Prog. Theor. Phys. Vol. 66, No. 5, November 1981, Progress Letters

Ising Ferromagnets with Random Anisotropy

Ikuo TAMURA and Takahito KANEYOSHI
Department of Physics, Nagoya University, Nagoya 464
(Received May 9, 1981)

Ising ferromagnets with random anisotropy are studied by a new effective field theory beyond the usual Weiss molecular field approximation. Magnetization curves and variation of Curie temperatures are discussed for two random anisotropic cases. The results are compared with those of the usual mean field theory.

Effects of uniaxial anisotropy field on magnets have been studied in a number of recent papers by means of the molecular-field theory and Green-function technique. By the use of Green-function method Devlin has discussed the problem for the case of Heisenberg model.[1] In contrast with the Heisenberg model it is interesting to investigate the problem for the case of Ising model, especially beyond the mean field results.

On the other hand, the system in which uniaxial anisotropy fields on each magnetic site have random orientations has been investigated only within a simple molecular-field theory.[2),3)] Mizuno studied a model in which each spin is subjected to a random local magnetic field for some disordered ferromagnets.[4)]

In this work, the Ising ferromagnetic systems for $S=1$ with uniform and random uniaxial anisotropy are studied by making use of the newly developed effective field theory with correlations,[5)] in which spin correlations are partly taken into account.

The Hamiltonian of the system is given by

$$H=-\frac{1}{2}\sum_{ij}J_{ij}\cdot S_i^z S_j^z-\sum_i D_i(S_i^z)^2\,, \qquad (1)$$

where J_{ij} is the exchange interaction and D_i the magnitude of the uniaxial anisotropy.

Following Suzuki,[7)] we can obtain an exact spin correlation function,

$$\langle S_i^z\rangle=\left\langle\frac{2\sinh(\beta E_i)}{\exp(-\beta D_i)+2\cosh(\beta E_i)}\right\rangle, \qquad (2)$$

where $E_i=\sum_j J_{ij}S_j^z$ and $\langle\cdots\rangle$ indicates an ensemble average $\langle A\rangle=\mathrm{Tr}\,A\exp(-\beta H)/\mathrm{Tr}\exp(-\beta H)$.

Let us introduce the differential operator into Eq. (2) as follows:

$$\langle S_i^z\rangle=\left\langle\exp\left(\frac{\partial}{\partial x}\beta E_i\right)\right\rangle f_i(x)\Big|_{x=0} \qquad (3)$$

with

$$f_i(x)=\frac{2\sinh x}{\exp(-\beta D_i)+2\cosh x}\,. \qquad (4)$$

Introducing a decoupling approximation $\langle S_iS_j\rangle\cong\langle S_i\rangle\langle S_j\rangle$ for $i\neq j$, which corresponds to the Zernike approximation for $S=\frac{1}{2}$ Ising model,[5),6)] and using an identity for $S=1$ Ising system,

$$\exp(\alpha S^z)=(S^z)^2\cosh\alpha+S^z\sinh\alpha+1-(S^z)^2$$

Eq. (3) is given by

$$\langle S_i^z\rangle=\prod_j\left[\langle(S_j^z)^2\rangle\cosh\left(\frac{\partial}{\partial x}t_{ij}\right)+\langle S_j^z\rangle\sinh\left(\frac{\partial}{\partial x}t_{ij}\right)+1-\langle(S_j^z)^2\rangle\right]f_i(x)\Big|_{x=0}\,, \qquad (5)$$

where $t_{ij}=J_{ij}/k_BT$.

In order to evaluate $\langle S_i^z\rangle$, it is necessary to evaluate $\langle(S_i^z)^2\rangle$. By the same procedure as Eq. (5), we have

$$\langle (S_i^z)^2\rangle = \prod_j \Big[\langle (S_j^z)^2\rangle \cosh\Big(\frac{\partial}{\partial x} t_{ij}\Big) + \langle S_j^z\rangle \sinh\Big(\frac{\partial}{\partial x} t_{ij}\Big) + 1 - \langle (S_j^z)^2\rangle \Big] g_i(x)\Big|_{x=0} \tag{6}$$

with

$$g_i(x) = \frac{2\cosh x}{\exp(-\beta D_i) + 2\cosh x} . \tag{7}$$

For the case of random uniaxial anisotropy, it is necessary to take an average over all possible configurations for random variables. Using a decoupling approximation for a random average, (for example, see the Appendix of Ref. 6)) we obtain for a system with nearest-neighbor interaction J,

$$\sigma = \Big[q\cosh\Big(\frac{\partial}{\partial x} t\Big) + \sigma \sinh\Big(\frac{\partial}{\partial x} t\Big) + 1 - q \Big]^z \langle f_i(x)\rangle_r \Big|_{x=0} , \tag{8}$$

$$q = \Big[q\cosh\Big(\frac{\partial}{\partial x} t\Big) + \sigma \sinh\Big(\frac{\partial}{\partial x} t\Big) + 1 - q \Big]^z \langle g_i(x)\rangle_r \Big|_{x=0} \tag{9}$$

with

$$\langle f_i(x)\rangle_r = \int P(D_i) f_i(x)\, dD_i ,$$

$$\langle g_i(x)\rangle_r = \int P(D_i) g_i(x)\, dD_i ,$$

where σ and q are the random averages of $\langle S_i^z\rangle$ and $\langle (S_i^z)^2\rangle$ respectively, $t = J/k_B T$, z is the coordination number, and $P(D_i)$ is the probability function of the anisotropy fields. For simplicity $P(D_i)$ is chosen in the following two forms:

case (I) $P(D_i) = c\delta(D_i - D_1) + (1-c) \times \delta(D_i - D_2)$

with

$$0 \le D_2 \le D_1 , \qquad 0 \le c \le 1 ,$$

case (II) $P(D_i) = \dfrac{1}{2\Delta}$ for $0 \le D_0 - \Delta \le D_i \le D_0 + \Delta$,

0 , elsewhere .

Here we solve the coupled equations (8) and (9) numerically for $z = 6$. At first, let us discuss the case of uniform anisotropy (crystalline case). Magnetization curve and q vs T/T_c curve in Fig. 1 are above those in molecular-field approximation (MFA) and Curie temperatures also depicted in Fig. 1 are smaller in our formalism than those in MFA. These results are all in contrast to those of Heisenberg model.[1] This is probably due to the facts that in Ising systems with uniaxial anisotropy the thermal fluctuations at low temperatures are restricted in comparison with those in Heisenberg systems and the

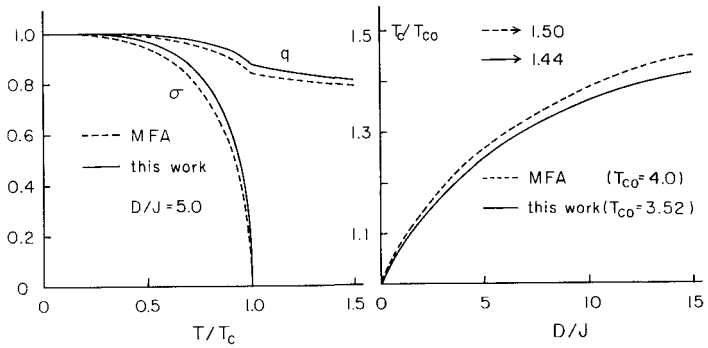

Fig. 1. Magnetization curves, q vs T and Curie temperatures vs D in crystalline case in comparison with MFA result. Arrows show the values of T_c/T_{co} at $D = \infty$.

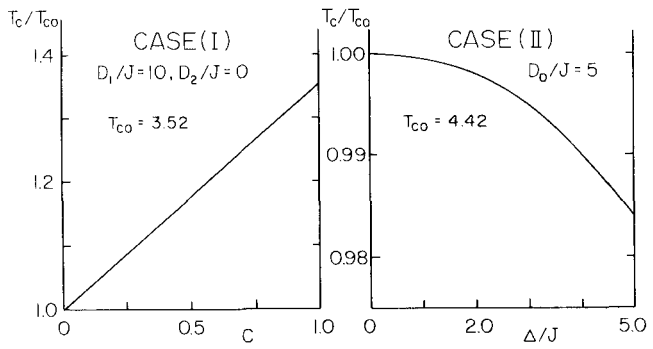

Fig. 2. Curie temperatures in random anisotropy case as a function of c (case I) and Δ (case II).

effect of anisotropy is partly taken into account in Ising systems.

This method corresponds to the Zernike approximation for a limit of infinite anisotropy ($S=\frac{1}{2}$), as already discussed in Ref. 6). (The arrow in Fig. 1 shows the value of T_c in the Zernike approximation.)

Next we turn to the case of random anisotropy. In Fig. 2 Curie temperatures are plotted as a function of c and Δ for cases (I) and (II). Figures 3 and 4 represent σ and q as a function of T/T_c respectively. From these results we can see that the existence of the random anisotropy fields makes σ and q for $T/T_c<1$ decrease faster than those of crystalline case as the temperature increases, whereas q for $T/T_c>1$ decreases slower. Those qualitatively agree with the previous MFA results.[2),3)] Finally the advantage of our method is that its formulations are rather simpler than other approximations beyond MFA and its applications to the various random spin systems are facile.

1) J. F. Devlin, Phys. Rev. **B4** (1971), 136.
2) G. B. Taggart, R. A. Tahir-Kheli and E. Shiles, Physica **75** (1974), 234.
3) J. D. Patterson, Phys. Stat. Sol. (b) **79** (1977), K165.
4) J. Mizuno, J. of Phys. **C7** (1974), 3755.
5) R. Honmura and T. Kaneyoshi, Prog. Theor. Phys. **60** (1978), 635.
6) T. Kaneyoshi and H. Beyer, J. Phys. Soc. Japan **49** (1980), 1306.
7) M. Suzuki, Phys. Letters **19** (1965), 267.

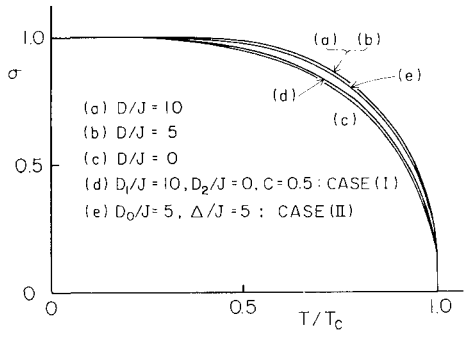

Fig. 3. Magnetization curves in crystalline case [(a), (b), (c)] and in random anisotropy case [(d), (e)].

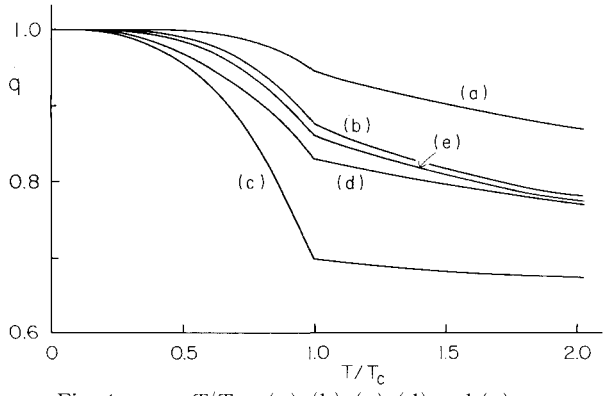

Fig. 4. q vs T/T_c. (a), (b), (c), (d) and (e) are the same as in Fig. 3.