Is there contextuality in behavioral and social systems?

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Abstract

Most behavioral and social experiments aimed at revealing contextuality are confined to cyclic systems with binary outcomes. In quantum physics, this broad class of systems includes as special cases Klyachko-Can-Binicioglu-Shumovsky-type, Einstein-Podolsky-Rosen-Bell-type, and Suppes-Zanotti-Leggett-Garg-type systems. The theory of contextuality known as Contextuality-by-Default allows one to define and measure contextuality in all such system, even if there are context-dependent errors in measurements, or if something in the contexts directly interacts with the measurements. This makes the theory especially suitable for behavioral and social systems, where direct interactions of "everything with everything" are ubiquitous. For cyclic systems with binary outcomes the theory provides necessary and sufficient conditions for noncontextuality, and these conditions are known to be breached in certain quantum systems. We review several behavioral and social data sets (from polls of public opinion to visual illusions to conjoint choices to word combinations to psychophysical matching), and none of these data provides any evidence for contextuality. Our working hypothesis is that this may be a broadly applicable rule: behavioral and social systems are noncontextual, i.e., all "contextual effects" in them result from the ubiquitous dependence of response distributions on the elements of contexts other than the ones to which the response is presumably or normatively directed.

KEYWORDS:contextuality, cyclic systems, inconsistent connectedness

1 Introduction

Although the word is widely used in linguistics, psychology, and philosophy, the notion of contextuality as it is used in this paper comes from quantum mechanics, where in turn it came from logic [1]. The reason for the prominence of this notion in quantum theory is that classical-mechanical systems are not contextual while some quantum-mechanical systems are. Contextuality is sometimes even presented as one of the "paradoxes" of quantum mechanics. In psychology, as it turns out, a certain variety of (non)contextuality has been prominent too, but it is known under different name: selectiveness of influences, or lack thereof (for details, see Refs. [2, 3]).

The term "contextuality" refers to properties of systems of random variables each of which can be viewed (sometimes artificially) as a measurement of some "object" in some *context*. For instance, an object q may be a question, and the context may be defined by what other question q' it is asked in combination with. Then the answer to this question is a random variable $R_q^{(q,q')}$ that can be interpreted as the measurement of q in the context (q, q'). If the same question q is then asked in combination with some other question q'', then the measurement is a different random variable, $R_q^{(q,q'')}$. More generally, context in which q is measured is defined by the conditions c under which the measurement is made, yielding random variable R_q^c. This notation (or one of numerous variants thereof) is called *contextual notation* for random variables: it codifies the idea that the identity of a measurement is defined both by what is measured and by the conditions under it is measured [4–11].

Within each context the measurements are made "together", because of which they have an empirically defined *joint distribution.* Thus, in context (q, q') we have two jointly distributed random variables $R_q^{(q,q')}$ and $R_{q'}^{(q,q')}$. We call the set of all random variables jointly recorded in a given context a *bunch* (of random variables, or of measurements). Two different bunches have no joint distribution, because there is no empirically defined way of coupling the values of one bunch with those of another. We say that they are *stochastically unrelated.* Thus, in

$$R^{(q,q')} = (R_q^{(q,q')}, R_{q'}^{(q,q')}) \text{ and } R^{(q,q'')} = (R_q^{(q,q'')}, R_{q''}^{(q,q'')}) \tag{1}$$

any component of $R^{(q,q')}$ is stochastically unrelated to any component of $R^{(q,q'')}$, including $R_q^{(q,q')}$ and $R_q^{(q,q'')}$.

This work is based on the theory of contextuality dubbed Contextuality-by-Default (CbD) [5,6,12–17] (for precursors of this theory, see Refs. [9–11]). On a very general level, its main idea is that

> *a system of different, stochastically unrelated bunches of random variables can be characterized by considering all possible ways in which they can be coupled under well-chosen constraints imposed, for each object, on the relationship between the measurements of this object in different contexts.*

To *couple* different bunches simply means to impose a joint distribution on them. In the example above, this means finding four jointly distributed random variables (A, B, X, Y) such that, in reference to (1),

$$(A, B) \sim R^{(q,q')} \text{ and } (X, Y) \sim R^{(q,q'')}, \tag{2}$$

$\sim$ standing for "is distributed as". The quadruple (A, B, X, Y) is then called a *coupling* for the bunches $R^{(q,q')}$ and $R^{(q,q'')}$. The "well-chosen constraints" is a key notion in the formulation above. In our example, these constraints should apply to A and X, the coupling counterparts of $R_q^{(q,q')}$ and $R_q^{(q,q'')}$ measuring (answering) the same question q in two different contexts.

Intuitively, "noncontextuality" means "independence of context", and because of this it is tempting to say that the system of two bunches in (1) is noncontextual if we can consider $R_q^{(q,q')}$ and $R_q^{(q,q'')}$ as "one and the same" random variable, R_q. This may appear simple, but in fact it is logically impossible: since $R_q^{(q,q')}$ and $R_q^{(q,q'')}$ are stochastically unrelated, they cannot be "the same". A random variable cannot be stochastically unrelated to itself. The precise meaning here comes from considering couplings (A, B, X, Y) for the two bunches. Clearly, in every such a coupling $A \sim R_q^{(q,q')}$ and $X \sim R_q^{(q,q'')}$. We can say that the measurement of q in the system is context-independent if among all possible couplings (A, B, X, Y) there is at least one in which $\Pr[A \neq X] = 0$. In this particular example, due to its simplicity (only three random variables involved in two contexts) it can be shown that such a coupling does exist, provided $R_q^{(q,q')} \sim R_q^{(q,q'')}$. In a more complex system, such a coupling may not exists even if the system is *consistently connected*: which means that in this system the measurements of one and the same "object" always have the same distribution.

The traditional approaches to contextuality were confined to consistent connectedness, but this condition is too restrictive in quantum physics [14, 16, 18] and virtually inapplicable in social and behavioral sciences: almost always, a response to question (or stimulus) q will depend on the context in which it is asked, which may translate into $R_q^{(q,q')}$ and $R_q^{(q,q'')}$ having different distributions. There is nothing wrong in calling any such a case contextual, and this is done by many (see Sections 3 and 6 below). It is, however, more informative to separate inconsistent connectedness from contextuality, and this is what is done in the CbD theory. We use the term *inconsistently connected* for the systems that are not necessarily consistently connected (but may be so, as a special or limit case).

The logic of the CbD approach is as follows. We first consider separately the random variables measuring the same object in different contexts, in our example $R_q^{(q,q')}$ and $R_q^{(q,q'')}$. We call this set of random variables the *connection* (for the measured object, in our case q). Among all possible couplings (A', X') for the connection $\{R_q^{(q,q')}, R_q^{(q,q'')}\}$, i.e., among all jointly distributed (A', X') such that $A' \sim R_q^{(q,q')}$ and $X' \sim R_q^{(q,q'')}$, we find the minimal value m' of $\Pr[A' \neq X']$. Then we look at the entire system of the bunches, in our case (1), and among all possible couplings (A, B, X, Y) for this system we find the minimal value m for $\Pr[A \neq X]$. It should be clear that m' cannot exceed m, because in every coupling (A, B, X, Y) for (1) the part (A, X) forms a coupling for the connection $\{R_q^{(q,q')}, R_q^{(q,q'')}\}$. But they can be equal, $m = m'$, and then we say that the system is noncontextual. If $m > m'$, the system is contextual. Again, due to its simplicity, the system consisting of the two bunches (1) cannot be contextual, but this may very well be the case in more complex systems.

As an example of the latter, consider a system with two bunches

$$R^{(q,q')} = (R_q^{(q,q')}, R_{q'}^{(q,q')}) \text{ and } R^{(q',q)} = (R_q^{(q',q)}, R_{q'}^{(q',q)}) \tag{3}$$

in which there are only two "objects" q, q', and the two contexts differ in the order in which these objects are measured. We have two connections here,

$$\{R_q^{(q,q')}, R_q^{(q',q)}\} \text{ and } \{R_{q'}^{(q,q')}, R_{q'}^{(q',q)}\}. \tag{4}$$

Let us assume the measurements are binary, with values $+1$ and -1 (e.g., corresponding to answers Yes and No), and let us further assume that all four random variables are "fair coins", with equal probabilities of +1 and -1. Then the distribution of the bunches $R^{(q,q')}$ and $R^{(q',q)}$ in (3) are uniquely defined by the product expected values $\langle R_q^{(q,q')} R_{q'}^{(q,q')} \rangle$ and $\langle R_q^{(q',q)} R_{q'}^{(q',q)} \rangle$.

It easy to see that, across all possible couplings (A', X') for $\{R_q^{(q,q')}, R_q^{(q',q)}\}$, the minimum value m'_1 of $\Pr[A' \neq X']$ is 0, and the same is true for the minimum value m'_2 of $\Pr[B' \neq Y']$ across all possible couplings (B', Y') for $\{R_{q'}^{(q,q')}, R_{q'}^{(q',q)}\}$. However, it follows from the general theory that across all possible couplings (A, B, X, Y) for the entire system (3) the values m_1 of $\Pr[A \neq X]$ and m_2 of $\Pr[B \neq Y]$ cannot be both zero unless $\langle R_q^{(q,q')} R_{q'}^{(q,q')} \rangle = \langle R_q^{(q',q)} R_{q'}^{(q',q)} \rangle$. The latter need not be the case: it may, e.g., very well be that $\langle R_q^{(q,q')} R_{q'}^{(q,q')} \rangle = 1$ (perfect correlation) and $\langle R_q^{(q',q)} R_{q'}^{(q',q)} \rangle = -1$ (perfect anti-correlation). In this case $m_1 + m_2 \geq 1$, whence either $m_1 > m'_1 = 0$ or $m_2 > m'_2 = 0$, indicating that the system is contextual.

As we show in this paper, the general rule for a broad spectrum of behavioral and social systems of measurements seems to be that *they are all noncontextual in the sense of CbD.*

2 Cyclic systems of arbitrary rank

In this section and throughout the rest of the paper we assume that all our measurements are binary random variables, with values ± 1.

We apply the logic of the CbD theory to systems in which all objects are measured in pairs so that each object belongs to precisely two pairs. We call such systems *cyclic*, because we can enumerate the objects in such a system $q_1, \ldots, q_n$ and arrange them in a cycle

$$q_1 \longrightarrow q_2 \longrightarrow \cdots \longrightarrow q_{n-1} \longrightarrow q_n \quad (q_n \to q_1) \tag{5}$$

in which any two successive objects form a context. The number n is referred to as the *rank* of the system. Our last example in the previous section is a cyclic system of rank 2, the smallest possible.

In accordance with our notation, each object q_i in a cyclic system is measured by two random variables: $R_{q_i}^{(q_i, q_{i\oplus 1})}$ and $R_{q_i}^{(q_{i\ominus 1}, q_i)}$, where the operations $\oplus$ and $\ominus$ are cyclic addition and subtraction (so that $n \oplus 1 = 1$ and $1 \ominus 1 = n$). Since there are no other random variables involved, we can simplify notation: we will denote $R_{q_i}^{(q_i, q_{i\oplus 1})}$, measuring the first object in the context, by V_i, and $R_{q_i}^{(q_{i\ominus 1}, q_i)}$, measuring the second object in the context, by W_i. As a result each bunch in a cyclic system has the form $(V_i, W_{i\oplus 1})$; e.g., the bunch of measurements for (q_1, q_2) is (V_1, W_2), for (q_n, q_1) the bunch is (V_n, W_1), etc.

Now we can represent a cyclic system of measurements in the form of a $V - W$ cycle:

$$V_1 \text{ --- } W_2 \cdots V_2 \text{ --- } W_3 \cdots \cdots \cdots V_n \text{ --- } W_1 \quad (W_1 \cdots V_1) \tag{6}$$

where solid lines indicate bunches (joint measurements) and point lines indicate connections (measurements of the object in different contexts).

It is proved in Refs. [14, 16, 17] that such a system is noncontextual if and only if its bunches satisfy the following inequality:

$$\Delta C = \mathsf{s}_1(\langle V_1 W_2 \rangle, \ldots, \langle V_{n-1} W_n \rangle, \langle V_n W_1 \rangle) - (n-2) - \sum_{i=1}^{n} |\langle V_i \rangle - \langle W_i \rangle| \leq 0, \tag{7}$$

where $\langle \cdot \rangle$ denotes expected value, and the s_1-part is the maximum of all linear combinations $\pm \langle V_1 W_2 \rangle \pm \ldots \pm \langle V_{n-1} W_n \rangle \pm \langle V_n W_1 \rangle$ with the proviso that the number of minuses is odd. Note that the criterion is written entirely in terms of the expectations of V_i, W_i and of the products $V_i, W_{i\oplus 1}$ $(i = 1, \ldots, n)$. This means that the information about a cyclic system we need can be presented in the form of the diagram

$$\langle V_1 \rangle \xrightarrow{\langle V_1 W_2 \rangle} \langle W_2 \rangle \cdots \langle V_2 \rangle \xrightarrow{\langle V_2 W_3 \rangle} \cdots \cdots \langle V_n \rangle \xrightarrow{\langle V_n W_1 \rangle} \langle W_1 \rangle \tag{8}$$

We will use such diagrams to discuss experimental data in the subsequent sections.

This criterion of noncontextuality is generally breached by quantum-mechanical systems. Thus, for consistently connected systems, for $n = 3$, the inequality reduces to Suppes-Zanotti-Leggett-Garg inequality [19, 20], for $n = 4$ it acquires the form of the Clauser-Horn-Shimony-Holt inequalities for the Einstein-Podolsky-Rosen-Bell paradigm [21–23], and for $n = 5$ (with an additional constraint) it becomes what is known as Klyachko-Can-Binicioglu-Shumovsky inequality [24]. All of them are predicted by quantum theory and supported by experiments to be violated by some quantum-mechanical systems. For $n = 3$, using the criterion (7), violations are also predicted for inconsistently connected systems [18]; and for $n = 5$ violations of (7) were demonstrated experimentally [25] (as analyzed in Ref. [16]).

By contrast, we find no violations of (7) in all known to us behavioral and social experiments aimed at revealing contextuality: ΔC never exceeds zero. In the subsequent sections we demonstrate this "failure to fail" the noncontextuality criterion on several experimental studies, for cyclic systems of rank 2, 3, and 4.

3 Question order effect (cyclic systems of rank 2)

Wang, Solloway, Shiffrin, and Busemeyer [26] considered 73 polls in which two questions, A and B (playing the role of "objects" q_1, q_2 being measured), were asked in two possible orders, $A \to B$ and $B \to A$ (forming two contexts). The possible answers to each question, random variables

$$V_1 = R_A^{A\to B}, W_2 = R_B^{A\to B}, V_2 = R_B^{B\to A}, W_1 = R_A^{B\to A}, \tag{9}$$

were binary: +1 (Yes) or −1 (No). For instance, in the Gallup poll results used in Ref. [27], one pair of questions was (paraphrasing)

> A: Do you think many white people dislike black people?
>
> B: Do you think many black people dislike white people?

with the resulting estimates of joint and marginal probabilities

$A \to B$	Yes to B	
Yes to A	.3987	.4161
	.5599	$N \doteq 500$

	Yes to B	$B \to A$
.5391	.4012	Yes to A
$N \doteq 500$	.4609	

We translate "Yes to A" into $V_1 = 1$ in $A \to B$ and into $W_1 = 1$ in $B \to A$; correspondingly, "Yes to B" translates into $W_2 = 1$ in $A \to B$ and into $V_2 = 1$ in $B \to A$. Using the notation (8), we deal here with the system

$$\begin{array}{ccc} \langle V_1 \rangle & \overline{\langle V_1 W_2 \rangle} & \langle W_2 \rangle \\ \vdots & & \vdots \\ \langle W_1 \rangle & \overline{\langle V_2 W_1 \rangle} & \langle V_2 \rangle \end{array} = \begin{array}{ccc} -.1678 & \overline{.6428} & .1198 \\ \vdots & & \vdots \\ .0782 & \overline{.6048} & -.0782 \end{array}$$

To make sure the calculations are clear, for any ±1 random variables X, Y,

$$\langle X \rangle = 2\Pr[X = 1] - 1,$$
$$\langle XY \rangle = \Pr[X = Y] - \Pr[X \neq Y] = 4\Pr[X = 1, Y = 1] - 2\Pr[X = 1] - 2\Pr[Y = 1] + 1.$$

The noncontextuality criterion (7) for cyclic systems of rank 2 specializes to the form

$$\Delta C = |\langle V_1 W_2 \rangle - \langle V_2 W_1 \rangle| - (|\langle V_1 \rangle - \langle W_1 \rangle| + |\langle V_2 \rangle - \langle W_2 \rangle|) \leq 0. \tag{10}$$

For the values in the diagram above, $\Delta C = -0.406$, so there is no evidence the system is contextual.

Ref. [26] contains analysis of 73 such pairs of questions, including 66 taken from PEW polls (with N ranging from 125 to 927), four taken from Gallup polls reported by Moore [27] (with N about 500), and three pairs of questions with N ranging from 106 to 305. (The data were kindly provided to us by the authors of Ref. [26]; our computations based of these data are shown in supplementary file S1.)

The analysis is simplified if we accept the empirical regularity discovered by Wang and Busemeyer [28] and convincingly corroborated in Ref. [26]: using our notation, the discovery is that for vast majority of question pairs,

$$\langle V_1 W_2 \rangle = \langle V_2 W_1 \rangle, \tag{11}$$

while

$$|\langle V_1 \rangle - \langle W_1 \rangle| + |\langle V_2 \rangle - \langle W_2 \rangle| \neq 0. \tag{12}$$

The last inequality is what traditionally called the question order effect [27], and (11) is dubbed by Wang and Busemeyer the *quantum question* (QQ) equality. Wang and Busemeyer [28] theoretically justify the QQ equality by positing that the process of answering two successive questions can be modeled by orthogonally projecting a state vector ψ twice in a succession in a Hilbert space. Denoting the projectors corresponding to response Yes to the questions A and B by P and Q, respectively, we have $P^2 = P$, $Q^2 = Q$. The orthogonal projectors corresponding to response No to the same two questions are then $I - P$ and $I - Q$, with I denoting the identity operator. We have, for the question order $A \to B$,

$$\frac{1 + \langle V_1 W_2 \rangle}{2} = \|QP\psi\|^2 + \|(I-Q)(I-P)\psi\|^2 = \langle (PQP + (I-P)(I-Q)(I-P))\psi \,|\, \psi \rangle,$$

and it is readily shown that

$$PQP + (I-P)(I-Q)(I-P) = I - (P+Q) + (PQ + QP).$$

As P and Q enter in this expression symmetrically, the expression is precisely the same for

$$\frac{1 + \langle V_2 W_1 \rangle}{2} = \|PQ\psi\|^2 + \|(I-P)(I-Q)\psi\|^2.$$

The empirical QQ effect now follows from the assumption that the operators P, Q do not vary across respondents (being determined by the questions alone), whereas the mixture of the initial states ψ has the same distribution in any two large groups of respondents. At the same time, the question order effect follows from the fact that $\|QP\psi\|^2$ is not the generally the same as $\|PQ\psi\|^2$.

The QQ equality trivially implies (10), i.e., lack of contextuality. Therefore, to the extent the QQ equality can be viewed as an empirical law (and Ref. [26] demonstrates this convincingly for 72 out of 73 question pairs), the criterion of noncontextuality should be satisfied for any $\langle V_1 \rangle, \langle W_1 \rangle, \langle V_2 \rangle, \langle W_2 \rangle$. We can confirm and complement the statistical analysis presented in Ref. [26] of the 72 questions by pointing out that the overall chi-square test of the equality (11) over all of them yields $p > 0.35$, $df = 72$.

The singled out pair of questions that violates the QQ equality is taken from the Gallup poll study reported in Ref. [27]: paraphrasing,

> A: Should Pete Rose be admitted to the baseball hall of fame?
>
> B: Should shoeless Joe Jackson be admitted to the baseball hall of fame?

Refs. [26, 28] provide an explanation for why the double-projection model should not apply to this particular pair of questions, but we need not be concerned with it. The diagram of the results for this pair is

$$\begin{array}{ccc} \langle V_1 \rangle & \overline{\langle V_1 W_2 \rangle} & \langle W_2 \rangle \\ \vdots & & \vdots \\ \langle W_1 \rangle & \underline{\langle V_2 W_1 \rangle} & \langle V_2 \rangle \end{array} = \begin{array}{ccc} .3241 & \overline{.6190} & -.2886 \\ \vdots & & \vdots \\ -.0346 & \underline{.3162} & .0780 \end{array},$$

and it is readily seen to violate the equality $\langle V_1 W_2 \rangle = \langle V_2 W_1 \rangle$ ($p < 10^{-7}$, chi-square test with $df = 1$). At the same time the diagram yields $\Delta C = -0.422$, no evidence of contextuality. This example serves as a good demonstration for the fact that while the QQ equality is a sufficient condition for lack of contextuality, it is by no means necessary.

Considering the question pairs one by one, all but six ΔC values out of 73 are negative. In five of these six cases, the QQ equality $|\langle V_1 W_2 \rangle - \langle V_2 W_1 \rangle| = 0$ cannot be rejected with p-values ranging from 0.06 to 0.47. Therefore (10) cannot be rejected either. In the remaining case, p-value for the QQ equality is 0.008, and $\Delta C = 0.063$. While this case is suspicious, we do not think it warrants a special

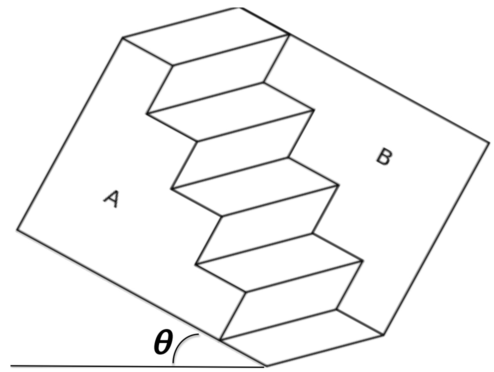


Figure 1: Shröder's staircases used in the experiments reported in Ref. [29]

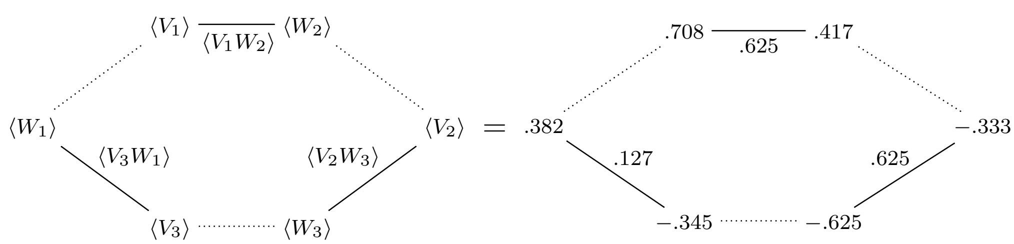
investigation: using conventional significance values, say, 0.01, for 73 similar cases we get the probability of at least one rejection inflated to 0.52.

Note that in the literature cited, including Refs. [26, 28], the term "contextual effect" is used to designate the question order effect (12). This meaning of contextuality corresponds to what we call here inconsistent connectedness (or violations of marginal selectivity), and it should not be confused with the meaning of contextuality as defined in Sections 1 and 2 and indicated by the sign of ΔC.

4 Schröder's staircase illusion (a cyclic system of rank 3)

Asano, Hashimoto, Khrennikov, Ohya, and Tanaka [29] studied a cyclic system of rank 3, using as "objects" q_1, q_2, q_3 Shröder's staircases tilted at three different angles, $\theta = 40, 45, 50$ degrees, as shown in Figure 1. In fact, these three angles formed the middle part of a set of 11 angles ranging from 0 to 90 degrees and presented either in the descending order (context c_1), or in the ascending order (context c_2), or else in a random order (context c_3). Each context involved a separate set of about 50 participants, and each participant in response to each of 11 angles had to indicate whether she/he sees the surface A in front of B (+1) or B in front of A (−1). From these 11 responses, in each context, the authors selected two. In context c_1 the selected responses where those to $\theta = 40, 45$ deg, so, formally, c_1 can be identified with (q_1, q_2); in contexts c_2 and c_3 the selected responses were those to $\theta = 45, 50$ deg and to $\theta = 50, 40$ deg, respectively, making $c_2 = (q_2, q_3)$ and $c_3 = (q_3, q_1)$. It is irrelevant to the logic of the analysis that each context in fact contained all three tilts q_1, q_2, q_3, as well as eight other tilts. (Ref. [29] includes a variety of other combinations of three objects and three contexts extracted from the experiment in question. The data set for the combination described here was kindly made available to us by the authors of Ref. [29].)

The results of the experiment are shown in the diagram of expected values below:



The criterion of noncontextuality for a rank 3 cyclic system has the form

$$\Delta C = \mathsf{s}_1(\langle V_1 W_2\rangle, \langle V_2 W_3\rangle, \langle V_3 W_1\rangle) - 1 - \sum_{i-1}^{3} |\langle V_i\rangle - \langle W_i\rangle| \leq 0 \tag{13}$$

where

$$\mathsf{s}_1(x, y, z) = \max(x + y - z, x - y + z, -x + y + z, -x - y - z).$$

The calculation shows $\Delta C = -1.233$, no evidence for contextuality.

Search for contextuality is the specific goal of Ref. [29], but the meaning of the concept there is different from ours: there, it means violations of the Suppes-Zanotti-Leggett-Garg inequality (which is the consistently connected case of (13)), irrespective of whether these violations are due to inconsistent connectedness or due to contextuality in our sense.

5 Conjoint choices: Animals and sounds they make (a cyclic system of rank 4)

Aerts, Gabora, and Sozzo [30] present results of an experiment in which each of 81 participants had to choose between two animals and between two animal sounds, under four conditions c_1, c_2, c_3, c_4 (contexts), as shown below:

	c_1	Growls (W_2)	Whinnies (W_2)	
V_1	Horse	.049	.630	.679
V_1	Bear	.259	.062	.321
		.308	.692	

c_4	Snorts (V_4)	Meows (V_4)		
Horse	.593	.025	.618	W_1
Bear	.296	.086	.382	W_1
	.889	.111		

	c_2	Growls (V_2)	Whinnies (V_2)	
W_3	Tiger	.778	.086	.864
W_3	Cat	.086	.049	.135
		.864	.135	

c_3	Snorts (W_4)	Meows (W_4)		
Tiger	.148	.086	.234	V_3
Cat	.099	.667	.766	V_3
	.247	.753		

The "objects" to be measured here are the choices offered:

$$\begin{array}{ll} q_1 = \text{Horse or Bear?} & q_2 = \text{Growls or Whinnies?} \\ q_3 = \text{Tiger or Cat?} & q_4 = \text{Snorts or Meows?} \end{array}$$

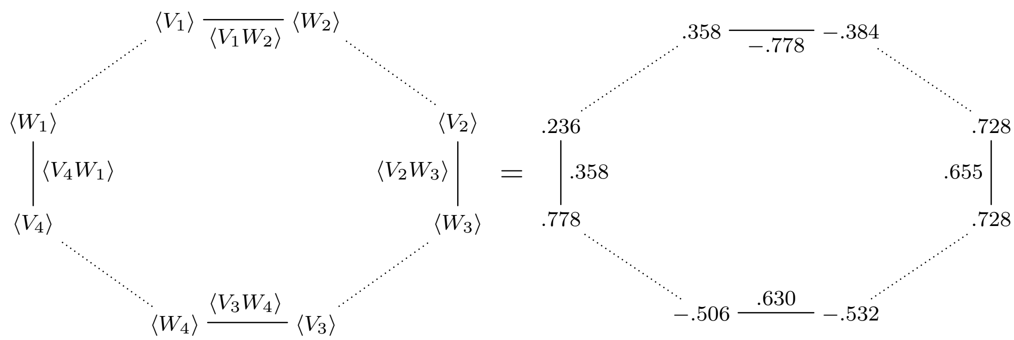
Each of the four contexts corresponds to a pair of these objects,

$$c_1 = (q_1, q_2), c_2 = (q_2, q_3), c_3 = (q_3, q_4), c_4 = (q_4, q_1),$$

and the choices made are binary measurements (random variables)

$$\begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \\ (V_1, W_2) & (V_2, W_3) & (V_3, W_4) & (V_4, W_1) \end{array}.$$

The table of the results above translates into the diagram of expected values



The noncontextuality criterion for rank 4 has the form

$$\Delta C = \mathsf{s}_1(\langle V_1W_2\rangle, \langle V_2W_3\rangle, \langle V_3W_4\rangle, \langle V_4W_1\rangle) - 2 - \sum_{i=1}^{4} |\langle V_i\rangle - \langle W_i\rangle| \leq 0, \tag{14}$$

where

$$\mathsf{s}_1(w, x, y, z) = \max(|w + x + y - z|, |w + x - y + z|, |w - x + y + z|, |-w + x + y + z|).$$

The value computed from the data is $\Delta C = -3.357$, providing no evidence for contextuality.

Ref. [30] reports that contextuality in this data set is present because

$$\mathsf{s}_1(\langle V_1W_2\rangle, \langle V_2W_3\rangle, \langle V_3W_4\rangle, \langle V_4W_1\rangle) - 2 > 0, \tag{15}$$

i.e., the data violate the classical CHSH inequalities [22, 23]. As pointed out in Ref. [31], the CHSH inequalities are predicated on the assumption of consistent connectedness (marginal selectivity). Without this assumption they cannot be derived as a necessary or sufficient condition of noncontextuality, and this assumption is clearly violated in the data. Aerts [32] has developed a theory which allows for inconsistent connectedness, but it is unclear to us how this justifies the use of CHSH inequalties in Ref. [30].

6 Word combinations and priming (cyclic systems of rank 4)

Bruza, Kitto, Ramm, and Sitbon [33] studied ambiguous two-word combinations, such as “apple chip”. One can understand this word combination to refer to an edible chip made of apples or to an apple computer component. It is even possible to imagine such meanings as a piece chipped off of an apple computer, or a computer component made of apples. In the experiments referred to the participants were asked to explain how they understood the first and the second word in a combination: one meaning of each word (e.g., the fruit meaning for “apple”, the edible product meaning for “chip”, etc.) can be taken for $+1$, any other meaning being classified as -1. The meanings were determined by asking the participants to explain how they understood the words. For each two-word combination the experimenters used one of four pairs of priming words presumably affecting the meanings. For the “apple chip” combination, the priming words could be

$$\begin{array}{ll} q_1 = \text{banana} & q_2 = \text{potato} \\ q_3 = \text{computer} & q_4 = \text{circuit} \end{array},$$

forming four contexts

$$\begin{array}{ll} c_1 = (\text{banana, potato}) & c_2 = (\text{potato, computer}) \\ c_3 = (\text{computer, circuit}) & c_4 = (\text{circuit, banana}) \end{array}.$$

The order in which we list the words in a context is chosen to create a cycle: $(q_1, q_2), (q_2, q_3)$, etc. Although this is not intuitive, formally, the measured "objects" here are the priming words q_1, q_2, q_3, q_4, while the measurements are binary random variables indicating in what meaning (± 1) the participant understood "apple" and "chip". In (V_1, W_2) and (V_3, W_4) the V's are meanings of "apple" and W's the meanings of "chip"; in (V_2, W_3) and (V_4, W_1) it is vice versa. (This is no more than a notational convention, purely for the purposes of using the cyclic indexation.)

Ref. [33] presents data on 23 word combinations preceded by priming words (each combination in each context being shown to each of 61-65 participants). In all 23 cases the computed values of ΔC are negative, ranging from -2.882 to -0.418 (for the "apple chip" example the value is -1.640). We conclude, once again, that there is no evidence in favor of contextuality. (The authors of Ref. [33] kindly provided to us the word pairs and priming words, with the computed values of s_1 and equivalents of $|\langle V_i \rangle - \langle W_i \rangle|$ $(i = 1, \ldots, 4)$, for all 23 word combinations; they are presented, with permission, in the supplementary file S2, with the computation of ΔC added.)

The aim of Ref. [33] was not to study contextuality. Rather they were interested in the property called *compositionality*, defined, in our terms, as consistent connectedness together with lack of contextuality. Violations of this condition therefore amount to either inconsistent connectedness or, if connectedness is consistent, to contextuality in our sense.

7 Psychophysical matching (cyclic systems of rank 4)

All experiments discussed so far use participants as replicants: the estimate of $\Pr[V = v, W = w]$ in a given context is the proportion of participants who responded (v, w), $v = \pm 1$, $w = \pm 1$. In the question order effect and Schröder's staircase illusion studies different groups of people participated in different contexts, whereas the conjoint choices and word combinations studies employed repeated measures design: each participant made one choice in each of the four contexts.

In our laboratory, we searched for possible contextual effects in a large series of psychophysical experiments where each of very few (usually, three) participants were repeatedly "measuring" the same four "objects" in the same four contexts. In each of the seven experiments the number of replications per participant was 1000-2000, evenly divided between different contexts.

The logic of an experiment was as follows. The participant was shown two stimuli, target one (T) and adjustable one (A), both completely specified by two parameters. In each trial, the values α and β of these parameters (real numbers) in the target stimulus $T(\alpha, \beta)$ are fixed at one of several values, each pair of values determining a context; in the adjustable stimulus the two parameters can be simultaneously or (in some experiments) successively changed by the participant rotating a trackball. At the end of each trial the participant reaches some values X and Y of these parameters that she/he judges to make $A(X, Y)$ match (i.e., look the same as) $T(\alpha, \beta)$. In most experiments α and β vary on several levels each (or even quasi-continuously within certain ranges), and we always choose four specific values or subranges of their values: q_1, q_3 for α and q_2, q_4 for β. They form four contexts that can be cyclically arranged as $(q_1, q_2), (q_2, q_3), (q_3, q_4), (q_4, q_1)$, and for each of them we get empirical distributions of X and Y: (X_{12}, Y_{12}) for context (q_1, q_2), (X_{41}, Y_{41}) for context (q_4, q_1), etc. In this notation, of the two objects q_i, q_j, the random variable X_{ij} "measures" the q with an odd index (1 or 3), whether i or j; analogously, Y_{ij} "measures" the q with the even index.

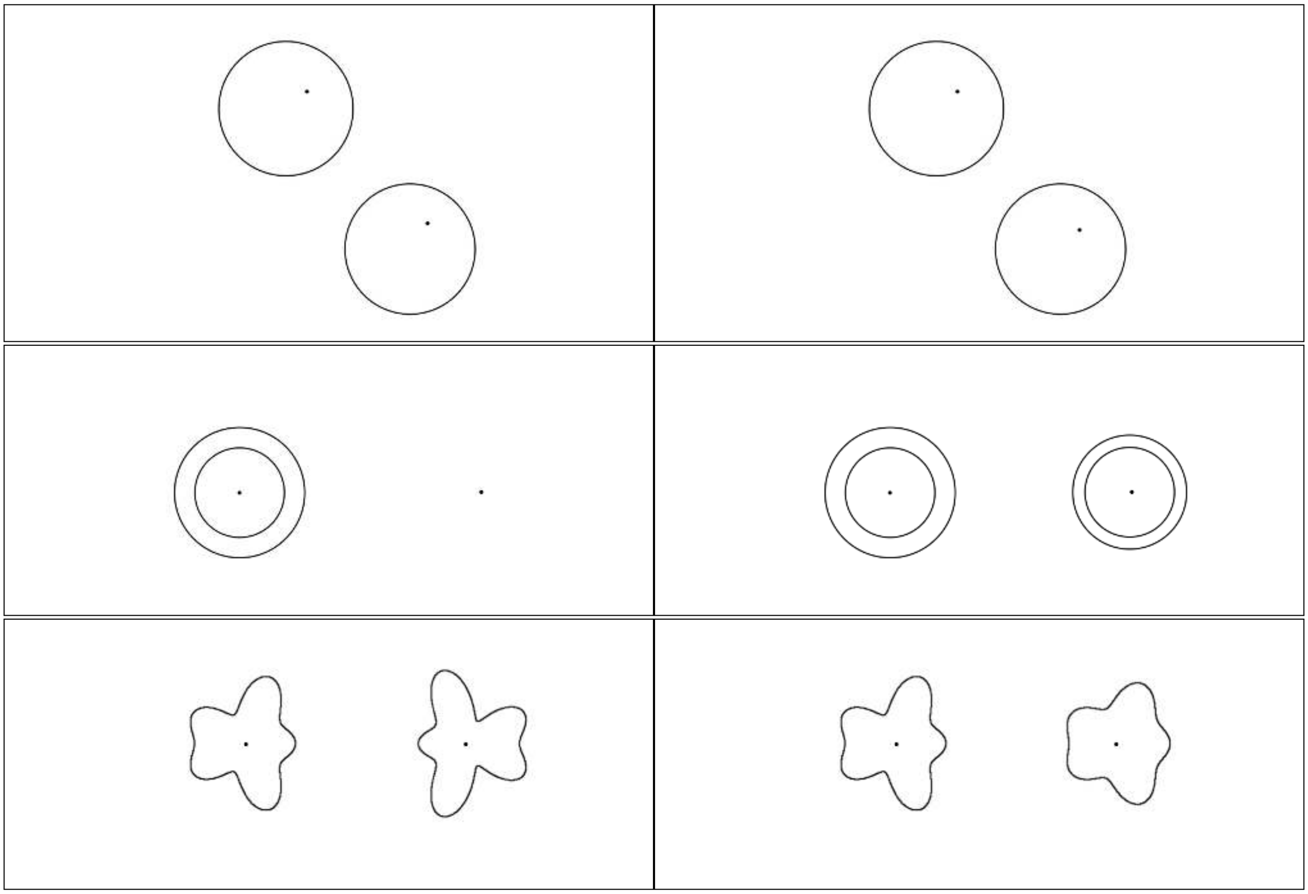


Figure 2: Stimuli used in the matching experiments. The left panels show pairs of stimuli at the beginning of a trial, the right panels show an intermediate stage in the matching process. Top panels: in Experiments 1a-b there participants adjusted the position of the dot within a lower-right circle to match a fixed position of the target dot in the upper-left circle. Middle panels: in Experiments 2a-c they adjusted the radii of two concentric circles on the right to match two fixed concentric circles on the left. Bottom panels: in Experiments 3a-b they adjusted the amplitudes of two Fourier harmonics of a floral shape on the right to match a fixed floral shape on the left. For details, see the supplementary file S3.

The values of X and Y are then dichotomized in the following way: we choose a value x_i and a value y_j ($i = 1, 3,\ j = 2, 4$) and define

$$V_i = \begin{cases} +1 & if \quad X_{i,i\oplus 1} > x_i \\ -1 & if \quad X_{i,i\oplus 1} \leq x_i \end{cases}, \quad V_j = \begin{cases} +1 & if \quad Y_{j,j\oplus 1} > y_j \\ -1 & if \quad Y_{j,j\oplus 1} \leq y_j \end{cases}. \tag{16}$$

$$W_i = \begin{cases} +1 & if \quad X_{i\ominus 1,i} > x_i \\ -1 & if \quad X_{i\ominus 1,i} \leq x_i \end{cases}, \quad W_j = \begin{cases} +1 & if \quad Y_{j\ominus 1,j} > y_j \\ -1 & if \quad Y_{j\ominus 1,j} \leq y_j \end{cases}. \tag{17}$$

The values of (x_1, x_3, y_2, y_4) can be chosen in a variety of ways, and for each choice we apply to the obtained V and W variables the criterion (14).

As an example, in one of the experiments the stimuli T and A were two dots in two circles, like the ones shown in Figure 2, top, with a dot's position within a circle described in polar coordinates (α and X denoting distance from the center in pixels, β and Y denoting angle in degrees measured counterclockwise from the horizontal rightward radius-vector). We extract from this

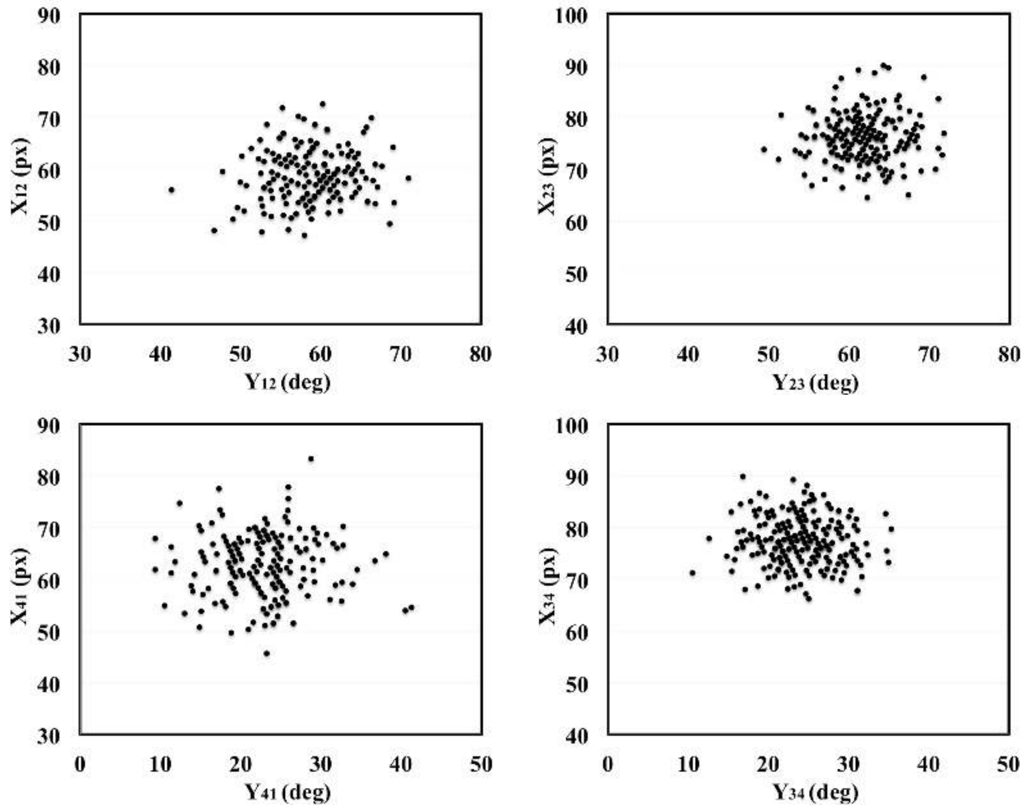
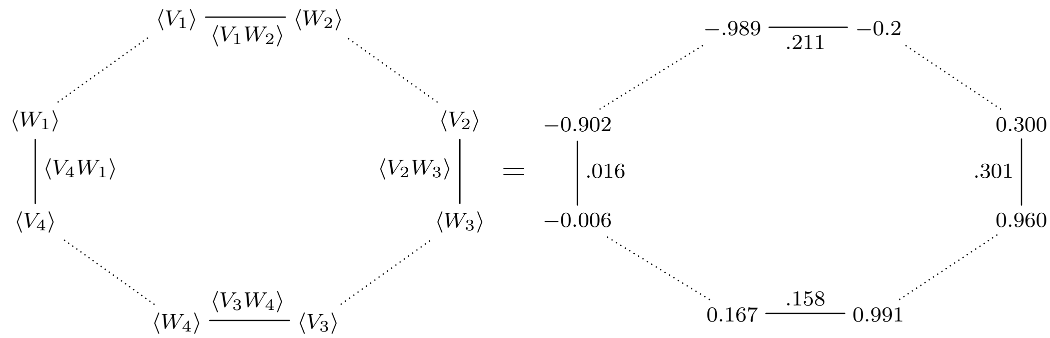


Figure 3: Results for four contexts α (px) $\times\, \beta$ (deg) $= \{q_1 = 53.67, q_3 = 71.55\} \times \{q_2 = 63.43, q_4 = 26.57\}$ extracted from Experiment 1a, participant P3, about 200 replications per context.

experiment a 2×2 subdesign as shown in Figure 3. Then we choose a value of x_1 as any integer (in pixels) between $\max[\min X_{12}, \min X_{41}]$ and $\min[\max X_{12}, \max X_{41}]$, we choose y_2 as any integer (in degrees) between $\max[\min Y_{12}, \min Y_{23}]$ and $\min[\max Y_{12}, \max Y_{23}]$, and analogously for x_3 and y_4. This yields $25 \times 23 \times 21 \times 79$ quadruples of (x_1, x_3, y_2, y_4), and the corresponding number of cyclic systems of binary random variables $(V_1, W_2, V_2, W_3, V_3, W_4, V_4, W_1)$. Consider, e.g., one such choice: $(x_1, x_3, y_2, y_4) = (72 \text{ px}, 67 \text{ px}, 60 \text{ deg}, 23 \text{ deg})$. The diagram of this system is



and the value of $\Delta C = -2.137$, no evidence of contextuality. In fact negative values of ΔC are obtained for all $25 \times 23 \times 21 \times 79$ dichotomizations. Clearly, different dichotomizations of the same random variables are not stochastically independent, but there is no mathematical reason for ΔC to be of the same sign in all of them.

In the supplementary file S3 we describe in detail how the dichotomizations were made, their number ranging from 3024 to 11,663,568 per 2×2 (sub)design in each experiment for each participant. The outcome is: not a single case with positive ΔC observed.

8 Conclusion

The empirical data analyzed above suggest that the noncontextuality boundaries, that are generally breached in quantum physics, are not breached by behavioral and social systems. This may seem a disappointing conclusion for some. With the realization that quantum formalisms may be used to construct models in various areas outside physics [34–37], the expectation was created that human behavior should exhibit contextuality, perhaps even on a greater scale than allowed by quantum theory. However, if the no-contextuality conclusion of the present paper is proved to be a rule for a very broad class of behavioral and social systems, it is rather fortunate for behavioral and social sciences. Noncontextuality means more constrained behavior, and constraints allow one to make predictions. The power of quantum mechanics is not in that quantum systems breach the classical-mechanical bounds of noncontextuality, but in the theory that imposes other, equally strict constraints. Presence of contextuality, in the absence of a general theory like quantum mechanics, translates into unpredictability.

It must be noted that absence of contextuality in behavioral and social systems does not mean that quantum formalisms are not applicable to them. A good argument for why this conclusion would be groundless is provided by the question order effect discussed in Section 3: it is precisely the applicability of a quantum-mechanical model in the question order effect analysis [26, 28] that allows one to predict the lack of contextuality in this paradigm.

When discussing contextuality, one should be aware of the likelihood of purely terminological confusions. It is clear that in the behavioral and social systems a context generally influences the measurement of an object within it. For instance, the distribution of answers to a question depends on a question asked and answered before it. One could call this contextuality, and many do. This is, however, a trivial sense of contextuality, on a par with the fact that the distribution of answers to a question depends on what this question is. One should not be surprised that other factors (such as temperature in the lab or questions asked and answered previously) can influence this distribution too. We call this inconsistent connectedness, and we offer a theory that distinguishes this ubiquitous feature from contextuality in a different, one could argue more interesting meaning.

Acknowledgments

This research has been supported by NSF grant SES-1155956, AFOSR grant FA9550-14-1-0318, and A. von Humboldt Foundation. We are grateful to the authors of Refs. [29], [33], and [26] for kindly providing data sets for our analysis. We have benefited from discussions with Jan-Åke Larsson and Victor H. Cervantes (who pointed out a mistake in an earlier version of the paper). The computations discussed in Sections 3 and 6 are presented in the supplementary files S1 and S2, respectively. The original data sets are available from the authors of Refs. [33] and [26]. Details of the experiments discussed in Section 7 are presented in the supplementary file S3; the data sets are available as "Contextuality in Psychophysical Matching", http://dx.doi.org/10.7910/DVN/OJZKKP, Harvard Dataverse, V1.

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Resp to A		Resp to B				s1-value	|<V1>-<W1>|+|<V2>-<W2>|	Delta C	Delta C > 0?	QQ chi^2	QQ p<0.05?	QQ p<0.01?	Data set #	
order A -> B	order B -> A	order A -> B	order B -> A											
<V1>	<W1>	<W2>	<V2>	<V1*W2>	<V2*W1>									
0.2105	0.0482	-0.1100	-0.1491	0.5359	0.4781	0.0578	0.2014	-0.1435	0	0.9830	0	0	1	PEW
0.2584	0.2081	0.0096	0.0407	0.5694	0.5882	0.0189	0.0814	-0.0625	0	0.1150	0	0	2	PEW
0.3398	0.2705	0.1359	0.0984	0.6214	0.5820	0.0394	0.1069	-0.0675	0	0.2736	0	0	3	PEW
-0.0484	-0.0324	-0.2823	-0.2146	0.5806	0.5749	0.0057	0.0837	-0.0779	0	0.0123	0	0	4	PEW
0.1239	0.1367	-0.1193	0.0325	0.6193	0.6529	0.0337	0.1646	-0.1309	0	0.4268	0	0	5	PEW
0.0136	0.1198	-0.1900	-0.0645	0.6606	0.6406	0.0201	0.2318	-0.2117	0	0.1531	0	0	6	PEW
0.0411	0.0206	-0.2055	-0.1512	0.6370	0.5945	0.0425	0.0748	-0.0323	0	0.8474	0	0	7	PEW
-0.1442	-0.0946	-0.3759	-0.2634	0.5981	0.6675	0.0694	0.1620	-0.0926	0	1.6353	0	0	8	PEW
-0.0494	-0.0847	-0.2565	-0.2037	0.6424	0.6156	0.0268	0.0881	-0.0613	0	0.2559	0	0	9	PEW
-0.0432	-0.1231	-0.2815	-0.3588	0.6304	0.5633	0.0671	0.1571	-0.0900	0	2.0235	0	0	10	PEW
-0.2070	-0.1544	-0.3815	-0.2354	0.6259	0.5139	0.1120	0.1987	-0.0866	0	3.6979	0	0	11	PEW
0.3515	0.3293	-0.4387	-0.4622	0.1662	0.1118	0.0544	0.0457	0.0087	1	0.5272	0	0	12	PEW
0.3099	0.1750	-0.3568	-0.3650	0.2770	0.3700	0.0930	0.1431	-0.0500	0	1.9950	0	0	13	PEW
0.1979	0.2245	-0.3298	-0.2857	0.3562	0.3878	0.0316	0.0707	-0.0391	0	0.2227	0	0	14	PEW
0.1715	0.0633	-0.3487	-0.3321	0.3950	0.4894	0.0945	0.1248	-0.0304	0	2.8834	0	0	15	PEW
0.1789	0.0691	-0.4252	-0.4362	0.3372	0.3351	0.0021	0.1207	-0.1185	0	0.0009	0	0	16	PEW
0.0651	0.0815	-0.4943	-0.3718	0.3870	0.4115	0.0246	0.1389	-0.1143	0	0.1838	0	0	17	PEW
0.0805	-0.0056	-0.5862	-0.5562	0.3103	0.2921	0.0182	0.1161	-0.0979	0	0.0642	0	0	18	PEW
0.0291	0.0132	-0.5058	-0.5145	0.3721	0.3140	0.0581	0.0246	0.0335	1	0.6917	0	0	19	PEW
0.1358	-0.0171	-0.2741	-0.3888	0.5012	0.4621	0.0391	0.2676	-0.2285	0	0.4058	0	0	20	PEW
0.0074	-0.0076	-0.4604	-0.4582	0.4629	0.4025	0.0603	0.0172	0.0432	1	1.7896	0	0	21	PEW
0.0030	-0.0877	-0.5137	-0.4854	0.4225	0.3567	0.0658	0.1191	-0.0533	0	0.8554	0	0	22	PEW
-0.0620	-0.0392	-0.5682	-0.5196	0.4342	0.3107	0.1235	0.0715	0.0520	1	3.4820	0	0	23	PEW
0.0278	0.0997	-0.4889	-0.4394	0.4056	0.3100	0.0956	0.1215	-0.0259	0	1.9147	0	0	24	PEW
0.0290	-0.0261	-0.5411	-0.4774	0.3816	0.3682	0.0135	0.1187	-0.1053	0	0.0441	0	0	25	PEW
0.0230	-0.0300	0.0138	-0.0700	-0.4286	-0.3000	0.1286	0.1369	-0.0083	0	1.9870	0	0	26	PEW
-0.0876	-0.0729	-0.0046	-0.0729	-0.2258	-0.1667	0.0591	0.0829	-0.0238	0	0.3719	0	0	27	PEW
0.0486	-0.0960	-0.1243	-0.1818	-0.2000	-0.1263	0.0737	0.2021	-0.1284	0	1.0697	0	0	28	PEW
-0.0611	-0.1761	-0.1444	-0.2159	0.0056	-0.0398	0.0453	0.1865	-0.1412	0	0.3658	0	0	29	PEW
-0.1940	-0.1705	-0.2239	-0.1705	0.1343	0.0650	0.0693	0.0770	-0.0077	0	0.9338	0	0	30	PEW
-0.1279	-0.2046	-0.0444	-0.1124	-0.0548	-0.1182	0.0633	0.1447	-0.0814	0	0.7374	0	0	31	PEW
-0.2185	-0.3063	-0.2185	-0.1848	0.0231	-0.0228	0.0459	0.1215	-0.0756	0	0.4133	0	0	32	PEW
-0.2204	-0.4018	0.0863	0.1239	-0.1757	-0.1178	0.0579	0.2190	-0.1611	0	0.5515	0	0	33	PEW
-0.3199	-0.3651	-0.0428	-0.0370	-0.0428	-0.0794	0.0365	0.0510	-0.0144	0	0.2597	0	0	34	PEW
-0.4142	-0.3803	-0.1420	-0.2169	0.0414	0.0817	0.0403	0.1088	-0.0685	0	0.2820	0	0	35	PEW
-0.2845	-0.2044	-0.0264	-0.2044	-0.1261	-0.0899	0.0362	0.2581	-0.2219	0	0.2344	0	0	36	PEW
-0.3433	-0.3008	-0.1164	-0.0786	-0.0388	0.0081	0.0469	0.0803	-0.0334	0	0.3878	0	0	37	PEW
-0.3598	-0.4525	-0.2691	-0.1844	0.0255	-0.0503	0.0758	0.1775	-0.1017	0	1.0208	0	0	38	PEW
-0.2978	-0.2213	-0.2089	-0.1762	-0.1022	-0.0451	0.0571	0.1091	-0.0520	0	0.7698	0	0	39	PEW
-0.4000	-0.3265	-0.3158	-0.1877	-0.0526	-0.1465	0.0939	0.2017	-0.1078	0	1.7120	0	0	40	PEW
-0.4677	-0.4707	-0.4160	-0.2977	0.0801	-0.0382	0.1183	0.1213	-0.0031	0	2.7289	0	0	41	PEW
0.0576	0.0335	-0.0262	0.1196	0.6859	0.6842	0.0017	0.1699	-0.1682	0	0.0005	0	0	42	PEW
-0.0343	0.0554	-0.0218	0.0321	0.7632	0.7901	0.0268	0.1435	-0.1167	0	0.3015	0	0	43	PEW
-0.1736	-0.1304	-0.1460	-0.1237	0.7741	0.7659	0.0082	0.0654	-0.0572	0	0.0275	0	0	44	PEW
0.0360	-0.0455	0.4676	0.5649	-0.2230	-0.2208	0.0022	0.1787	-0.1765	0	0.0008	0	0	45	PEW
0.4067	0.3800	-0.2575	-0.0019	0.0089	0.0888	0.0800	0.2824	-0.2025	0	1.7499	0	0	46	PEW
0.0846	0.0777	0.1741	0.3368	0.6318	0.5337	0.0982	0.1695	-0.0714	0	1.4372	0	0	47	PEW
0.1481	0.1743	-0.0617	0.0642	0.6914	0.7798	0.0885	0.1521	-0.0637	0	1.6198	0	0	48	PEW
0.3838	0.2394	0.1616	0.1737	0.6768	0.8216	0.1448	0.1565	-0.0117	0	4.9124	1	0	49	PEW
-0.5987	-0.6622	-0.1973	-0.1824	0.3043	0.2500	0.0543	0.0784	-0.0240	0	0.4759	0	0	50	PEW
-0.0236	-0.1360	-0.0866	-0.2960	0.5906	0.6160	0.0254	0.3218	-0.2963	0	0.0641	0	0	51	PEW
-0.3111	-0.4415	-0.4833	-0.5847	0.5278	0.5990	0.0713	0.2318	-0.1605	0	1.4492	0	0	52	PEW
0.2000	0.3135	-0.0173	0.0618	-0.1654	-0.1304	0.0350	0.1926	-0.1576	0	0.2636	0	0	53	PEW
0.5371	0.5601	0.2800	0.3314	0.0686	0.0557	0.0129	0.0744	-0.0615	0	0.0286	0	0	54	PEW
-0.4947	-0.5130	-0.5211	-0.4404	0.2368	0.1917	0.0451	0.0989	-0.0537	0	0.4088	0	0	55	PEW
0.0684	0.0957	-0.0769	-0.0522	0.6325	0.6957	0.0632	0.0520	0.0111	1	0.8281	0	0	56	PEW
0.2874	0.3117	-0.1494	-0.0931	0.3487	0.3522	0.0036	0.0807	-0.0771	0	0.0018	0	0	57	PEW
-0.8069	-0.6839	-0.8414	-0.7871	0.7310	0.6129	0.1181	0.1773	-0.0592	0	3.8166	0	0	58	PEW
0.3173	0.3548	0.0481	-0.0215	0.2885	0.3656	0.0771	0.1071	-0.0300	0	0.6562	0	0	59	PEW
-0.6582	-0.5960	0.0408	0.1010	-0.1276	-0.3131	0.1856	0.1224	0.0632	1	7.1309	1	1	60	PEW
-0.1048	0.1181	-0.4516	-0.3307	0.3629	0.3622	0.0007	0.3439	-0.3432	0	0.0001	0	0	61	PEW
-0.0797	-0.0842	0.3768	0.5421	-0.2536	-0.2997	0.0460	0.1697	-0.1237	0	0.3288	0	0	62	PEW
0.2600	0.1906	-0.0680	-0.0650	0.4822	0.5291	0.0469	0.0723	-0.0254	0	1.3466	0	0	63	PEW
-0.1441	-0.0091	-0.7205	-0.7455	0.3013	0.2273	0.0740	0.1599	-0.0859	0	0.6615	0	0	64	PEW
-0.3260	-0.2665	-0.0313	-0.0621	0.0645	0.0421	0.0224	0.0903	-0.0679	0	0.1308	0	0	65	PEW
-0.2343	-0.3559	0.1033	-0.0056	-0.0730	-0.0056	0.0674	0.2306	-0.1632	0	0.8542	0	0	66	PEW
0.0694	0.1759	0.3333	0.5231	0.5570	0.5509	0.0061	0.2964	-0.2903	0	0.0119	0	0	67	Gallup
-0.0481	-0.1830	0.4279	0.3740	0.1945	0.1883	0.0062	0.1889	-0.1827	0	0.0081	0	0	68	Gallup
-0.1678	0.0782	0.1198	-0.0782	0.6427	0.6049	0.0378	0.4440	-0.4062	0	0.5515	0	0	69	Gallup
-0.3793	0.0741	0.0862	-0.2222	0.2931	0.4444	0.1513	0.7618	-0.6105	0	1.4845	0	0	70	see PNAS 2014 paper
0.0508	0.1887	0.2881	0.3962	0.5932	0.6792	0.0860	0.2459	-0.1599	0	0.6963	0	0	71	see PNAS 2014 paper
0.6792	0.2131	-0.0375	0.6590	0.2423	0.4230	0.1806	1.1626	-0.9820	0	5.4846	1	0	72	see PNAS 2014 paper
0.4111	0.5606	0.6166	0.3333	0.1680	0.1537	0.0143	0.8656	-0.8513	0	25.4164	1	1	73	Gallup: Rose-Jackson

Supplementary Information S1: Question Order Effect
Computed, with permission, from the collection of data sets analyzed by
Wang, Z., Solloway, T., Shiffrin, R.M., Busemeyer, J.R. 2014
and Wang, Z., Busemeyer, J.R. 2013.
The 73d data set is the "Rose-Jackson" question pair showing a significant violation of QQ equality.

word combination	prime q1	prime q3	prime q2	prime q4	s1-value	$\lvert\langle V1\rangle-\langle W1\rangle\rvert$	$\lvert\langle V2\rangle-\langle W2\rangle\rvert$	$\lvert\langle V3\rangle-\langle W3\rangle\rvert$	$\lvert\langle V4\rangle-\langle W4\rangle\rvert$	N	Delta C
boxer bat	dog	fighter	ball	vampire	0.740	0.350	0.676	0.280	0.316	64.000	-2.882
table file	chair	chart	nail	folder	0.330	0.116	0.228	0.470	0.226	63.000	-2.710
star suit	moon	movie	vest	law	1.180	0.616	0.108	0.326	0.116	62.000	-1.986
mole pen	dig	face	pig	ink	1.180	0.250	0.126	0.042	0.600	63.000	-1.838
crane hatch	lift	bird	door	egg	1.920	0.282	0.298	0.592	0.466	63.000	-1.718
stag yarn	party	deer	story	wool	1.770	0.750	0.208	0.438	0.090	61.000	-1.716
apple chip	banana	computer	potato	circuit	2.110	0.500	0.588	0.228	0.434	65.000	-1.640
web bug	spider	internet	beetle	computer	2.000	0.420	0.592	0.134	0.306	63.000	-1.452
bank log	money	river	journal	tree	2.130	0.110	0.676	0.184	0.514	65.000	-1.354
port vessel	harbour	wine	ship	bottle	1.560	0.212	0.226	0.170	0.236	65.000	-1.284
count watch	number	dracula	time	look	1.400	0.390	0.022	0.126	0.126	65.000	-1.264
rock strike	stone	music	hit	union	2.010	0.376	0.626	0.234	0.026	64.000	-1.252
fan post	football	cool	mail	light	2.130	0.700	0.050	0.250	0.376	63.000	-1.246
match bowl	flame	contest	disk	throw	1.750	0.274	0.150	0.500	0.044	64.000	-1.218
seal pack	walrus	envelop	leader	suitcase	2.140	0.166	0.324	0.426	0.442	64.000	-1.218
spring plant	summer	coil	leaf	factory	2.020	0.588	0.000	0.266	0.346	64.000	-1.180
slug duck	snail	punch	quack	dodge	1.830	0.192	0.266	0.306	0.052	63.000	-0.986
bill scale	phone	pelican	weight	fish	1.630	0.162	0.108	0.226	0.108	64.000	-0.974
toast gag	jam	speech	choke	joke	1.230	0.000	0.036	0.016	0.052	63.000	-0.874
net cap	gain	volleyball	limit	hat	1.860	0.070	0.118	0.184	0.350	65.000	-0.862
battery charge	car	assault	volt	prosecute	2.010	0.134	0.234	0.096	0.240	63.000	-0.694
club bar	member	golf	pub	handle	2.280	0.266	0.250	0.000	0.276	64.000	-0.512
poker spade	card	fire	ace	shovel	2.150	0.272	0.000	0.070	0.226	65.000	-0.418

Supplemntary Information S2: Word Pairs with Priming

Results of the experiment by
Bruza, P.D., Kitto, K., Ramm, B.J., & Sitbon, L.,
analyzed in Section 6 of the main text;
all the computations are made by Bruza et al., except for the last column
and coversion of probabilities into expectations

Supplementary Information S3: Details of the Matching Experiment

1 Participants

All the participants were students at Purdue University. The second author of this paper, labeled as P3, participated in all the experiments. Two persons (P1 and P2) participated in Experiments 1(a) and 2(a), and two other persons (P4 and P5) in Experiments 1(b), 2(b), 2(c), 3(a), and 3(b). All participants were aged around 25 and had normal or corrected to normal vision.

2 Stimuli and Procedure

Visual stimuli consisting of curves and (sometimes) dots were presented on a flat-panel monitor. They were grayish-white on a comfortably low intensity background. The diameter of the dots and the width of the curves was 5 pixels (px). The participants viewed the stimuli in darkness using a chin rest with a forehead support at the distance of 90 from the monitor, making 1 screen pixel approximately 62 sec arc. In each trial the participants were asked to match a fixed stimulus by adjusting a variable stimulus by rotating a trackball using their dominant hand. Once a response was made to the participant's satisfaction, she or he clicked a button on the trackball device to end this trial, and a new stimulus appeared half a second later. Each experiment took several days, each of which consisted of about 200 trials conducted with a 10-min break in the middle; each such session was preceded by a practice series of 10 trials (which were not recorded).

2.1 Experiment 1(a)

Each trial began with presenting two circles with a dot in the first quadrant of each circle (as shown in Figure 1, top panels). The radius of each circle was 160 px. The dot in the upper left circle was fixed at one of randomly chosen six positions. Using the center of its circle as the origin, they can be represented equivalently using the rectangular coordinates: {(24 px, 48 px), (32 px, 32 px), (32 px, 64 px), (48 px, 24 px), (64 px, 32 px), (64 px, 64 px)} or the polar coordinates: {(53.67 px, 63.43 deg), (45.25 px, 45 deg), (71.55 px, 63.43 deg), (53.67 px, 26.57 deg), (71.55 px, 26.56 deg), (90.51 px, 45 deg)}. Hence the experimental design contained a 2×2 "rectangular" subdesign, {32 px, 64 px}×{32 px, 64 px}, and a 2×2 "polar" subdesign {53.67 px, 71.55 px}×{63.43 deg, 26.57 deg}.

The position of the dot in the bottom right circle was controlled by the trackball, until its location matched that of the fixed one. Once a response was made, the program recorded the locations of the target dot and the matching dot in both rectangular coordinates and polar coordinates. There were 1200 trials overall with approximately 200 trials per treatment.

2.2 Experiment 1(b)

The horizontal coordinate and vertical coordinate of the target dot were random integers drawn before each trial from the the rectangle [20 px, 80 px)×[20 px, 80 px). This Cartesian rectangle contained the polar-coordinate rectangle [40 px, 90 px)×[30 deg, 60 deg), allowing us to analyze the data falling within it separately. The overall number of trials was 1800, of which 900 fell within the polar-coordinate rectangle. In all other respect, the procedure was the same as in Experiment 1(a).

2.3 Experiment 2(a)

Each trial began as shown in Figure 1, middle left panel. The target figure, on the left, consisted of two concentric circles together with their center. The radii of circle 1 and circle 2 were randomly chosen from the sets {16 px, 56 px, 64 px} and {48 px, 72 px, 80 px}, respectively, in a 3×3 factorial design. On the right, in the beginning of the trial, there was a dot located at (250 px, 0 px) relative to the center of the target figure. By rotating the trackball the participant aimed at matching the target figure by "blowing up" two circles from the dot on the right, one by one. Once the first matching circle was produced (inner or outer, the person could choose), the participant clicked a button on the trackball to stabilize this circle and then the program enabled him or her to "blow" the other circle. After the second match was made, the trial was terminated by clicking the same button on the trackball. The program recorded the radii of the target and matching concentric circles in each trial. There were 1800 trials overall, approximately 200 trials per treatment.

2.4 Experiment 2(b)

Experiment 2(b) was identical to Experiment 2(a) except that in each trial the radii of the target circle 1 and circle 2 were randomly chosen from four possibilities $\{12 \text{ px}, 24 \text{ px}\}\times\{18 \text{ px}, 30 \text{ px}\}$. There were 1600 trials overall, about 400 trials per treatment.

2.5 Experiment 2(c)

Experiment 2(c) was identical to Experiment 2(a) except that in each trial the radius of the target circle 1 was a number randomly chosen from the uniform distribution on the interval [18 px, 48 px) and the radius of the target circle 2 was randomly chosen from the interval [56 px, 86 px). There were 1800 trials overall.

2.6 Experiment 3(a)

Examples of two floral shapes together with their centers are shown in Figure 1, bottom panels. Two such configurations were presented simultaneously in each trial. The target one was on the left, the variable one on the right. The floral shape was generated using the function

$$x = cos(.02\pi\Delta)[70 + \alpha cos(.06\pi\Delta) + \beta cos(.1\pi\Delta)], \quad (1)$$

$$y = sin(.02\pi\Delta)[70 + \alpha cos(.06\pi\Delta) + \beta cos(.1\pi\Delta)],$$

where x, y stand for the rectangular coordinates. Amplitude α and amplitude β of the left floral shape were randomly chosen from the sets {-18 px, 10 px, 14 px} and {-16 px, -12 px, 20 px}, respectively. Δ was varied from 0 to 99 with an increment of 1 at each step. At each step, a point with coordinates (x, y) was drawn to the screen and each floral shape was composed of 100 such points. The α and β for the shape on the right was controlled by rotating the trackball. The program converted the horizontal (vertical) component of the rotation to the change of α (respectively, β). The initial values for these amplitudes were randomly picked from the interval [-35 px, 35 px). There were 1800 trials overall, about 200 trials per treatment.

2.7 Experiment 3(b)

Experiment 3(b) was identical to Experiment 3(a) except that the two amplitudes for the target shape were randomly chosen numbers from the interval [-30 px, 30 px).

3 Results

The data are available as "Contextuality in Psychophysical Matching" dataset (Excel files) in http://dx.doi.org/10.7910/DVN/OJZKKP, Harvard Dataverse, V1.

4 Analysis

In each experiment we deleted outliers, defined, rather informally, as the matching values that were too far from the target values. The outliers made less than 1% of all data. (Note that the files in the repository do not have the outliers deleted.)

In Experiment 2b the design was 2×2. In Experiment 1a there were two 2×2 subdesigns, the “rectangular” and “polar” ones. In Experiment 2a and 3a the design was 3×3 and the analysis was made for each of the nine possible 2×2 subdesigns. In Experiments 1b, 2c, 3b the values of the target stimulus were first dichotomized into below-median and above-median values, forming a 2×2 factorial design in each of them.

Once a 2×2 design was formed, the responses (matching values of the variable stimuli) were dichotomized as described in Section 7 of the main text, by choosing all possible combinations of four integer values in the intervals

$$x_1 \in \{\max[\min X_{12}, \min X_{41}], \min[\max X_{12}, \max X_{41}]\},$$
$$x_3 \in \{\max[\min X_{23}, \min X_{34}], \min[\max X_{23}, \max X_{34}]\},$$
$$y_2 \in \{\max[\min Y_{12}, \min Y_{23}], \min[\max Y_{12}, \max Y_{23}]\},$$
$$y_4 \in \{\max[\min Y_{41}, \min Y_{34}], \min[\max Y_{41}, \max Y_{34}]\}$$

The analysis afterwards consisted in computing the value of ΔC. The number of dichotomizations in each 2×2 (sub)design was between 3024 and 11,663,568.