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IS UTILITY ADDITIVE?
The Case of Alcohol*

by

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Abstract

The hypothesis of additive utility (or preference independence) is often applied to the demand for broad aggregates. Recent testing provides some evidence favourable to the hypothesis, thus overturning the older results based on the standard asymptotic tests which are seriously biased against the null in small samples. Using data for seven countries and a variety of tests, this paper shows that preference independence also cannot be rejected for more narrowly-defined commodities -- beer, wine and spirits. The implication of the results for efficient taxation of alcoholic beverages are also explored.

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I. Introduction

Preferences are said to be additive if the consumer's utility function can be written as the sum of n sub-utility functions, one for each good:

$$(1) \qquad u(q_1,\ldots,q_n) = \sum_{i=1}^{n} u_i(q_i) \ ,$$

where q_i is the quantity consumed of good i. As equation (1) implies that the marginal utility of good i is independent of the consumption of j, $i \neq j$, it is also known as preference independence. It is frequently argued that if the n goods are broad aggregates (such as food, clothing, housing, etc.), then as they could be interpreted as referring to the consumer's basic wants which would be unlikely to exhibit much utility interactions, the assumption of preference independence may be adequate.

When preference independence has been tested, until recently, however, it was frequently rejected, even for the broad aggregates (see Barten, 1977, for a survey). But most of these previous tests suffer from using asymptotic procedures which, in small samples, are seriously biased against the null hypothesis, a problem that also applies to the standard asymptotic tests of homogeneity and symmetry (Theil, 1987). S. Selvanathan (1993) avoids the problems of asymptotics entirely by employing Monte Carlo testing and, when applied to data from 18 OECD countries, she finds that the evidence is quite favourable to the hypothesis of preference independence.

If tastes with respect to the broad aggregates can be characterised by preference independence (PI), then the question arises what constitutes a broad aggregate? In other words, how far can we push the PI hypothesis? In this paper we pursue this topic by testing PI with data from seven countries on the consumption of three alcoholic beverages, beer, wine and spirits. As these commodities would probably not be regarded as broad aggregates by most, we should expect to reject the hypothesis. Remarkably, using a variety of tests, the results indicate that PI cannot be rejected. Consequently, our results, when added to those of S. Selvanathan (1993), help to rehabilitate preference independence.

II. Demand Equations

Let p_{it}, q_{it} be the price and quantity demanded of alcoholic beverage i (i = 1, 2, 3, denoting beer, wine and spirits) at time t, $M_t = \sum_{i=1}^{3} p_{it} q_{it}$ be the total expenditure on alcohol and $w_{it} = p_{it} q_{it} / M_t$ be the conditional budget share of i. Further, let D be the log-change operator ($Dx_t = \log x_t - \log x_{t-1}$) and $\overline{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1})$, so that $DQ_t = \sum_{i=1}^{3} \overline{w}_{it} Dq_{it}$ is the Divisia volume index. Consider the following conditional demand equation for beverage i:

$$\overline{w}_{it} Dq_{it} = \alpha_i + \theta_i DQ_t + \sum_{j=1}^{3} \pi_{ij} Dp_{jt} + \varepsilon_{it} , \tag{2}$$

where α_i is the intercept for i; θ_i is the marginal share of i; π_{ij} the $(i,j)^{th}$ Slutsky coefficient; and ε_{it} is a disturbance term. Note that equation (2) holds under the assumption of weak separability between alcohol and all other goods, and is known as the Rotterdam model (Barten, 1964, Theil, 1965). A feature of differential demand equations such as (2) is that they can be aggregated consistently over consumers under weak conditions; see Barnett (1979) and E. A. Selvanathan (1991b, 1995).

The assumption of preference independence within alcohol implies that the Slutsky coefficients satisfy (see, e.g., Clements et al., 1995)

$$\pi_{ij} = \phi \theta_i (\delta_{ij} - \theta_j) , \tag{3}$$

where ϕ is the own-price elasticity of demand for alcohol as a whole; and δ_{ij} is the Kronecker delta. Substituting the right-hand side of equation (3) for π_{ij} in (2) and defining $DP'_t = \sum_{i=1}^{3} \theta_i Dp_{it}$ as the Frisch price index, we obtain

$$\overline{w}_{it} Dq_{it} = \alpha_i + \theta_i DQ_t + \phi \theta_i \left[Dp_{it} - DP'_t \right] + \varepsilon_{it} . \tag{4}$$

As the term $Dp_{it} - DP'_t$ is interpreted as the change in the relative price of beverage i, the implication of (4) is that under preference independence only the own-relative price appears in each demand equation.[1]

The income elasticity implied by equation (4) takes the form $\eta_i = \theta_i / \overline{w}_{it}$, while the Frisch (or marginal-utility-constant) own-price elasticity is $\eta'_{ii} = \phi \eta_i$. Accordingly, price elasticities are proportional to income elasticities under PI. The $(i, j)^{th}$ Slutsky (or real-income-constant) price elasticity is $\eta^*_{ij} = \pi_{ij} / \overline{w}_{it}$. It follows from (3) that under PI $\eta^*_{ii} = \phi \eta_i (1 - \theta_i)$. As both $|\phi|$ and θ_i are likely to be positive fractions, while the income elasticity η_i is centred around unity, the term $\phi \eta_i \theta_i \approx 0$. This implies that the Slutsky elasticity is approximately proportional to the corresponding income elasticity:

(5) $$\eta^*_{ii} \approx \phi \eta_i .$$

In words, PI implies that luxuries are more price elastic than necessities.

III. The First Test

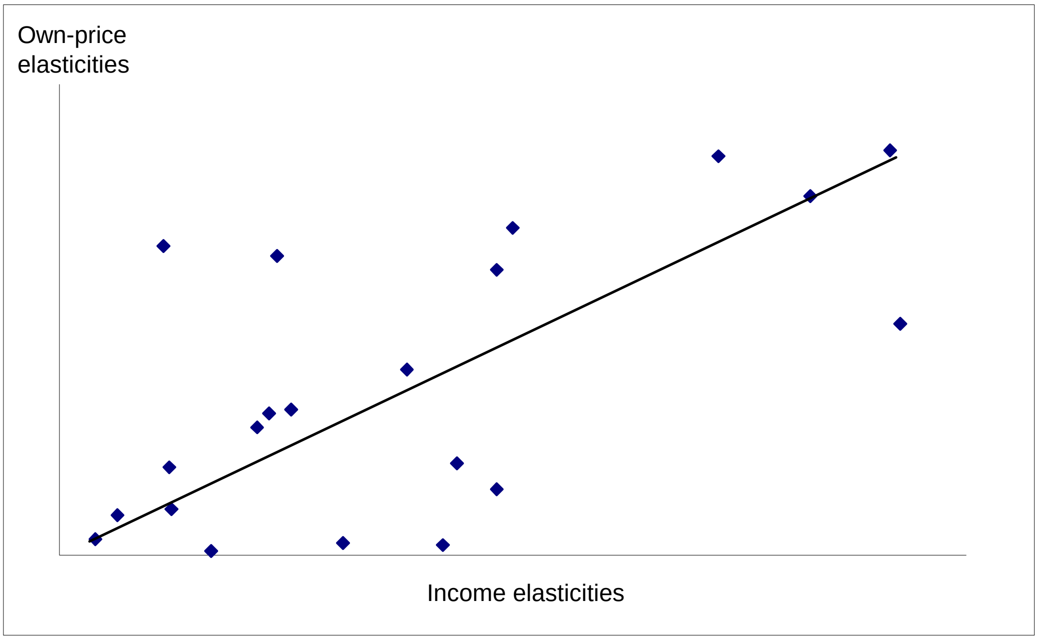
The Slutsky coefficients in equation (2) are subject to homogeneity and symmetry constraints ($\sum_j \pi_{ij} = 0$, $\pi_{ij} = \pi_{ji}$). We estimate by GLS equation (2) under homogeneity and symmetry for i = 1, 2, 3 with annual data for seven countries listed in Table 1.[2] Figure 1 plots the absolute values of the associated $3 \times 7 = 21$ Slutsky own-price elasticities against the corresponding income elasticities. To allow for sampling fluctuations in the price elasticities, observations are weighted by the reciprocals of the standard errors of the price elasticities. As can be seen, there is a distinct tendency for those beverages with higher income elasticities to be more price elastic and vice versa. The solid line in this figure is the LS regression line with the intercept suppressed. Using these weighted elasticities to estimate the equation $|\eta^*_{ii}| = \alpha + |\phi| \eta_i$ yields (with standard errors in parentheses):

	Intercept, α	Slope, $\lvert\phi\rvert$	R^2
With intercept	.62 (.53)	.23 (.06)	.41
Without intercept	-	.29 (.04)	.36

These results support the proportionality hypothesis (5) which points in the direction of the alcoholic beverages being preference independent.

FIGURE 1

WEIGHTED PRICE AND INCOME ELASTICITIES
FOR ALCOHOLIC BEVERAGES



IV. The Second Test

Next, we proceed more formally and estimate equation (4) for i = 1, 2, 3 by maximum likelihood under the assumption that the disturbances ε_{it} are serial-uncorrelated, multivariate normal. The estimates are given in Table 1.[3] Note that as the demand equations are formulated in first differences, the intercepts play the role of autonomous trends. The estimated intercepts (given in columns 1-3 of the table)

TABLE 1

ESTIMATES OF CONDITIONAL DEMAND EQUATIONS FOR ALCOHOLIC BEVERAGES UNDER PREFERENCE INDEPENDENCE

(Asymptotic standard errors in parentheses)

Intercepts $\alpha_i \times 100$			Marginal shares θ_i			Own-price elasticity of alcohol ϕ	Likelihood ratio test statistic	Rank of test statistic in 1000 trials	
Beer (1)	Wine (2)	Spirits (3)	Beer (4)	Wine (5)	Spirits (6)	(7)	(8)	Normal errors (9)	Bootstrap errors (10)
Australia, 1955-85									
-.392	.562	.170	.553	.154	.294	-.498	5.39	885	905
(.137)	(.120)	(.136)	(.045)	(.037)	(.044)	(.085)			
Canada, 1953-82									
-.240	.377	-.137	.329	.096	.575	-.419	6.13	933	937
(.269)	(.104)	(.261)	(.072)	(.029)	(.067)	(.266)			
Finland, 1970-83									
.536	.024	-.560	.166	.177	.656	-1.347	1.52	407	409
(.247)	(.210)	(.203)	(.042)	(.029)	(.032)	(.250)			
New Zealand, 1965-82									
-.350	.733	-.383	.527	.109	.364	-.438	.70	249	251
(.419)	(.207)	(.465)	(.114)	(.058)	(.110)	(.205)			
Norway, 1960-86									
.923	.283	-1.206	.150	.196	.654	-.080	.49	184	185
(.312)	(.150)	(.229)	(.057)	(.026)	(.042)	(.136)			
Sweden, 1967-84									
.081	.729	-.809	.061	.110	.829	-1.433	9.43	978	974
(.391)	(.152)	(.328)	(.076)	(.032)	(.076)	(.465)			
United Kingdom, 1955-85									
-.299	.436	-.137	.467	.165	.367	-.538	2.08	569	582
(.233)	(.137)	(.181)	(.046)	(.030)	(.035)	(.084)			

indicate a trend into wine in all countries, out of the other two beverages. All the marginal shares (columns 4-6) for beer, wine and spirits are positive and mostly significant. The column 7 estimates of ϕ, the own-price elasticity of alcohol as a whole, are always negative and significant except for Canada and Norway.

Column 8 of Table 1 reports the likelihood ratio test statistics of the hypothesis of preference independence.[4] Asymptotically, this statistic is distributed as $\chi^2(2)$. As the critical value of $\chi^2(2)$ at the 5 percent level is 5.99, PI can be rejected for only two of the seven countries, Canada and Sweden. Next, we employ a Monte Carlo test for PI which involves comparing the observed value of the test statistic with its empirical distribution, rather than its asymptotic counterpart (S. Selvanathan, 1987, 1993, Taylor et al., 1986, Theil et al., 1985).[5] Column 9 contains the results in the form of the ranks of the observed likelihood ratio test statistics (given in column 8) among 1,000 simulated values under the assumption of normal error terms. As only one of these ranks is greater than 950 (the "critical value" at the 5 percent level), again we are unable to reject PI. Column 10 contains similar rankings, but now with error terms generated with the bootstrap procedure (Efron, 1979). The rankings in column 10 are similar to those in column 9, indicating that the results are not overly sensitive to the normality assumption.

Taken as a whole, the results of this section favour the PI hypothesis.[6]

V. The Third Test

A third way of testing preference independence is via a Wald test of the restrictions on the Slutsky coefficients. Let $\hat{\pi}_{ij}$ and $\hat{\theta}_i$ be the homogeneity- and symmetry-constrained estimates. In addition to being the price elasticity of demand for alcohol as a whole, under the plausible assumption of a unity income elasticity for the group, the coefficient ϕ also has the interpretation as the income flexibility, the reciprocal of the income elasticity of the marginal utility of income. As the central value of previous estimates of the income flexibility is $-\frac{1}{2}$ (Clements and

S. Selvanathan, 1994), we set $\phi = -\frac{1}{2}$ and equation (3) then implies that

$$z_{ij} \equiv \hat{\pi}_{ij} + \frac{1}{2}\,\hat{\theta}_i(\delta_{ij} - \hat{\theta}_j) = 0.$$

As $\sum_j \hat{\pi}_{ij} = 0$, $\sum_j \delta_{ij} = 1$ and $\sum_j \hat{\theta}_j = 1$, it follows that $\sum_j z_{ij} = 0$. The test of PI then becomes a t-test of $z_{ij} = 0$. Table 2 contains the results. As the 3×3 matrix $[z_{ij}]$ is symmetric, it is sufficient to present the diagonal and upper triangle. As can be seen, among the $7 \times 6 = 42$ values of z_{ij} only 5 are significant at the 5 percent level. Accordingly, this test tends to reinforce the previous results of supporting PI.

TABLE 2

THIRD TEST OF PREFERENCE INDEPENDENCE

(Asymptotic standard errors in parentheses)

	Beer	Wine	Spirits	Beer	Wine	Spirits
		Australia			Norway	
Beer	.196 (.328)	.178 (.246)	-.374 (.226)	-.027 (1.383)	-.101 (.550)	.129 (.943)
Wine		-.445 (.360)	.267 (.175)		.732 (.388)	-.631 (.409)
Spirits			.108 (.263)			.502 (.794)
		Canada			Sweden	
Beer	-.285 (.763)	*.604 (.270)	-.319 (.699)	1.002 (.787)	-.567 (.413)	-.435 (.782)
Wine		.162 (.385)	*-.767 (.388)		-1.031 (.565)	*1.598 (.443)
Spirits			1.086 (.782)			-1.160 (.900)
		Finland			United Kingdom	
Beer	.179 (1.507)	-.934 (1.117)	.755 (.939)	-.461 (.348)	.209 (.189)	.252 (.249)
Wine		-.262 (1.187)	*1.196 (.591)		-.008 (.213)	-.202 (.168)
Spirits			*-1.952 (.737)			-.050 (.266)
		New Zealand				
Beer	.430 (.743)	-.362 (.424)	-.068 (.784)			
Wine		.040 (.521)	.322 (.365)			
Spirits			-.254 (1.141)			

Notes: 1. All entries are to be divided by 10.

2. An asterisk (*) indicates significant at the 5 percent level.

VI. Interpretations and Policy Implications

Table 3 gives the elasticities, at sample means, implied by the Table 1 estimates. As can be seen, beer is always a necessity and spirits a luxury. In most cases, the price elasticities are less than one in absolute value and, on average, spirits is the most price elastic beverage, then wine, and then beer.

TABLE 3

DEMAND ELASTICITIES FOR ALCOHOLIC BEVERAGES

	Conditional income elasticities			Frisch own-price elasticities		
Country	Beer	Wine	Spirits	Beer	Wine	Spirits
Australia	.81	1.00	1.83	-.40	-.50	-.91
Canada	.74	1.05	1.25	-.31	-.44	-.52
Finland	.45	1.32	1.32	-.61	-1.78	-1.78
New Zealand	.84	.88	1.45	-.37	-.39	-.64
Norway	.34	1.48	1.55	-.03	-.12	-.12
Sweden	.21	.69	1.52	-.30	-.99	-2.18
United Kingdom	.82	1.06	1.34	-.44	-.57	-.72
Mean	.60	1.07	1.47	-.35	-.68	-.98

The finding that the three alcoholic beverages are preference independent means that the marginal utility of one beverage is unaffected by changes in the consumption of the other two. This could be interpreted as saying that consumers do not mix their drinks, which is not unreasonable. Alternatively, consider a social function where there are groups of beer drinkers, wine drinkers and spirits drinkers. In this context, preference independence could imply that as utility at the margin of the beer drinkers is independent of how much the wine and spirits drinkers consume, beer drinkers just talk to themselves; and similarly for the other two groups of drinkers. Hence, alcohol consumption facilitates social interaction within the same groups of drinkers, not among them.

The policy implications of our results are clear-cut. One objective of taxing alcohol consumption is to raise revenue at minimum cost. In this case, the "inverse elasticity" rule is appropriate whereby tax rates are inversely proportional to the own-price elasticity. Using the mean elasticities given in the bottom row of Table 3, this rule implies that tax rates should be proportional to:

$$\text{Beer } \frac{1}{.35} = 2.86, \quad \text{Wine } \frac{1}{.68} = 1.47, \quad \text{Spirits } \frac{1}{.98} = 1.02 .$$

Accordingly, beer should be taxed at almost three times the spirits rate, while the wine tax should be about 50 percent higher than spirits. Note that beer has the lowest income elasticity and the highest tax rate, while spirits is the most income elastic and has the lowest tax rate. As the poor consume proportionately more necessities and less luxuries, this implies that the "optimal tax" structure for alcohol is regressive; this is a general implication of preference independence.

APPENDIX

RESULTS WITH WORKING'S MODEL

The Rotterdam model used in the text specifies the marginal share (θ_i) as a constant. An alternative assumption is that this share differs from the corresponding conditional budget share ($\overline{w}_{it}$) by a constant (β_i), so that the marginal share now varies with time:

$$\text{(A1)} \qquad \theta_{it} = \overline{w}_{it} + \beta_i \ .$$

This parameterization is known as Working's (1943) model and β_i is known as the income coefficient of i. We can examine the effects of this different functional form by substituting the right-hand side of (A1) for θ_i everywhere it appears in the text and then re-do the computations.

Figure A1 presents the graph of price elasticities against income elasticities with observations weighted by the reciprocals of the standard errors of the price elasticities, as in the text. The solid line is the LS regression line with the intercept suppressed. Estimation of equation $|\eta_{ii}^*| = \alpha + |\phi|\eta_i$ gives (with standard errors in parentheses):

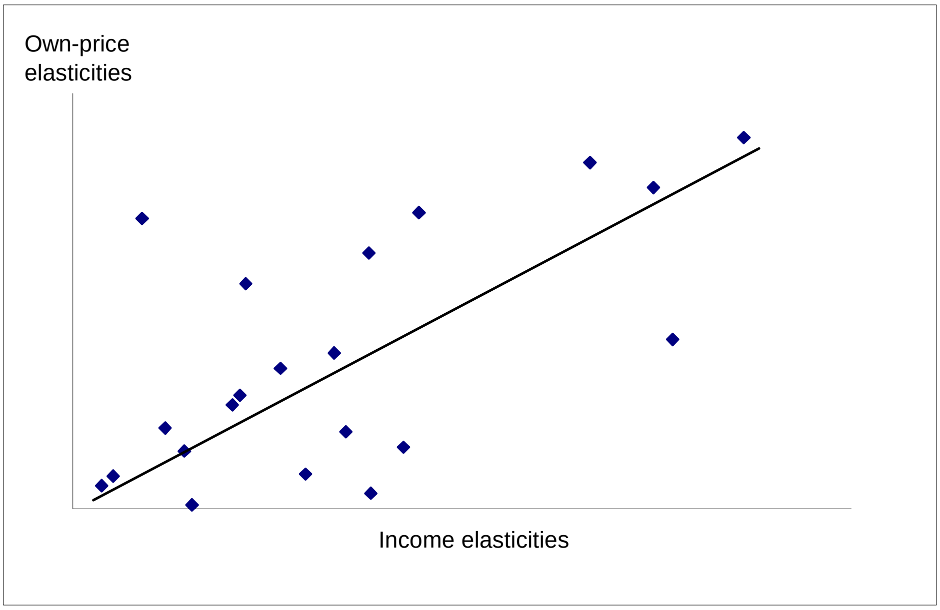
	Intercept, α	Slope, $\lvert\phi\rvert$	R^2
With intercept	.63 (.53)	.23 (.06)	.40
Without intercept	-	.29 (.04)	.36

As can be seen, these results are almost identical to those in the text and support the proportionality relationship associated with preference independence.

Table A1 contains the ML estimates of the conditional demand equations under Working's model. Here the intercepts (α_i) and the own-price elasticity of alcohol (ϕ) are directly comparable with those of the Rotterdam model and the estimates of these coefficients in Tables 1 and A1 are quite similar. The likelihood ratio test statistics and the rankings of those statistics are also similar for the two models.[7]

FIGURE A1

WEIGHTED PRICE AND INCOME ELASTICITIES
FOR ALCOHOLIC BEVERAGES: WORKING'S MODEL



The Wald test results for Working's model are reported in Table A2. Among the 42 values of z_{ij} only 3 are significant at the 5 percent level. Accordingly, PI can not be rejected again.

Taken as a whole, the results from the two models are quite consistent with one another.

TABLE A1

ESTIMATES OF CONDITIONAL DEMAND EQUATIONS FOR ALCOHOLIC BEVERAGES UNDER PREFERENCE INDEPENDENCE: WORKING'S MODEL

(Asymptotic standard errors in parentheses)

Intercepts $\alpha_i \times 100$			Income coefficients β_i			Own-price elasticity of alcohol ϕ	Likelihood ratio test statistic	Rank of test statistic in 1000 trials	
								Normal errors	Bootstrap errors
Beer (1)	Wine (2)	Spirits (3)	Beer (4)	Wine (5)	Spirits (6)	(7)	(8)	(9)	(10)
Australia, 1955-85									
-.380	.538	-.158	-.140	-.001	.141	-.471	3.97	821	806
(.140)	(.122)	(.137)	(.046)	(.039)	(.045)	(.084)			
Canada, 1953-82									
-.240	.356	-.116	-.119	.005	.114	-.271	5.89	921	915
(.265)	(.100)	(.253)	(.071)	(.028)	(.066)	(.235)			
Finland, 1970-83									
.493	.061	-.554	-.198	.051	.146	-1.313	2.43	531	536
(.247)	(.210)	(.203)	(.042)	(.029)	(.032)	(.250)			
New Zealand, 1965-82									
-.385	.761	-.376	-.112	-.000	.112	-.453	.28	271	220
(.429)	(.217)	(.472)	(.117)	(.060)	(.111)	(.204)			
Norway, 1960-86									
.961	.291	-1.252	-.303	.061	.243	-.122	.66	237	237
(.333)	(.144)	(.244)	(.060)	(.025)	(.044)	(.127)			
Sweden, 1967-84									
.057	.828	-.885	-.256	-.045	.301	-1.711	7.86	936	945
(.403)	(.143)	(.343)	(.066)	(.025)	(.068)	(.474)			
United Kingdom, 1955-85									
-.337	.469	-.133	-.079	-.006	.085	-.584	-1.88	95	85
(.230)	(.128)	(.178)	(.045)	(.027)	(.034)	(.086)			

TABLE A2

THIRD TEST OF PREFERENCE INDEPENDENCE: WORKING'S MODEL

(Asymptotic standard errors in parentheses)

	Beer	Wine	Spirits	Beer	Wine	Spirits
		Australia			Norway	
Beer	.313 (.349)	.057 (.263)	-.370 (.233)	-.045 (1.485)	-.085 (.545)	.131 (1.011)
Wine		-.327 (.375)	.270 (.184)		.620 (.360)	-.535 (.408)
Spirits			.100 (.262)			.404 (.829)
		Canada			Sweden	
Beer	-.118 (.754)	.507 (.261)	-.390 (.693)	.951 (.802)	-.468 (.411)	-.483 (.803)
Wine		.142 (.370)	-.650 (.374)		*-1.139 (.562)	*1.607 (.444)
Spirits			1.039 (.769)			-1.124 (.920)
		Finland			United Kingdom	
Beer	.003 (1.440)	-.900 (1.064)	.897 (.914)	-.469 (.351)	.267 (.193)	.203 (.246)
Wine		-.229 (1.142)	1.128 (.586)		-.095 (.226)	-.172 (.167)
Spirits			*-2.025 (.735)			-.031 (.255)
		New Zealand				
Beer	.448 (.738)	-.319 (.423)	-.130 (.780)			
Wine		.068 (.497)	.251 (.370)			
Spirits			-.121 (1.106)			

Notes: 1. All entries are to be divided by 10.

2. An asterisk (*) indicates significant at the 5 percent level.

Footnotes

1. Other implications of PI are that inferior goods are ruled out and all goods are Slutsky substitutes.

2. The data were kindly supplied by E. A. Selvanathan; see E. A. Selvanathan (1991a) for details. Japan and the USA are omitted due to the unavailability of data.

3. Regarding serial correlation, the first-order autocorrelation coefficients are significant in only 3 out of the 21 cases -- beer and wine in Canada, and beer in Norway. This indicates that serial correlation is not a major problem, which probably reflects the first-difference formulation of the equations. Re-estimating with AR(1) disturbances for Canada and Norway did not lead to substantial changes in the estimates.

4. For this test, we use as the unrestricted model equation (2) for i = 1, 2, 3 with homogeneity and symmetry imposed. For most countries, these data satisfy the homogeneity and symmetry restrictions (E. A. Selvanathan, 1991a).

5. The Monte Carlo test works as follows: (1) Generate disturbances from a multivariate normal distribution with zero mean and covariance matrix S, the mean squares and cross products of the restricted residuals. (2) Generate the simulated values of the dependent variables by using in equation (4) for i = 1, 2, 3 (a) the generated disturbances; (b) the observed values of the independent variables; and (c) the ML estimates of the coefficients under PI. (3) Estimate the unrestricted and restricted models, equations (2) and (4) for i = 1, 2, 3, with the simulated values of the dependent variables and the observed values of the independent variables. (4) Compute the simulated value of the likelihood ratio test statistic. (5) Repeat 1,000 times steps 1-4 to generate the simulated distribution of the test statistic under null. (6) Reject the PI hypothesis at the 5 percent level if the observed value of the test statistic is larger than 950 simulated values.

6. Similar results have been obtained with Working's (1943) model. Details are given in the Appendix.

7. Note that the likelihood ratio test statistic for the United Kingdom is negative. A Monte Carlo simulation with Working's model shows that among 1000 simulated values of this, the proportion of negative values is 1.8 percent in Australia, 0.7 percent in Canada, 2.0 percent in Finland, 4.4 percent in New Zealand, 23.7 percent in Sweden and 8.9 percent in United Kingdom. This indicates that the likelihood ratio is not distributed as χ^2 and, in part at least, relates to the non-nested nature of the preference independence hypothesis in the context of Working's model. This may also account for the apparently different rankings of the likelihood ratio test statistics in the Monte Carlo tests in Tables 1 and A1 for the UK. It is to be noted that this problem does not arise with the Rotterdam model.

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