

Is the Value Premium a Puzzle ?

JOB MARKET PAPER

Dana Kiku *

Current Draft: January 17, 2006

Abstract

This paper provides an economic explanation of the value premium puzzle, differences in price/dividend and Sharpe ratios of value and growth assets, volatilities of ex-post returns on the two stocks and their correlation. I consider a model that features two equally important ingredients: a small persistent component in cash-flow growth dynamics and the Epstein-Zin recursive utility preferences. In the model, as in the data, cash flows of value firms are highly exposed to low-frequency fluctuations in aggregate consumption, whereas growth firms' dividends are mainly driven by short-lived consumption news and risks related to fluctuating economic uncertainty. I show that the dispersion in long-run risks is the key mechanism that allows the model to quantitatively replicate the magnitude of the historical value premium, resolving the puzzle. Furthermore, heterogeneity in systematic risks across firms helps account for the whole transitional dynamics of value and growth returns, as well as the empirical failure of the CAPM and C-CAPM. In addition, the model is able to successfully accommodate the time-series behavior of the aggregate equity market.

*Department of Economics, Duke University, email: dak6@duke.edu. I would like to thank my advisor Ravi Bansal, and the members of my dissertation committee, David A. Hsieh, Albert S. "Pete" Kyle, and George Tauchen, for their invaluable comments, encouragement and guidance. All mistakes are my own.

1 Introduction

One of the well-established features of financial data is the fact that firms with high book-to-market ratios (commonly referred to as value firms) tend to consistently deliver higher returns than firms with low book-to-market ratios (or growth firms). First documented in Graham and Dodd (1934), this finding, known as the value premium, turns out to be quite robust to alternative definitions of value, and, in contrast to the size effect, does not disappear over time. The latter observation naturally calls for a risk-based interpretation of the value phenomenon. It is reasonable to believe that investors engaged in the value strategy are exposed to some systematic risks, and the value premium is simply a reward required for risk-bearing. Although appealing, this story is strongly rejected if these fundamental risks are measured by the commonly-used CAPM betas (see Fama and French (1992, 1993)). Empirically, the dispersion in market risks between value and growth stocks is too small to generate sizable spread in average returns, making the value premium a puzzle. It is further confronted by the poor performance of the consumption-based CAPM (Mankiw and Shapiro (1986)).

This paper offers an economic explanation of the value premium phenomena. I introduce value, growth and market portfolios into a general equilibrium model that features long-run consumption risks and show that it can successfully account for the differences in their expected returns, valuation and Sharpe ratios, as well as volatilities, cross-correlations and time-variation in assets' risk premia. Specifically, I provide a fundamental explanation for the scale and the joint transitional density of the three assets within the long-run risks model of Bansal and Yaron (2004). I show that the model goes a long way towards resolving the value premium puzzle — it quantitatively replicates the observed magnitude of the value premium and, at the same time, accommodates the empirical failure of the CAPM and C-CAPM.

The existing empirical literature as yet has studied cross-sectional and time-series dimensions of financial data in isolation. On the one hand, the works of Fama and French (1992, 1993), Jagannathan and Wang (1996), Lettau and Ludvigson (2001b), Campbell and Vuolteenaho (2004), Parker and Julliard (2005), Lettau and Wachter (2005), Jagannathan and Wang (2005)¹ have focused on measuring risk exposures (i.e., betas) and applying them to the cross-section of mean returns to explain the differences in risk premia. On the other hand, Campbell and Cochrane (1999), Barberis, Huang, and Santos (2001), Menzly, Santos, and Veronesi (2004), and Bansal and Yaron (2004) have been primarily interested in understanding the time-series behavior of asset markets. My research, in essence, merges the two strands of the literature. The novel aspect of this paper is to explore the ability of the general equilibrium model to simultaneously account for both cross-sectional and time-series puzzles of value, growth and market returns.

¹See Cochrane (2005) for an updated survey of the cross-sectional literature.

This paper is based on the model of Bansal and Yaron (2004) that incorporates long-run risks in aggregate and asset-specific cash flows. This choice is motivated by two reasons. First, a number of recent studies, including Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2005), and Bansal, Dittmar, and Kiku (2005), document empirically the importance of long-run properties of assets' cash flows for understanding the risk-return tradeoff in financial markets and, in particular, the value spread. Second, the long-run risks model provides a balance between growth- and discount-rate risks that enhances its potential to capture various features of asset pricing data.

Long-run risks in the model are captured by a small but highly persistent component that governs the evolution of consumption growth. In addition to long-run growth risks, the model allows for time-variation in the conditional volatility of consumption. Firms are distinguished by the exposure of their dividends to low- and high-frequency shocks in consumption, as well as news about future economic uncertainty. A complementary ingredient in the model is time-non-separable preferences of Epstein and Zin (1989) type that break the link between agents' attitude towards smoothing consumption over time and across different states of nature. This separation is very important as it makes the marginal rate of substitution depend not only on present and future consumption (as in the standard power utility case), but also on the forward-looking return on the aggregate wealth portfolio. Consequently, predictable variations in all sources of systematic risks have a significant bearing for the implied dynamics of asset returns and, as I show, are able to quantitatively replicate a wide spectrum of value/growth/market phenomena under reasonable configurations of investors' preferences.

What drives the value premium in the model? Risks related to long-term consumption growth, coupled with Epstein-Zin preferences, entail a significant risk premium. The intuition behind this finding is straightforward — shocks to the persistent growth-rate component significantly alter investors' expectations about consumption growth far into the future, leading to large reactions in stock prices and sizable risk compensations. Assets' valuations and risk premia, therefore, by and large depend on the amount of low-frequency risks embodied in assets' cash flows. I document that in the data, value firms are highly exposed to long-run consumption shocks. Growth firms, on the other hand, are mostly driven by short-lived fluctuations in consumption and risks related to future economic uncertainty. Consequently, value firms exhibit higher elasticity of their price/dividend ratios to long-run consumption news (relative to growth assets) and have to provide investors with high ex-ante compensation.

To evaluate the model's ability to quantitatively capture various phenomena of value, growth and market equities, I solve it numerically using the quadrature-based method of Tauchen and Hussey (1991). I show that once time-series dynamics of aggregate and asset-specific cash flows are calibrated so as to match the observed annual data on consumption and dividends, the model is

able to successfully account for both time-series and cross-sectional properties of assets' prices and returns. In particular, the model generates the value premium of about 5.3% per annum that is fairly comparable to 6% in the data. In the model, as in the data, the CAPM and C-CAPM fail to justify the spread in average returns — the model-implied market and consumption betas of value stocks, on average, are lower than those for growth firms. More importantly, the model is able to simultaneously replicate the value-growth spread in Sharpe ratios (0.34 versus 0.20), differences in their price/dividend ratios (24.7 for value versus 39.8 for growth), high persistence in assets' valuations (the first-order autocorrelation of price/dividend ratios varies between 0.8 and 0.9), and high volatilities of stock returns (of about 20-30% per annum). In addition, the model reproduces long-horizon predictability of returns, countercyclical variation in the value and aggregate equity premia (both increase during times of high economic uncertainty), and largely accounts for the high contemporaneous correlation in assets' returns. Finally, the model generates high premium on the market portfolio of about 6% per annum, along with low and fairly stable risk-free rates; the average return and volatility of the riskless asset are about 1.5% and 1%, respectively.

The rest of the paper is organized as follows. The next section presents an overview of the stylized empirical features of value, growth and aggregate stock market data. Section 3 provides details of the long-run risks model and highlights its intuition. The choice of preferences and time-series parameters is discussed in Section 4. Section 5 summarizes asset pricing implications of the model. Finally, Section 6 provides concluding remarks.

2 Empirical Evidence: Value/Growth/Market

This section reviews the historical performance of high and low book-to-market firms and highlights some intriguing patterns in value and growth strategies. In addition, it summarizes the time-series dynamics of the market portfolio and real interest rates. I focus on the long-term behavior of financial markets, employing for this reason the longest available set of data that spans the period from 1929 to 2003.

2.1 Data Construction

Asset market data consist of annual observations on value-weighted real returns and cash-flows for portfolios sorted by book-to-market ratios, as well as for the aggregate stock market. I construct 5 portfolios on a monthly basis as in Fama and French (1993), using data from the Center for Research in Securities Prices (CRSP) and the Compustat database. The book-to-market ratio is calculated as book equity at the last fiscal year end of the prior calendar year divided by market equity at the end of December of the previous year. Following Fama and French, I define book

equity as the stockholders' equity, plus balance sheet deferred taxes and investment tax credit, minus the book value of preferred stock. Depending on availability, I use redemption, liquidation or par value for the book value of preferred stock. Portfolios are formed for NYSE, AMEX and NASDAQ stocks at the end of June of each year using NYSE breakpoints. For each portfolio, I construct the per-share dividend series as in Campbell and Shiller (1988b) and Bansal, Dittmar, and Lundblad (2005), extracting dividend yields for a given portfolio, y_{t+1} , using CRSP returns with and without dividends. Portfolio dividends are created as $D_{t+1} = y_{t+1}V_t$, where the value of the portfolio, V_t , is computed using the price gain series h_{t+1} , as $V_{t+1} = h_{t+1}V_t$ and $V_0 = 100$. Monthly returns and dividends are time-aggregated to an annual frequency and converted to real quantities using the personal consumption deflator. Empirical findings discussed below are fairly robust to an alternative measure of payouts that adjusts dividends by share repurchases. Therefore, in what follows, I only report evidence based on the former, i.e., conventional per-share dividend series.

I focus on cash flows and returns on two portfolios with opposite book-to-market characteristics — firms in the bottom quintile that I refer to as *growth* firms, and firms in the top book-to-market quintile that I correspondingly refer to as *value* firms.

I use the 90-day T-bill as a measure of the return on the riskless asset. To construct the real rate of interest, I subtract a 12-month moving average of inflation from the observed nominal rate. Treasury and inflation data are taken from the CRSP dataset. Finally, I construct the growth rate of aggregate consumption using seasonally adjusted data on real per capita consumption of nondurables and services from the NIPA tables available from the Bureau of Economic Analysis. Consumption data, as well as all asset pricing data, are sampled on an annual basis.

2.2 The Value Premium in the Data

One of the most robust features of financial data is the finding that value firms, on average, have higher returns than growth firms. Figure 1 visualizes this evidence by plotting the spread in realized returns on high and low book-to-market portfolios. It can be seen that over the course of the last 74 years, the value strategy delivered superior returns about 70% of the time. Numerically, the value effect is illustrated in the top panel of Table I, which reports descriptive statistics for returns, cash-flow growth rates, and logarithms of price/dividend ratios of the two portfolios along with their robust standard errors.² The first column of the table shows that growth firms on average offer about 8% to investors (comparable to that for the market portfolio), whereas value stocks deliver an impressive 14% per annum. The difference in average compensations or the value premium is about 6% over this time period. Value investing seems to be somewhat riskier in the

²Robust standard errors are calculated using the Newey-West variance-covariance estimator with 8 lags.

traditional sense as the standard deviation of value stocks exceeds that for growth stocks. The volatility spread, however, does not quantitatively overweight the difference in average returns — value stocks provide a much better deal to investors even in terms of average compensation per unit of risks as measured by the Sharpe ratio. The ratio of average excess return to standard deviation is equal to 0.43 for value versus 0.34 for growth stocks.

What causes such a remarkable difference in mean returns on value and growth stocks? It is sensible to argue that high book-to-market firms are subject to some systematic risks, and the extra premium is the appropriate compensation required by value investors. This interpretation, however, finds no empirical support within standard asset pricing paradigms. In particular, according to the CAPM, the difference in mean returns should be entirely accounted for by the difference in market risks measured by the covariation of asset returns with the market portfolio. There is, however, too little dispersion in market betas of value and growth assets (in our sample, both are virtually the same, equal to 1.03) to justify the observed magnitude of the value premium. The consumption-based CAPM developed in Breeden (1979) and Grossman and Shiller (1981) similarly fails to explain the value-growth spread, as well as variation in risk premia across a wider asset menu (see, for example, Mankiw and Shapiro (1986), Campbell (1996), and Lettau and Ludvigson (2001b)).

The violation of traditional asset pricing models in the cross-section of book-to-market sorted assets has spurred extensive academic research during the last two decades. This research was pioneered by a series of papers by Fama and French (1992, 1993, 1996) who argue that the spread in returns on high and low book-to-market firms proxies for some common sources of risks not captured by the CAPM betas, and use it as a risk-factor to explain the variation in risk premia across a broader set of assets. Their view was subsequently challenged by Daniel and Titman (1997) who show that an alternative, characteristics-based explanation is equally consistent with the observed asset pricing data. Lakonishok, Shleifer, and Vishny (1994) also depart from a risk-based premise, arguing that the value strategy delivered an extra reward in the past because naive market participants appeared to be overly optimistic about long-term growth of low book-to-market assets relative to value stocks. This conclusion, however, is hard to reconcile with the fact that the value anomaly has persisted for such a long time. It seems quite unlikely that growth investors have systematically failed in their forecasts of future growth differences between growth and value firms. Consequently, subsequent studies have tried to restore the validity of the risk-based argument. This literature relies on either conditional versions of traditional models as in Jagannathan and Wang (1996) and Lettau and Ludvigson (2001b), the ICAPM ideas (Campbell and Vuolteenaho (2004)), a duration-based premise (Lettau and Wachter (2005)), or simply on risk measures that allegedly are more robust to measurement errors in consumption data and slow adjustment of consumption decisions to economic news (Jagannathan and Wang (2005), Parker and Julliard (2005)). A number of recent studies highlight the role of low-frequency properties

of assets' cash flows in explaining the cross-sectional risk-return relation. Relying on different specifications for the joint dynamics of aggregate consumption and assets' dividends, Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2005), and Bansal, Dittmar, and Kiku (2005) find that value firms exhibit much higher exposure to permanent consumption risks than firms with low book-to-market characteristics. Thus, investors are willing to trade their holdings of growth stocks for value only if they are compensated for the extra long-run risk-bearing.

2.3 Other Phenomena of Value and Growth Data

Although the existing cross-sectional literature provides valuable insights about the origins of the value premium, it does not address other dimensions of value and growth data that are just as important for investment decision-making as is the spread in expected returns. These include differences in valuations of the two portfolios, volatilities of assets' returns and their correlation.

P/D Ratios

As Table I shows, even though value firms, on average, have quite sizeable growth of their cash flows, they usually sell at prices that are fairly low relative to current dividends. The mean of the log price/dividend ratio of value stocks is equal to 3.25, while that for growth assets is significantly higher, of about 3.61.³ According to the present value relation, asset valuations reflect expected dividend growth and the riskiness of the future dividend stream. Hence, any explanation of the cross-sectional price dispersion requires a clear understanding of time-series properties of assets' cash-flows, as well as agents' concerns about risks encoded in these cash flows.⁴

P/D Variance Decomposition

It is often argued in the literature that growth firms are highly exposed to discount-rate variation since their cash flows are delayed more into the future. Consequently, price/dividend ratios of low book-to-market firms are much more sensitive to variation in future expected returns than valuations of high book-to-market firms. The data, however, do not strongly support this view. I find that the percentage of variation in price/dividend ratios due to variation in future expected returns is very similar across the two portfolios. In particular, it is estimated at 0.39 (SE=0.27) for growth, compared to 0.32 (SE=0.20) for value firms. The decomposition is performed as in Cochrane (1992). Specifically, the fraction of variance of the log price/dividend ratio that comes

³In terms of levels, the average price/dividend ratio is about 27.6 and 43.2 for value and growth firms, respectively.

⁴Throughout the paper, the term "valuation" refers to the ratio of price to dividends.

from the variation in future expected returns is estimated by $\sum_{j=1}^J \varrho^j \frac{Cov(\log(P_t/D_t), -r_{t+1+j})}{Var(\log(P_t/D_t))}$, where the discount factor $\varrho = 1/[1+E(r)]$, and the lag length, J , is set to 15 years. Notice that even though quantitatively growth firms do exhibit somewhat higher exposure to discount-rate fluctuations relative to value firms, this dispersion is not significant due to the high degree of uncertainty in point estimates.

Volatilities and Correlations

It is well known from the volatility literature (see Shiller (1981), LeRoy and Porter (1981)) that prices are highly volatile relative to fundamentals. For example, as shown in the top panel of Table I, the sample standard deviation of value and growth returns varies from 20% to almost 30%, whereas the volatility of dividend growth rates is about half as low. The latter is about 14% for growth and 18% for value firms. Another pertinent aspect of the data is the finding that the unconditional correlation in ex-post assets' returns substantially exceeds that in assets' cash-flow growth rates. In particular, the correlation between growth rates of dividends of high and low book-to-market firms is about 32% compared to 75% for returns.

2.4 The Market-Equity Premium and the Risk-Free Rate

The behavior of the overall stock market is also known to exhibit some puzzling features. First, the average return on the market portfolio over the sample period is about 8.5%, which is much higher than the return on the short-term T-bill equal to 0.9% per annum. As shown in Mehra and Prescott (1985), the standard consumption-based model fails to simultaneously rationalize the observed high equity premium and low interest rates under any reasonable values of risk aversion and time discount factor.

2.5 Time-Varying Premia

Much empirical literature has documented that the premium on the market tends to be higher in recessions than during economic booms. There is, in fact, ample evidence that aggregate stock returns are forecasted by variables that either describe current or predict future economic activity (Fama and French (1989), Lettau and Ludvigson (2001a)). The spread in expected compensations on value and growth portfolios displays similar countercyclical fluctuations, especially in the post-war period. Figure 2 illustrates this evidence by plotting the spread in expected returns on value-minus-growth investment strategy along with the realized volatility of consumption. The latter is measured by the 3-year moving average of squared residuals from an AR(1) process fitted to

consumption growth data. The value premium is constructed by regressing the spread in realized returns on value and growth firms on lagged price/dividend ratios and dividend growth rates of the two stocks. In order to facilitate the comparison, the measure of consumption uncertainty is rescaled so that it has the same mean and standard deviation as the value premium. Notice that excluding several episodes in the 1950-60's, the spread in expected returns increases during "bad" times — when the uncertainty about consumption realizations is high. On the other hand, during times of low economic uncertainty, investors seem to reverse their expectations of the relative future performance of growth and value firms. The correlation between the value premium and the volatility of consumption for the post-war period is about 40%. For the expected excess return on the market, constructed in an analogous way as the value spread, this correlation is approximately the same, equal to 37%.

Traditional asset pricing models that assume time-invariant risk preferences of a representative agent along with constant ex-ante volatility of underlying cash flows are not able to accommodate these findings. Either premise has to be relaxed in order to account for the cyclical variation in asset prices. This is done in Campbell and Cochrane (1999), who allow for the time-varying risk aversion generated inside habit-formation preferences, as well as in Bansal and Yaron (2004), who instead depart from the i.i.d. assumption for dividend growth rates.

3 The Long-Run Risks Model

To provide a rational explanation for the above-mentioned stylized features of value, growth and aggregate equity portfolios, I adopt the long-run risk model of Bansal and Yaron (2004). The model is built on Epstein and Zin (1989) preferences. These are a generalization of the standard time-separable utility that relaxes the link between risk aversion and the elasticity of intertemporal substitution of a representative investor. Below, I will discuss in detail the importance of this separation; in short, it allows the model to assign distinct nontrivial prices to different sources of systematic risks. Another key ingredient of the model is the assumption that growth rates in the economy are driven by a small but highly persistent component. Shocks to this expected growth component are risks that investors fear the most — although quantitatively small, they have a long-lasting, near permanent effect on future levels of consumption. The amount of low-frequency risks embodied in assets' cash flows, therefore, is a major determinant of compensation in financial markets. In addition, to capture predictable variations in the observed risk premia, Bansal and Yaron assume a GARCH-type process for the conditional volatility of consumption and dividend growth rates.

3.1 Epstein-Zin Preferences

A representative agent in the Epstein-Zin framework maximizes her life-time utility, which is defined recursively as

$$V_t = \left[(1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t[V_{t+1}^{1-\gamma}] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \quad (1)$$

where C_t is consumption at time t , $0 < \delta < 1$ reflects the agent's time preferences, γ is the coefficient of risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ is the elasticity of intertemporal substitution (IES). Utility maximization is subject to the budget constraint,

$$W_{t+1} = (W_t - C_t) R_{c,t+1}, \quad (2)$$

where W_t is the wealth of the agent, and $R_{c,t}$ is the return on all invested wealth.

Given the preference structure, the intertemporal marginal rate of substitution (IMRS) for this economy is driven by the growth rate of consumption and the wealth return,

$$M_{t+1} = \delta^\theta (C_{t+1}/C_t)^{-\theta/\psi} R_{c,t+1}^{\theta-1}. \quad (3)$$

In equilibrium, dividends sum up to the consumption of the agent, $C_t = \sum_{i=1}^I D_{i,t}$, and the price of any security i is derived through the standard Euler equation:

$$E_t[M_{t+1} R_{i,t+1}] = 1. \quad (4)$$

It is implicitly assumed that human capital is a tradeable asset that delivers labor income as its dividends each time period. The time-series dynamics of labor income, therefore, can be inferred as the residual between aggregate consumption and the dividend stream on financial stock holding.

Taking the logarithm of (3), the pricing kernel can be written as

$$m_{t+1} \equiv \log(M_{t+1}) = \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \quad (5)$$

where Δc_t is consumption growth defined as the first difference of the log consumption, and $r_{c,t} \equiv \log(R_{c,t+1})$. Notice that the IMRS of Epstein and Zin preferences, in addition to consumption growth (as in the standard power utility), includes the endogenous return on the wealth portfolio.⁵ Thus, the notion of “good” and “bad” times in this framework may be quite different from the one in the time-separable specification. Here, the state of the economy depends not only on today's and tomorrow's consumption, but also on future investment and growth opportunities subsumed

⁵In the case of power utility, $\gamma = \frac{1}{\psi}$, consequently $\theta = 1$. The second term in (5) disappears and the IMRS is solely determined by consumption dynamics. Further, if $\gamma = 1$, the preferences collapse to the log utility.

in $r_{c,t+1}$. Consequently, predictable variations in all state variables that determine the time-series dynamics of consumption growth will importantly affect the current level of marginal utility, and, therefore, will be priced in equilibrium.

3.2 Cash-Flow Growth Rates

It is assumed that the conditional distribution of consumption and dividend growth rates varies over time. Specifically, I assume that predictable fluctuations in growth rates are governed by an AR(1) process x_t , while fluctuations in their second moments are driven by a common variance component σ_t^2 . Let Δd_t denote the growth rate of a given asset's cash flows. The joint dynamics of consumption and dividend growth rates is described as follows,

$$\begin{aligned}
 \Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1} \\
 \Delta d_{t+1} &= \mu + \phi x_t + \varphi \sigma_t u_{t+1} \\
 x_{t+1} &= \rho x_t + \varphi_x \sigma_t \epsilon_{t+1} \\
 \sigma_{t+1}^2 &= \sigma^2(1 - \nu) + \nu \sigma_t^2 + \sigma_w w_{t+1} \\
 \eta_{t+1}, u_{t+1}, \epsilon_{t+1}, w_{t+1} &\sim iid N(0, 1),
 \end{aligned} \tag{6}$$

where μ_c and μ are average growth rates of consumption and dividends, respectively. For simplicity, it is assumed that all shocks are orthogonal to each other, except I allow for the contemporaneous correlation between news in realized growth rates of consumption and dividends, which I denote by $\alpha \equiv Corr(\eta_t, u_t)$. In this specification, the two state variables, x_t and σ_t^2 , govern the dynamics of the conditional mean and variance of consumption growth, and φ_x and σ_w allow us to calibrate the amount of predictable variation in these moments. Parameter ϕ in the dividend growth equation reflects the degree of leverage on expected consumption growth, while φ captures the exposure of cash flows to volatility, as well as realized shocks in consumption.

3.3 Solving for Equilibrium Asset Prices

Given that growth rates are specified exogenously, finding solutions for price/consumption and price/dividend ratios is sufficient to describe equilibrium stock prices and returns in this economy. I solve for valuation ratios of consumption- and dividend-paying assets numerically using the quadrature-based method proposed by Tauchen and Hussey (1991). The idea of the method is to approximate the dynamics of the state variables with discrete Markov chains (details are provided in the Appendix). Asset valuations for each pair of $\{x_t, \sigma_t^2\}$ are then derived by exploiting the

Euler condition.⁶ I discretize the process for the expected growth component using a 30-point Gauss-Hermite quadrature, and assume that the volatility of consumption growth takes on 4 possible values.

To highlight the model's intuition, I first discuss some key analytical expressions for the implied moments of asset returns. After that, I present quantitative implications of the model based on numerical solutions.

3.4 Model Intuition

3.4.1 Assets' Valuations

Quasi-analytical solutions for the model can be obtained by recognizing that the log of the price/consumption ratio is approximately linear in the state variables:

$$z_{c,t} \equiv P_t/C_t = A_{c,0} + A_{c,1}x_t + A_{c,2}\sigma_t^2 . \quad (7)$$

The solution coefficients can be obtained by the method of undetermined coefficients using the Campbell and Shiller (1988b) approximation for the continuously compounded wealth return,

$$r_{c,t+1} = \kappa_{c,0} + \Delta c_{t+1} + \kappa_{c,1}z_{c,t+1} - z_{c,t} , \quad (8)$$

where $\kappa_{c,0}$ and $\kappa_{c,1}$ are constants of linearization, together with the log-linear equivalent of the Euler equation,

$$E_t[\exp(m_{t+1} + r_{c,t+1})] = 1 . \quad (9)$$

In particular, the elasticities of the price/consumption ratio with respect to expected growth and volatility shocks are given, respectively, by

$$A_{c,1} = \frac{1 - \frac{1}{\psi}}{1 - \kappa_{c,1}\rho} , \quad A_{c,2} = (1 - \gamma) \left(1 - \frac{1}{\psi}\right) \left[\frac{1 + \left(\frac{\kappa_{c,1}\varphi x}{1 - \kappa_{c,1}\rho}\right)^2}{2(1 - \kappa_{c,1}\nu)} \right] . \quad (10)$$

Notice that the effect of the expected growth component, x_t , on the valuation ratio is positive ($A_{c,1} > 0$) as long as the IES parameter, ψ , is greater than one. In this case, the substitution effect dominates, and, in response to good news about future economic growth, investors increase their demand for consumption asset driving up its price. Moreover, the higher the persistence in the expected growth component (captured by ρ), the larger the effect. Intuitively, if ρ is close to 1, shocks in x_t are perceived to have a long-lasting (near permanent) impact on future levels of

⁶The Euler equation can similarly be used to solve for the prices of constant maturity discount bonds that allows us to characterize the whole term structure of interest rates.

consumption, leading to a greater reaction in current prices.

The expression for $A_{c,2}$ is more complex and involves two preference parameters: the IES and the risk aversion of the representative agent. If both are greater than 1, an increase in economic uncertainty will lower asset valuations. This scenario is consistent with the empirical evidence in Bansal, Khatchatrian, and Yaron (2005) that asset prices fall during times of high consumption uncertainty. It should be emphasized that this parameter configuration, which is easily accommodated in the Epstein and Zin framework, would not be feasible in the case of time-separable utility that restricts $\gamma = \frac{1}{\psi}$. The expression for $A_{c,2}$ suggests that while preference parameters primarily determine the sign of the volatility effect on the price/consumption ratio, its magnitude is largely determined by the permanence of volatility shocks.

Solution coefficients for the valuation ratio on a dividend-paying asset can be derived analogously. In particular,

$$A_1 = \frac{\phi - \frac{1}{\psi}}{1 - \kappa_1 \rho}, \quad (11)$$

where κ_1 is the parameter of log-linearization for an asset's return. Notice that the effect of the expected growth news on the price/dividend ratio is further magnified by the leverage parameter ϕ . Similarly, high exposure of dividends to volatility shocks reinforces the impact of economic uncertainty on the price of a dividend-paying equity relative to that for consumption asset,

$$A_2 = \frac{(1 - \theta)A_{c,2}(1 - \kappa_{c,1}\nu) + 0.5[H_1 + H_2]}{1 - \kappa_1\nu}, \quad (12)$$

where $H_1 = \gamma^2 + \varphi^2 - 2\gamma\varphi\alpha$, and $H_2 = \left[\left((\theta - 1)\kappa_{c,1}A_{c,1} + \kappa_1A_1 \right) \varphi_x \right]^2$.

3.4.2 Systematic Risks and Their Pricing

The approximate analytical solution for the price/consumption ratio allows us to express the innovation in the pricing kernel in terms of the underlying risks,

$$m_{t+1} - E_t[m_{t+1}] = -\Lambda_\eta \sigma_t \eta_{t+1} - \Lambda_\epsilon \sigma_t \epsilon_{t+1} - \Lambda_w \sigma_w w_{t+1}, \quad (13)$$

where:

$$\begin{aligned} \Lambda_\eta &= \gamma \\ \Lambda_\epsilon &= \left(\gamma - \frac{1}{\psi} \right) \left[\frac{\kappa_{c,1} \varphi_x}{1 - \kappa_{c,1} \rho} \right] \\ \Lambda_w &= (1 - \gamma) \left(\gamma - \frac{1}{\psi} \right) \left[\frac{\kappa_{c,1} \left(1 + \left(\frac{\kappa_{c,1} \varphi_x}{1 - \kappa_{c,1} \rho} \right)^2 \right)}{2(1 - \kappa_{c,1} \nu)} \right]. \end{aligned} \quad (14)$$

There are three systematic shocks in the economy that, in general, command different risk compensations as shown in (14). The first risk comprises news about the realized growth rate of consumption, η_t . I will refer to these shocks as *short-run* risks since their effect on growth rates is purely transient. In contrast, the impact of expected growth rate shocks extends far beyond the current level of consumption — today’s news about expected growth rates will affect both short- and long-term consumption decisions of the agent. These risks, therefore, are labeled *long-run* risks. Finally, I call risks related to fluctuations in economic uncertainty *volatility* risks.

If preferences are constrained to the standard power utility (that sets $\gamma = \frac{1}{\psi}$), risks related to long-term growth and fluctuating economic uncertainty are not reflected in the innovation in the pricing kernel. Shocks in x_t and σ_t^2 still affect price/dividend ratios, however, in equilibrium, they do not carry a separate risk compensation. The price of consumption risks, in this case, is always positive and equivalent to the price in the standard C-CAPM.

By breaking the link between risk aversion and intertemporal substitution, non-expected utility preferences of Epstein and Zin allow the model to assign nontrivial distinct prices to all sources of risk. Intuitively, in the time-additive setting, agents are indifferent to when the uncertainty about future consumption is resolved. In essence, they have the same attitude (preferences) towards all systematic risks, independent of their intertemporal nature. In sharp contrast, investors’ concerns in the Epstein and Zin economy critically depend on the time-series properties of various consumption risks. In particular, if $(\gamma - \frac{1}{\psi}) > 0$, agents are more concerned with long-run growth risks, i.e., risks realized far into the future. The price of low-frequency consumption risks for this configuration of preference parameters, therefore, is positive. Moreover, the higher is the duration of these risks, the higher is the price required by investors. For ρ sufficiently close to 1, the magnitude of the long-run risks compensation may far exceed that for short-run fluctuations in consumption.

3.4.3 Equity and Value Premia in the Model

Given that asset returns and the IMRS are conditionally log-normal, the risk premium on an asset is determined by the conditional covariation of the asset return with the pricing kernel,

$$E_t[r_{t+1} - r_{f,t}] + \frac{\sigma_{r,t}^2}{2} = -Cov_t(m_{t+1}, r_{t+1}) , \quad (15)$$

where $r_{f,t}$ is the risk-free rate, and the second term on the left-hand side is a Jensen’s inequality adjustment. Using the solution for the price/dividend ratio, the premium can be expressed as follows,

$$E_t[r_{t+1} - r_{f,t}] + \frac{\sigma_{r,t}^2}{2} = \beta_\eta \Lambda_\eta \sigma_t^2 + \beta_\epsilon \Lambda_\epsilon \sigma_t^2 + \beta_w \Lambda_w \sigma_w^2, \quad (16)$$

where

$$\begin{aligned} \beta_\eta &= \varphi \alpha \\ \beta_\epsilon &= \kappa_1 A_1 \varphi_x \\ \beta_w &= \kappa_1 A_2. \end{aligned} \quad (17)$$

The expected excess return is determined by the loading on each risk factor (the beta) multiplied by the corresponding risk price. Assets' betas with respect to the three risks are determined endogenously by preference parameters and parameters that govern time-series dynamics of consumption and cash-flow growth rates. Further, since the volatility of consumption is time-varying, the implied risk premium fluctuates over time. Similarly, the cross-sectional spread in expected returns on any two assets varies across business cycles. In particular, let V and G label value and growth stocks, respectively. Using (16), the value premium can be approximately expressed as,

$$E_t[R_{V,t+1} - R_{G,t+1}] \simeq (\beta_{V,\eta} - \beta_{G,\eta}) \Lambda_\eta \sigma_t^2 + (\beta_{V,\epsilon} - \beta_{G,\epsilon}) \Lambda_\epsilon \sigma_t^2 + (\beta_{V,w} - \beta_{G,w}) \Lambda_w \sigma_w^2. \quad (18)$$

Equations (17) and (18) allow us to analyze the contribution of different risks to the cross-sectional spread in risk premia. Notice first that the difference in dividend exposures to short-run risks translates one-for-one into the difference in expected returns, as $\beta_{V,\eta} - \beta_{G,\eta} = \varphi_V \alpha_V - \varphi_G \alpha_G$. In contrast, the cross-sectional heterogeneity in cash-flow loadings on the expected growth component is amplified by the persistence of long-run risks: $\beta_{V,\epsilon} - \beta_{G,\epsilon} \approx \frac{\phi_V - \phi_G}{1 - \rho} \varphi_x$. Even if news about future expected growth in the economy is small ($\varphi_x \ll 1$) but highly persistent (ρ close to 1), a modest difference in dividend exposures to long-run consumption risks may be transmitted into quite a sizeable spread in expected returns. On top of this, the difference in expected rewards for long-run risk-bearing is magnified through the price channel as the price of low-frequency consumption risks, Λ_ϵ , also increases in the permanence parameter ρ .

3.4.4 Second Moments and Cross-Moments of Asset Returns

Predictable variations in systematic risks have an important bearing on the implied properties of second moments and cross-moments of asset returns. If consumption and cash-flow growth were i.i.d., the model would produce constant volatilities of equity returns simply equal to volatilities of dividend growth rates. Similarly, with time-invariant cost of capital, cross-sectional correlations in asset returns would exactly match correlations in asset's cash-flow growth. Both

outcomes, however, are fairly inconsistent with the stock market data. First, there is by now vast, irrefutable evidence of time-variation in conditional volatilities of financial returns (see, for example, Bollerslev, Chou, and Kroner (1992)). Second, as discussed in Section 2.3, empirical second moments and cross-moments of asset returns are significantly higher than the corresponding moments of dividend growth rates. These salient features of the data are easily accommodated once the i.i.d. assumption is relaxed. While persistent changes in expected growth rates allow the model to resolve the volatility puzzle, the channel of fluctuating economic uncertainty is required to justify high cross-sectional correlations in asset returns, and account for the time-varying volatility of stock returns. The intuition behind the first effect is revealed by expression (11). Shocks to the persistent expected growth component lead to a significant revision in agents' expectations of future economic growth and, consequently, result in large elasticity of stock prices with respect to growth rate news. Further, time-varying uncertainty about future growth prospects introduces an additional common source of risks in asset prices leading to more pronounced co-movements in ex-post assets' returns.

4 Calibration of the Model

To examine the ability of the model to quantitatively account for various phenomena of value, growth and market prices and returns, I solve it numerically for a chosen configuration of preference and time-series parameters.

I calibrate the model at a monthly frequency but evaluate its implications for the time-averaged annual data. This approach is consistent with the calibration exercises in Bansal and Yaron (2004), Campbell and Cochrane (1999) and Kandel and Stambaugh (1991), who likewise aim to match various features of annual data but assume that the decision interval of the agent is one month. I simulate $(74 * 12)$ months of artificial consumption and dividend data from the model. This corresponds to 74 years in the targeted sample discussed in Section 2. Simulated monthly observations are then aggregated to an annual frequency to calculate the implied annual moments of interest. I repeat this exercise 1000 times and report the empirical distribution of the estimated statistics.

The values of preference and time series parameters, reported in Table II, are chosen so as to capture various aspects of actual consumption and dividend data. I first motivate the choice of preferences and parameters that describe time-series dynamics of consumption growth, then provide details and empirical validation for the calibration of the cross-section of assets' cash flows.

4.1 Preferences

I set the time-discount factor of the agent to 0.999 and, consistent with admissible values of risk aversion considered by Mehra and Prescott (1985), I choose $\gamma = 10$. The remaining preference parameter is the elasticity of intertemporal substitution. The magnitude of the IES parameter has been a subject of intense debate in the financial literature. Hansen and Singleton (1982), Vissing-Jorgensen (2002), Vissing-Jorgensen and Attanasio (2003), and Guvenen (2005) estimate it well above 1, whereas Hall (1988) and Campbell (1999) advocate for much lower values of intertemporal substitution. However, as argued in Bansal and Yaron (2004), Hall and Campbell's estimates may suffer a significant downward bias since their models ignore possible fluctuations in the conditional volatility of consumption. Further, small values of the IES parameter would imply a negative response of the price of consumption claim to good growth prospects, and coupled with the risk aversion coefficient above 1 would lead to a positive elasticity of prices with respect to volatility shocks (see discussion in Section 3.4.1). To rule out these counterfactual outcomes, I choose the IES in excess of 1; specifically I set $\psi = 1.5$. As shown below, the IES above one is also required to resolve the risk-free rate puzzle as it allows the model to generate plausible level and volatility of interest rates.

4.2 Consumption Growth

4.2.1 Motivation and Empirical Support

It is quite common in the literature to assume that consumption growth is simply an i.i.d. process (e.g., Campbell and Cochrane (1999), Barberis, Huang, and Santos (2001)). This paper departs from the i.i.d. assumption and argues that long-run predictability of growth rates is essential for the model to quantitatively replicate a wide spectrum of time-series and cross-sectional asset market phenomena. It is, therefore, legitimate to ask whether the presumption of time-varying growth rates is consistent with observed data. This issue has been carefully examined in a number of recent studies including Bansal and Yaron (2000), Hansen, Heaton, and Li (2005), and Bansal, Gallant, and Tauchen (2004). Exploring different econometric techniques, these papers provide pervasive empirical evidence that there is indeed an important low-frequency component that helps account for the variation in realized consumption growth. I briefly illustrate this point in Figure 3. It plots two estimates of the spectral density of consumption growth: a parametric one, constructed by fitting an ARMA(1,1) process to the observed data (thin line),⁷ and a more flexible non-parametric estimate based on the Bartlett kernel (thick line). If consumption growth were i.i.d., the spectral density would be constant across all the frequencies. Figure 3, however, shows quite

⁷An ARMA(1,1) specification is nested in (6) as a special case when $\eta_t \equiv \epsilon_t$.

an opposite picture — the spectral density of consumption growth exhibits a pronounced peak at frequencies close to zero. In fact, the contribution to the sample variance of consumption growth of low-frequency component is several times larger than the contribution of fluctuations at higher (business cycles) frequencies. The i.i.d. assumption is further rejected in Bansal and Yaron (2004), and Bansal, Khatchatrian, and Yaron (2005), who document time-variation in realized volatility of consumption growth and show that it is significantly predicted by past price/dividend ratios. Taken together, this evidence suggests that specification of consumption growth that incorporates long-run predictability in the first two moments provides a more adequate description of the underlying transitional dynamics of consumption data than does a simplistic i.i.d. model.

4.2.2 Calibration of Consumption Growth

The dynamics of monthly consumption in equation (6) is calibrated so that the implied moments of annual growth rates match the corresponding statistics of actual consumption data. I set the unconditional mean and standard deviation of monthly growth rates equal to 0.0015 and 0.0064, respectively. The conditional mean of consumption growth is assumed to be fairly persistent ($\rho = 0.98$), but the amount of predictable variation in consumption growth is quite small ($\varphi_x = 0.032$). In particular, this configuration implies that, on a monthly basis, the variation in expected consumption growth accounts for less than 3% of the overall variation in realized consumption growth. I further assume that the volatility of consumption growth changes very slowly over time by setting the autoregressive coefficient in the variance dynamics to 0.99. Finally, I choose σ_w so as to approximately match the volatility of volatility of annual consumption growth. It should be pointed out, though, that this parameter is quite difficult to calibrate since the variation in the conditional volatility of consumption is hardly detectable once that data are time-averaged to annual frequency. The calibration of consumption growth is consistent with Bansal and Yaron (2004), and Bansal, Gallant, and Tauchen (2004).

4.2.3 Implied Consumption Dynamics

To assess how well the calibrated model is able to capture time-series properties of annual consumption data, in Table III I present the empirical distribution of various annual statistics computed from the simulated data. To facilitate the comparison, the first two columns report the corresponding statistics estimated from the actual data along with their robust standard errors. As the table shows, the distribution of the first two moments of consumption growth is well centered around data estimates. In particular, the volatility of consumption growth implied by the model is 2.16% compared to 2.20% in the data. The first and the second-order autocorrelations of annual consumption growth in the sample are about 0.44 and 0.16, respectively. Both values are easily

replicated by the model. Higher order serial correlations are statistically negligible in the data and in simulations.

4.3 Dividend Growth

I consider a small cross-section of assets consisting of growth and value stocks, plus the aggregate market. To reiterate, I use terms “growth” and “value” to refer to the extreme (low and high) book-to-market sorted portfolios. Since together these two do not span the whole market, the conditional distribution of the market portfolio is calibrated separately. I first report the chosen parameter values and evaluate the implied moments of annual dividend growth rates. After that, I will discuss in detail the empirical evidence that motivates the calibration of firms’ cash flows.

4.3.1 Calibration of Dividend Growth Rates

The calibration, details of which are provided in the bottom panel of Table II, is performed so as to match key unconditional moments of annual growth rates, as well as to replicate the degree of consumption leverage of assets’ dividends identified in the data. The unconditional means of monthly growth rates, μ ’s, are chosen to ensure that implied annual average growth rates match their data counterparts. The loading on the expected growth component, ϕ , is calibrated at 2.6 for growth stock, 6.2 for value stock and 2.8 for the market. The two remaining parameters, φ and α , govern the exposure of dividends to high-frequency consumption risks and risks coming from fluctuating economic uncertainty. I assign a somewhat higher value to $\varphi_{Growth}(= 8.4)$ and correspondingly lower values to $\varphi_{Value}(= 7.4)$ and $\varphi_{Market}(= 7.5)$. The correlation between realized consumption and dividend news is set to 0.27, 0.15 and 0.55 for growth, value and the market portfolios, respectively. As shown below, the choice of α ’s allows the model to replicate sample correlations in annual growth rates of consumption and assets’ cash flows. Finally, to adequately capture the correlation in realized growth rates across assets, I allow for the contemporaneous correlation in idiosyncratic dividend news. The correlation between dividend shocks orthogonal to realized news in consumption is set to 0.20 for growth and value assets, 0.80 for growth and the market portfolio, and 0.45 for value and aggregate market.

4.3.2 Implied Dynamics of Assets’ Cash Flows

Table IV reports the implied moments of time-averaged dividend growth rates along with their counterparts computed from the data. Overall, the model has no difficulties in capturing time-series properties of annual growth rates. There is one exception, however. The model generates somewhat excessive serial correlation in growth rates, especially for value stocks. High first-order

autocorrelation emerges as a result of the assumed high loading of value dividends on the persistent growth component. The conditional variation in x is quite small, so that the amount of predictable variation in monthly cash-flow growth is almost negligible. In particular, the implied predictability of monthly growth rates (i.e., R^2) is equal to about 1.7% for value stock and essentially zero for the other two assets. However, once the data are aggregated to lower frequencies, the effect of realized dividend news diminishes due to its transitory nature, whereas the impact of long-lasting expected growth shocks strengthens, leading to a much more pronounced predictability of time-averaged growth rates. A modification of dividend growth dynamics that allows for time-dependencies in realized dividend innovations may help the model lower the degree of serial correlation in simulated annual data. This, however, would require the introduction of an additional state variable. Notice that, although the first-order autoregressive parameters for growth and value stocks are not matched on average, their point estimates are still within the 95% confidence interval of the empirical distribution implied by the model. All the other moments are matched quite well, including the correlation between cash-flow and consumption growth rates. The model reproduces the same consumption-dividend correlation pattern as in the data with market portfolio having the highest correlation, followed by value and growth assets. In addition, the model is able to match cross-sectional sample correlations of assets' growth rates. As illustrated in Table V, cash flows of growth and market portfolios move quite closely over time — the correlation coefficient between their growth rates is about 80% in the data and in the model. In contrast, cash-flows of value firms are much less related to the time-series of market and growth dividends. The correlation between value-market and value-growth growth rates is 50% and about 30%, respectively.

4.3.3 Risks Exposures: Motivation and Empirical Support

In this section, I document that in the data, value firms exhibit higher exposure to long-run consumption risks, whereas growth firms load more strongly on short-run and discount-rate risks. I further show that the calibration of assets' cash flows in the model takes into account this empirical evidence.

Long-Run Risk Exposures

I estimate dividend exposures to long-run consumption risks by the projection coefficient from regressing cash-flow growth rates on the moving average of lagged consumption growth, i.e.,

$$\Delta d_t = d_0 + \tilde{\phi} \sum_{k=1}^2 \Delta c_{t-k} + \varepsilon_t . \quad (19)$$

I smooth consumption growth over the previous two years in order to capture risks related to

low-frequency (rather than short-term) fluctuations in consumption. Regression results, reported in the top panel of Table VI, show that the performance of value stocks is tightly linked to persistent fluctuations in consumption, whereas dividends paid by growth firms show virtually no exposure to long-run risks in aggregate consumption. The point estimate for high book-to-market stocks is equal to 2.16 compared to -0.38 for low book-to-market firms.⁸ Figure 4 helps visualize the differences in low-frequency properties of assets' dividends. It plots the 3-year moving average of asset-specific growth rates along with the corresponding smoothed growth in aggregate consumption.⁹ Consistent with the reported estimates, value firms closely track underlying persistent components in aggregate consumption. Low book-to-market firms, on the other hand, seem to contain much less information about long-run evolution of consumption growth. The cross-sectional heterogeneity in aggregate risks may naturally arise inside production economies with nonlinear adjustment costs as argued in Kyle (2004) and Zhang (2005). In particular, Zhang (2005) considers an economy where firms face higher cost of reducing than expanding capital. In his model, costly reversibility limits the ability of value firms (i.e., firms with high capital stock) to adjust their cash flows to technological shocks. Dividends paid out by value firms, therefore, strongly covary with aggregate output. Growth firms, on the other hand, are less constrained in smoothing their dividends over time as they have more incentives to invest than disinvest, and expanding capital is relatively cheap. Consequently, their dividends have much lower exposure to aggregate shocks.

The leverage parameter ϕ for growth, value and market assets in the model is chosen so as to capture the documented cross-sectional dispersion in long-run risk exposures. The Model panel of Table VI reports projection coefficients from equation (19) estimated using time-averaged simulated data. Long-run risk loadings implied by the model exhibit the same cross-sectional pattern and magnitude as in the data, confirming the calibration of this dimension of firms' dynamics.

Short-Run and Volatility Risks

The bottom panel of Table VI reveals that cash-flow exposures to short-run consumption risks and risks related to fluctuating economic uncertainty have a different cross-sectional pattern. These loadings are estimated by the correlation coefficient between residuals from regression (19) and contemporaneous innovations in consumption. The latter are computed by filtering consumption growth data through an AR(1) process. In the data and in the model, the market portfolio has the highest exposure to realized consumption shocks. Cash flows of value and growth firms seem to be

⁸Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2004), and Bansal, Dittmar, and Kiku (2005) similarly find that the amount of low-frequency growth risks increases from low to high book-to-market sorted portfolios.

⁹In order to facilitate the comparison, consumption data are rescaled in the two subfigures.

less sensitive to contemporaneous news in consumption. This is not surprising, as innovations in individual asset dividends are also driven by the firm-specific news not necessarily related to the current state of the economy. Between the two, growth firms have somewhat higher exposure to innovation risks. In the model, the innovation correlation for growth and value firms is centered around 0.33 and 0.28, respectively, which is quite comparable to 0.37 and 0.30 estimated in the data.

5 Asset Pricing Implications

Table VII presents key asset pricing implications of the model — expected returns, standard deviations and log price/dividend ratios for value, growth and market equity.

The Unconditional Value Premium

The average return on growth stock implied by the model is about 6.1%, while for value stock it is about 11.4% per annum. Consequently, the model-implied value premium is about 5.3%, which is fairly comparable to 6.1% observed in the data.

What drives the value premium in the model? As discussed in Section 3, assets are subject to three types of systematic risks. Risks that investors dislike the most and, therefore, demand higher reward for, are risks related to the long-run growth in the economy. Consequently, the risk premium on any asset is largely determined by the exposure of the asset's cash flows to low-frequency fluctuations in consumption.

Since value stocks load more strongly on the persistent growth component, they have to provide investors with high ex-ante compensation. In contrast, growth firms have higher exposure to realized consumption news and risks related to fluctuating economic uncertainty. This, however, does not quantitatively overweight the effect of low-frequency risks. As discussed in Bansal (2004), the magnitude of prices assigned to short-run and volatility shocks is much lower than that for slow-moving fluctuations in consumption. This is quite intuitive. First, investors require quite modest compensation for business cycle risks, as such risks do not alter growth prospects in the economy. Second, bad news about future economic uncertainty raises discount rates, and this increase has a twofold effect. A rise in the cost of capital lowers asset valuations as future cash flows are now discounted at a higher rate. The decrease in wealth, however, gets partially compensated by improved future opportunity set. As the result of these offsetting forces, volatility risks receive a low, in our calibration even negative, price. Time-varying economic uncertainty still contributes positively towards the risk premium since assets' betas with respect to volatility shocks are also negative. Moreover, even though growth stocks in the calibration have higher exposure to risks

pertaining to consumption uncertainty, the volatility-risk premium they provide does not exceed that offered by value firms as volatility betas are also nontrivially affected by assets' exposures to low-frequency consumption risks.

The contribution of each source of risk to the value premium can be roughly quantified using the approximate analytical solutions. I find that about 85% of the value spread is attributed to the value-growth dispersion in long-run consumption risks, about -10% is due to the difference in loadings on short-run risks, and the remaining 25% is due to the dispersion in volatility risks.¹⁰ Thus, the documented differential exposure to long-run growth risks is the key mechanism that accounts for an extra premium on value relative to growth stocks.

Value-Growth P/D Ratios

The bottom panel of Table VII shows that the model is also able to capture the observed differences in assets' price/dividend ratios. As can be seen, growth stocks, on average, have significantly higher ratios of prices to fundamentals than value stocks. The mean of the log valuation ratio across simulations is about 3.61 for growth stocks and only 3.25 for value stocks.

In the model, as in the data, value firms, on average, have substantially higher growth rate of dividends than the other two assets. Value strategies, however, are perceived by investors as quite risky, especially in the long run, as cash flows of value firms are tightly linked to persistent fluctuations in consumption. The future expected flow of their dividends is therefore discounted at a relatively high rate leading to low present values. Investing in growth firms, on the other hand, is considered less risky as the variation of their cash flows is dominated by idiosyncratic and transient news rather than long-lasting aggregate shocks. Hence, investors are willing to buy growth stocks at prices that are fairly high relative to current dividends, even despite low average growth of the firms' cash flows.

Consistent with the data, the model-implied price/dividend ratios are highly persistent. The first-order autocorrelation coefficient ranges from 0.78 for value asset to 0.89 for growth and market equities.

Value-Growth P/D Variance Decomposition

The variation in assets' valuation ratios in the model is mostly driven by the variation in expected cash-flow growth rather than future discount rates. In particular, the fraction of variance of price/dividend ratios due to fluctuations in expected returns, on average, is equal to about 0.18

¹⁰In other words, the overall value premium can be approximately decomposed as $5.3\% \approx -0.6\% + 4.6\% + 1.3\%$, where the three components represent the contribution of short-, long-run and volatility risk, respectively (see equation (18)).

for growth and 0.22 for value stocks. These are somewhat lower than the corresponding quantities in the data. Furthermore, in contrast to the empirical evidence, the model-implied valuations of high book-to-market firms, to some extent, are more sensitive to the variation in future discount rates than price/dividend ratios of growth firms. These statistics, however, should be treated with great caution as their standard deviations across simulations are very large, as well as is the uncertainty in data estimates.

Value-Growth Sharpe Ratios

It is quite common among practitioners to view value stocks as providing a better deal to investors relative to growth assets. Indeed, the historical Sharpe ratio of value firms is significantly higher than the compensation per unit of risks offered by growth firms. This measure of relative performance, however, may be very misleading as it does not account for the composition of idiosyncratic and various systematic risks in assets' returns.

The long-run risk model also implies that value firms have higher Sharpe ratio than do firms with low book-to-market characteristics. In particular, the Sharpe ratio, across simulations, averages about 0.34 for value stocks versus only 0.20 for growth assets. Given this difference, orthodox mean-variance investors would find it advantageous to tilt their portfolio holdings towards value firms. In contrast, risk-averse Epstein-Zin investors would not follow such a delusory strategy, especially if the investment horizon is long. Instead, they would optimally mix value and growth firms so as to balance the exposure of their portfolios to different consumption risks.

Violation of the CAPM/C-CAPM

In the model, as in the data, both the CAPM and the C-CAPM fail to explain the difference in risk premia on value and growth assets. The model-implied market beta of value stocks, on average, is lower than that for growth stocks. Similarly, high book-to-market firms have lower correlation of their returns with contemporaneous consumption growth relative to growth firms. Table VIII reports the ratio of the CAPM betas of value and growth firms, $\beta_{\text{Value}}/\beta_{\text{Growth}}$, and the corresponding ratio of their consumption betas. Both quantities fall well below one; the first is equal around 0.92, while the second is about 0.85.

What allows the model to encompass the violation of the standard betas? As discussed above, the risk premium on any asset in the model is determined by the asset's betas with respect to the three consumption risks (short-run, long-run risks and risks related to fluctuating economic uncertainty), each one of which carries a separate compensation. Once the model is compressed to a one-factor model, such as the CAPM or C-CAPM, this important risk pricing information gets

entirely distorted. While the risk premium arises largely as a reward for long-run risks bearing, a single risk-measure is dominated by high-frequency properties of assets' cash flows and, therefore, fails to account for the cross-sectional differences in average returns.

The Unconditional Market-Equity Premium and the Risk-Free Rate

The model also does a decent job in capturing the dynamics of the overall equity market and the riskless asset. In particular, the average return on the market portfolio over the risk-free rate, in the model, is about 6% per annum, which is reasonably close to the observed market premium. Importantly, the expected excess return on the market, as well as the value spread are not driven by the term premium as the latter is small, in fact, slightly negative in the model.

The average risk-free rate across simulations is about 1.58%. Although somewhat higher than its sample counterpart (0.91%), it lies within two standard errors bounds of the data estimate. The model-implied volatility of the interest rate is about 0.90%, compared to 1.22% in the data. Both the mean and the volatility of the risk-free rate depend on the elasticity of intertemporal substitution. Increasing the IES would help further lower the level of the interest rate; however, it would also decrease its variability. The chosen value of the IES parameter seems a reasonable compromise between these two ends.

Volatilities and Correlations of Assets' Returns

The model also correctly predicts the spread in return volatilities as shown in Table VII. The standard deviation of low book-to-market returns is about 10% smaller than that for firms with high book-to-market characteristics; the implied volatility of the market portfolio matches that in the data.

As discussed in Section 2.3, the correlation in realized returns on value and growth is significantly larger than the correlation in assets' dividend growth rates. This also holds for value-market and growth-market pairs of assets. Table IX examines the model's ability to account for the differences in correlation structures of firms' cash flows and returns. The contemporaneous correlation between high and low book-to-market returns in the model is centered around 44%. Although this is much smaller than 75% in the data, the model makes an important step towards capturing the gap between dividend and return dynamics in the cross-section. Recall that inside the model, the correlation in cash-flow growth rates of value and growth firms is about 30%. This figure is amplified to a significantly higher 44%. Similarly, the model-implied correlations between value-market and growth-market returns are larger than correlations in their dividend growth rates. This magnification is carried through the channel of time-varying economic uncertainty.

Predictable shifts in future discount rates introduce an additional source of common variation in asset prices, inducing higher covariation in ex-post returns across assets. A richer specification of cash-flow dynamics would allow the model to even better fit this aspect of the data, however, it would require additional state variables.

Conditional Market-Equity and Value Premia

The long-run risks model implies that both the market-equity premium and the value premium rise during times of high economic uncertainty and shrink in “good” times. In order to quantitatively assess this implication, I simulate 1000 years of monthly data, aggregate this artificial sample to an annual frequency and use it to estimate the volatility of consumption growth, expected excess returns on the market, and the value premium in the same way as has been done in Section 2.5 for the observed data. Specifically, I fit an AR(1) process to annualized consumption growth and construct a measure of realized consumption volatility by averaging the AR(1) squared residuals over the previous 3 years. The value and market-equity premia are estimated by regressing simulated annual returns on lagged price/dividend ratios and dividend growth rates of the corresponding assets. Figure 5 plots the model-implied value premium along with the implied volatility of consumption. Consistent with the empirical evidence illustrated previously in Figure 2, the model-implied spread between expected returns on value and growth firms exhibit countercyclical fluctuations. The correlation between the value premium and realized consumption volatility in the model is equal to 36%. For the market-equity premium this correlation is about 32%. Both numbers are quite close to the post-war data estimates — 40% and 37% for the value and aggregate equity premia, respectively.

Predictability Evidence

Price/dividend ratios in the model are negatively related to future asset returns, especially at long investment horizons. This is in line with empirical evidence on return predictability in Campbell and Shiller (1988b, 1988a), Fama and French (1988) and Hodrick (1992). The left set of columns of Table X reports estimated projection coefficients and adjusted- R^2 's from regressing 1- and 5-year ahead returns on the log of current price/dividend ratios for each asset. The two right columns present the corresponding estimates averaged over simulated samples. The model-implied slope coefficients are negative and increase in magnitude as the horizon lengthens. As in the data, the predictive power of price/dividend ratios seem to improve with the horizon. The magnitude of both R^2 and regression coefficients in the model, though, is somewhat smaller than in the data. However, given the high degree of uncertainty in point estimates and R^2 's, the model performs quite well in capturing this important dimension of the data.

6 Conclusions

This paper emphasizes the importance of intertemporal, specifically long-run, properties of assets' cash flows for understanding various intriguing phenomena of asset market data along both time-series and cross-sectional dimensions. The empirical investigation focuses on three stocks — value, growth and market portfolios, and relies on the model proposed by Bansal and Yaron (2004) that incorporates long-run consumption risks within the Epstein-Zin utility specification. I show that when growth rate dynamics are calibrated so as to fit the time-series behavior of consumption and firms' cash flows, the model is able to quantitatively replicate differences in average returns, price/dividend and Sharpe ratios of value and growth firms, the high premium on the market and the low risk-free rate, high volatilities and correlations of stock returns, as well as to capture cyclical variation in the market-equity and value premia.

The paper argues that the riskiness and, consequently, the premium on value and growth stocks are largely driven by the amount of long-run consumption risks embodied in firms' cash flows. In the model, as in the data, value firms exhibit high exposure to low-frequency fluctuations in aggregate consumption, and, therefore, have to provide investors with high ex-ante compensation. Growth firms, on the other hand, carry a significantly smaller premium as their cash flows are less sensitive to persistent movements in consumption. The paper thereby provides a rational explanation of the observed value premium, which traditional CAPM and C-CAPM frameworks have failed to resolve.

A final remark concerns the two underlying assumptions of the model. It should be emphasized that both ingredients, a persistent component in consumption-dividend growth dynamics and recursive utility preferences, are equally responsible for the empirical success of the model. While the first accommodates investors' concerns about future growth prospects, the second allows for the reflection and manifestation of these concerns in financial markets. If one of the assumptions were simplified, the model would not be able to pass either time-series or cross-sectional tests raised by the data.

Appendix: Quadrature Approximation

We solve the model using the method of Tauchen and Hussey (1991), based on a discrete approximation of the conditional density of a continuous stochastic process. To illustrate the method, consider a continuous random variable y with the law of motion given by the density $f(y|x)$ conditional on the observed history of y ,

$$1 = \int f(y|x)dy , \quad (20)$$

where x denotes the set of conditioning values. Our objective is obtain an “equivalent” discrete representation of y . Consider the following transformation of (20),

$$1 = \int f(y|x)dy = \int \frac{f(y|x)}{\omega(y)} \omega(y)dy , \quad (21)$$

where ω is some strictly positive weighting function. This integral can now be approximated with a finite sum,

$$1 \approx \sum_{k=1}^N \frac{f(y_k|x)}{\omega(y_k)} w_k = \sum_{k=1}^N \pi(y_k|x) , \quad (22)$$

where y_k and w_k are the nodes and the weights of an N -point quadrature rule for the density $\omega(y)$, and

$$\pi(y_k|x) = \frac{f(y_k|x)}{\omega(y_k)} w_k . \quad (23)$$

The nodes and weights are chosen in such a way that the approximation is exact when the function $F(y|x) \equiv \frac{f(y|x)}{\omega(y)}$ is a polynomial of order $(2N - 1)$. Notice that expression (22) implies that $\pi(y_k|x)$ is an estimate of the transitional probability from the current state $x = \{y_j\}$ to state y_k . However, since the quadrature is just an approximation, $\pi(y_k|x)$ may not necessarily sum up to 1. To facilitate the interpretation, Tauchen and Hussey (1991) propose re-normalize $\pi(y_k|x)$,

$$\tilde{\pi}(y_k|x) = \frac{\pi(y_k|x)}{s(x)} , \quad s(x) = \sum_{k=1}^N \pi(y_k|x) , \quad (24)$$

so that $\sum_{k=1}^N \tilde{\pi}(y_k|x) = 1$. This now allows us to interpret $\{y_k, \tilde{\pi}(y_k|x)\}_{k=1}^N$ as a Markov chain that approximates the continuous process y .

The accuracy of the approximation hinges on several important issues. The first one involves the weighting function ω . Although different alternatives are plausible, in our empirical work we use $\omega(y) = f(y|\bar{y})$, where \bar{y} is the unconditional mean of the process. As argued in Tauchen and Hussey (1991) this choice, in general, allows to achieve a better quality of approximation. Another pertinent issue concerns the choice of N . There is, however, no common rule as to how many grid points to use to achieve a desired level of accuracy. The optimal N depends on dynamic properties of the process that is discretized and should be chosen in each case individually.

References

- Bansal, Ravi, 2004, Long run risks and risk compensation in equity markets, in Rajnish Mehra, eds.: *Handbook of Investments: Equity Risk Premium*, Blackwell, North Holland, Amsterdam.
- Bansal, Ravi, Robert F. Dittmar, and Dana Kiku, 2005, Long run risks and equity returns, Working paper, Duke University.
- Bansal, Ravi, Robert F. Dittmar, and Christian Lundblad, 2005, Consumption, dividends, and the cross-section of equity returns, *Journal of Finance* 60, 1639–1672.
- Bansal, Ravi, A. Ronald Gallant, and George Tauchen, 2004, Rational pessimism, rational exuberance, and markets for macro risks, Working paper, Duke University.
- Bansal, Ravi, Varoujan Khatchatrian, and Amir Yaron, 2005, Interpretable asset markets?, *European Economic Review* 49, 531–560.
- Bansal, Ravi, and Amir Yaron, 2000, Risks for the long run: A potential resolution of asset pricing puzzles, *NBER working paper* 8059.
- Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.
- Barberis, Nicholas, Ming Huang, and Tano Santos, 2001, Prospect theory and asset prices, *Quarterly Journal of Economics* 116, 1–53.
- Bollerslev, Tim, Ray Y. Chou, and Kenneth F. Kroner, 1992, ARCH modeling in finance: A review of the theory and empirical evidence, *Journal of Econometrics* 52, 5–59.
- Breeden, Douglas T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–296.
- Campbell, John, 1996, Understanding risk and return, *Journal of Political Economy* 104, 298–345.
- Campbell, John, and Robert Shiller, 1988a, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–227.
- Campbell, John, and Robert Shiller, 1988b, Stock prices, earnings, and expected Dividends, *Journal of Finance* 43, 661–676.
- Campbell, John Y., 1999, Asset prices, consumption and the business cycle, in John B. Taylor, and Michael Woodford, eds.: *Handbook of Macroeconomics*, Elsevier Science, North-Holland, Amsterdam.
- Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–255.
- Campbell, John Y., and Tuomo Vuolteenaho, 2004, Bad beta, good beta, *American Economic Review* 94, 1249–1275.

- Cochrane, John H., 1992, Explaining the variance of price-dividend ratios, *The Review of Financial Studies* 5, 243–280.
- Cochrane, John H., 2005, Financial markets and the real economy, Working paper, University of Chicago.
- Daniel, Kent, and Sheridan Titman, 1997, Evidence on the characteristics of cross sectional variation in stock returns, *Journal of Finance* 52, 1–33.
- Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, risk aversion, and the intertemporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.
- Fama, Eugene, and Kenneth French, 1988, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3–27.
- Fama, Eugene, and Kenneth French, 1989, Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics* 25, 23–49.
- Fama, Eugene, and Kenneth French, 1992, The cross-section of expected stock returns, *Journal of Finance* 59, 427–465.
- Fama, Eugene, and Kenneth French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.
- Fama, Eugene, and Kenneth French, 1996, Multifactor explanations of asset pricing anomalies, *Journal of Finance* 51, 55–84.
- Graham, Benjamin, and David L. Dodd, 1934, *Security Analysis*. (McGraw Hill New York).
- Grossman, Sanford J., and Robert J. Shiller, 1981, The determinants of the variability of stock market prices, *American Economic Review* 71, 222–227.
- Guvenen, Fatih, 2005, Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective, *Journal of Monetary Economics*, forthcoming.
- Hall, Robert E., 1988, Intertemporal substitution in consumption, *Journal of Political Economy* 96, 339–357.
- Hansen, Lars, John Heaton, and Nan Li, 2004, Consumption strikes back?, Working paper, University of Chicago.
- Hansen, Lars, John Heaton, and Nan Li, 2005, Consumption strikes back?: Measuring long-run risks, *NBER working paper* 11476.
- Hansen, Lars Peter, and Kenneth Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica* 63, 767–804.
- Hodrick, Robert, 1992, Dividend yields and expected stock returns: Alternative procedures for inference and measurement, *Review of Financial Studies* 5, 357–386.

- Jagannathan, Ravi, and Yong Wang, 2005, Consumption risk and the cost of equity capital, *NBER working paper* 11026.
- Jagannathan, Ravi, and Zhenyu Wang, 1996, The conditional CAPM and the cross-section of expected returns, *Journal of Finance* 51, 3–54.
- Kandel, Eugene, and Robert Stambaugh, 1991, Asset returns and intertemporal preferences, *Journal of Monetary Economics* 27, 39–71.
- Kyle, Albert S., 2004, A two-factor model of value and growth with adjustment costs, Working paper, Duke University.
- Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny, 1994, Contrarian investment, extrapolations, and risk, *Journal of Finance* 49, 1541–1578.
- LeRoy, Stephen, and Richard Porter, 1981, The present value relation: Tests based on implied variance bounds, *Econometrica* 49, 555–574.
- Lettau, Martin, and Sydney Ludvigson, 2001a, Consumption, aggregate wealth, and expected stock returns, *Journal of Finance* 56, 815–850.
- Lettau, Martin, and Sydney Ludvigson, 2001b, Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying, *Journal of Political Economy* 109, 1238–1287.
- Lettau, Martin, and Jessica A. Wachter, 2005, Why is long-horizon equity less risky? A duration-based explanation of the value premium, Working paper, Stern School of Business, Warton School.
- Mankiw, Gregory N., and Matthew D. Shapiro, 1986, Risk and return: consumption beta versus market beta, *Review of Economics and Statistics* 68, 452–459.
- Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–162.
- Menzly, Lior, Tano Santos, and Pietro Veronesi, 2004, Understanding predictability, *Journal of Political Economy* 112(1), 1–47.
- Parker, Jonathan A., and Christian Julliard, 2005, Consumption risk and the cross-section of expected returns, *Journal of Political Economy* 113(1), 185–222.
- Shiller, Robert J., 1981, Do stock prices move too much to be justified by subsequent changes in dividends?, *American Economic Review* 71, 421–436.
- Tauchen, George, and Robert Hussey, 1991, Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models, *Econometrica* 59, 371–396.
- Vissing-Jorgensen, Annette, 2002, Limited asset market participation and the elasticity of intertemporal substitution, *Journal of Political Economy* 110, 825–853.

Vissing-Jorgensen, Annette, and Orazio P. Attanasio, 2003, Stock-market participation, intertemporal substitution, and risk-aversion, *American Economic Review, Papers and Proceedings* pp. 383–391.

Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103.

Table I
Summary Statistics

Panel A: MEANS / VOLATILITIES

	Returns		Div. Growth		Log(P/D)
	Mean	StDev	Mean	StDev	Mean
Growth	7.81 (1.98)	20.2 (2.00)	0.68 (1.25)	13.9 (2.24)	3.61 (0.18)
Value	13.88 (1.74)	29.9 (4.34)	3.63 (3.06)	18.1 (2.69)	3.25 (0.12)
Market	8.56 (1.79)	20.1 (2.23)	0.85 (0.95)	10.9 (2.41)	3.34 (0.13)

Panel B: CORRELATIONS

	Returns			Dividend Growth		
	Growth	Value	Market	Growth	Value	Market
Growth	1			1		
Value	0.75 (0.05)	1		0.32 (0.17)	1	
Market	0.95 (0.01)	0.87 (0.04)	1	0.80 (0.09)	0.53 (0.10)	1

Table I presents descriptive statistics for returns, dividend growth rates and logarithms of price/dividend ratios on value and growth firms, and the aggregate stock market. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. Returns are value-weighted, price/dividend ratios are constructed by dividing the end-of-year price by the cumulative annual dividend, growth rates are constructed by taking the first difference of the logarithm of dividend series. All data are annual, expressed in real terms, and cover the period from 1930 to 2003. Panel A reports means and standard deviations. Panel B presents the correlation structure of returns and cash-flow growth rates for the three stocks. Robust standard errors, reported in parentheses, are calculated using the Newey-West variance-covariance estimator with 8 lags.

Table II
Model Parameters

PREFERENCES					
δ	γ	ψ			
0.999	10	1.5			

CONSUMPTION					
μ	ρ	φ_x	σ	ν	σ_w
0.0015	0.98	0.032	0.0064	0.99	0.0000017

DIVIDENDS				
	μ_i	ϕ_i	φ_i	α_i
Growth	0.0009	2.6	8.4	0.27
Value	0.0019	6.2	7.4	0.15
Market	0.0012	2.8	7.5	0.55

Table II reports the chosen configuration of preferences: the time-discount factor δ , the coefficient of risk aversion γ , the elasticity of intertemporal substitution ψ , along with time-series parameters of the model:

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1} \\ x_{t+1} &= \rho x_t + \varphi_x \sigma_t \epsilon_{t+1} \\ \sigma_{t+1}^2 &= \sigma^2(1 - \nu) + \nu \sigma_t^2 + \sigma_w w_{t+1} \\ \Delta d_{i,t+1} &= \mu_i + \phi_i x_t + \varphi_i \sigma_t u_{i,t+1} \\ \alpha_i &= \text{Cov}(\eta, u_i) \\ i &= \{Growth, Value, Market\} \end{aligned}$$

The decision interval of the investor is one month.

Table III
Implied Moments of Consumption Growth

Statistic	— Data —	— Model —
$E[\Delta c]$	1.96 (0.32)	1.86 (0.64)
$\sigma(\Delta c)$	2.20 (0.45)	2.16 (0.48)
$AC(1)$	0.44 (0.12)	0.43 (0.12)
$AC(2)$	0.16 (0.15)	0.20 (0.15)

Table III reports various moments of annual consumption growth implied by the model and from the data. $E[\Delta c]$ and $\sigma(\Delta c)$ are the mean and the volatility of consumption growth, $AC(1)$ and $AC(2)$ are the 1st and 2nd-order autocorrelation coefficients. Consumption data, taken from the BEA, are real, annual per-capita expenditure of nondurables and services that cover the period from 1930 to 2003. Growth rates are constructed by taking the first difference of the logarithm of consumption. Robust standard errors of data estimates, reported in parentheses, are calculated using the Newey-West variance-covariance estimator with 8 lags. Model-implied statistics are based on 1000 simulated samples, each with 74×12 monthly observations, aggregated to an annual frequency. The entries in the Model column represent means and standard deviations (in parentheses) of the corresponding statistics across simulations.

Table IV
Implied Moments of Dividend Growth Rates

Asset	Statistic	— Data —	— Model —
Growth	$E[\Delta d]$	0.68 (1.25)	1.05 (2.63)
	$\sigma(\Delta d)$	13.9 (2.24)	14.1 (3.37)
	$AC(1)$	0.08 (0.14)	0.19 (0.13)
	$Corr(\Delta c, \Delta d)$	0.33 (0.14)	0.33 (0.13)
Value	$E[\Delta d]$	3.63 (3.06)	2.38 (3.83)
	$\sigma(\Delta d)$	18.1 (2.69)	15.2 (3.15)
	$AC(1)$	0.06 (0.16)	0.29 (0.14)
	$Corr(\Delta c, \Delta d)$	0.42 (0.06)	0.38 (0.13)
Market	$E[\Delta d]$	0.85 (0.95)	1.51 (2.50)
	$\sigma(\Delta d)$	10.9 (2.41)	13.2 (3.36)
	$AC(1)$	0.18 (0.13)	0.20 (0.13)
	$Corr(\Delta c, \Delta d)$	0.59 (0.16)	0.57 (0.10)

Table IV reports various moments of annual dividend growth of value and growth firms, and the aggregate market implied by the model and from the data. $E[\Delta d]$ and $\sigma(\Delta d)$ are the mean and the volatility of dividend growth, $AC(1)$ is the 1st-order autocorrelation, and $Corr(\Delta c, \Delta d)$ denotes the correlation between dividend and consumption growth rates. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. Consumption data, taken from the BEA, are per-capita expenditure of nondurables and services. All data are annual, expressed in real terms, and cover the period from 1930 to 2003. Growth rates are constructed by taking the first difference of the logarithm of dividend and consumption series. Robust standard errors of data estimates, reported in parentheses, are calculated using the Newey-West variance-covariance estimator with 8 lags. Model-implied statistics are based on 1000 simulated samples, each with 74×12 monthly observations, aggregated to an annual frequency. The entries in the Model column represent means and standard deviations (in parentheses) of the corresponding statistics across simulations.

Table V
Implied Correlations in Dividend Growth Rates

	Data			Model		
	Growth	Value	Market	Growth	Value	Market
Growth	1			1		
Value	0.32 (0.17)	1		0.31 (0.14)	1	
Market	0.80 (0.09)	0.53 (0.10)	1	0.80 (0.06)	0.50 (0.16)	1

Table V reports pair-wise correlations between dividend growth rates of value and growth firms, and the aggregate market implied by the model and from the data. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. The data are annual, expressed in real terms, and cover the period from 1930 to 2003. Growth rates are constructed by taking the first difference of the logarithm of dividend series. Robust standard errors of data estimates, reported in parentheses, are calculated using the Newey-West variance-covariance estimator with 8 lags. Model-implied statistics are based on 1000 simulated samples, each with 74×12 monthly observations, aggregated to an annual frequency. The entries in the Model panel represent means and standard deviations (in parentheses) of the corresponding statistics across simulations.

Table VI
Risks Exposures

Panel A: LONG-RUN RISK EXPOSURES				
Asset	— Data —		— Model —	
Growth	-0.38	(1.34)	1.01	(1.41)
Value	2.16	(1.44)	2.43	(1.29)
Market	0.66	(1.20)	1.19	(1.35)

Panel B: INNOVATION RISK EXPOSURES				
Asset	— Data —		— Model —	
Growth	0.37	(0.14)	0.33	(0.14)
Value	0.30	(0.07)	0.28	(0.13)
Market	0.58	(0.15)	0.53	(0.11)

Table VI presents risk exposures of value and growth firms, and the aggregate market estimated from the data along with the corresponding estimates implied by the model. Panel A reports estimated projection coefficients from regressing the growth rate of dividends on the two-year moving average of past consumption growth, i.e.,

$$\Delta d_t = d_0 + \tilde{\phi} \sum_{k=1}^2 \Delta c_{t-k} + \varepsilon_t.$$

Panel B presents the correlation between dividend innovations from the above regression with the contemporaneous news in consumption growth, computed by fitting an AR(1) process to consumption growth. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. Consumption data, taken from the BEA, are per-capita expenditure of nondurables and services. All data are annual, expressed in real terms, and cover the period from 1930 to 2003. Growth rates are constructed by taking the first difference of the logarithm of dividend and consumption series. Robust standard errors of data estimates, reported in parentheses, are calculated using the Newey-West variance-covariance estimator with 4 lags. Model-implied statistics are based on 1000 simulated samples, each with 74×12 monthly observations, aggregated to an annual frequency. The entries in the Model column represent means and standard deviations (in parentheses) of the corresponding estimates across simulations.

Table VII
Asset Pricing Implications

Statistic	Asset	— Data —		— Model —	
$E(R)$	Growth	7.81	(1.98)	6.07	(2.91)
	Value	13.88	(1.74)	11.36	(4.30)
	Market	8.56	(1.79)	7.53	(2.69)
	Risk-free	0.91	(0.39)	1.58	(0.01)
$\sigma(r)$	Growth	20.2	(2.00)	21.5	(4.90)
	Value	29.9	(4.34)	29.0	(6.13)
	Market	20.1	(2.23)	20.1	(4.35)
	Risk-free	1.22	(0.31)	0.90	(0.00)
$E(pd)$	Growth	3.61	(0.18)	3.65	(0.06)
	Value	3.25	(0.12)	3.10	(0.15)
	Market	3.34	(0.13)	3.24	(0.07)

Table VII presents asset pricing implications of the model for value and growth firms, and the aggregate market along with corresponding statistics computed from the data. $E(R)$, $\sigma(r)$ and $E(pd)$ denote expected returns, volatilities of returns and means of the log price/dividend ratios respectively. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. All data are annual, expressed in real terms, and cover the period from 1930 to 2003. Returns are value-weighted, price/dividend ratios are constructed by dividing the end-of-year price by the cumulative annual dividend. Robust standard errors of data estimates, reported in parentheses, are calculated using the Newey-West variance-covariance estimator with 8 lags. Model-implied statistics are based on 1000 simulated samples, each with 74×12 monthly observations, aggregated to an annual frequency. The entries in the Model column represent means and standard deviations (in parentheses) of the corresponding statistics across simulations.

Table VIII
Model Implications for CAPM / C-CAPM

<i>$\beta_{\text{Value}}/\beta_{\text{Growth}}$</i>	
CAPM	C-CAPM
0.92	0.85

Table VIII presents the model-implied ratio of the CAPM betas of value and growth stocks, and the corresponding ratio for the consumption-based CAPM betas computed using a sample of 1000 annual observations simulated from the model.

Table IX
Implied Correlations in Realized Returns

	Data			Model		
	Growth	Value	Market	Growth	Value	Market
Growth	1			1		
Value	0.75 (0.05)	1		0.44 (0.13)	1	
Market	0.95 (0.01)	0.87 (0.04)	1	0.82 (0.06)	0.60 (0.11)	1

Table IX reports pair-wise correlations between realized returns on value and growth firms, and the aggregate market implied by the model and from the data. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. Returns are value-weighted. The data are annual, expressed in real terms, and cover the period from 1930 to 2003. Robust standard errors of data estimates, reported in parentheses, are calculated using the Newey-West variance-covariance estimator with 8 lags. Model-implied statistics are based on 1000 simulated samples, each with 74×12 monthly observations, aggregated to an annual frequency. The entries in the Model panel represent means and standard deviations (in parentheses) of the corresponding statistics across simulations.

Table X
Predictability of Asset Returns

Asset	Horizon	Data		Model		
		\hat{b}	\bar{R}^2	\hat{b}	\bar{R}^2	
Growth	1	-0.07	(0.047)	0.03	-0.00	0.00
	5	-0.33	(0.145)	0.22	-0.17	0.08
Value	1	-0.13	(0.062)	0.05	-0.05	0.01
	5	-0.41	(0.087)	0.21	-0.19	0.09
Market	1	-0.07	(0.057)	0.02	-0.07	0.01
	5	-0.42	(0.158)	0.23	-0.38	0.14

Table X presents predictability evidence for 1- and 5-year returns on value and growth firms, and the aggregate stock market from the data and from the model. The entries correspond to the estimated projection coefficients and adjusted- R^2 s from regressing future multi-horizon returns on the log of the current price/dividend ratios, i.e.,

$$r_{t+1} + \dots + r_{t+h} = b_0 + b \log(P_t/D_t) + e_{t,t+h},$$

where h denotes the forecasting horizon in years. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. All data are annual, expressed in real terms, and cover the period from 1930 to 2003. Returns are value-weighted, price/dividend ratios are constructed by dividing the end-of-year price by the cumulative annual dividend. Robust standard errors of data estimates, reported in parentheses, are calculated using the Newey-West variance-covariance estimator with 8 lags. Model-implied statistics are based on 1000 simulated samples, each with 74×12 monthly observations, aggregated to an annual frequency. The entries in the Model column represent means of the estimates across simulations.

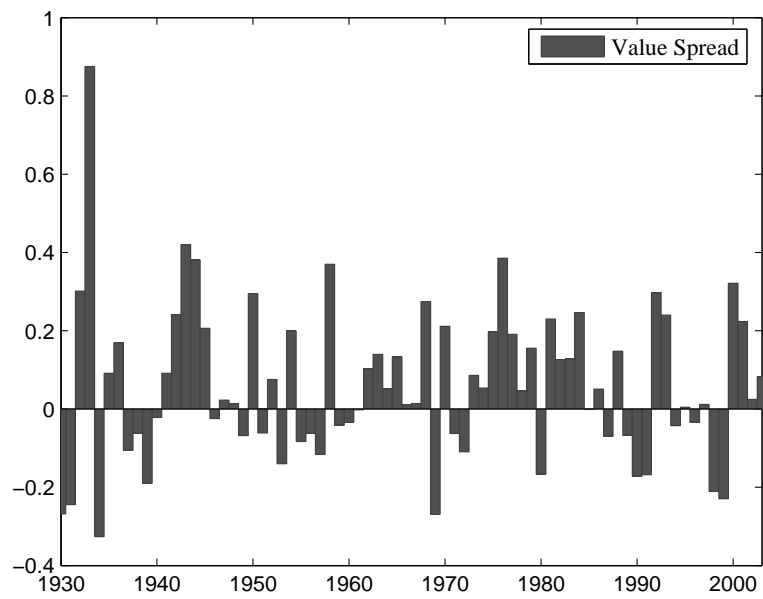


Figure 1. Spread in Realized Returns

Figure 1 plots the spread in realized returns on value and growth firms. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. Returns are value-weighted, annual and cover the period from 1930 to 2003.

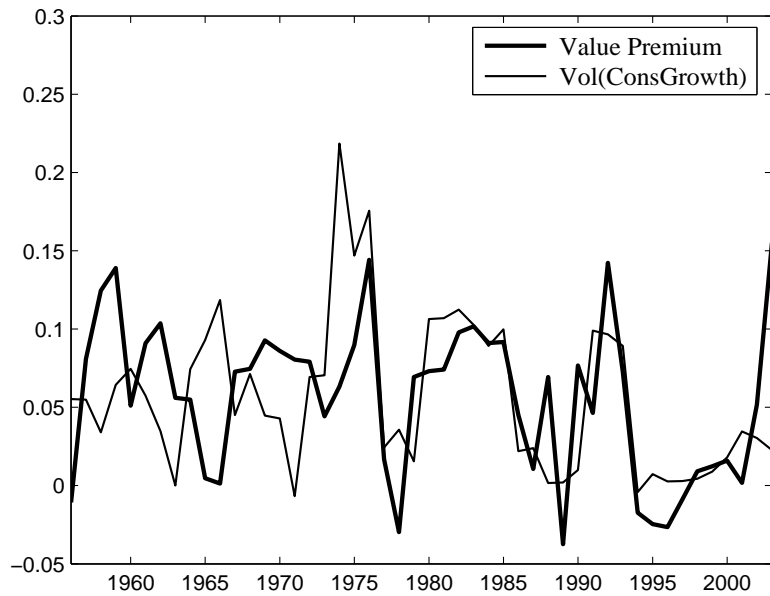


Figure 2. Value Premium and Economic Uncertainty

Figure 2 plots the value premium along with the realized volatility of consumption. The latter is measured by the 3-year moving average of squared residuals from an AR(1) process fitted to consumption growth data. The value premium is constructed by projecting the spread in realized returns on value and growth stocks on lagged price/dividend ratios and dividend growth rates of the two stocks. Consumption volatility is rescaled so that it has the same mean and standard deviation as the value spread. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. Consumption data, taken from the BEA, are per-capita expenditure of nondurables and services. All data are annual, expressed in real terms, and cover the period from 1952 to 2003.

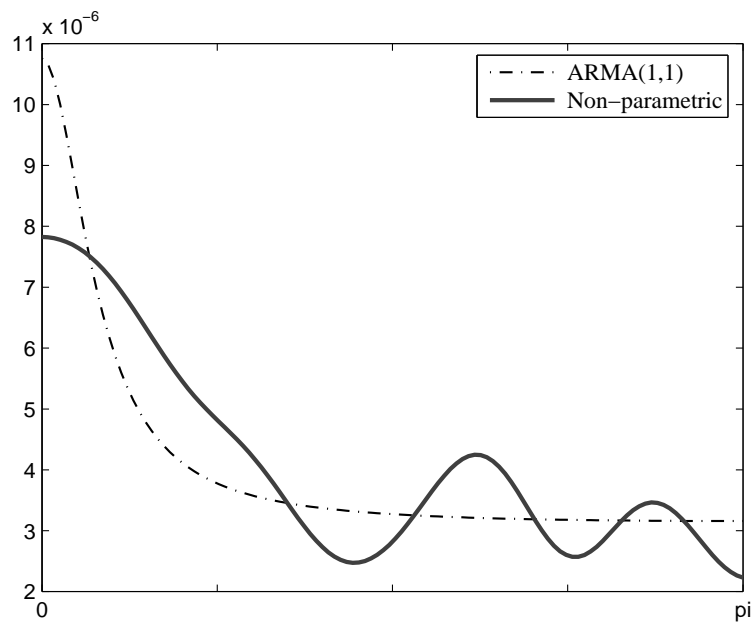
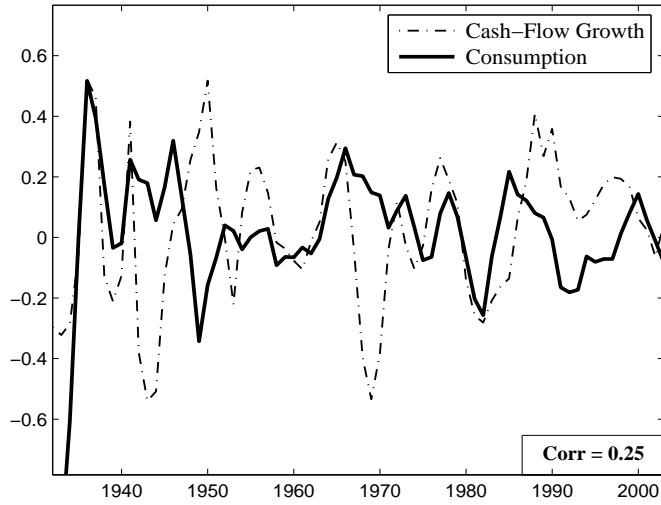
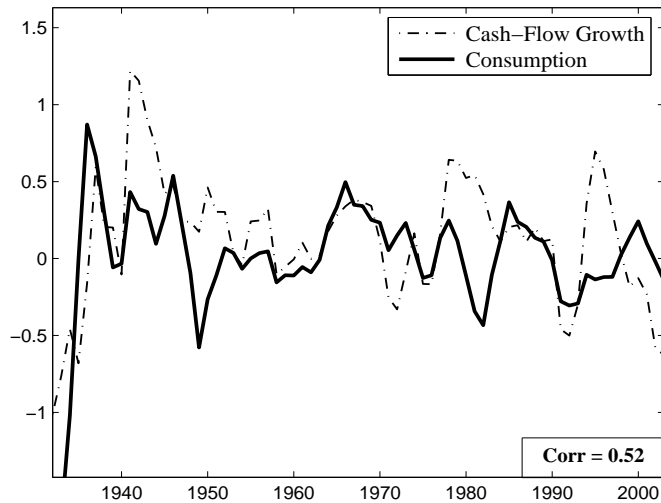


Figure 3. Spectral Density of Consumption Growth

Figure 3 plots the spectral density for consumption growth implied by an ARMA(1,1) model, and a non-parametric estimate based on the Bartlett kernel. Consumption data, taken from the BEA, are real, annual per-capita expenditure of nondurables and services that cover the period from 1930 to 2003. Growth rates are constructed by taking the first difference of the logarithm of consumption.



(a) Growth Firms



(b) Value Firms

Figure 4. Cash-Flow and Consumption Growth Rates

Figure 4 plots the 3-year moving average of dividend growth rates of value and growth firms along with the corresponding smoothed growth in aggregate consumption. Value firms represent companies in the highest book-to-market quintile of all NYSE, AMEX and NASDAQ firms. Growth firms correspond to the lowest book-to-market quintile. Consumption data, taken from the BEA, are per-capita expenditure of nondurables and services. All data are annual, expressed in real terms, and cover the period from 1930 to 2003. Growth rates are constructed by taking the first difference of the logarithm of dividend and consumption series. In order to facilitate the comparison, consumption growth is rescaled in the two subfigures.

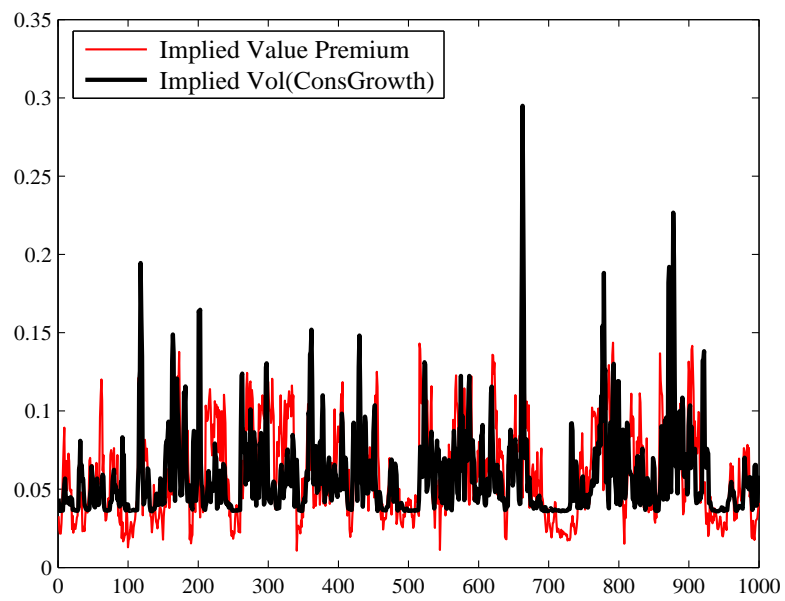


Figure 5. Model-Implied Value Premium and Economic Uncertainty

Figure 5 plots the value premium along with the realized volatility of consumption growth, estimated as described in the footnote to Figure 2 using a sample of 1000 annual observations simulated from the model.