs. Quantities." *Review of Economic Studies* (October 1974).