# Isobaric multiplet mass equation for A=7 and 8

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# Isobaric multiplet mass equation for $A$=7 and 8.

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Deviations from the isobaric multiplet mass equation are presented and discussed for the $A$=7, $T$=3/2 quartet and the $A$=8, $T$=2 quintet.

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## I. INTRODUCTION

The isobaric multiplet mass equation (IMME) relates the mass excesses of the members of an isobaric multiplet in terms of their isospin projection $T_Z$. To the extent that isospin is a good quantum number, the energies of the multiplet should be independent of $T_Z$ in the absence of Coulomb forces. If two-body forces are responsible for charge-dependent effects, Wigner found

$$M(T, T_Z) = a + b\,T_Z + c\,T_Z^2. \tag{1}$$

Typically cubic and quartic terms ($d\,T_Z^3 + e\,T_Z^4$) are added to the IMME to provide a measure of any deviation from the quadratic form associated with isospin symmetry. In general the quadratic form ($d$=0 and $e$=0) provides a good description of isospin quartets and quintets for $A$ < 40 where the appropriate experimental masses are known [1]. The success of IMME has lead people to use it to predict the mass where no measurement is available and thus it is important to understand the magnitude of deviations when they occur.

Experimentally the $A$=9 ($T$=3/2) quartet is well documented as violating the IMME [1, 2]. A purely quadratic fit gives a $\chi^2/n$=10.2 and one requires $d$=5.5±1.8 keV to fit the multiplet. The use of more recent data from the AME2011 [3] and the ENSDF [4] databases gives an enhanced values of $\chi^2/n$=15.3 and a consistent value of $d$=6.3±1.6 keV.

For heavier systems, there is good evidence of a violation for the $A$=32 ($T$=2) quintet [5–7] which requires a small but statistical significant cubic term ($d$ ∼1 keV) to fit the multiplet. This small $d$ term can be explained from isospin mixing with $T$=1 states [8]. A deviation for the $A$=33 quartet has also been reported with $d$=-2.95±0.90 keV [9].

While a quantitative understanding of deviations to the IMME for the lighter systems has not been achieved, such deviations can be expected from either isospin mixing or from the expansion of the single-particle wavefunctions near threshold [2]. The $A$=9 quartet has two members which are particle unbound. If continuum effects were the only isospin symmetry breaking mechanism, then one would expect larger deviations for the multiplet associated with the first excited state where all levels are ∼2.3 MeV less bound. However, the cubic coefficient in this case ($d$=3.5±3.4 keV) is less than the ground-state value. In this work, we will investigate the IMME for two other multiplets with particle-unbound members: the $A$=8 quintet and the $A$=7 quartet. In both of these cases, the proton rich members have negative binding energies and so the single-particle wavefunctions change considerable across the respective multiplets.

In the 1998 systematic study of isospin multiplets in Ref. [1], the $A$=8 quintet was also found to deviate from the quadratic IMME with $\chi^2/n$=15.5. However in this 1998 study, an accidental error led to the use of an uncertainty for the $^8$Li$_{IAS}$ mass which is much smaller than the correct experimental value [10]. In view of this and new mass measurements since 1998, it is useful to reevaluate the IMME for $A$=8. In the present work, we incorporate the new mass measurement for $^8$C$_{g.s.}$ determined from the invariant mass of its five decay products (4$p$+$\alpha$) [11]. In the same experimental study, a new measurement was also made for $^7$B$_{g.s.}$ allowing us to also reexamine the $A$=7 quartet. For the $^7$B case the mass excess was determined from the invariant mass of its four decay products (3$p$+$\alpha$). The accuracy of both of these measurements can

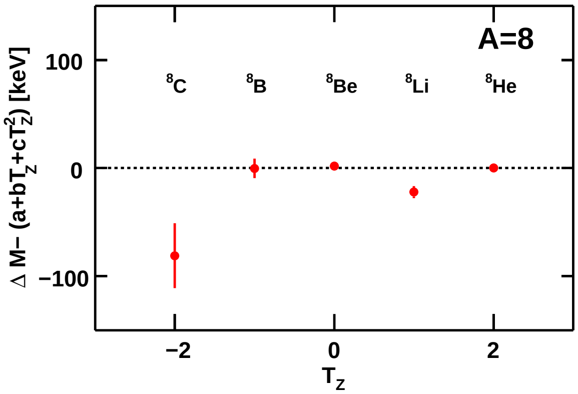


FIG. 1. (Color online) Deviation from the fitted quadratic form of the IMME for the $A$=8 quintet.

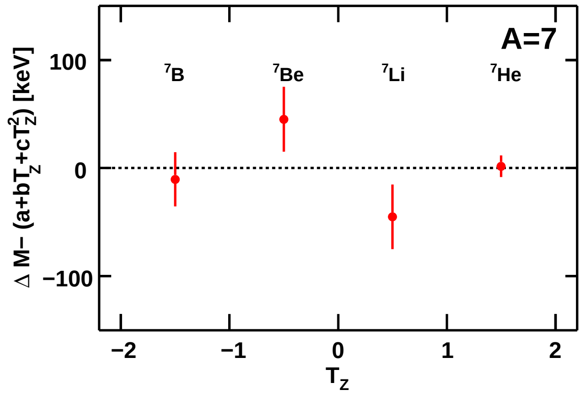


FIG. 2. (Color online) Deviation from the fitted quadratic form of the IMME for the $A$=7 quartet.

be judged from the excellent reproduction of the mass excess of $^6$Be associated with the $2p+\alpha$ channel. These new mass measurements are significantly smaller than the previous values listed in the AME2011 data base, by 64 and 190 keV for the $^8$C and $^7$B nuclei respectively. In addition to these data from invariant mass determinations, we have also included a very recent, highly accurate, mass measurement of $^8$He [12] and the average of two recent measurements of the $^7$He$_{g.s.}$ mass [13, 14]. Apart from these masses, the remaining mass are identical to those used in the 1998 study. The experimental mass excesses for the $A$=8 quintet and the $A$=7 quartet are summarized in Tables I and II.

The results of fits to these data as also listed in these Tables and the residuals from quadratic fits are plotted in Figs. 1 and 2. For $A$=8 case, the quadratic fit does not reproduce the data and gives a $\chi^2/n$=12.3. The statistical probability that this is consistent with a quadratic fit is $\sim10^{-5}$. To fit this data, one requires a cubic term with $d$=11.1±2.3 keV. The addition of a quartic term does not improve the fit to any significant extent (see Table I). The magnitude of $d$ is about twice as large as that for the $A$=9 quartet indicating that the size of the deviation from the IMME is larger. If the old mass excess from the 1998 review is used for $^8$C, then we obtain $\chi^2/n$=8.5 and $d$ = 7.7±2.0 keV.

For the $A$=7 quartet, the quadratic fit gives $\chi^2/n$=4.7 and the significance of the deviation is not as strong as for the $A$=8. However, if the mass dependence is truly quadratic, the statistical probability of finding a $\chi^2/n$ equal or greater than this value is only 3%. The cubic coefficient required to fit the $A$=7 data is $d$=47 (22) keV, which is very large, however the error is also large.

Although the possible deviation for the $A$=7 quartet is large, there are uncertainties in applying the IMME as both the $^7$B$_{g.s.}$ and $^7$He$_{g.s.}$ line shapes are asymmetric [11, 13, 14] and the $^7$B width is quite wide $\Gamma$=801 keV [11]. It is not clear what characteristic mass associated with these distributions should be used in the IMME. Some theoretical guidance on this isse would be useful. The values listed in Table II are the resonance energies obtained from $R$-matrix fits [15] with the background term chosen such that the shift term $\Delta$ is zero at the resonance energy.

In summary, we demonstrate that the $A$=7 quartet and the $A$=8 quintet show significant deviations from quadratic isobaric multiplet mass equation. The case for the $A$=8 is particularly strong. Large cubic coefficients, $d$=11.1±2.3 ($A$=8) and 47±22 keV ($A$=7), are required to fit the experimental masses. Together with the $A$=9 quartet where deviations from the IMME are well known, these multiplets contain particle-unstable members suggesting that this may be an important ingredient in understanding the deviations.

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TABLE I. Mass excesses for the $A$=8 isospin $T$=2 quintet and the coefficients obtained from quadric, cubic, and quartic fits.

| Nucl. | $T_Z$ | Mass Excess [keV] | $a, b, c$ [keV] | $a, b, c, d$ [keV] | $a, b, c, d, e$ [keV] |
|---|---|---|---|---|---|
| He | 2 | 31609.7(1) | $a$=32433.9 (17) | $a$=32435.5 (17) | $a$=32435.7 (18) |
| Li | 1 | 31768.0(55) | $b$=-875.3 (43) | $b$=-898.1 (63) | $b$=-896.6 (75) |
| Be | 0 | 32435.7 (18) | $c$=231.6 (22) | $c$=220.3 (32) | $c$=217.7 (75) |
| B | -1 | 33540.5 (90) | $\chi^2/n$=12.3 | $d$=11.1 (2.3) | $d$=10.4 (3.1) |
| C | -2 | 35030 (30) | | $\chi^2/n$=0.1 | $e$=0.8 (2.2) |

TABLE II. Mass excesses for the $A$=7 isospin $T$=3/2 quartet and the coefficients obtained from quadric and cubic fits.

| Nucl. | $T_Z$ | Mass Excess [keV] | $a, b, c$ [keV] | $a, b, c, d$ [keV] |
|---|---|---|---|---|
| He | 3/2 | 26506 (10) | $a$=26411 (24) | $a$=26412 (24) |
| Li | 1/2 | 26148 (30) | $b$=-540 (9) | $b$=-642 (48) |
| Be | -1/2 | 26779 (30) | $c$=206 (13) | $c$=204 (13) |
| B | -3/2 | 27677 (25) | $\chi^2/n$=4.7 | $d$=47 (22) |

[1] J. Britz, A. Pape, and M. S. Antony, At. Data Nucl. Data Tables **69**, 125 (1998).
[2] W. Benenson and E. Kashy, Rev. Mod. Phys. **51**, 527 (1979).
[3] G. Audi and W. Meng, (2011), private communication.
[4] Evaluated Nuclear Structure Data File (ENSDF), http://www.nndc.bnl.gov/ensdf/.
[5] A. A. Kwiatkowski, B. R. Barquest, G. Bollen, C. M. Campbell, D. L. Lincoln, D. J. Morrissey, G. K. Pang, A. M. Prinke, J. Savory, S. Schwarz, C. M. Folden, D. Melconian, S. K. L. Sjue, and M. Block, Phys. Rev. C **80**, 051302 (2009).
[6] A. Kankainen, T. Eronen, D. Gorelov, J. Hakala, A. Jokinen, V. S. Kolhinen, M. Reponen, J. Rissanen, A. Saastamoinen, V. Sonnenschein, and J. Äystö, Phys. Rev. C **82**, 052501 (2010).
[7] C. Wrede, J. A. Clark, C. M. Deibel, T. Faestermann, R. Hertenberger, A. Parikh, H.-F. Wirth, S. Bishop, A. A. Chen, K. Eppinger, A. García, R. Krücken, O. Lepyoshkina, G. Rugel, and K. Setoodehnia, Phys. Rev. C **81**, 055503 (2010).
[8] A. Signoracci and B. A. Brown, Phys. Rev. C **84**, 031301 (2011).
[9] F. Herfurth, J. Dilling, A. Kellerbauer, G. Audi, D. Beck, G. Bollen, H.-J. Kluge, D. Lunney, R. B. Moore, C. Scheidenberger, S. Schwarz, G. Sikler, J. Szerypo, and I. Collaboration, Phys. Rev. Lett. **87**, 142501 (2001).
[10] J. Britz, A. Pape, and M. S. Antony, (2011), private communication.
[11] R. J. Charity, J. M. Elson, J. Manfredi, R. Shane, L. G. Sobotka, B. A. Brown, Z. Chajecki, D. Coupland, H. Iwasaki, M. Kilburn, J. Lee, W. G. Lynch, A. Sanetullaev, M. B. Tsang, J. Winkelbauer, M. Youngs, S. T. Marley, D. V. Shetty, A. H. Wuosmaa, T. K. Ghosh, and M. E. Howard, Phys. Rev. C **84**, 014320 (2011).
[12] M. Brodeur, T. Brunner, C. Champagne, S. Ettenauer, M. J. Smith, A. Lapierre, R. Ringle, V. L. Ryjkov, S. Bacca, P. Delheij, G. W. F. Drake, D. Lunney, A. Schwenk, and J. Dilling, (2011), arXiv:nucl-ex/1107.1684v1.
[13] D. H. Denby, P. A. DeYoung, T. Baumann, D. Bazin, E. Breitbach, J. Brown, N. Frank, A. Gade, C. C. Hall, J. Hinnefeld, C. R. Hoffman, R. Howes, R. A. Jenson, B. Luther, S. M. Mosby, C. W. Olson, W. A. Peters, A. Schiller, A. Spyrou, and M. Thoennessen, Phys. Rev. C **78**, 044303 (2008).
[14] Y. Aksyutina, H. Johansson, T. Aumann, K. Boretzky, M. Borge, A. Chatillon, L. Chulkov, D. Cortina-Gil, U. D. Pramanik, H. Emling, C. Forssén, H. Fynbo, H. Geissel, G. Ickert, B. Jonson, R. Kulessa, C. Langer, M. Lantz, T. LeBleis, A. Lindahl, K. Mahata, M. Meister, G. Münzenberg, T. Nilsson, G. Nyman, R. Palit, S. Paschalis, W. Prokopowicz, R. Reifarth, A. Richter, K. Riisager, G. Schrieder, H. Simon, K. Sümmerer, O. Tengblad, H. Weick, and M. Zhukov, Physics Letters B **679**, 191 (2009).
[15] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).