

ISRM Suggested Method for Determining the Mode I Static Fracture Toughness Using Semi-Circular Bend Specimen

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Abstract

The International Society for Rock Mechanics has so far developed two standard methods for the determination of static fracture toughness of rock. They used three different core based specimens and tests were to be performed on a typical laboratory compression or tension load frame. Another method to determine the mode I fracture toughness of rock using semicircular bend specimen is herein presented. The specimen is semicircular in shape and made from typical cores taken from the rock with any relative material directions noted. The specimens are tested in three-point bending using a laboratory compression test instrument. The failure load along with its dimensions is used to determine the fracture toughness. Most sedimentary rocks which are layered in structure may exhibit fracture properties that depend on the orientation and therefore measurements in more than one material direction may be necessary. The fracture toughness measurements are expected to yield a size-independent material property if certain minimum specimen size requirements are satisfied.

Keywords Rock fracture mechanics, Mode I fracture toughness, Semicircular bend specimen, Fracture testing, Sedimentary rock, In-situ environment

1 Introduction

Rock fracture mechanics can be used to identify and predict the imminent failure of rock mass structures thereby providing guidelines to improve the stability and the safety of these structures. Another application is for the exploitation of mineral resources by adopting

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1 techniques such as mechanical mining, blasting and hydraulic fracturing. In fracture
2 processes which are not associated with high strain rates, the Mode I plane strain static
3 fracture toughness gives the critical value of the stress intensity factor leading to the onset of
4 crack growth in that mode (Liu 1983). Some of the applications of fracture toughness include
5 index of fragmentation processes like those used in tunnel boring, a modelling parameter in
6 processes such as rock cutting and hydraulic fracturing and for the stability analysis of civil,
7 mining and earthen structures (Whittaker et al. 1992).
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10 A number of standard methods have been proposed to determine the Mode I fracture
11 toughness of rock. They include those based on short rod (SR) specimen, chevron bend (CB)
12 specimen and cracked chevron-notched Brazilian disk (CCNBD) specimen (ISRM 2007).
13 The semi-circular bend (SCB) specimen has been widely used for fracture toughness
14 determination of geomaterials owing to inherent favourable properties such as its simplicity,
15 minimal requirement of machining and the convenience of testing that can be accomplished
16 by applying 3-point compressive loading using a common laboratory load frame (Chong and
17 Kuruppu 1984; Chong et al. 1987; Lim et al. 1993, 1994; Ayatollahi and Aliha 2007; Aliha et
18 al. 2012; Karfakis and Akram 1993; Obara et al. 2006, 2007a, 2007b, 2009; Molenaar et al.
19 2002). As geomaterials are weak in tension, fracture tests should preferably be conducted
20 with compressive loading in such a way that tensile fractures are induced. The CB and
21 CCNBD specimens used for the standard methods as well as the SCB specimen satisfy those
22 requirements. Use of more than one type of specimen is regarded as appropriate when it is
23 required to measure the fracture toughness of anisotropic materials in different material
24 directions of a rock sample (Chong et al. 1987). An ISRM Suggested Method for Mode I
25 static fracture toughness determination of rock and other geomaterials using SCB specimen is
26 herein presented.
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37 **2 Scope**

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40 This test method is intended to measure the Mode I static fracture toughness K_{Ic} under slow
41 and steady loading where dynamic effects are negligible. However, another suggested
42 method developed by the ISRM should be followed if the loading rate is high, as in the case
43 of explosive fragmentation of rock (Zhou et al. 2012). The geometry of the test specimen is
44 designed to use standard core material. A minimum specimen diameter D_{min} is suggested to
45 be used in order to satisfy the minimum size requirement as explained in section 7. If the rock
46 material is known to be anisotropic, the core axis should be oriented either parallel or
47 perpendicular to any anisotropic feature, such as a bedding plane. If required, the remaining
48 material from Mode I fracture toughness tests performed using CB and SR methods can be
49 used to find fracture toughness in orthogonal directions (Chong et al. 1987). For example, for
50 sedimentary rocks that exhibit transversely isotropic material properties, a combination of
51 tests performed using SR, CB, CCNBD and/or SCB specimen with cores taken perpendicular
52 and parallel to bedding planes, will give the complete information of fracture toughness.
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Alternatively, SCB specimens themselves can be made from cores such that the notch directions are either aligned or perpendicular to the bedding planes.

The advantages of using the SCB specimens are (a) material requirement per specimen is small (b) machining is relatively simple and (c) only the maximum compressive load is required to determine the fracture toughness.

3 Specimen Preparation

3.1 The circular disks required to make the SCB specimen are prepared by sawing or slicing standard rock cores using a high precision diamond tool. The geometry of the SCB specimen is shown in Figure 1. The specimen diameter ($D = 2R$) should be related to the average grain size in the rock by a ratio of at least 10:1 or should be at least 76 mm and the minimum specimen thickness shall be the larger of $0.4D$ or 30 mm. Caution should be exercised to minimise the micromechanical damage of the specimens as it can affect the fracture toughness. Water or other coolant should be used while machining, in order to avoid heat damage that can alter the fracture toughness. Slow drilling is recommended in case that the cores are drilled from a large rock sample.

3.2 Each of the circular disks should be sawn into two halves which may be carried out using the same cutting tool used to make the circular disks. The final operation of introducing a notch should be performed using a thin cutting blade of thickness $\leq 1.5 \pm 0.2$ mm, or preferably, using a diamond-impregnated fine wire saw that will produce a straight notch of the required length. The radius of the notch tip should be less than the average grain size of the rock material. The notch length should be such that $0.4 \leq a/R \leq 0.6$.

3.3 The plane surface along the thickness direction should be flat to 0.01 mm. The plane of the notch shall not depart from perpendicularity to the plane surface in the thickness direction by more than 0.5° . The dimensions of the test specimen are given in Table 1.

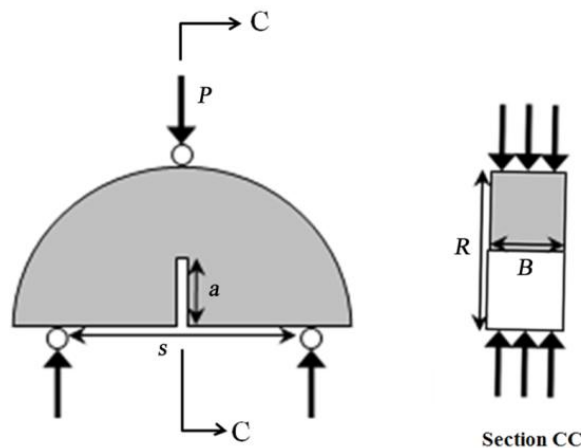


Fig. 1 SCB specimen geometry and schematic loading arrangement (R : radius of the specimen; B : thickness; a : notch length; s : distance between the two supporting cylindrical rollers; P : monotonically increasing compressive load applied at the central loading roller of the 3-point bend loading)

Table 1 Recommended geometrical dimensions of SCB specimen (see Fig. 1)

Descriptions	Values or range
Diameter, D	Larger of 10 x grain size or 76 mm
Thickness, B	Larger of 0.4D or 30 mm
Crack length, a	$0.4 \leq \frac{a}{R} (= \beta) \leq 0.6$
Span length, s	$0.5 \leq \frac{s}{2R} \leq 0.8$

3.4 When slicing a core, the plane of the resulting disks should not deviate from the perpendicularity to the core axis by more than 0.5°.

3.5 When cutting a disk into two halves to form two semi-circular disks, care must be given not to deviate the cutting plane from a diametral plane by more than 0.2 mm. Also, the perpendicularity to the plane of the disk should be assured to be within 0.5°.

3.6 The specimens must be marked with a reference that gives the details of its orientation with respect to any directions of material anisotropy (e.g. inclination of the notch plane to bedding planes). Specimens of the same sample should have identical notch orientation.

3.7 The notch length should be measured as an average taken on both the semi-circular planar surfaces which are perpendicular to the core axis. The two readings should be within 2% of each other.

3.8 The thickness should be uniform and shall not deviate by more than 0.2 mm.

3.9 The dimensions of the specimens should be measured to the nearest 0.1 mm. The required dimensions are the radius R , the thickness B and the notch length a .

3.10 If the thickness of the saw blade used to cut the disks into semi-circular specimens is greater than 0.05 D , where D is the disk diameter, then the values of the measured radius R' and the measured notch length a' should be corrected as shown in Figure 2 (i.e. the corrected radius $R = R' + \Delta r$ and corrected notch length $a = a' + \Delta r$). Note that the radius measurement shall be taken aligned with the notch direction.

3.11 The specimen should be stored after specimen preparation for an appropriate period of time that is sufficient to achieve the desired conditions (e.g. moisture content). The conditions of storage, moisture adjustment or drying, as well as any macroscopically noticeable features of the specimen surfaces, shall be reported with fracture toughness results.

3.12 The tensile strength of the material should be known (or measured) (ISRM 2007).

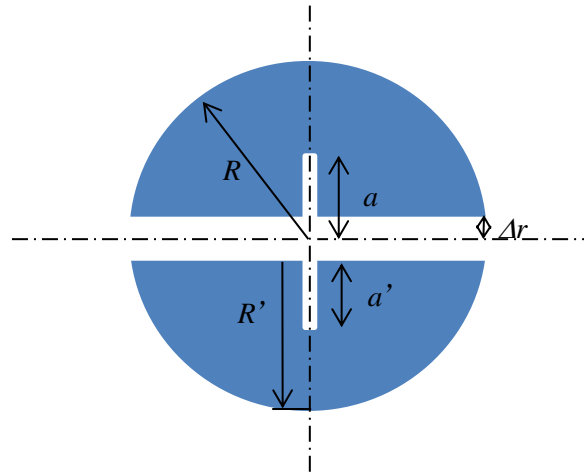


Fig. 2 Correction for a , R when the thickness of the saw blade is not negligible (i.e. $2\Delta r \geq 0.05D$, where Δr is the half thickness of the saw blade used for cutting)

4 Experimental Set-up

4.1 The test should be performed using a standard compressive or universal test frame commonly available in most rock mechanics laboratories. While a servo-hydraulic test system is preferable, a mechanically-driven compressive testing machine may be adequate if the capacity and the precision of the load measurement is as given in section 4.5 below. The load frame should be equipped with a system to record the load, the axial displacement and any other measuring signal of interest.

4.2 The load application is performed via a conventional three-point bend fixture. The specimen is to be placed on the two bottom loading cylindrical rollers which are kept apart at a pre-determined distance commensurate with the size of the specimen as shown in Figure 3. The rollers shall be placed on the bottom loading plate so that they can rotate and move apart slightly when the specimen is loaded, thus permitting roller contact at supports offering no frictional resistance. A suitable span length should be selected within the range of span (s) to diameter ratio (D), s/D of $0.5 \leq s/D \leq 0.8$. The parallel positioning of the two bottom support rollers should be ensured. It may help to mark the positions of the two bottom support rollers on either side of the semi-circular faces of the specimen prior to its positioning on the support rollers. These positions should be drawn symmetrical to the plane of the notch. A top loading cylindrical roller is attached to the top loading plate so that the load application occurs symmetrically between the two bottom support rollers. A suitable recess made on the top loading plate may be required to hold the roller in position.

4.3 The diameter of the rollers should be chosen in relation to the specimen diameter. A ratio of 1:20 is recommended (i.e. 10 mm diameter rollers are used for testing 200 mm diameter

specimens). However, the minimum diameter of the rollers used for testing specimens less than 100 mm diameter should be 5 mm.

4.4 Alignment of the notch plane with the loading direction should be carefully controlled.

4.5 The load frame should be equipped with a load cell of having a resolution of 0.01kN or greater. A linear variable displacement transducer (LVDT) set up between the top and bottom loading roller positions is the preferred arrangement for measuring the displacement. Crack opening displacement measurement by a clip gauge is also useful (Karfakis and Akram 1993). A successful test is usually associated with a monotonically increasing and continuous load-displacement graph. Moreover, the load versus displacement behaviour reveals the degree of nonlinearity of the rock material.

4.6 If testing is required to be performed at conditions other than the ambient, then the specimen may be kept inside an environment chamber that will provide those conditions. For example, moisture content measured by water vapour pressure may be set at a predetermined level and maintained until reaching saturation under that condition (Obara et al. 2010). Temperature may be set at a predetermined level and maintained until the specimen is heated uniformly (Funatsu et al. 2004; Kuruppu and Seto 2001). They may be controlled independently from the axial load application that would perform the 3-point bend loading of the specimen.



Fig. 3 SCB specimen loading fixture

5 Testing Procedure

5.1 The minimum data required during testing is the peak load P_{max} and any other environmental conditions, if applicable. However, a continuous measurement of the load and

the displacement between the top and bottom loading roller positions during the test is recommended in order to verify that the load has increased continuously with increasing displacement until reaching the point of fracture as shown in Figure 4 (Kataoka et al. 2010, 2011). It is appropriate to gather data at a rate of 4 data sets per second if digital data acquisition is used.

5.2 The testing should be done at a constant displacement rate of not greater than 0.2 mm/min to avoid any dynamic effect (Backers and Stephansson 2012; Khan and Al-Shayea 2000). Data acquisition should begin prior to closing the gap between the specimen and the top loading roller and continue until the specimen fails completely.

5.3 When the gap is closed and a small load is applied, the test may be stopped momentarily to check the alignment of support/loading rollers and that they are in touch with the specimen along the entire thickness of the specimen.

5.4 After the test is completed, the two parts of the broken specimen should be kept for further observation of failure mode. The results shall be considered invalid if the plane of the cracked ligament deviates from the notch plane by more than $0.05D$.

5.5 The number of specimens tested per sample should be determined by practical considerations. A minimum of 5 specimens are recommended. All specimens of the sample ought to be tested subjected to the same conditions.

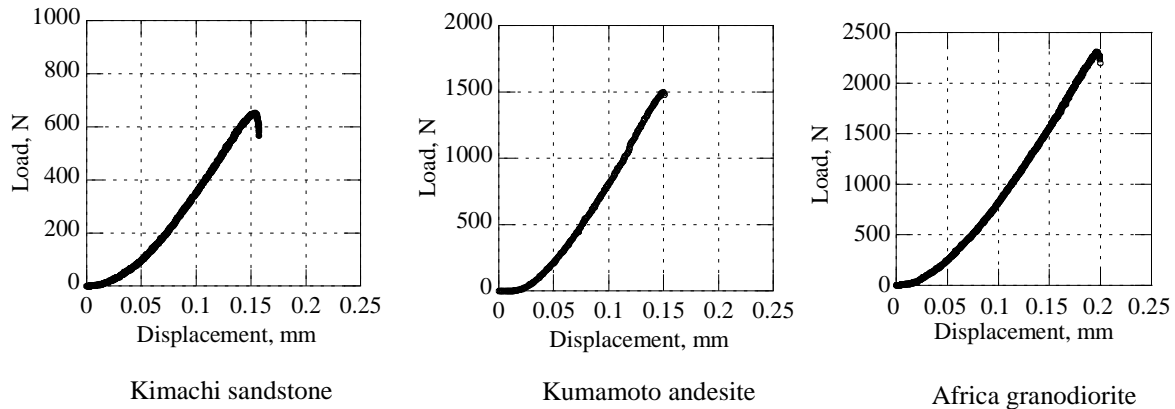


Fig. 4 Typical load versus displacement plots showing the critical fracture point

6 Calculations

6.1 Mode I fracture toughness K_{Ic} shall be determined using the observed peak load P_{max} such that:

$$K_{Ic} = Y' \frac{P_{max} \sqrt{\pi a}}{2RB} \quad (1)$$

where,

$$Y' = -1.297 + 9.516(s/2R) - (0.47 + 16.457(s/2R))\beta + (1.071 + 34.401(s/2R))\beta^2 \quad (2)$$

and $\beta = a/R$. Equation (2) gives the non-dimensional stress intensity factor Y' derived using the finite element method while assuming plane strain conditions. Further details about the numerical analyses performed for deriving Eq. (2) can be found in the Appendix. Table 2 gives some of the values calculated for Y' . Eq. (2) is valid for $\beta \geq 0.2$. However, a relatively deep notch is required for the bending effect to produce a strong mode I stress field near the tip of the notch. Hence a normalised length β in the range $0.4 \leq \beta \leq 0.6$ is recommended to be used.

Table 2 Non-dimensional stress intensity factor Y'

$s/2R$	$\beta = 0.4$	$\beta = 0.5$	$\beta = 0.6$
0.5	2.905	3.679	4.819
0.6	3.748	4.668	6.022
0.7	4.592	5.657	7.224
0.8	5.436	6.645	8.427

6.2 The suggested range of $s/2R$ is $0.5 \leq s/2R \leq 0.8$. For testing strong materials, it is preferable to use a value approaching 0.8. However, this may not be practical for specimens made of weak geomaterials in which case a value at the lower end of the range should be used.

6.3 Some of the previously published suggested methods are meant to determine 2 levels of fracture toughness of rock. Level I is based on the maximum failure load and level II further incorporates a non-linearity correction to take any non-linear material behaviour into account. However, this suggested method only addresses level I fracture toughness.

6.4 As described in section 2 the SCB specimen can be used to determine the fracture toughness of sedimentary rock in which major planes of anisotropy can be found. Those rocks mostly exhibit transversely isotropic properties. For complete characterization, specimens with their notches aligned in 3 mutually perpendicular directions should be tested (Fig. 5). One possibility is to use 3 sets of SCB specimens having the notches oriented in each of the arrester, divider and short transverse directions. However, it may be more practical to use a combination of fracture toughness test specimens. While it is up to the user to decide which combination of specimens to be employed, one combination is to use the straight edge cracked round bar in bending (SECRBB), SCB and centrally cracked Brazilian disc (CCBD) specimens made with their notches aligned to form divider, arrester and short transverse configurations respectively. These specimens must be made with cores taken in the direction of bedding planes.

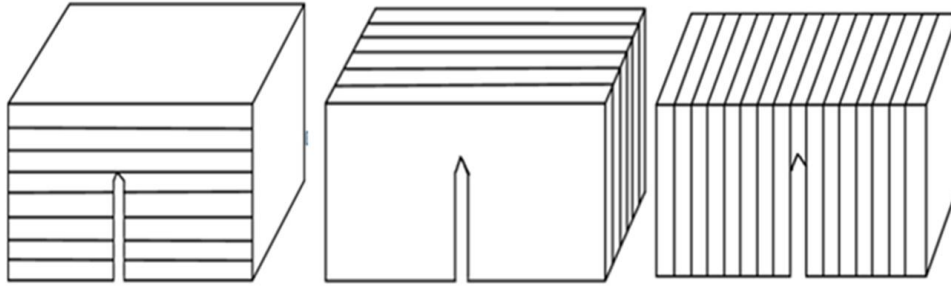


Fig. 5 Principal crack orientations with respect to bedding planes (left to right: arrester, divider and, short transverse configurations)

7 Size Effects

As geomaterials tend to form relatively large process zones prior to fracture, certain minimum specimen sizes need to be satisfied to achieve the requirements for linear elastic fracture mechanics, according to which the concept of K_{Ic} is defined. The process zone is largely affected by the grain size of the material; those consisting of relatively small grains have small size requirements and vice versa (Ouchterlony 1990; Kuruppu and Chong 2012; Bazant 1984).

7.1 Chong et al. (1987) suggested the following size requirement for the SCB specimen:

$$D \geq 2.0 \left(\frac{K_{Ic}}{\sigma_t} \right)^2 \quad (3)$$

where σ_t is the tensile strength of the material. However, this may be a conservative estimate as the size requirements applicable for chevron notched CB and SR specimens are much lower (Ouchterlony 1989). No definitive size requirement can be given for SCB specimens. One way to determine the size requirement for a particular material is by comparing K_{Ic} values deduced using a number of specimens of different diameters D . The smallest specimen diameter D_{min} , that generates K_{Ic} value consistent with larger diameter specimens, will be the minimum size required to give a valid fracture toughness test.

8 Reporting of Results

The report should include the following:

- 8.1 Source of specimens as precisely as possible (e.g. material, location, date and orientation).
- 8.2 Lithological description of the rock type including grain size.

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- 8.3 Dimensions of the specimens. Any particular observations about macroscopic appearance of the specimen surface.
 - 8.4 If applicable, the orientation of the notch with respect to the specimen anisotropy (e.g. direction of bedding planes etc.).
 - 8.5 History and environment of test specimen storage (e.g. temperature, water vapour pressure).
 - 8.6 Conditions at the time of test (e.g. temperature, water vapour pressure).
 - 8.7 Details of the test equipment and test procedure used, particularly if the method employed deviated from the suggested method and the reasons for such deviation.
 - 8.8 Record of all signals measured, loading rate and any other relevant parameters not included in this list.
 - 8.9 The calculated value of fracture toughness of each specimen.
 - 8.10 Description of the broken specimens after testing. If there are fractures other than the near-symmetric split of the specimens then the results of those specimens will not be valid.
 - 8.11 The average value of Mode I fracture toughness of each sample disregarding any invalid results. Statement of any associated environmental conditions.

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- Abaqus Unified FEA (2012) Dassault Systems.
- Ayatollahi MR, Aliha MRM (2007) Wide range data for crack tip parameters in two disc-type specimens under mixed mode loading. *Computational Materials Sci* 38:660-670
- Aliha MRM, Sistaninia M, Smith DJ, Pavier MJ, Ayatollahi MR (2012) Geometry effects and statistical analysis of mode I fracture in guiting limestone. *Int J Rock Mech Min Sci* 51:128-135
- Backers T, Stephansson O (2012) ISRM suggested method for the determination of Mode II fracture toughness. *Rock Mech Rock Engng* 45:1011-1022
- Basham KD (1989) Nonlinear fracture mechanics using semi-circular specimens and tension softening behaviour. PhD Dissertation, Dept of Civil Eng, The University of Wyoming
- Bazant ZP (1984) Size effect in blunt fracture: concrete, rock, metal. *J Engng Mech Div ASCE* 110:518-535
- Chong KP, Kuruppu MD (1984) New specimen for fracture toughness determination of rock and other materials. *Int J Fract* 26:R59-R62
- Chong KP, Kuruppu MD, Kuszmaul JS (1987) Fracture toughness determination of layered materials. *Eng Fract Mech* 28:43-54
- Funatsu T, Seto M, Shimada H, Matsui K, Kuruppu M (2004) Combined effects of increasing temperature and pressure on the fracture toughness of clay bearing rocks. *Int J Rock Mech Min Sci* 41: 927-938

- 1 ISRM (2007) The complete ISRM suggested methods for rock characterization, testing and
2 monitoring: 1974-2006. R. Ulusay and J.A. Hudson (eds), Suggested Methods Prepared
3 by the Commission on Testing Methods, International Society for Rock Mechanics,
4 Compilation Arranged by the ISRM Turkish National Group, Kozan Ofset, Ankara,
5 Turkey
- 6 Karfakis MG, Akram M (1993) Effects of chemical solutions on rock fracturing. *Int J Rock*
7 *Mech Min Sci Geomech Abstr* 30 (7): 1253-1259
- 8 Kataoka M, Obara Y, Yoshinaga T, Kuruppu M (2010) Fracture toughness of rock under
9 water vapour pressure. In: Proceedings of the ISRM Int Symp on Rock Mechanics and 6th
10 Asian Rock Mechanics Symp, New Delhi, Paper No.12 (on CD)
- 11 Kataoka M, Obara Y, Kuruppu M (2011) Estimation of fracture toughness of anisotropic
12 rocks by SCB test and visualization of fracture by means of X-ray CT. In: Proceedings of
13 the ISRM 12th International Congress on Rock Mechanics, Q Qian and Y Zhou (eds),
14 Beijing, pp 667-670
- 15 Khan K, Al-Shayea NA (2000) Effect of specimen geometry and testing method on mixed
16 mode I-II fracture toughness of a limestone rock from Saudi Arabia. *Rock Mech Rock*
17 *Engng* 33 (3): 179-206
- 18 Kuruppu MD, Seto M (2001) Determination of fracture toughness of rock under in-situ
19 conditions using semi-circular specimen. In: Proceedings of ICF10, 10th Int Conf on
20 Fracture, Hawaii, pp 651 (abstracts vol)
- 21 Kuruppu MD, Chong KP (2012) Fracture toughness testing of brittle materials using semi-
22 circular bend (SCB) specimen. *Eng Fract Mech* 91: 133-150
- 23 Lim IL, Johnston IW, Choi SK (1993) Stress intensity factors for semi-circular specimens
24 under three-point bending. *Eng Fract Mech* 44 (3): 363-382
- 25 Lim IL, Johnston IW, Choi SK, Boland JN (1994) Fracture testing of a soft rock with semi-
26 circular specimens under three-point loading, Part 1-Mode I. *Int J Rock Mech Min Sci* 31:
27 185-197
- 28 Liu HW (1983) On the fundamental basis of fracture mechanics. *Eng Fract Mech* 17: 425-
29 438
- 30 Molenar AAA, Scarpas A, Liu X, Erkens SMJG (2002) Semi-circular bending test; Simple
31 but useful? *J Association of Asphalt Paving Technologists* 71: 794-815
- 32 Obara Y, Kuruppu M, Kataoka, M (2010) Determination of fracture toughness of anisotropic
33 rocks under water vapour pressure by semi-circular bend test. In: Proceedings of Mine
34 Planning and Equipment Selection, E Topal and MD Kuruppu (eds), The Australasian
35 Institute of Mining and Metallurgy, Victoria, Australia, pp 599-610
- 36 Obara Y, Sasaki K, Matusyama T, Yoshinaga T (2006) Influence of water vapour pressure of
37 surrounding environment on fracture toughness of rock. In: Proceedings of ARMS 2006,
38 Asian Rock Mechanics Symposium, Singapore, 7th chapter (on CD)
- 39 Obara Y, Sasaki K, Yoshinaga T (2007a) Estimation of fracture toughness of rocks under
40 water vapour pressure by Semi-Circular Bend (SCB) test. *J of MMIJ* 123: 145-151
- 41 Obara Y, Sasaki K, Yoshinaga T (2007b) Influence of water vapour pressure of surrounding
42 environment on fracture toughness and crack velocity of rocks. In: Proceedings of 11th
43 Congress of ISRM, Lisbon, Volume 1, pp 51-54

- 1 Obara Y, Yoshinaga T, Hirata A (2009) Fracture toughness in Mode I and II of rock under
2 water vapour pressure. In: Proceedings of ISRM Regional Symp EUROCK 2009, I
3 Vrkljan (ed), Cavtat, pp 333-338
- 4 Ouchterlony F (1989) Fracture toughness testing of rock with core based specimens, the
5 development of an ISRM standard. In: Fracture Toughness and Fracture Energy, H
6 Mihashi and H Takahashi (eds), Balkema, pp 231-251
- 7 Ouchterlony F (1990) Fracture toughness testing of rock with core based specimens. Eng
8 Fract Mech 35: 351-366
- 9 Tutluoglu L, Keles C (2011) Mode I fracture toughness determination with straight notched
10 disk bending method. Int J Rock Mech Min Sci 48:1248-1261
- 11 Whittaker BN, Singh RN, Sun G (1992) Rock Fracture Mechanics – Principles, Design and
12 Applications. Elsevier Sci Publisher, Amsterdam
- 13 Zhou YX, Xia K, Li XB, Li HB, Ma GW, Zhao J, Zhou ZL, Dai F (2012) Suggested methods
14 for determining the dynamic strength parameters and mode-I fracture toughness of rock
15 materials. Int J Rock Mech Min Sci 49: 105-112
- 16
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21
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Appendix – Details of numerical analysis used for deriving equation (2)

The SCB specimens of different crack lengths were simulated and analyzed using 8-node plane-strain elements in the finite element code ABAQUS (2012). The loading, the boundary conditions and a typical finite element mesh used for the simulations are shown in Fig. A1. Singular elements with nodes at quarter-point positions were used for the first ring of elements around the crack tip. In the circular partitions surrounding the crack tip where the contour integrals are calculated, the mesh was biased toward the crack tip. The stress intensity factors K_I were extracted directly from ABAQUS which makes use of the J-integral method to compute the stress intensity factors. The numerical results showed that there was negligible variation in the J-integral values calculated for successive contours surrounding the crack tip.

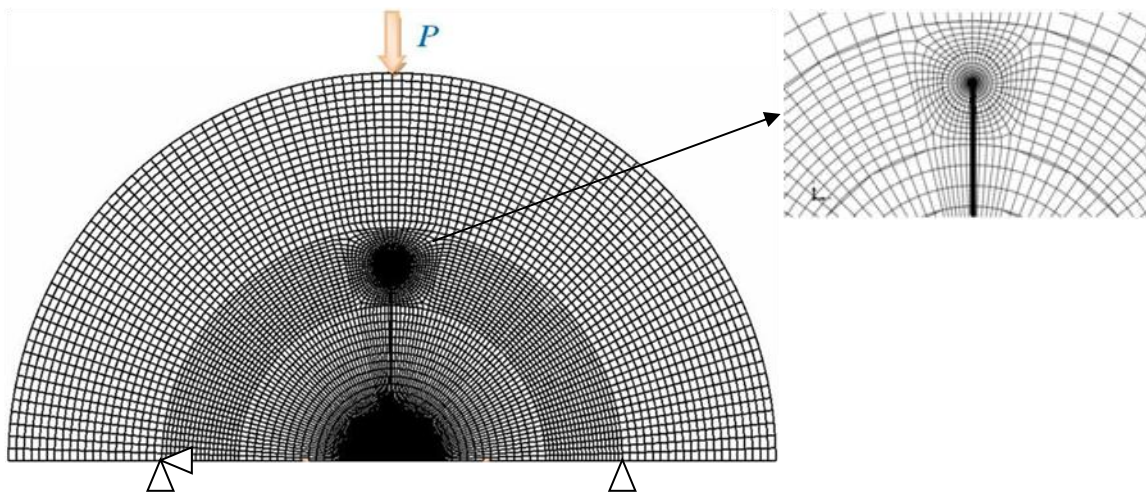


Fig. A1 A sample mesh pattern used for simulating the SCB specimen

Using a fixed arbitrary load P , the stress intensity factor K_I was determined for each set of β and $\frac{s}{2R}$, and the non-dimensional stress intensity factor Y' was calculated from

$$Y'(\beta, \frac{s}{2R}) = \frac{2RBK_I}{P\sqrt{\pi a}} \quad (\text{A1})$$

Then Eq (2) was derived by fitting a second order polynomial to the numerical results obtained for Y' . Tutluoglu and Keles (2011) recently reported limited numerical results for Y' in the SCB specimen. As shown in Table A1, very good agreement exists between the present results and those reported by Tutluoglu and Keles (2011). Table A1 can also be

considered as validation for the finite element results obtained in this study, particularly for the ranges $0.4 \leq \beta \leq 0.6$ and $0.5 \leq s/2R \leq 0.8$, as suggested in section 6.

Table A1 Numerical values of Y' , present results compared with those of Tutluoglu and Keles (2011)

$\beta=a/R$	$s/2R$	Present results	Tutluoglu and Keles (2011)
0.3	0.5	2.495	2.538
0.5	0.5	3.679	3.550
0.67	0.5	5.835	6.209

It is noteworthy that a number of investigators have presented mode I stress intensity factors of the SCB specimen (Chong et al. 1987; Lim et al. 1994; Basham 1989). For instance, Lim et al. (1994) extracted the stress intensity factors of the SCB specimen from finite element analysis and suggested a fifth order polynomial for Y' as

$$Y' = \frac{s}{2R} (2.91 + 54.39\beta - 391.4\beta^2 + 1210.6\beta^3 - 1650\beta^4 + 875.9\beta^5) \quad (A2)$$

Fig. A2 shows a comparison between the curves plotted based on Eq (2) and Eq (A2) for different values of β and $\frac{s}{2R}$. Significant discrepancies can be seen between these two sets of results.

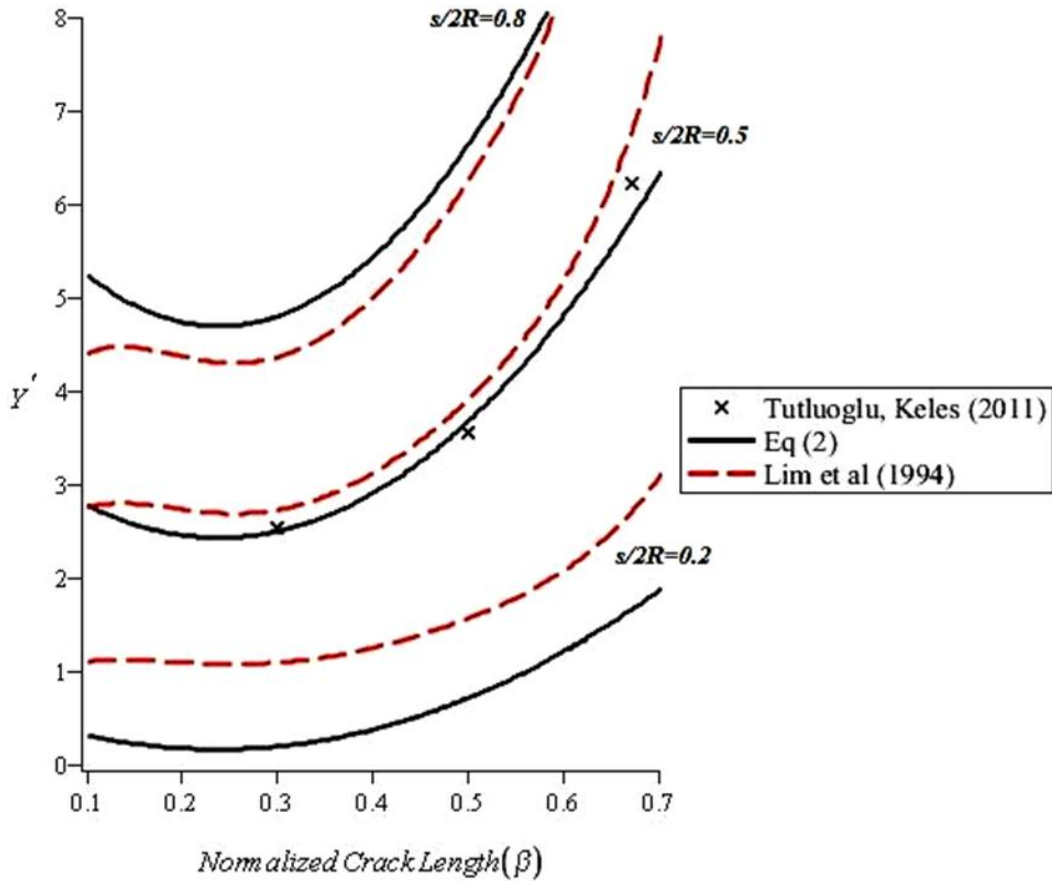


Fig. A2 The curves plotted based on Eq (2) and Eq (A2)

Having checked our finite element results by different mesh designs and element numbers, we concluded that the observed discrepancy can be due to less accurate method used by Lim et al. (1994) for determining the stress intensity factors of the SCB specimen. The displacement/stress extrapolation method employed by Lim et al. was a common technique in the 1990s for deriving stress intensity factors from finite element results. But, later more accurate methods were proposed like the contour integral techniques (e.g. J-integral method). It is now well established that the numerical errors in the region of high stress gradient around the crack tip affects the J-integral method much less than the displacement/stress extrapolation technique.

Figure Captions

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3
4 **Fig. 1** SCB specimen geometry and schematic loading arrangement (R : radius of the
5 specimen; B : thickness; a : notch length; s : distance between the two supporting
6 cylindrical rollers; P : monotonically increasing compressive load applied at the central
7 loading roller of the 3-point bend loading)
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10 **Fig. 2** Correction for a , R when the thickness of the saw blade is not negligible (i.e. $2\Delta r \geq$
11 $0.05D$, where Δr is the half thickness of the saw blade used for cutting)
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14 **Fig. 3** SCB specimen loading fixture
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16 **Fig. 4** Typical load versus displacement plots showing the critical fracture point
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19 **Fig. 5** Principal crack orientations with respect to bedding planes (left to right: arrester,
20 divider and, short transverse configurations)
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22 **Fig. A1** A sample mesh pattern used for simulating the SCB specimen
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25 **Fig. A2** The curves plotted based on Eq (2) and Eq (A2)
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