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Is There a Positive Relationship between Stock Market Volatility and the Equity Premium?

This paper investigates whether evidence for a positive relationship between stock market volatility and the equity premium is more decisive when the volatility feedback effects of large and persistent changes in market volatility are taken into account. The analysis has two components. First, a log–linear present value framework is employed to derive a formal model of volatility feedback under the assumption of Markov-switching market volatility. Second, the model is estimated for a variety of assumptions about information available to economic agents. The empirical results suggest the existence of a negative and significant volatility feedback effect, supporting a positive relationship between stock market volatility and the equity premium.

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To the extent that aggregate risk in the stock market is captured by the conditional variance of the return on a market portfolio, it seems reasonable to expect a positive empirical relationship between market volatility and the equity premium.[1] Studies by French, Schwert, and Stambaugh (1987), Bollerslev,

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1. The equity premium is the expected excess return on a market portfolio over the risk-free interest rate. It is sometimes referred to as the "equity risk premium," the "market risk premium," or simply the "risk premium." While a positive relationship between volatility and the equity premium is a reasonable partial equilibrium hypothesis, it is not a strict implication of modern general equilibrium models of asset prices. See Abel (1988), Backus and Gregory (1993), and Gennotte and Marsh (1993).

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Engle, and Wooldrige (1988), Turner, Startz, and Nelson (1989), Harvey (1989), Campbell and Hentschel (1992), Scruggs (1998), and Veronesi (1999) find some support for a positive relationship. However, other studies by Campbell (1987), Breen, Glosten, and Jagannathan (1989), Nelson (1991), Glosten, Jagannathan, and Runkle (1993), and Whitelaw (1994) find otherwise.

In this paper, we revisit the question of whether the relationship between market volatility and the equity premium is positive. First, we employ Campbell and Shiller's (1988) log–linear present value framework to derive an empirical model of stock returns under the assumption of Markov-switching market volatility. Then, we use monthly excess return data for a value-weighted portfolio of all NYSE stocks from CRSP for the sample periods of 1926–1951 and 1952–2000 to estimate the model under different assumptions about information available to economic agents. We find that when we consider information assumptions that allow for volatility feedback, there is statistically significant evidence of a positive relationship between market volatility and the equity premium.

Volatility feedback is the idea that an exogenous change in the level of market volatility initially generates additional return volatility as stock prices adjust in response to new information about future discounted expected returns. In particular, if market volatility is persistent and positively related to the equity premium, then stock prices should immediately move in the opposite direction to the level of market volatility. Thus, accounting for volatility feedback is important to avoid confusing a negative relationship between return volatility and *realized* returns with the underlying relationship between market volatility and the equity premium. Furthermore, since volatility feedback captures the effects of market volatility on all future discounted expected returns, and not just the contemporaneous expected return, it potentially provides a more powerful way to estimate the true sign of the relationship between market volatility and the equity premium.

Of the past studies supporting a positive relationship, French, Schwert, and Stambaugh (1987) (FSS), Turner, Startz, and Nelson (1989) (TSN), and Campbell and Hentschel (1992) (CH) also account for volatility feedback. We synthesize and build on these studies in three important ways. First, following TSN, we consider a Markov-switching specification for market volatility instead of the integrated autoregressive moving average (ARIMA) specification used in FSS or the quadratic generalized autoregressive conditional heteroskedasticity (QGARCH) specification used in CH. Second, following CH, we employ the log–linear present value framework to derive an analytical expression for the volatility feedback parameter in terms of the other parameters of the model, including the parameter associated with the underlying relationship between market volatility and the equity premium. Thus, unlike FSS and TSN, we are able to directly interpret the economic content of our volatility feedback parameter estimates. Third, we extend the Markov-switching approach used in TSN by considering alternative assumptions about what information is available to economic agents. In particular, we assume, in one case, that agents know the prevailing volatility regime by the end of each month. We believe this assumption is plausible given the wide availability of higher frequency stock return data.

There are three reasons why we pursue a Markov-switching specification for market volatility instead of an ARIMA- or ARCH-type specification. First, in a study of weekly stock returns that allows ARCH parameters to undergo Markov-switching regime changes, Hamilton and Susmel (1994) find that most of the ARCH dynamics die out at the monthly return horizon consider here. Only the Markov-switching regime changes persist over longer periods of time. Numerous other studies, including Schwert (1989), Schaller and van Norden (1997), Kim, Nelson, and Startz (1998), Kim and Nelson (1998), and Mayfield (1999), have successfully used a Markov-switching specification to model monthly stock return volatility. Second, the fact that the Markov-switching specification only captures large discrete changes in market volatility gives us more confidence than with an ARIMA- or ARCH-type specification that we have modeled volatility feedback, rather than a leverage effect. In particular, under the leverage hypothesis, a large movement in stock prices alters the debt/equity ratios of firms, changing their risk profiles and, therefore, the volatility of future returns. In this case, the direction of causality runs opposite to that of volatility feedback, with the size of the change in volatility being somewhat proportional to the size of the price movement. Thus, if the leverage hypothesis were the driving force behind the negative relationship between return volatility and realized returns, we would expect to find lingering ARCH effects in the residuals from a model that only captures large discrete changes in market volatility. Yet, we do not.[2] Third, as discussed in Hamilton (1993), an MS specification allows us to make a distinction between what economic agents know and what econometricians can infer about the volatility regime from observed returns alone, while at the same time maintaining an assumption of rational expectations. It is by making this important distinction that we are able to estimate a volatility feedback effect using Hamilton's (1989) filter for Markov-switching models. Thus, we avoid the complications that arise in CH from estimation of a nonlinear relationship between returns and regression errors under the QGARCH specification. Also, we avoid the statistical problems that arise from the two-step estimation procedure employed in FSS.

The rest of this paper is organized as follows. Section 1 presents the formal derivation of the model used in this paper. Section 2 reports empirical results using CRSP data for the model under different assumptions about what information is available to economic agents. Section 3 concludes.

1. MODEL

In this section, we present a brief discussion of the log–linear present value framework, derive a model of stock returns with volatility feedback under the

2. This result is consistent with Bekaert and Wu (2000), who directly test both the leverage hypothesis and volatility feedback and find support for volatility feedback.

assumption of Markov-switching market volatility, and discuss the information assumptions considered in our empirical analysis.

1.1 The Log–Linear Present Value Framework

Campbell and Shiller (1988) use a first-order Taylor series approximation to derive the following log–linear present value relationship:

$$p_t = \frac{\kappa}{1-\rho} + \mathrm{E}\left[\sum_{j=0}^{\infty} \rho^j [(1-\rho) d_{t+1+j} - r_{t+1+j}] \,\middle|\, \psi_t \right], \tag{1}$$

where p_t is the (ex-dividend) log price of a stock at the end of time t, d_{t+1+j} is the log dividend at time $t + 1 + j$ claimed at the beginning of the period, r_{t+1+j} is the log return on a stock or portfolio held from $t + j$ to $t + 1 + j$, E[·] is the expectations operator, ψ_t is conditioning information set available at time t, ρ is the average ratio of the stock price to the sum of the stock price and the dividend, and κ is a nonlinear function of ρ.

A pertinent question is how reasonable is the approximation used to derive the log–linear present value relation in Equation (1). Campbell, Lo, and MacKinlay (1997) provide evidence that, for the data we examine, the approximation error is quite small and relatively constant. In particular, the correlation between actual monthly returns and the approximate returns based on a first-order Taylor expansion is 0.99991, while the variance of the approximation error is only 0.0008 compared with a variance of 0.0555 for actual monthly returns. The intuition is that the Taylor expansion would be exact if the ratio of the stock price to the sum of the stock price and the dividend were constant. Since the ratio has an extremely small variance in practice, the approximation is quite accurate. Meanwhile, the benefit of linearization is that it allows us to simultaneously examine the effects on stock prices of changes in expected future dividends and changes in expected future returns.

1.2 A Markov-Switching Model of Stock Returns with Volatility Feedback

Following CH, we develop a partial equilibrium model of volatility feedback that relies on the log–linear present value relation given in Equation (1) and two simple assumptions. First, by contrast to the QGARCH assumption employed in CH, we assume that news about future dividends is subject to a two-state Markov-switching variance:

$$\varepsilon_t \sim N(0, \sigma^2_{S_t}), \tag{2}$$

$$\sigma^2_{S_t} = \sigma^2_0(1 - S_t) + \sigma^2_1 S_t, \ \sigma^2_0 < \sigma^2_1,$$

$$\Pr[S_t = 0 \mid S_{t-1} = 0] = q \text{ and } \Pr[S_t = 1 \mid S_{t-1} = 1] = p,$$

where ε_t denotes new information about future dividends that arrives during trading period t, $\sigma^2_{S_t}$ is the variance of ε_t, S_t is a Markov-switching state variable that takes on discrete values of 0 or 1 according to the prevailing volatility regime, and q and

p are the transition probabilities governing the evolution of S_t. Second, following CH, we assume that the expected return for a given period $t + j$ is a linear function of the market expectation, formed rationally in the sense of Muth (1960), about the volatility of news. Given this assumption and the Markov-switching specification for volatility, the expected return can be expressed as a linear function of the conditional probability of the high volatility regime:

$$E[r_{t+j} \mid \psi_t] = \mu_0 + \mu_1 \Pr[S_{t+j} = 1 \mid \psi_t] , \tag{3}$$

where μ_0 is the expected return in a perfectly anticipated low variance regime and μ_1 reflects the marginal effect on the expected return of a perfectly anticipated high variance regime. The linear specification in Equation (3) is important since it allows the average price of risk to be different from the marginal price. For our analysis, this is necessary since there is little doubt that the average price of risk is positive given a positive historical mean for stock returns.

As discussed in CH, the log–linear present value model given in Equation (1) can be rearranged to show that a realized return is determined by the expected return, volatility feedback, and news:

$$r_t = E[r_t \mid \psi_{t-1}] + f_t + \varepsilon_t , \tag{4}$$

where f_t is the volatility feedback term that reflects revisions in future expected returns.

$$f_t \equiv -\left\{ E\left[\sum_{j=1}^{\infty} \rho^j r_{t+j} \mid \psi'_t \right] - E\left[\sum_{j=1}^{\infty} \rho^j r_{t+j} \mid \psi_{t-1} \right] \right\} ,$$

and ε_t reflects news about dividends:

$$\varepsilon_t \equiv E\left[\sum_{j=0}^{\infty} \rho^j \Delta d_{t+j} \mid \psi'_t \right] - E\left[\sum_{j=0}^{\infty} \rho^j \Delta d_{t+j} \mid \psi_{t-1} \right] .$$

The information set ψ'_t contains all elements of ψ_t except the final realized value of r_t. Conceptually, it is necessary to make this distinction between ψ'_t and $\psi_t = \{\psi'_t, r_t)$ if Equation (4) is to describe a meaningful causal relationship from dividend news ε_t and volatility feedback f_t to the final realized return r_t.

In terms of the Markov-switching volatility feedback model, we can use the assumptions given in Equations (2) and (3) to obtain empirically tractable expressions for the expected return and news terms in Equation (4). Meanwhile, to find the tractable expression for the volatility feedback term for our model, it is helpful to first note that the expected return in Equation (3) can also be represented by

$$E[r_{t+j} \mid \psi_t] = \mu_0 + \mu_1 \Pr[S_t = 1] + \mu_1 \lambda^j (\Pr[S_t = 1 \mid \psi_t] - \Pr[S_t = 1]) , \tag{3′}$$

where $\lambda \equiv p + q - 1$ (see Hamilton 1989). Then, given recurring volatility regimes (i.e., $|\lambda| < 1$), it is straightforward to show that the discounted sum of future expected returns is

$$E\left[\sum_{j=1}^{\infty}\rho^j r_{t+j} \,|\, \psi_t\right] = \frac{\mu_0}{1-\rho} + \frac{\mu_1}{1-\rho}\Pr[S_t = 1] + \frac{\mu_1}{1-\rho\lambda}\left(\Pr[S_t = 1 \,|\, \psi_t] - \Pr[S_t = 1]\right), \qquad (5)$$

which, in turn, implies the volatility feedback term is

$$f_t = -\frac{\mu_1}{1-\rho\lambda}\left(\Pr[S_t = 1 \,|\, \psi_t'] - \Pr[S_t = 1 \,|\, \psi_{t-1}]\right). \qquad (6)$$

Thus, substituting in the empirically tractable expressions for the elements of Equation (4), the Markov-switching model of stock returns with volatility feedback is

$$r_t = \mu_0 + \mu_1 \Pr[S_t = 1 \,|\, \psi_{t-1}] + \delta\{\Pr[S_t = 1 \,|\, \psi'_t] - \Pr[S_t = 1 \,|\, \psi_{t-1}]\} + \varepsilon_t, \qquad (7)$$

where $\varepsilon_t \sim N(0, \sigma^2_{S_t})$ is Markov switching as given in Equation (2) and the volatility feedback coefficient $\delta = -\mu_1/(1-\rho\lambda)$ as implied by Equation (6).

In order to interpret the restriction $\delta = -\mu_1/(1-\rho\lambda)$, note that the parameter of linearization, ρ, which is the average ratio of the stock price to the sum of the stock price and the dividend, is slightly less than 1 (0.997) in practice. Thus, a positive price of risk implies that, as long as volatility regimes are persistent (i.e., $\lambda \equiv p + q - 1 > 0$), the coefficient δ on the volatility feedback term will be negative. Conversely, any evidence of a negative volatility feedback effect implies a positive relationship between market volatility and the equity premium.

1.3 Information Assumptions

The last issue that needs to be addressed before estimation of the parameters in Equation (7) is the assumptions about information available to economic agents during each trading period. Note that rational expectations implies that both information sets, ψ_t' and ψ_{t-1}, in Equation (7) should, at the very least, contain information directly observable by econometricians prior to the realization of r_t. For the Markov-switching specification of volatility given in Equation (2), the relevant information that is observable by econometricians is past returns $\{r_{t-1}, r_{t-2}, \ldots\}$. Meanwhile, volatility feedback occurs in period t if agents obtain new information *during* period t (i.e., $\psi_t' \neq \psi_{t-1}$). For example, volatility feedback would occur if the true state, S_t, were not known at the beginning of the trading period but was revealed to agents during the period through the process of trading.

We consider five different information assumptions in our empirical analysis:

1. $\psi_{t-1} = \psi_t' = \{\}$. Agents do not observe (or, equivalently, they do not react to) information about the volatility regime. In this case, the equity premium remains constant ($\mu_1 = 0$) and there is no volatility feedback ($\delta = 0$);

2. $\psi_{t-1} = \psi'_t = \{r_{t-1}, r_{t-2}, ...\}$. Agents observe past returns throughout the trading period t and do not update their inferences about volatility until the next trading period *after* they have observed r_t. That is, agents act like econometricians in making inferences about an unobserved volatility regime. Under this assumption, there is no volatility feedback ($\delta = 0$);

3. $\psi_{t-1} = \psi'_t = \{S_t\}$. Agents know the true volatility regime throughout the trading period t. Again, there is no volatility feedback ($\delta = 0$);

4. $\psi_{t-1} = \{r_{t-1}, r_{t-2}, ...\}$, $\psi'_t \approx \{S_t\}$. Agents observe past returns at the beginning of the trading period but obtain information through the process of trading about the volatility regime that we proxy using the true regime. This new information potentially produces volatility feedback ($\delta \neq 0$). We refer to this particular assumption as "partial revelation" since, given the information available at the beginning of the trading period, agents do not fully observe S_t but only discover information about the volatility regime that is implicit in r_t.[3] While using the actual value of S_t to proxy for this information creates measurement error, it has the advantage that it nests Assumptions (2) and (3). Also, it is the assumption used in TSN, allowing us to directly compare our results to theirs;

5. $\psi_{t-1} = \{S_{t-1}\}$, $\psi'_t = \{S_t\}$. Agents observe the previous volatility regime at the beginning of the trading period but discover what the current volatility regime is by the end of the trading period. We refer to this as the "full revelation" case, and it also corresponds to volatility feedback ($\delta \neq 0$). While this assumption only nests Assumption (3), it has the advantage over Assumption (4) that it involves no measurement error. Note that both this assumption and Assumption (4) are equivalent to Assumption (3) if $\mu_1 - \delta = 0$.

The first three assumptions provide benchmarks that allow us to motivate the Markov-switching specification for market volatility and test whether there is a time-varying equity premium. The last two assumptions allow us to test volatility feedback and investigate whether there is a positive relationship between market volatility and the equity premium. For each assumption, estimation is a straightforward application of maximum likelihood and the filter discussed in Hamilton (1989). Details of estimation using the Hamilton filter can be found in the appendix.

2. EMPIRICAL RESULTS

In this section, we describe the data and report estimation results for different versions of the model developed in the previous section in order to answer the following questions: Is there Markov-switching market volatility? Is there a time-varying equity premium? Is there volatility feedback? Is there a positive relationship between market volatility and the equity premium?

3. We cannot directly include the current return in the information set since it is the current return we seek to model in Equation (7). However, we assume that the return, r_t, realized at the end of the period is a sufficient statistic for the information about S_t revealed during the trading period t. Thus, for every t, $\psi_t = \{r_t, r_{t-1}, r_{t-2}, ...\}$.

2.1 Data

The data are excess stock returns on a market portfolio. In particular, we employ continuously compounded total monthly returns for a value-weighted portfolio of all NYSE-listed stocks in excess of continuously compounded one-month U.S. Treasury bill yields. The data are drawn from the CRSP files for the sample period of January 1926 to December 2000. Total returns represent capital gains plus dividend yields. Continuously compounded returns are calculated by taking natural logarithms of simple gross returns.

The use of excess returns means that "news" technically refers to information about future dividends *relative* to future interest rates. A relative measure of this kind makes sense since the theoretical effects of volatility on real returns alone are ambiguous, even if we assume a positive price of risk. For example, an increase in risk could cause investors to substitute away from riskier assets, putting downward pressure on interest rates. Thus, the expected real return could be lower, even if the equity premium is larger.

For estimation, we consider the 1926–51 and 1952–2000 periods separately. The breakpoint corresponds to the Fed-Treasury Accord and is also used in CH. We consider these periods separately since the underlying behavior of volatility has likely changed over the full sample (see, for example, Pagan and Schwert 1990) and we want the Markov-switching specification to capture persistent, but temporary, changes in volatility, not one-time structural breaks.[4]

2.2 Is There Markov-Switching Market Volatility?

Panels A and B of Table 1 report maximum likelihood estimates for constant, independent-switching, and Markov-switching models of stock market volatility for the sample periods of 1926–51 and 1952–2000, respectively.[5] For both samples, there is a huge improvement in log likelihood values when volatility is allowed to change over time. For the 1926–51 sample, the likelihood ratio statistic for the null hypothesis of a constant variance, $H_0 : \sigma_0 = \sigma_1$, is 86.39 when we consider the alternative of independent switching.[6] For the 1952–2000 sample, the likelihood ratio statistic for the same hypothesis is 37.69. While these likelihood ratio statistics are large, their distribution is nonstandard since the volatility regime probability q is not identified under the null (see Hansen 1992 and Garcia 1998). Thus, these results alone are only suggestive of time variation in the level of stock return

4. A one-time structural break in volatility could be modeled by allowing four states in total, with two states being "absorbing" in the sense that, once in one of them, the probability of returning to either of the other two "original" states is zero. However, such additional complexity in our model should have no direct implications for the tradeoff between market volatility and the equity premium, and so a possible structural break is more simply dealt with by examining the prespecified sample periods separately.

5. All maximum likelihood estimation was conducted using the OPTMUM procedure of the GAUSS programming language. Numerical derivatives were used in estimation, as well as for calculation of asymptotic standard errors. Parameters were appropriately constrained (e.g., variances were constrained to be nonnegative). Inferences appear robust to a variety of starting values.

6. Independent switching corresponds to an i.i.d. mixture of Normals model of stock market volatility where the prevailing volatility regime is independent of past volatility regimes.

TABLE 1

STOCK MARKET VOLATILITY: CONSTANT, INDEPENDENT-SWITCHING, AND MARKOV-SWITCHING MODELS

A. 1926–1951

	Model		
	Constant variance	Independent-switching variance	Markov-switching variance
Parameters[a]	$\sigma_0 = \sigma_1$	$\sigma_0 < \sigma_1, p = 1 - q$	$\sigma_0 < \sigma_1$
μ_0	0.00520	0.01114	0.01200
	(0.00411)	(0.00310)	(0.00268)
σ_0	0.07274	0.04214	0.03929
	(0.00291)	(0.00353)	(0.00210)
σ_1	—	0.13947	0.12087
	—	(0.01835)	(0.01058)
q	—	0.79917	0.98597
	—	(0.05697)	(0.01057)
p	—	—	0.96051
	—	—	(0.02921)
Log likelihood	375.01141	418.20464	447.94903

Residual diagnostics[b]	χ^2 statistic	p-value	χ^2 statistic	p-value	χ^2 statistic	p-value
ARCH-LM(k = 1)	14.08412	(0.0002)	10.43505	(0.0012)	0.007827	(0.9298)
ARCH-LM(k = 12)	32.3852	(0.0000)	27.36162	(0.0001)	4.234272	(0.6450)
ARCH-LM(k = 12)	66.44054	(0.0000)	54.43882	(0.0000)	15.65923	(0.2073)
JB Normality Test	321.5245	(0.0000)	0.866037	(0.6485)	1.683562	(0.4309)

B. 1952–2000

	Model		
	Constant variance	Independent-switching variance	Markov-switching variance
Parameters[a]	$\sigma_0 = \sigma_1$	$\sigma_0 < \sigma_1, p = 1 - q$	$\sigma_0 < \sigma_1$
μ_0	0.00533	0.00655	0.00716
	(0.00169)	(0.00159)	(0.00156)
σ_0	0.04114	0.03470	0.03390
	(0.00120)	(0.00200)	(0.00184)
σ_1	—	0.08109	0.07454
	—	(0.01582)	(0.01328)
q	—	0.90882	0.95924
	—	(0.05641)	(0.02193)
p	—	—	0.71091
	—	—	(0.17695)
Log likelihood	1041.90219	1060.74719	1066.08958

Residual diagnostics[b]	χ^2 statistic	p-value	χ^2 statistic	p-value	χ^2 statistic	p-value
ARCH-LM(k = 1)	5.98453	(0.0144)	3.637642	(0.0568)	6.326119	(0.0119)
ARCH-LM(k = 6)	8.781417	(0.1862)	16.94141	(0.0095)	9.654509	(0.1400)
ARCH-LM(k = 12)	15.94667	(0.1937)	23.88503	(0.0211)	15.17554	(0.2320)
JB Normality Test	269.5322	(0.0000)	16.00337	(0.0003)	7.849926	(0.0197)

[a]The models are restricted versions of Equation (7), with μ_1 and δ set to zero. Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of (A) January 1926 to December 1951; (B) January 1952 to December 2000. Asymptotic standard errors are reported in parentheses.

[b]The ARCH-LM statistics have $\chi^2(k)$ asymptotic distributions under the null of no ARCH effects and are constructed using TR^2 from an auxiliary regression of squared standardized residuals on k lags of squared standardized residuals. The JB (Jarque and Bera, 1980) Normality test statistics have $\chi^2(2)$ asymptotic distributions under the null of Normality and are constructed using sample skewness and kurtosis for the standardized residuals.

volatility. However, conditional on market volatility changing over time, the test statistic for a Markov-switching specification should have an asymptotic $\chi^2(1)$ distribution. In particular, when we consider the alternative of Markov-switching market volatility, the likelihood ratio statistic for the null hypothesis of independent switching, $H_0 : p = 1 - q$, is 59.49 for the 1926–51 sample and 10.68 for the 1952–2000 samples. Both statistics are significant, with p-values of <0.01. Thus, the evidence supports the presence of persistent Markov-switching regimes (i.e., $\lambda \equiv p + q - 1 > 0$) necessary to generate volatility feedback.

In addition to supporting Markov-switching market volatility, the estimates in Table 1 suggest that prewar returns were much more volatile than postwar returns. The standard deviation of monthly returns in the high volatility regime is 0.12 in the 1926–51 sample period versus 0.07 in the 1952–2000 sample period. Meanwhile, the probability of staying in the high volatility regime drops from 0.96 to 0.71, corresponding to a drop in expected regime duration from about 25 months to only three months. These changes in volatility behavior are also evident in Figures 1A and 1B, which display excess returns and the smoothed probability of a high volatility regime for the sample periods of 1926–51 and 1952–2000, respectively. Thus, it appears that the separation of the 1926–2000 sample period into the two subsamples is appropriate, although the lack of high volatility episodes throughout the 1940s and 1950s makes it difficult to identify exactly when the underlying change in behavior occurred. The Fed–Treasury Accord merely provides a convenient dividing line.

The estimates in Table 1 also provide an initial basis for our claim, discussed in further detail below, that any evidence of Markov-switching market volatility for the subsequent models is not a consequence of allowing for a time-varying mean. This claim is important since the volatility feedback model is based on the assumption that changes in the mean return are generated by exogenous changes in volatility, not vice versa.

2.3 Is There a Time-Varying Equity Premium?

Table 2 reports maximum likelihood estimates for Markov-switching models of stock returns with benchmark assumptions about what information is available to economic agents. When agents are assumed to observe only past returns, $\psi_{t-1} = \psi'_t = \{r_{t-1}, r_{t-2}, \ldots\}$, there is mixed evidence of a time-varying mean. The likelihood ratio statistic for the null hypothesis of a constant mean, $H_0 : \mu_1 = 0$, is 0.78, with a p-value of 0.38, for the 1926–51 sample but is 5.26, with a p-value of 0.02, for the 1952–2000 sample. When agents are assumed to observe the true volatility regime, $\psi_{t-1} = \psi'_t = \{S_t\}$, there is strong evidence of a time-varying mean. The likelihood ratio statistics are 4.74, with a p-value of 0.03, for the 1926–51 sample and 7.89, with a p-value of <0.01, for the 1952–2000 sample. However, in this case, the estimated tradeoff between the mean and variance is significantly negative, with t-statistics for μ_1 of -2.17 and -1.94 for the 1926–51 and 1952–2000 samples, respectively.[7] Overall, these results support a time-varying equity

7. The estimates for the 1952–2000 sample are qualitatively very similar to those in TSN, although they use excess returns for the Standard and Poor's composite index for the period of January 1946 to December 1987.

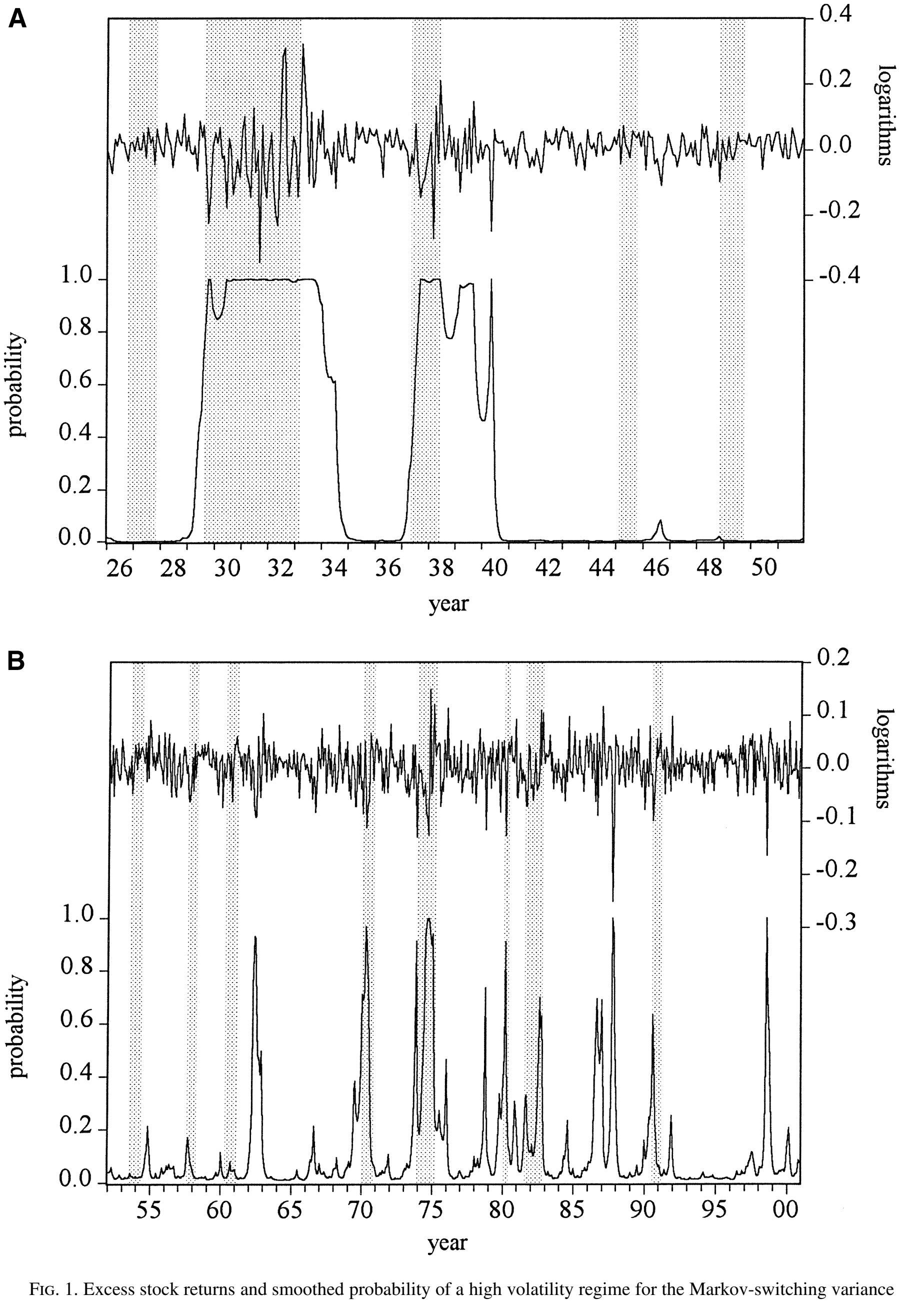

FIG. 1. Excess stock returns and smoothed probability of a high volatility regime for the Markov-switching variance model: (A) 1926–51; (B) 1952–2000. Source: Excess stock returns are continuously compounded total monthly value-weighted returns of all NYSE-listed stocks from CRSP in excess of one-month Treasury bill yields from CRSP for the sample period of (A) January 1926 to December 1951; (B) January 1952 to December 2000. Smoothed probability inferences are calculated using Kim's (1994) smoothing algorithm and maximum likelihood estimates for the Markov-switching variance model presented in Table 1. NBER-dated recessions are shaded.

TABLE 2

VOLATILITY AND THE EQUITY PREMIUM: BENCHMARK INFORMATION SPECIFICATIONS

A. 1926–1951

	Model specification			
	Agents observe past returns		Agents observe true regime	
Parameters[a]	$\psi_{t-1} = \psi'_t = \{r_{t-1}, r_{t-2}, \ldots\}$		$\psi_{t-1} = \psi'_t = \{S_t\}$	
μ_0	0.01317		0.01352	
	(0.00301)		(0.00278)	
μ_1	−0.01159		−0.02820	
	(0.01291)		(0.01297)	
σ_0	0.03929		0.03901	
	(0.00206)		(0.00212)	
σ_1	0.11888		0.11723	
	(0.01007)		(0.01005)	
q	0.98702		0.98534)	
	(0.00955)		(0.01150)	
p	0.96708		0.95980	
	(0.02591)		(0.03079)	
Log likelihood	448.33808		450.31835	
Residual diagnostics[b]	χ^2 statistic	p-value	χ^2 statistic	p-value
ARCH-LM(k = 1)	0.1307	(0.7177)	0.57116	(0.4498)
ARCH-LM(k = 6)	4.3721	(0.6265)	4.72839	(0.5791)
ARCH-LM(k = 12)	16.1325	(0.1852)	18.6107	(0.0984)
JB Normality Test	1.89601	(0.3875)	0.87686	(0.6450)

B. 1952–2000

	Model specification			
	Agents observe past returns		Agents observe true regime	
Parameters[a]	$\psi_{t-1} = \psi'_t = \{r_{t-1}, r_{t-2}, \ldots\}$		$\psi_{t-1} = \psi'_t = \{S_t\}$	
μ_0	0.00407		0.00920	
	(0.00207)		(0.00183)	
μ_1	0.03912		−0.03135	
	(0.02233)		(0.01614)	
σ_0	0.03476		0.03377	
	(0.00173)		(0.00186)	
σ_1	0.08077		0.06893	
	(0.01472)		(0.00995)	
q	0.97565		0.95537	
	(0.01571)		(0.02370)	
p	0.75759		0.68192	
	(0.12789)		(0.17008)	
Log likelihood	1068.72159		1070.03630	
Residual diagnostics[b]	χ^2 statistic	p-value	χ^2 statistic	p-value
ARCH-LM(k = 1)	5.16658	(0.0230)	7.92311	(0.0049)
ARCH-LM(k = 6)	8.64643	(0.1945)	12.4663	(0.0523)
ARCH-LM(k = 12)	14.5032	(0.2697)	19.0451	(0.0874)
JB Normality Test	6.49528	(0.0389)	2.77132	(0.2502)

[a]Both specifications are restricted versions of Equation (7), with δ set to zero. Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of (A) January 1926 to December 1951; (B) January 1952 to December 2000. Asymptotic standard errors are reported in parentheses.

[b]The ARCH-LM statistics have $\chi^2(k)$ asymptotic distributions under the null of no ARCH effects and are constructed using TR^2 from an auxiliary regression of squared standardized residuals on k lags of squared standardized residuals. The JB (Jarque and Bera, 1980) Normality test statistics have $\chi^2(2)$ asymptotic distributions under the null of Normality and are constructed using sample skewness and kurtosis for the standardized residuals.

premium. They also suggest that agents act upon information inherent in the true volatility regime. However, these findings do not answer the question of whether the negative correlation between the mean and variance is the result of a negative price of risk or the presence of volatility feedback. The remaining model specifications are designed to help us answer this question.

2.4 Is There Volatility Feedback?

Table 3 reports maximum likelihood estimates for the Markov-switching volatility feedback model with partial revelation, $\psi_{t-1} = \{r_{t-1}, r_{t-2}, ...\}$ and $\psi_t' \approx \{S_t\}$. The model nests both benchmark information assumptions. For the null hypothesis of no feedback with agents observing only past returns, $H_0 : \delta = 0$, the likelihood ratio statistics are 4.83, with a p-value of 0.03, for the 1926–51 sample and 23.50, with a p-value of <0.01, for the 1952–2000 sample. The t-statistics for the feedback term δ are -2.23 and -7.09 for the 1926–51 and 1952–2000 samples, respectively.[8] These results support the existence of volatility feedback. For the null hypothesis of no feedback with agents observing the true volatility regime throughout the trading period, $H_0 : \mu_1 - \delta = 0$, the likelihood ratio statistics are 0.87, with a p-value of 0.35, for the 1926–51 sample and 20.87, with a p-value of <0.01, for the 1952–2000 sample. These results also support the existence of volatility feedback in postwar returns, although the evidence for prewar returns is weaker.

Table 4 reports maximum likelihood estimates for the Markov-switching volatility feedback model with full revelation, $\psi_{t-1} = \{S_{t-1}\}$ and $\psi_t' = \{S_t\}$. This model only nests the benchmark assumption that agents observe the true volatility regime, $\psi_{t-1} = \psi_t' = \{S_t\}$. For the null of no feedback with agents observing the true volatility regime throughout the trading period, $H_0 : \mu_1 - \delta = 0$, the likelihood ratio statistics are 2.44, with a p-value of 0.12, for the 1926–51 sample and 15.60, with a p-value of <0.01, for the 1952–2000 sample. As in the partial revelation case, these results support the existence of volatility feedback in postwar returns, with somewhat weaker results for prewar returns.

2.5 Is There a Positive Relationship between Market Volatility and the Equity Premium?

Having established the existence of volatility feedback, at least for postwar returns, we examine the evidence of a positive relationship between volatility and the equity premium. Returning again to the results in Table 3 for the model with feedback due to partial revelation, it is apparent from the t-statistics on the feedback term, δ (-2.23 and -7.09 for the 1926–51 and 1952–2000 samples, respectively), that the volatility feedback effect is significantly negative for both sample periods when the feedback parameter is unrestricted. The estimated partial effect, μ_1, is even positive, though not significant (t-statistic is 0.53), for the 1952–2000 sample.[9] Since volatility regimes appear to be very persistent (i.e., $\lambda \equiv p + q - 1 > 0$), these results

8. Again, the estimates for the 1952–2000 sample are qualitatively similar to those in TSN.
9. By contrast, TSN find a negative partial effect. However, their estimate is also not significant.

TABLE 3

VOLATILITY AND THE EQUITY PREMIUM: FEEDBACK DUE TO PARTIAL REVELATION

	Model specification			
	δ is freely estimated. Agents observe past returns, but regime is partially revealed during period *t*		δ is restricted.[a] Agents observe past returns, but regime is partially revealed during period *t*	
Parameters[b]	$\psi_{t-1} = \{r_{t-1}, r_{t-2}, \ldots\}$, $\psi'_t = \{S_t\}$		$\psi_{t-1} = \{r_{t-1}, r_{t-2}, \ldots\}$, $\psi'_t = \{S_t\}$	
A. 1926–1951				
μ_0	0.01241		0.01056	
	(0.00301)		(0.00285)	
μ_1	−0.02308		0.00161	
	(0.01402)		(0.00146)	
δ	−0.04014		−0.02657	
	(0.01809)		(0.01718)	
σ_0	0.03899		0.03922	
	(0.00214)		(0.00210)	
σ_1	0.11591		0.11705	
	(0.01024)		(0.01017)	
q	0.98607		0.98751	
	(0.01275)		(0.01145)	
p	0.95712		0.95458	
	(0.03392)		(0.03191)	
Log likelihood	450.75542		449.15260	
Residual diagnostics[c]	χ^2 statistic	p-value	χ^2 statistic	p-value
ARCH-LM(k = 1)	0.58017	(0.4462)	0.08496	(0.7707)
ARCH-LM(k = 6)	4.94683	(0.5507)	4.92673	(0.5532)
ARCH-LM(k = 12)	19.194	(0.0840)	17.4789	(0.1325)
JB Normality Test	0.68842	(0.7088)	1.01212	(0.6029)

B. 1921–2000

	Model specification			
	δ is freely estimated. Agents observe past returns, but regime is partially revealed during period *t*		δ is restricted.[a] Agents observe past returns, but regime is partially revealed during period *t*	
Parameters[b]	$\psi_{t-1} = \{r_{t-1}, r_{t-2}, \ldots\}$, $\psi'_t = \{S_t\}$		$\psi_{t-1} = \{r_{t-1}, r_{t-2}, \ldots\}$, $\psi'_t = \{S_t\}$	
μ_0	0.00484		0.00444	
	(0.00185)		(0.00164)	
μ_1	0.00374		0.00665	
	(0.00708)		(0.00199)	
δ	−0.05549		−0.05361	
	(0.00783)		(0.00631)	
σ_0	0.03182		0.03190	
	(0.00100)		(0.00098)	
σ_1	0.05436		0.05437	
	(0.00289)		(0.00286)	
q	0.97243		0.97217	
	(0.00649)		(0.00651)	
p	0.91019		0.90638	
	(0.02507)		(0.02441)	
Log Likelihood	1080.47246		1080.38524	
Residual diagnostics[c]	χ^2 statistic	p-value	χ^2 statistic	p-value
ARCH-LM(k = 1)	0.02237	(0.8811)	0.0613	(0.8049)
ARCH-LM(k = 6)	3.34021	(0.7651)	3.24581	(0.7774)
ARCH-LM(k = 12)	9.90094	(0.6247)	9.88648	(0.6259)
JB Normality Test	1.23805	(0.5385)	1.24966	(0.5354)

[a]The restriction is $\delta = -\mu_1/(1 - \rho\lambda)$, where $\rho = 0.997$ and $\lambda = p + q - 1$.

[b]The model is given by Equation (7). Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of (A) January 1926 to December 1951; (B) January 1952 to December 2000. Asymptotic standard errors are reported in parentheses below the estimates.

[c]The ARCH-LM statistics have $\chi^2(k)$ asymptotic distributions under the null of no ARCH effects and are constructed using TR^2 from an auxiliary regression of squared standardized residuals on k lags of squared standardized residuals. The JB (Jarque and Bera, 1980) Normality test statistics have $\chi^2(2)$ asymptotic distributions under the null of Normality and are constructed using sample skewness and kurtosis for the standardized residuals.

TABLE 4

VOLATILITY AND THE EQUITY PREMIUM: FEEDBACK DUE TO FULL REVELATION

A. 1926–1951

	Model specification			
	δ is freely estimated. Agents observe past returns, but regime is partially revealed during period t		δ is restricted.[a] Agents observe past returns, but regime is partially revealed during period t	
Parameters[b]	$\psi_{t-1} = \{S_{t-1}\}, \psi'_t = \{S_t\}$		$\psi_{t-1} = \{S_{t-1}\}, \psi'_t = \{S_t\}$	
μ_0	0.01116		0.00979	
	(0.00301)		(0.00289)	
μ_1	−0.02265		0.00613	
	(0.01446)		(0.00367)	
δ	−0.10623		−0.08546	
	(0.03457)		(0.03026)	
σ_0	0.03957		0.03974	
	(0.00209)		(0.00205)	
σ_1	0.12033		0.12400	
	(0.01066)		(0.01103)	
q	0.98439		0.98418	
	(0.00983)		(0.00978)	
p	0.94952		0.94683	
	(0.03216)		*(0.03332)*	
Log likelihood	451.53924		449.56636	
Residual diagnostics[c]	χ^2 statistic	p-value	χ^2 statistic	p-value
ARCH-LM(k = 1)	0.3076	(0.5792)	0.04888	(0.8250)
ARCH-LM(k = 6)	4.32004	(0.6335)	4.13853	(0.6579)
ARCH-LM(k = 12)	17.9532	(0.1171)	15.5373	(0.2134)
JB Normality Test	0.22216	(0.8949)	0.9598	(0.6188)

B. 1951–2000

	Model specification			
	δ is freely estimated. Agents observe past returns, but regime is partially revealed during period t		δ is restricted.[a] Agents observe past returns, but regime is partially revealed during period t	
Parameters[b]	$\psi_{t-1} = \{S_{t-1}\}, \psi'_t = \{S_t\}$		$\psi_{t-1} = \{S_{t-1}\}, \psi'_t = \{S_t\}$	
μ_0	0.00653		0.00417	
	(0.00186)		(0.00197)	
μ_1	−0.00673		0.01541	
	(0.00888)		(0.00829)	
δ	−0.08518		−0.06670	
	(0.01373)		(0.01594)	
σ_0	0.03174		0.03187	
	(0.00154)		(0.00169)	
σ_1	0.05438		0.05803	
	(0.00521)		(0.00703)	
q	0.96921		0.96405	
	(0.01269)		(0.01831)	
p	0.85091		0.80718	
	(0.06523)		(0.13100)	
Log likelihood	1077.83798		1074.56168	
Residual diagnostics[c]	χ^2 statistic	p-value	χ^2 statistic	p-value
ARCH-LM(k = 1)	1.62165	(0.2029)	2.03654	(0.1536)
ARCH-LM(k = 6)	4.68932	(0.5842)	3.48677	(0.7457)
ARCH-LM(k = 12)	13.8797	(0.3085)	12.0758	(0.4396)
JB Normality test	3.38955	(0.1836)	0.29181	(0.8642)

[a]The restriction is $\delta = -\mu_1/(1-\rho\lambda)$, where $\rho = 0.997$ and $\lambda = p + q - 1$.

[b]The model is given by Equation (7). Maximum likelihood estimates are calculated using continuously compounded total monthly value-weighted excess returns of all NYSE-listed stocks for the sample period of (A) January 1926 to December 1951; (B) January 1952 to December 2000. Asymptotic standard errors are reported in parentheses below the estimates.

[c]The ARCH-LM statistics have $\chi^2(k)$ asymptotic distributions under the null of no ARCH effects and are constructed using TR^2 from an auxiliary regression of squared standardized residuals on k lags of squared standardized residuals. The JB (Jarque and Bera, 1980) Normality test statistics have $\chi^2(2)$ asymptotic distributions under the null of Normality and are constructed using sample skewness and kurtosis for the standardized residuals.

provide strong support for a positive relationship between market volatility and the equity premium. Similarly, when the restriction $\delta = -\mu_1/(1 - \rho\lambda)$ is imposed, the estimated relationship is always positive, with *t*-statistics for μ_1 of 1.10 and 3.34 for the 1926–51 and 1952–2000 samples, respectively. This restriction can be rejected at the 10% level for the 1926–51 sample. The likelihood ratio statistic is 3.21, with *p*-value of 0.07. However, the restriction cannot be rejected for the 1952–2000 sample. The likelihood ratio statistic is 0.17, with *p*-value 0.68. Thus, the results for the restricted model support a positive relationship between market volatility and the equity premium, especially for postwar returns.

Returning to the results in Table 4 for the model with feedback due to full revelation, the feedback effect is always negative and significant when the feedback parameter is unrestricted. The *t*-statistics are −3.07 and −6.20 for the 1926–51 and 1952–2000 samples, respectively. Similarly, when the restriction $\delta = -\mu_1/(1 - \rho\lambda)$ is imposed, the estimated relationship between market volatility and the equity premium is always positive, with *t*-statistics for μ_1 of 1.67 and 1.86 for the 1926–51 and 1952–2000 samples, respectively. However, in the full revelation case, the restriction on δ can be rejected at conventional levels for both sample periods. The likelihood ratio statistics are 3.95, with a *p*-value of 0.05, for the 1926–51 sample and 6.55, with a *p*-value of 0.01, for the 1952–2000 sample.

The rejections of the restriction $\delta = -\mu_1/(1 - \rho\lambda)$ for the full revelation specification raise the possibility that this information assumption, despite its conceptual advantages, may be too strong compared with the partial revelation assumption. Agents may only observe the true volatility regime with some error. Still, the overall results support a positive relationship between market volatility and the equity premium.

It should be noted that the estimated partial effect, μ_1, is always positive when δ is restricted, while it is negative in three of the four cases when μ_1 and δ are estimated separately. This result confirms our claim in the introduction that a volatility feedback effect, by summarizing the impact of an unanticipated change in market volatility on all future discounted expected returns, should be easier to detect than the partial effect on the contemporaneous expected return. In particular, when the restriction $\delta = -\mu_1/(1 - \rho\lambda)$ is imposed, the information from the negative and significant volatility feedback effect dominates the less significant partial effect. Thus, it is useful to account for volatility feedback, not just to avoid obscuring the true relationship between market volatility and the equity premium, but also because it brings new information about this relationship to estimation.

It should also be noted that the negative volatility feedback estimates are not only statistically significant but also imply economically significant changes in the equity premium. The magnitude of these changes are evident from the estimates of μ_1 when the restriction on $\delta = -\mu_1/(1 - \rho\lambda)$ is imposed. For example, the prewar estimates when feedback is due to full revelation suggest that a change in risk from $\sigma_0 = 0.0397$ to $\sigma_1 = 0.12400$ increases the equity premium, in annualized terms, from 12.4% to 20.9%. Meanwhile, the corresponding postwar estimates suggest that a change in risk from $\sigma_0 = 0.03187$ to $\sigma_1 = 0.05803$ increases the equity premium, again in annualized terms, from 5.1% to 26.2%. These estimates might appear unreasonably high, but further consideration suggests they are plausible. First, the *unconditional* estimates of the equity premium implied by the model are, in annualized terms, 14.3% and 8.2% for the two samples, respectively. Meanwhile, the average excess return for the two samples are, in annualized terms, 6.4% and 6.6%, respectively. For the prewar period, the large difference is due to a few large negative volatility feedback events (e.g., the 1929 stock market crash). In particular, unlike the sample average, our model does not attribute the large negative returns associated with an increase in volatility to a drop in the equity premium. Instead, the unconditional equity premium is estimated to be relatively high in the prewar period, consistent with the higher volatility of that era and the general uncertainty surrounding the Great Depression. For the postwar period, the unconditional equity premium is only somewhat higher than the sample average even though the equity premium increases to an estimated 26.2% during the high volatility episodes. This result arises because these episodes are expected to be very short in the postwar period. In particular, their unconditional expected duration is only five months.

2.6 The Exogeneity of Market Volatility

Underpinning the story behind volatility feedback is the idea that news volatility is exogenous, with returns endogenously reacting to its changes. The fact that the estimates of the Markov-switching process reported in the tables do not change dramatically when we allow the mean as well as the variance to switch provides some informal support for this idea. Further informal support comes from the similarity between the smoothed probability inferences across all of the models. For example, Figures 2A and 2B display excess returns and smoothed probability of a high volatility regime for the model with restricted feedback due to full revelation. The timing of changes in regime is virtually indistinguishable from the timing given in Figures 1A and 1B.

A more formal test of exogeneity involves checking the standardized residuals from our model for lingering ARCH effects.[10] In particular, if the leverage hypothesis holds, and stock price movements generate changes in future volatility, rather than vice versa, there should be persistent and continuous (i.e., ARCH-type) changes in the level of volatility that will not be captured by a discrete two-state Markov-switching specification. Yet, the residual diagnostics in the tables suggest that there are no significant ARCH effects when volatility feedback is taken into account.

10. A practical difficulty in examining the standardized residuals arises from the fact that the residuals and their conditional variance are dependent upon an unobservable state variable. To address this problem, we substitute the smoothed probabilities for the true state to calculate the residuals. Note that it is precisely because we do not observe the true residuals that we cannot accommodate ARCH and leverage effects directly in our model. By contrast, Hamilton and Susmel (1994) are able to incorporate these effects in their Markov-switching model since their assumption of *constant* expected returns makes the residuals of their model observable. However, we cannot make the same assumption since it is changes in expected returns that we are most interested in examining.

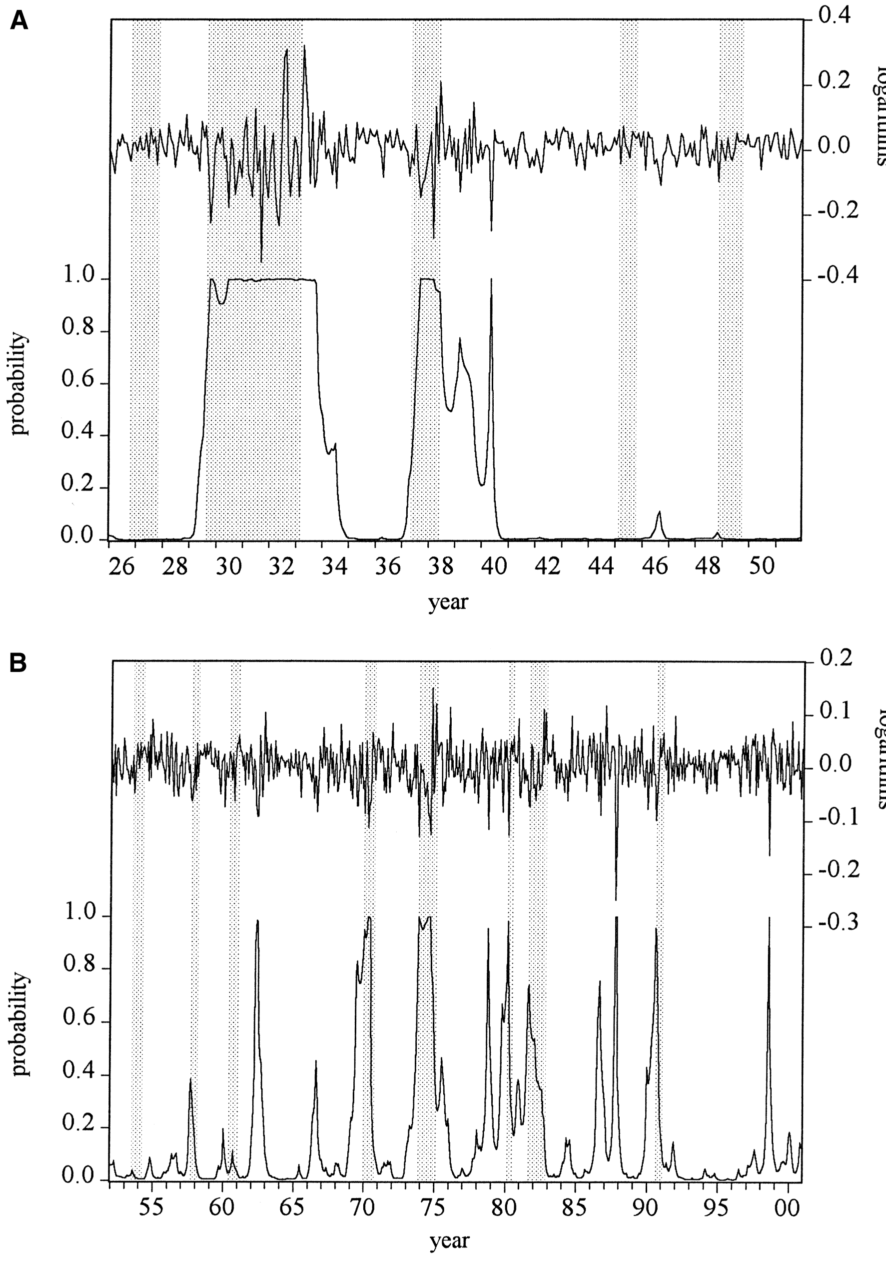

FIG. 2. Excess stock returns and smoothed probability of a high volatility regime for the model with restricted feedback due to full revelation: (A) 1926–51; (B) 1952–2000. Source: Excess stock returns are continuously compounded total monthly value-weighted returns of all NYSE-listed stocks from CRSP in excess of one-month Treasury bill yields from CRSP for the sample period of (A) January 1926 to December 1951; (B) January 1952 to December 2000. Smoothed probability inferences are calculated using Kim's (1994) smoothing algorithm and maximum likelihood estimates for the restricted feedback due to full revelation model presented in Table 4. NBER-dated recessions are shaded.

Specifically, the ARCH-LM $\chi^2(k)$ statistics for a k-lag autoregression of the standardized residuals ($k = 1, 6,$ and 12 months) reported in Tables 3 and 4 are all insignificant at conventional levels. Meanwhile, the Jarque and Bera (1980) $\chi^2(2)$ Normality test statistics based on the sample skewness and kurtosis for the standardized residuals are also insignificant at conventional levels and are much lower than for the raw data (reported in the first column of panels A and B of Table 1), suggesting that the two-regime model of market volatility with volatility feedback captures the heteroskedasticity in monthly stock returns.

3. CONCLUSIONS

When the effects of volatility feedback are fully taken into account, the empirical evidence supports a significant positive relationship between stock market volatility and the equity premium. We arrive at this conclusion in the following way. First, we employ the log–linear present value framework under an assumption of Markov-switching market volatility to show that evidence of a negative volatility feedback effect is equivalent to evidence of a positive relationship between market volatility and the equity premium. Second, using monthly excess returns on a value-weighted portfolio of all NYSE stocks, we find that a two-state Markov-switching specification captures changes in the level of market volatility for both prewar and postwar returns. Third, we find statistically significant evidence that the equity premium changes in response to the Markov-switching level of market volatility. Fourth, for postwar returns at least, we find statistically significant evidence in support of the existence of volatility feedback. Fifth, for both prewar and postwar returns, we find that estimates of the volatility feedback effect are negative and significant, implying economically significant changes in the equity premium over time. Meanwhile, we show that the standardized residuals for the volatility feedback models display none of the ARCH effects implied by the leverage hypothesis, suggesting that we can directly interpret the negative volatility feedback effect estimates as evidence for a positive relationship between market volatility and the equity premium.

APPENDIX

The four main equations of the Hamilton (1989) filter for estimating a Markov-switching model are given as follows:

$$\Pr[S_t = j, S_{t-1} = i \mid \tilde{r}_{t-1}] = \Pr[S_t = j \mid S_{t-1} = i] \cdot \Pr[S_{t-1} = i \mid \tilde{r}_{t-1}] \,, \tag{A1}$$

$$f(r_t \mid S_t = j, S_{t-1} = i, \tilde{r}_{t-1}) = \frac{1}{\sqrt{2\pi\sigma^2_{S_t}}} \exp\left\{ -\frac{1}{2\sigma^2_{S_t}} \varepsilon_t^2 \right\} , \tag{A2}$$

$$f(r_t \mid \tilde{r}_{t-1}) = \sum_{j=0}^{1}\sum_{i=0}^{1} f(r_t \mid S_t = j, S_{t-1} = i, \tilde{r}_{t-1}) \cdot \Pr[S_t = j, S_{t-1} = i \mid \tilde{r}_{t-1}] \text{ , and} \tag{A3}$$

$$\Pr[S_t = j \mid \tilde{r}_t] = \sum_{i=0}^{1} \frac{f(r_t \mid S_t = j, S_{t-1} = i, \tilde{r}_{t-1}) \cdot \Pr[S_t = j, S_{t-1} = i \mid \tilde{r}_{t-1}]}{f(r_t \mid \tilde{r}_{t-1})}, \qquad \text{(A4)}$$

where $j = 0, 1$, $i = 0,1$, $\tilde{r}_t \equiv \{r_t, r_{t-1}, r_{t-2}, ...\}$, and from Equation (7),

$$\varepsilon_t \equiv r_t - \mu_0 - \mu_1 \Pr[S_t = 1 \mid \psi_{t-1}] - \delta\{\Pr[S_t = 1 \mid \psi'_t] - \Pr[S_t = 1 \mid \psi_{t-1}]\}.$$

To initialize the filter, we use unconditional probabilities

$$\Pr[S_1 = 0] = \frac{1-p}{2-p-q} \text{ and} \qquad \text{(A5)}$$

$$\Pr[S_1 = 1] = \frac{1-q}{2-p-q}. \qquad \text{(A6)}$$

Then, we iterate through Equations (A1)–(A4) for $t = 1,...,T$ to obtain the density of r_t given in Equation (A3). This allows us to find maximum likelihood estimates of the parameters:

$$\max_{\theta} \left\{ l(\theta \mid \tilde{r}_T) = \sum_{t=1}^{T} \ln(f(r_t \mid \tilde{r}_{t-1})) \right\}. \qquad \text{(A7)}$$

Finally, we obtain smoothed probability inferences for S_t using Kim's (1994) smoothing algorithm. In particular, given the filtered probability in Equation (A4) and the conditional probability $\Pr[S_t = j \mid \tilde{r}_{t-1}]$, which can be found by collapsing across states for S_{t-1} in Equation (A1), we iterate backward for $t = T,...,1$ through the following two equations:

$$\Pr[S_{t+1} = l, S_t = j \mid \tilde{r}_T] = \frac{\Pr[S_{t+1} = l \mid \tilde{r}_T] \cdot \Pr[S_t = j \mid \tilde{r}_t] \cdot \Pr[S_{t+1} = l \mid S_t = j]}{\Pr[S_{t+1} = l \mid \tilde{r}_t]} \text{ and} \qquad \text{(A8)}$$

$$\Pr[S_{t+1} = j \mid \tilde{r}_T] = \sum_{l=0}^{1} \Pr[S_{t+1} = l, S_t = j \mid \tilde{r}_T], \qquad \text{(A9)}$$

where $l = 0,1$.

LITERATURE CITED

Abel, Andrew B. (1988). "Stock Prices under Time-Varying Dividend Risk: An Exact Solution in an Infinite-Horizon General Equilibrium Model." *Journal of Monetary Economics* 22, 375–393.

Backus, David K., and Allan W. Gregory. (1993). "Theoretical Relations between Risk Premiums and Conditional Variances." *Journal of Business and Economic Statistics* 11, 177–185.

Bekaert, Geert, and Guojun Wu (2000). "Asymmetric Volatility and Risk in Equity Markets." *Review of Financial Studies* 13, 1–42.

Bollerslev, Tim, Robert F. Engle, and J. M. Wooldrige (1988). "A Capital Asset Pricing Model with Time-Varying Covariances." *Journal of Political Economy* 96, 116–131.

Breen, William, Lawrence R. Glosten, and Ravi Jagannathan (1989). "Economic Significance of Predictable Variations in Stock Index Returns." *Journal of Finance* 44, 1177–1189.

Campbell, John Y. (1987). "Stock Returns and the Term Structure." *Journal of Financial Economics* 18, 373–399.

Campbell, John Y., and Ludger Hentschel (1992). "No News Is Good News: An Asymmetric Model of Changing Volatility in Stock Returns." *Journal of Financial Economics* 31, 281–318.

Campbell, John Y., Andrew C. Lo, and A. Craig MacKinlay (1997). *The Econometrics of Financial Markets*. Princeton, NJ: Princeton University Press.

Campbell, John Y., and Robert Shiller (1988). "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *Review of Financial Studies* 1, 195–227.

French, Kenneth R., G. William Schwert, and Robert F. Stambaugh (1987). "Expected Stock Returns and Volatility." *Journal of Financial Economics* 19, 3–29.

Garcia, Rene (1998). "Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models." *International Economic Review* 39, 763–788.

Gennotte, Garard, and Terry A. Marsh (1993). "Variations in Economic Uncertainty and Risk Premiums on Capital Assets." *European Economic Review* 37, 1021–1041.

Glosten, Lawrence R., Ravi Jagannathan, and David E. Runkle (1993). "On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks." *Journal of Finance* 48, 1779–1801.

Hamilton, James D. (1989). "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle." *Econometrica* 57, 357–384.

Hamilton, James D. (1993). "Estimation, Inference, and Forecasting of Time Series Subject to Changes in Regime." In *Handbook of Statistics*, Vol. 11, edited by G. S. Maddala, C. R. Rao, and H. D. Vinod. Amsterdam: North-Holland.

Hamilton, James D., and R. Susmel. (1994). "Autoregressive Conditional Heteroskedasticity and Changes in Regime." *Journal of Econometrics* 64, 307–333.

Hansen, Bruce E. (1992). "The Likelihood Ratio Test under Nonstandard Conditions: Testing the Markov Switching Model of GNP." *Journal of Applied Econometrics* 7, S61–S82.

Harvey, Campbell R. (1989). "Time-Varying Conditional Covariances in Tests of Asset Pricing Models." *Journal of Financial Economics* 24, 289–317.

Jarque, C., and A. Bera (1980). "Efficient Tests for Normality, Heteroskedasticity, and Serial Independence of Regression Residuals." *Economics Letters* 6, 255–259.

Kim, Chang-Jin (1994). "Dynamic Linear Models with Markov-switching." *Journal of Econometrics* 60, 1–22.

Kim, Chang-Jin, and Charles R. Nelson (1998). "Testing for Mean Reversion in Heteroskedastic Data II: Autoregression Tests Based on Gibbs-Sampling-Augmented Randomization." *Journal of Empirical Finance* 5, 385–396.

Kim, Chang-Jin, Charles R. Nelson, and Richard Startz (1998). "Testing for Mean Reversion in Heteroskedastic Data Based on Gibbs-Sampling-Augmented Randomization." *Journal of Empirical Finance* 5, 131–154.

Mayfield, E. Scott. (1999). "Estimating the Market Risk Premium." Working Paper, Harvard Business School.

Morley, James C. (1999). "Essays in Empirical Finance." Ph.D. dissertation, University of Washington.

Muth, John (1960). "Optimal Properties of Exponentially Weighted Forecasts." *Journal of the American Statistical Association* 55, 299–306.

Nelson, Daniel (1991). "Conditional Heteroskedasticity in Asset Returns: A New Approach." *Econometrica* 59, 347–370.

Pagan, Adrian R., and G. William Schwert (1990). "Alternative Models of Conditional Stock Volatility." *Journal of Econometrics* 45, 267–290.

Schaller, Huntley, and Simon van Norden (1997). "Regime Switching in Stock Market Returns." *Applied Financial Economics* 7, 177–191.

Schwert, G. William (1989). "Business Cycles, Financial Crises, and Stock Volatility." *Carnegie-Rochester Conference Series on Public Policy* 31, 83–126.

Scruggs, John T. (1998). "Resolving the Puzzling Intertemporal Relation between the Market Risk Premium and Conditional Market Variance: A Two-Factor Approach." *Journal of Finance* 53, 575–603.

Turner, Christopher M., Richard Startz, and Charles R. Nelson (1989). "A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market." *Journal of Financial Economics* 25, 3–22.

Veronesi, Pietro (1999). "Stock Market Overreaction to Bad News in Good Times: A Rational Expectations Equilibrium Model." *Review of Financial Studies* 12, 975–1007.

Whitelaw, Robert F. (1994). "Time Variations and Covariations in the Expectation and Volatility of Stock Market Returns." *Journal of Finance* 49, 515–541.