

## Research Article

# Ising Model of User Behavior Decision in Network Rumor Propagation

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The continuous breeding and rapid spread of rumors in social networks poses a severe challenge to the effective utilization and scientific management of social media. Therefore, it is of great theoretical significance and application value to study the decision-making behavior of users in rumors spread in social networks and to reveal the rumor transmission rules. Based on the Ising model, this paper constructs a social network rumor propagation dynamics model and then reveals the rumor transmission rules. In the model, the Monte Carlo method is used to simulate the interaction between the user's self-identity attribute (micropart), the user-user interaction (the middle part), and the social environment's influence (the macroscopic part) to study user decision behavior of rumor spread in a social network system. The results show that, in the rumor propagation system, von Neumann entropy can quantify well the phase transition of the system and is consistent with the phase transition information obtained by measuring the spontaneous magnetization and magnetic susceptibility of the system. In addition, the introduction of the self-identity characteristics of individual users into the Ising model has greatly changed the users' decision-making behavior of rumor spreading and changed the internal structure of the system, thus changing the type of phase transition. As the degree of self-identification increases, the need for lower temperatures can change the orderly state of the system, and user behavior changes more rapidly. For the rumor transmission system, the state of close order is conducive to the blocking of rumors, thus maintaining social stability.

## 1. Introduction

With the wide use of social networks and the convenience of the media, rumors permeate all aspects of social life. The importance, ambiguity, and potential dangers of rumors are often widely disseminated without confirmation [1, 2] which affects public attitudes, beliefs, and behaviors [3, 4]. In particular, the explosive development of online social networks such as Facebook, Twitter, and Microblog [5] make an increasingly rapid diffusion of rumors. A rumor can often bring a butterfly effect [6] and bring about huge social and network exchanges and normal social order. The negative impact can easily lead to serious social problems and even crisis social stability, economic development, and national security [7]. DiFonzo believes that Internet rumor has become an important issue that affects social stability.

The research on the rumors characteristics and propagation related to cyber rumor can effectively curb the negative effects caused by it [8]. The research [9–14] on identifying major depressive disorder (MDD) has also provided important ideas and methods for rumor identification and dissemination. Therefore, it is of great theoretical significance and practical value to study rumors spreading rules of social network and stop rumors immediately, which can effectively control rumors and maintain social stability.

The rumor spread in social media involves many factors such as information, users, and network environment. It not only embodies the network effect of information dissemination but also integrates the social psychological factors, the decision-making behaviors of human beings, and the interaction among users and the social environment influence. Rumor spread in social media will cause different

degrees of harm to people's lives and even the whole society. So the ultimate goal of rumor communication research is to suppress or even prevent rumors from spreading. Therefore, based on the user's attributes of the micropart, based on the interaction between the user parts of the micropart, and based on the environmental influence and positive and negative social reinforcement on the rumors of social media of the macropart has become an important research issue one.

In social media, the user's decision-making behavior is influenced by its factors and the influence of the neighborhood users on its impact and social environment. The dissemination users' intrinsic factors, specific purposes, and their authority, influence, and behavior characteristics determine the width and depth of rumor spread and the outbreak point of rumors. Scholars [15] have found that user differences can provide an important basis for rumor identification in the study of users' differences involved in the spread of rumors and nonrumors. Based on the Ising model, this paper combines the influence among users and the influence of social environment, that is, describing the decision-making behavior of users in the process of spreading rumors in the medium and macroaspects. At the same time, taking into account the user's attributes and introducing the individual identity characteristics (microcosmic part), the influence of the feature on the model is analyzed. Finally, the experiments show that the interaction of the user itself, interaction between a user and another, and the social environment in which the user is located can well maintain the orderly state of the cyberspace and provide the decision basis for decision-making of the network governance.

## 2. Related Works

In social media, the user's factors, the neighborhood users, and social environment influence the user's decision-making behavior. The study found that the spread of rumors is driven by many factors, including the internal factors that spread users, the influence of neighboring neighbors, social factors and environmental factors, and so on.

In the research of user decision behavior based on user's attributes, the internal factors of transmitting users (such as personality traits) and their specific purpose have influenced their behavior of spreading messages on social media. Chen et al. [16] have found that the Neuroticism and Openness in the user's personality have a significant impact on the spread of rumor news. In addition, the dissemination users' motivations for entertainment, socializing, and seeking status have also had a significant impact on the spread of rumor messages. In fact, whether users can spread rumors will be affected by a variety of factors, such as personal preference [17]. In recent years, many scholars have shifted their attention to the social attributes of human behavior [18].

In the research of user decision behavior based on the influence of neighbor nodes, since the nodes in social networks are users in real life, their propagation behavior is highly autonomous and independent. Therefore, it is necessary to evaluate the behavior decisions of individual nodes and their neighbor nodes. Simon et al. [19] found that, in

the case of infection, shopping, and social interaction, users' decision-making behavior is influenced by their neighbors or friends. People will make and modify their behavior decisions based on others' decisions. Jackson et al. [20] analyzed the game in social networks where each user would make a binary choice. And Lopez analyzes the influence of the neighbor nodes on the user's decision and gives the equation of the field average [21]. If a good reputation or influential person or a media participates in the spread of rumors, it will cause qualitative changes in rumors; that is to say, it will become quasi-news and make potential rumors and ballads increase significantly. Liao and Shi's [22] research on the contribution and influence of celebrities, certified users, mass media, organizations, websites, talent, and ordinary users in the process of rumor dissemination found that the users who played a major role in the dissemination process were dignitaries, celebrities, and the mass media. Considering that the decision-making in social networks is rational and autonomous, and the user's behavior decision-making is influenced by the neighbors' nodes. Many scholars use game theory to depict this kind of influence and regard the diffusion process of rumors in the network as the process of users choosing the best decision through the game and maximizing their interests.

In the analysis of user decision-making behavior based on the role of social environment, such as social strengthening mechanism, before an individual takes action, it will be affected by the superposition of multiple actions from his neighbors and even the society [23]. At present, some achievements have shown that external public opinion plays an important role in user communication [24]. But these studies only study the influence of positive or negative one of the social reinforcement mechanisms on the spread of rumors. However, in fact, due to the extremely vague nature of the rumors, people's life experience and education experience are different [25]. After the spread of rumors, it is easy to produce two opposing views at the same time, thus forming two powerful social reinforcing functions: positive and negative [26]. Most previous works focused on detecting rumors by shallow features of messages, including content and blogger features. Zhang et al. [27] also believe that shallow features cannot distinguish between rumors and nonrumors well in many situations. Therefore, they extracted four implicit features based on content: popularity orientation, internal and external consistency, emotional polarity, and comment point of view. The popularity orientation feature can obtain the correlation between the content of the message and a hot topic or event in the current society, and the feature value of popularity orientation can be obtained by calculating the Jaccard coefficient. In some cases, user proximity to events physically or emotionally has a significant impact on whether they publish or forward messages [28–30]. When people have physical proximity to or are emotionally associated with people or places (such as friends, family, or former residences) that may be affected by the incident, they can spread more news and sometimes think of helping others by spreading the message. However, these messages may contain a large number of rumors and their forwarding expanded the spread of rumors.

### 3. Ising Model of the Decision-Making of Rumor Propagation

**3.1. Ising Model.** Ising model is an important model in statistical physics. It describes the phase transition of a matter by characterizing the interaction between particles and the effect of external environment on the particles; that is, magnetization disappears above certain critical temperature, while magnetization appears below certain critical temperature. In the Ising model, a ferromagnetic material consists of a stack of regularly arranged small magnetic pins, each of which has only two directions (spin) up and down. Adjacent small needles interact with each other through energy constraints. At the same time, the random magnetic transition (upside down or vice versa) occurs due to the interference of ambient thermal noise [31].

Assuming that the  $i$ th node is a small magnetic pin, each small magnetic pin has two states up and down. We use  $s_i$  to indicate this state and  $s_i = \begin{cases} +1 \\ -1 \end{cases}$  means the magnetic pin is up or down. The two adjacent small magnetic needles on the grid can interact.

In the Ising model, there is an interaction between each pair of adjacent spins, and the energy of the system is

$$E_{\{s_i\}} = -J \sum_{\langle i,j \rangle} s_i s_j - H \sum_i s_i, \quad (1)$$

where  $J$  is energy coupling constant and  $E_{\{s_i\}}$  represents the total energy of the system under state combination  $\{s_i\}$ . The summing subscript  $\langle i, j \rangle$  indicates the summation of all two adjacent small magnetic needles. We see that if  $s_i = s_j$ , the total energy is reduced by  $J$ .  $H$  represents the strength of the external magnetic field; the magnetic field up then  $H$  is positive and otherwise negative. If the direction of a small needle coincides with the field, the total energy is reduced by one unit.

**3.2. Ising Decision Model.** Based on the Ising model, this paper constructs a behavioral decision-making model of rumor propagation. Each small magnetic needle is compared to a user in a social network. The upper and lower states of the small magnetic needle are compared to the user's decision-making behavior, propagation, and nonpropagation. The interaction between adjacent small needles is analogous to the effect of behavioral decision-making among users. The temperature of the environment is analogous to the influence of the environment (the heat of the social network rumor topic and the appeal or influence of a rumor). In this way, the entire Ising model can express the dynamic evolution of rumors made by different users in social networks. In addition, this paper will also add the self-identity characteristics of individuals to the rumor communication behavior decision-making model and then analyze the characteristics of the impact of the model.

This paper calculates the revenue function of the behavior strategy adopted by the user through the rumor propagation user behavior decision-making model and obtains the equilibrium state of the system, that is, the time point when the rumor no longer propagates and the system reaches

the steady state. The Nash equilibrium is used to judge the decision-making distribution of each user when the social network is in a steady state. By measuring the von Neumann entropy, we study and analyze the phase transitions and critical phenomena of the user behavior decision-making system and get the critical information. Finally, the Monte Carlo simulation of the Ising model is carried out.

**3.3. Ising Model Construction.** The Ising model was originally used in phase change studies and later with the development of complexity science. Because of its simple mechanism and rich dynamic behavior [32], it can effectively simulate the evolution of binary opinion; it has been widely used in the study of viewpoint kinematics.

**3.3.1. User Decision Behavior Model.** Let  $Z^2$  be represented as a square lattice; the element  $i$  in the lattice is a pair of integers  $(i_1, i_2)$ , for any finite symmetric subset  $\Lambda \subset Z^2$ ; let  $\Omega_\Lambda = \{-1, 1\}^\Lambda$  denote the spin configuration on  $\Lambda$ ; the element of  $\Omega_\Lambda$  may be denoted as  $\sigma_\Lambda = \{\sigma_i : i \in \Lambda\}$ . Without being obfuscated, the subscript  $\Lambda$  in  $\sigma_\Lambda$  can be omitted. Considering the Ising model with the following Hamiltonian system, the energy of an Ising model system on a complex network over any  $\sigma \in \Omega_\Lambda$  is defined:

$$H_{\Lambda,h}(\sigma) = - \sum_{i,j \in \Lambda} J_{ij} \sigma_i \sigma_j - h \sum_{i \in \Lambda} \sigma_i, \quad (2)$$

where  $h$  is a real number, which represents the environmental influence of the microblog ( $h = \xi h_1 + (1 - \xi) h_2$ , where  $h_1$  represents the heat of the rumor topic,  $h_2$  represents the attraction or influence of the rumor, and  $\xi$  represents the environmental factor, the value of which depends on the real environment).  $J_{ij}$  is the interaction between users, the view of the individual  $i$  is updated by the nearest neighbors, and the direction selection similar to spin is subject to the local spin interaction

**3.3.2. Calculate the User's Local Environment.** Based on Simon's research, the user's neighbors or friends influence his or her decision-making behavior. People make and modify their own behavioral decisions based on others' decisions. Therefore, we can calculate the individual user's local environment. Assume that the users in the social network are at a point in the  $\Lambda$  grid ( $\Lambda$  is a subset of square grids  $Z^2$ ). At each point of time, the user decides whether to spread the rumor ( $\sigma_i = +1$ ) or not to spread the rumor ( $\sigma_i = -1$ ). The decision of each user  $i$  depends on the local environment  $I_i(t)$  at the time  $t$ , and its formula is

$$I_i(t) = \frac{1}{|\Lambda|} \sum_{j \in \Lambda} J_{ij}(t) \sigma_j(t) - h_i(t). \quad (3)$$

The first term in (3)  $(1/|\Lambda|) \sum_{j \in \Lambda} J_{ij}(t) \sigma_j(t)$  of  $I_i(t)$  denotes the influence of the behavior decision attitude of other users on the decision-making of user  $i$  at time  $t$ . Due to the interaction between users,  $J_{ij}(t)$  is time-varying and has the form  $J_{ij}(t) = a\xi(t) + b\eta_{ij}(t)$ ,  $a\xi(t)$ , which reflects the average effect of the entire society on the user  $i$  and  $b\eta_{ij}(t)$  reflects the influence of all users on user  $i$  decision.

The second term in (3)  $h_i(t) = c_i \sum_{i=i}^{i-1} \alpha x_i + d_i B(t)$  of  $I_i(t)$  represents the social environment,  $c_i \sum_{i=i}^{i-1} \alpha x_i$  represents the influence of the total return of behavior decision-making on the user  $i$  over a period,  $B(t)$  is the standard Brownian motion, which represents the external random information into society, and  $d_i B(t)$  reflects the impact of stochastic information on the behavior decisions of different users  $i$ .

**3.3.3. Calculate User's Behavior Decision Probability.** Users change their behavior decisions based on changes in the environment. At each moment  $t$ , the user changes his attitude with a certain probability, the probability of spreading the rumor is  $P$ , and the probability of not spreading the rumor is  $1 - P$ . Then the formula is as follows:

$$P = \frac{1}{1 + \exp[-2I_i(t)]}. \quad (4)$$

**3.3.4. Calculate the User's Revenue Function.** In social networks, users' behavioral decisions are influenced not only by their own interests, but also by their neighbors or friends. Everyone makes and modifies their behavioral decisions based on the decisions of others. Users choose the best decision through the game to maximize their own interests. Based on the game in social networks analyzed by Jackson et al., we can calculate the user's revenue function and find the equilibrium state threshold. Therefore, in the Ising model, user decisions are rational and autonomous, and users always choose to make their own decisions. An individual achieves an equilibrium state during the interaction, in which no individual can increase returns by unilaterally changing his or her behavior strategy. Therefore, the Nash equilibrium is used to judge the decision-making distribution of each user when the social network is in a steady state.

Suppose the social network group user  $i$  will choose to spread rumors with a probability of  $p$  and with a probability of  $1 - p$  choosing not to spread rumors. The user  $j$  will choose to spread rumors with a probability of  $q$ , with a probability of  $1 - q$  choosing not to spread rumors. Then the probability matrix of the user  $i$  and  $j$  selection behavior is  $P = p, 1 - p$   $Q = q, 1 - q$ , respectively; among them,  $0 \leq p \leq 1, 0 \leq q \leq 1$ . In addition,  $\beta_{ij}$  denotes the trust degree of the neighbor node,  $V_d$  denotes the proceeds of the choice of spreading the rumor,  $V_a$  denotes the proceeds of receiving the rumor, and  $C$  is the cost of spreading the rumor.

The expected utility function for user  $i$ 's choice of spreading rumor behavior is

$$U_i^s = q[\beta_{ij}(V_d + V_a) - C] + (1 - q)(\beta_{ij}V_d - C). \quad (5)$$

The expected utility function when choosing not to spread rumors is

$$U_i^{ns} = q\beta_{ij}V_a + 0(1 - q). \quad (6)$$

Get the user  $i$ 's overall expected utility function

$$\begin{aligned} U_i(p, 1 - p) \\ = p \{ q[\beta_{ij}(V_d + V_a) - C] + (1 - q)(\beta_{ij}V_d - C) \} \\ + (1 - p)q\beta_{ij}V_a. \end{aligned} \quad (7)$$

Similarly, for the user  $j$ , its expected utility function is the same as  $i$ .

Differentiate the above-expected utility function to obtain the first-order condition:

$$\frac{\partial U_i}{\partial p} = \beta_{ij}V_d - C = 0, \quad \beta_{ij}V_d = C. \quad (8)$$

The user  $j$ 's hybrid strategy is  $(q, 1 - q)$ . If the user  $i$  chooses to propagate rumors, then the revenue is  $q\beta_{ij}V_a + \beta_{ij}V_d - C$ ; if user  $i$  chooses not to spread rumors, the revenue is  $q\beta_{ij}V_a$ . When  $\beta_{ij}V_d > C$ , the larger the value of  $p$ , the more the participants expected to spread the gains, at which point ( $p = 1$ ) represents the best strategy for  $i$ . When  $\beta_{ij}V_d < C$ , the user's expectation of the spread of earnings is inversely proportional to  $p$ . The smaller the  $p$  is, the less likely the user is to choose to spread the rumor. Do not spread rumors ( $p = 0$ ) to become the best strategy for  $i$ . When  $\beta_{ij}V_d = C$ , user  $i$  spreads that the benefits of nondissemination are the same. Therefore, every feasible  $p$  is the best strategy for user  $i$ . User  $j$  and user  $i$ 's analysis method are the same.

When  $\beta_{ij}V_d - C > 0$ , the game's mixed strategy Nash equilibrium is  $(1, 1)$ .

When  $\beta_{ij}V_d - C < 0$ , the game's mixed strategy Nash equilibrium is  $(0, 0)$ .

When  $\beta_{ij}V_d - C = 0$ , any feasible  $(p, q)$  is the mixed strategy Nash equilibrium of this game.

From this, we can conclude that the more the trustworthiness among users, the higher the rate of rumor propagation. If there is a massive spread of rumors and sensitive information on the Internet, the government and the media should timely release official messages to reduce the panic and anxiety in the society. The more important the rumor, the higher the spread rate. After the rumor incident, the public is easily influenced by herd mentality, and herd behavior can easily occur [33]. The common denominator of these events is the life, property, and economic interests of many people, even though the initial communicators are all very humble users. If the government clarifies the relevant laws and regulation to punish criminals who spread rumors, they can increase the risk of transmission of rumors, thus reducing the rate of spreading rumors.

**3.3.5. Calculate the Von Neumann Entropy.** In recent years, researchers have found that it is possible to quantify quantum phase transitions by measuring the entanglement of the quantum. The researchers found that von Neumann entropy shows finite-scale scaling behavior as a measure of quantum entanglement near the critical quantum point [34]. This method can effectively and directly obtain the quantum transition in the quantum model. By the predecessors, this paper uses the von Neumann entropy to study the Ising model and find out the transformation point and critical information of the rumor propagation system. The von Neumann's entropy formula is as follows:

$$\begin{aligned} S_p(T) &\equiv -\text{tr}_p \rho_p(T) \ln \rho_p(T) \\ &= 2 \ln 2 - \frac{1}{2} \left[ \gamma \ln \frac{1 + \gamma}{1 - \gamma} + \ln(1 - \gamma^2) \right]. \end{aligned} \quad (9)$$

Among them, the spin correlation is  $\gamma(T) = \langle \sigma_i \sigma_{i+1} \rangle$ . Where  $\rho_p(T)$  represents the reduced density matrix,  $tr_p \rho_p(T)$  represents the trace of the subsystem, and  $\gamma$  is a critical exponent.

#### 4. Monte Carlo Experimental Simulation

MC simulation refers to a calculation method that uses computer-generated pseudo-random numbers to deal with probability problems. In recent years, the MC method has become a standard method for scientific research and has been widely used in statistical physics and complex systems, nonlinear dynamic processes, chaos, and some chemical reactions and mathematical random processes.

The key to performing a Monte Carlo simulation is to first give the state transfer rate. First write the master equation

$$\frac{\partial P_s(t)}{\partial t} = \sum_{s \neq ns} (P_{ns} \Psi_{ns \rightarrow n} - P_s \Psi_{s \rightarrow ns}), \quad (10)$$

where  $P_s(t)$  represents the probability that the system is in the  $s$ -state at time  $t$ , and  $\Psi_{s \rightarrow ns}$  represents the transition rate from the state  $s$  to the state  $ns$ . When the system reaches equilibrium,  $\partial P_s(t)/\partial t = 0$ , so careful balance conditions can be written as follows:

$$P_{ns} \Psi_{ns \rightarrow s} = P_s \Psi_{s \rightarrow ns}. \quad (11)$$

In addition, the probability of balancing the temporal  $s$  can be given by the Boltzmann distribution:

$$P_s^{eq} = \frac{e^{-H_s/k_B T}}{Z}. \quad (12)$$

The denominator  $Z$  in the above formula is the distribution function of the system. In general, the partition function cannot be directly solved. For this reason, the method often used by people is to assume that the system evolution process is a Markov process, so that we can directly generate new states from the old state, and there is no time and other memory processes. The system transitions from one state to another, and its transition probability should satisfy the following formula:

$$\frac{\Psi_{s \rightarrow ns}}{\Psi_{ns \rightarrow s}} = \frac{P_{ns}^{eq}}{P_s^{eq}} = e^{-\Delta H/k_B T}, \quad \Delta H = H_{ns} - H_s. \quad (13)$$

Using the above method can avoid solving the complicated partition function. The ratio of the transition probability can be obtained by calculating the difference between the energy before and after the transition and the temperature. Only this one equation still cannot get the transition probability, so we introduce a fine balance condition to get another equation, for example, according to the rules of the Metropolis dynamics, you can write

$$\Psi_{ns \rightarrow s} = \begin{cases} \exp\left(-\frac{\Delta H}{k_B T}\right), & \Delta H > 0 \\ 1, & \Delta H < 0. \end{cases} \quad (14)$$

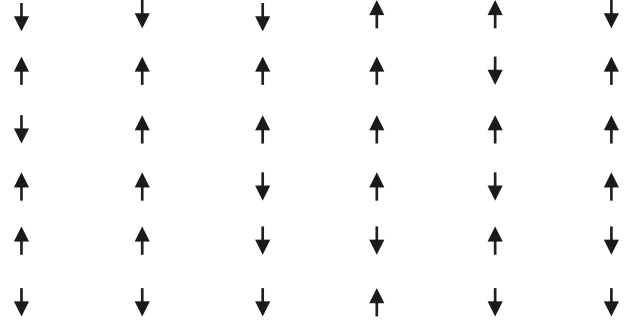


FIGURE 1: Ferromagnetic Ising model.

*4.1. Magnetization and Magnetic Susceptibility of Rumor Propagation Systems.* In order to prove the von Neumann entropy method to study the critical temperature  $T_c$  of the rumor propagation system and whether the phase transition type and some critical exponents the system experiences are effective and feasible. Therefore, the magnetization and magnetic susceptibility of the system are measured according to the traditional method. Magnetization is a physical quantity that describes the degree of magnetic properties of a macroscopic magnetic body,  $\langle |m| \rangle = (1/N^2) \sum_{i=1}^{N^2} m_i$ ,  $m = (1/N^2) \sum_{i=1}^{N^2} \sigma_i$ . Magnetic susceptibility is a physical quantity that characterizes the properties of a magnetic medium,  $\chi/N^2 = (1/N^2)(\partial m/\partial T) = \langle (m - \langle |m| \rangle)^2 \rangle = \langle m^2 \rangle - \langle |m| \rangle^2$ . The “spontaneous magnetization” average  $m = (1/N^2) \sum_{i=1}^{N^2} \sigma_i$  as the order parameter [35]; among them  $\sigma_i \in \{+1, -1\}$ ;  $N \times N$  is the size of the network. Here, when  $\langle |m| \rangle$  is close to 1, the rumor propagation system is in an orderly state; when  $\langle |m| \rangle$  is 0, the system is in a completely disordered state. Therefore, we use Monte Carlo method to carry out numerical simulation, respectively, in the size of  $4 \times 4$ ,  $6 \times 6$ ,  $8 \times 8$ ,  $10 \times 10$ ,  $12 \times 12$ ,  $14 \times 14$ ,  $20 \times 20$ , and  $50 \times 50$  network to simulate; take the temperature interval  $\Delta T = 0.03$ ; the results are as shown in Figure 2.

It can be seen from Figure 2 that, as the scale  $N \times N$  of the system increases,  $T_c(N)$  will be close to the critical point  $T_c$  at which the true rumor propagation system intersects. Figure 1 shows us some change rules, but in order to obtain the exact critical point and the critical index, we assume that the critical index  $\nu = 1$  ( $\nu$  is expressed as a critical index in the rumor propagation system) fitted to the case, from which the critical point  $T_c = 2.3075, \pm 0.00852$  can be obtained.

*4.2. Von Neumann Entropy of Rumor Propagation System.* In order to study the critical temperature of the rumor propagation system and the types of phase transitions experienced by the system and some critical indices, we measured the von Neumann entropy of the system in a conventional way. von Neumann entropy formula is as follows:

$$\begin{aligned} S_p(T) &\equiv -tr_p \rho_p(T) \ln \rho_p(T) \\ &= 2 \ln 2 - \frac{1}{2} \left[ \gamma \ln \frac{1+\gamma}{1-\gamma} + \ln(1-\gamma^2) \right]. \end{aligned} \quad (15)$$

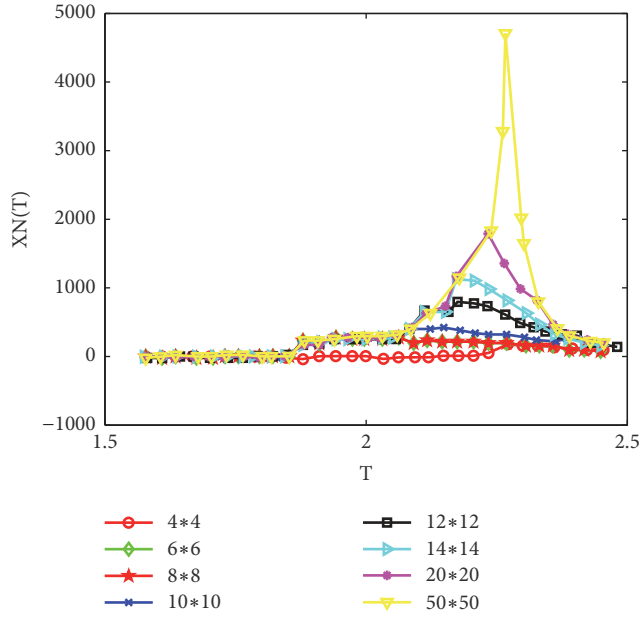


FIGURE 2: Functional images of magnetic susceptibility  $\chi_N(T)$  and temperature  $T$ .

Therefore, we performed simulations on  $15 \times 15$ ,  $20 \times 20$ ,  $30 \times 30$ ,  $35 \times 35$ ,  $50 \times 50$ ,  $80 \times 80$ , and  $100 \times 100$  networks, respectively; in the rumor propagation system, von Neumann entropy is measured, and the temperature interval of  $\Delta T = 0.03$ , according to finite-scale theory, and we got some information about the critical point; the results are shown in Figure 3.

From Figure 3, it can be easily found that the inflection point appears on the curve. This implies the existence of a critical point. Next, we observe the changes in these inflection points by deriving the curves.

From Figure 4, based on the finite-scale scale theory, we can determine that the critical temperature of the system is  $T_c = 2.30728$ .

**4.3. Comparison of Experimental Results.** Through the study of the Ising model of rumor propagation system from the aspects of measuring magnetization, magnetic susceptibility, and measuring von Neumann entropy, we get some conclusions, respectively.

(1) To study the magnetization and magnetic susceptibility, we obtain that the critical temperature of the system is  $T_c = 2.3075, \pm 0.00852$  and the critical length of the correlation length is  $\nu = 1$ .

(2) We study the von Neumann entropy of the rumor propagation system, and we obtain that the critical temperature of the system is  $T_c = 2.30728$  and the critical length of the correlation length is  $\nu = 1$ . The derivative of von Neumann entropy is logarithmically divergent at the critical point and reflects the secondary phase changing characteristics.

(3) Through comparative studies, von Neumann entropy of rumor propagation system can also be used to characterize the phase transition of user behavior decision in the system.

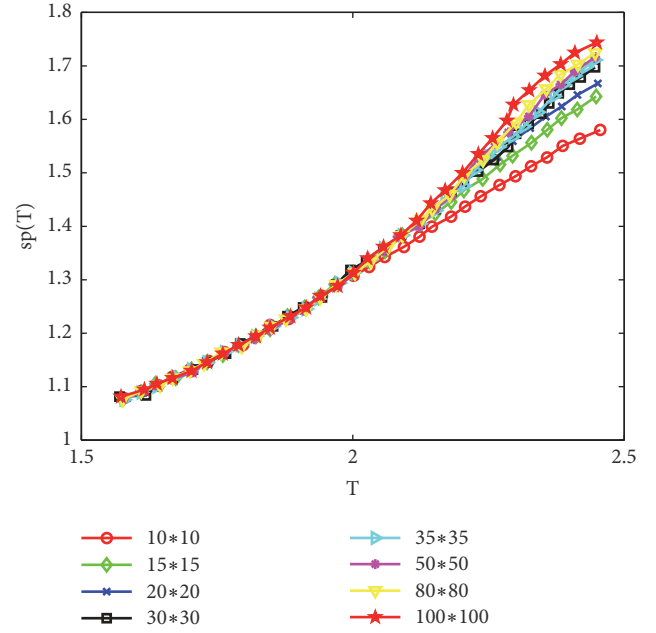


FIGURE 3: Variation of von Neumann entropy  $S_p(T)$  of different systems with temperature  $T$ .

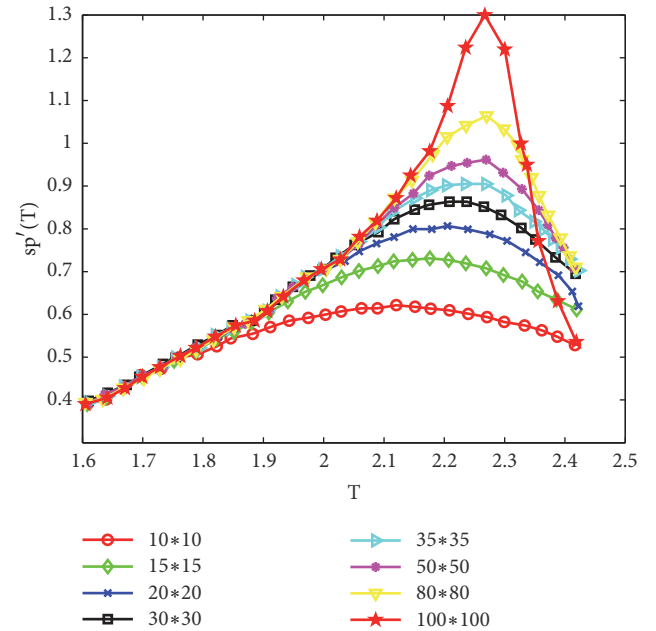


FIGURE 4: Variation of von Neumann entropy derivative  $S_p'(T)$  of different systems with temperature  $T$ .

## 5. Introducing Personal Self-Identity

The previous model ignores the individual differences in the system, such as differences in the self-identity characteristics of the population [36, 37]. Based on the Ising model, this paper introduces the self-identity into the rumor propagation dynamics. When this feature reaches a certain level of strength, the phase-shifting behavior of the user's rumor

decision in the rumor communication system changes from continuous to discontinuous.

**5.1. Construct Model.** In a homogeneous grid, each grid represents individual users in a social network, and each user has four neighbors individually. All user entities have one of two decision-making behaviors. Therefore, an individual's decision behavior can be described as one of two possible behaviors  $\sigma_i \in \{+1, -1\}$  at any moment. On the one hand, we introduce the function  $S(i) = 2\sigma_i \sum_{j=1}^4 \sigma_j$  to describe the relationship between the state of an individual  $i$  and the state of its companion  $j$  ( $j = 1, 2, 3, 4$ ). On the other hand, we use  $\varepsilon$  ( $0 < \varepsilon \leq 1$ ) to describe the individual's psychology of acceptance of the surrounding social environment, which allows the individual to follow the behavioral decisions of the majority of the neighbors. On the contrary,  $1 - \varepsilon$  describes the individual's self-identity psychology, which allows the individual to insist on his own behavior. The smaller the value of  $\varepsilon$  is, the more the individuals identifying with the self are, and the easier it is for individuals to adhere to their own behavioral decisions rather than following most of their neighbors' behavioral decisions.

Based on this, the following model is constructed as follows: when  $S(i) > 0$ ,  $\sigma_i$  is the same as most neighbors,  $\sigma_i$  changes its own behavior by the probability of  $\exp[-S(i)/T]$ , and  $T$  is a parameter of the class temperature associated with the internal fluctuation of the system; when  $S(i) < 0$ ,  $\sigma_i$  is contrary to the state of most neighbors, and  $\sigma_i$  changes its own state by the probability of  $\varepsilon$ ; when  $S(i) = 0$ , the number of states in the neighborhood is the same as and opposite to the state of the individual, so  $\sigma_i$  changes its own behavior decision according to its characteristics; that is,  $\sigma_i$  changes its own behavior decision with the probability of  $\varepsilon$ . From the dynamic evolution rules of the model, we find that, in the case of  $\varepsilon = 1$ , the model returns to the Ising model.

**5.2. Numerical Simulation Results.** Figure 5 shows that when the temperature is below the critical value, the final state of evolution of the rumor propagation system is ordered, and when the temperature is above the critical value, the final state of evolution of the system is disordered. As the degree of self-identity increases, the user behavior decision-making of the rumor communication system changes from an ordered state to an unordered state, from continuous phase change to discontinuous phase change, with  $\varepsilon_c$  being about 0.575. This shows that the individual's self-identity has greatly changed the dynamics of rumor transmission. As the level of self-identification increases, the need for lower temperatures will change the orderly status of the rumor transmission system and change more and more rapidly.

*Note.* In Figure 6(1), *a* is a graph of the probability density function of  $m$  in the case of  $\varepsilon = 0.8$ , where  $T$  is less than  $T_c$ . *b* is a graph of the probability density function of  $m$  in the case of  $\varepsilon = 0.8$ , where  $T$  is greater than  $T_c$ .

In Figure 6(2), *c* is a graph of the probability density function of  $m$  in the case of  $\varepsilon = 0.2$ , where  $T$  is less than  $T_c$ . *d*

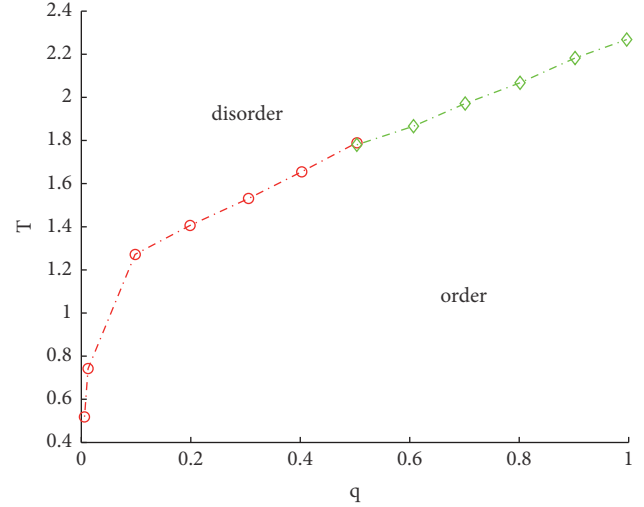


FIGURE 5: Phase diagram of model: discontinuous phase change occurs on the red representative system, and the continuous phase change occurs on the green representative system.  $q$  describes the individual's identity to the surrounding public opinion environment.

is a graph of the probability density function of  $m$  in the case of  $\varepsilon = 0.2$ , where  $T$  is greater than  $T_c$ .

In (1) and (2) of Figure 6, we show the variation of  $\langle |m| \rangle$  with temperature in the case of  $\varepsilon = 0.8$  and  $\varepsilon = 0.2$ , respectively. We find that the phase transitions at  $\varepsilon = 0.8$  are continuous and the phase transitions at  $\varepsilon = 0.2$  are discontinuous. The inset *a* and the inset *b* in Figure 6(1) are, respectively, the probability density functions of  $\langle |m| \rangle$  when less than  $T_c$  and greater than  $T_c$  in the case of  $\varepsilon = 0.8$ : when less than  $T_c$ , there are only two peaks on both sides of  $\langle |m| \rangle' = 0$ , but only one peak at  $\langle |m| \rangle' = 0$ . This is analogous to the continuous change of the ferromagnetic phase to the paramagnetic phase in a balanced phase transition. So when  $\varepsilon = 0.8$ , what happens to the rumor propagation system is a continuous phase change. The  $\langle |m| \rangle$  probability density function diagram in the case of illustration *c* and illustration *d* in Figure 6(2) in the case of  $\varepsilon = 0.2$  is less than  $T_c$  and greater than  $T_c$ , respectively: the most probable value of  $\langle |m| \rangle'$  in illustration *c* has two peaks of symmetry, and the most probable value of  $\langle |m| \rangle'$  in the illustration *d* has a peak. In the process of phase transition point  $T_c$ , the two peaks in the insertion of *c* become unstable, and, at  $T_c$ , the value of  $\langle |m| \rangle'$  rapidly changes to 0. Therefore, in the case of  $\varepsilon = 0.2$ , the system of rumor propagation is a discontinuous phase transition. Obviously, the characteristics of self-identity change the internal structure of rumor transmission system, which in turn changes the type of phase transition.

**5.3. Conclusion.** By introducing the self-identity of individuals into the nonequilibrium Ising model, the influence of individual self-identity on the dynamics of system rumor propagation is discussed. These studies have shown that the pervasive self-identity psychology of individuals in the social system has a significant impact on rumor propagation dynamics. The system presents a nonequilibrium phase transition from a state with a single public behavior decision

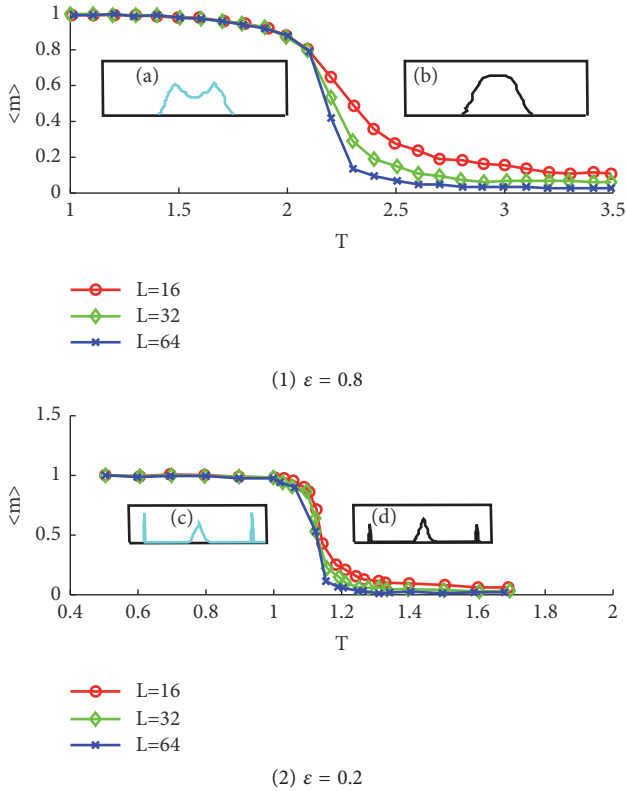


FIGURE 6: Order parameter changes with temperature (the illustration shows the distribution of probability density functions of order parameters).

to a state where a large number of behavioral decisions coexist. Moreover, with the strengthening of self-identity, the phase transition of the user decision in the rumor propagation system changes from the continuous phase to the discontinuous phase.

## 6. Conclusion and Outlook

**6.1. Conclusion.** Based on the Ising model, this paper combines the characteristics of the individual's self-identity (the micropart), the interaction between a user and another (the middle part), and the influence of the social environment (the macroscopic part) and then applies Monte Carlo simulation method to study the social network system rumor spreading user decision-making behavior, revealing that the law of rumor transmission has important theoretical significance and application value.

The results show that after the von Neumann entropy concept is used to study the phase transitions and the critical phenomena in the classical thermodynamic system, we use the Ising model to obtain the critical information of the rumor propagation system by measuring the von Neumann entropy of the system. The von Neumann entropy of the rumor propagation system can quantify well the phase transition of the system and is consistent with the phase change information obtained by measuring the spontaneous magnetization and the susceptibility of the system. The

successful application of von Neumann entropy in rumor propagation system proves that von Neumann entropy can indeed be used to quantify the phase transition of rumor propagation. This provides a good way to quantify the phase change of rumor communication. In addition, the study also shows that the introduction of self-identity characteristics of individual users into the Ising model has greatly changed the user decision-making behavior of rumor transmission and changed the internal structure of the system, thereby changing the type of phase transition. As the level of self-identification increases, the need for lower temperatures will change the orderly state of the system and the transition will change more rapidly. For the spread of rumors, the close order of the state is conducive to social stability.

**6.2. Outlook.** (1) Based on the Ising model, this paper only considers the feature of the user's personal self-identity but does not consider the positive and negative social reinforcement of the social environment. Double social enhancement plays a role in the spread of the rumor, and rumors vary in their attractiveness to users. The next step of research will be further combined with the differences in the ability of users to explore the impact of user influence on the spread of rumors. Through further analysis of rumors, we can find out its propagation rule, effectively reduce the final spread scale of rumors, weaken the biggest impact of rumors, and slow down the speed of rumor spreading and provide more time for relevant departments to take measures to control rumor spread.

(2) The user behavior decision model studied in this paper is individualized; that is, each node in the network represents each user, so it is not suitable for large-scale complex networks. In order to study the complex dynamic behavior of large-scale networks and their relationship with local structures, the next step is to use the theory and method of multiscale coarse-graining to look at multiple users as a node in the network. We merge nodes with similar strengths (i.e., user's friend nodes) into one coarse-grained node, because in weighted networks it is impractical to combine nodes of exactly the same strength. On the basis of modeling individual user behavior decision based on the Ising model, it is hoped that the coarse granulation method based on node strength merging can be proposed to make it suitable for large-scale complex networks. It is more widely used.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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