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Issues in Quantifying Variability from a Dynamical Systems Perspective

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Variability is a critical aspect of a dynamical systems analysis. Because there are a number of numerical techniques that can be used in such an analysis, the calculation of variability has several issues that must be addressed. The purpose of this paper is to present a variety of quantitative methods for investigating variability from a dynamical systems perspective. The paper is divided into two major sections covering discrete and continuous methods. Each of these sections is subdivided into two sections. Within discrete methods, we discuss, first, the calculation of the discrete relative phase from a time-series history of two parameters and, second, the use of return maps. Using continuous methods, we present procedures for using angle-angle plots in the evaluation of relative phase. We then discuss the use of phase plots in the calculation of the continuous relative phase. Each of these methods presents unique problems for the researcher and the method to be used is determined by the nature of the question asked.

Key Words: phase plot, angle-angle plot, continuous relative phase, discrete relative phase

Introduction

As noted in the previous paper, when addressing movement coordination from the dynamical systems perspective, measures of variability become critical to the understanding of the movement dynamics. However, there are a number of issues that must be addressed prior to the computation of the variability of the dependent measure. These issues include the different approaches used to evaluate movement coordination and the procedures necessary to correctly determine the variability of the coordination.

In this paper, we will discuss two approaches that have been used in the dynamical systems perspective to address movement coordination between two bio-physical oscillators. In this paper, the oscillators represent two segments or two joints or step frequency and breathing frequency. In each case we will then discuss the calculation of variability of the dependent measures. The first of these approaches will be referred to as discrete methods. The discrete methods section will also be subdivided into two techniques referred to as the time series and return map techniques. The second approach is referred to

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as continuous methods. Two categories of continuous methods will be presented. These are referred to as the angle-angle and continuous relative phase techniques.

Discrete Methods

The discrete methods under consideration are essentially two types of temporal analysis. These methods are point estimate approaches that illustrate the relative timing of key events in a movement cycle. That is, the approaches illustrate the latency of an event of an oscillator's motion with respect to an event of another oscillator. The disadvantage of these methods is that they evaluate coordination at only one point in each cycle. However, both methods are advantageous in that no further manipulations to the data must be made other than what normally would be done in the calculation of the joint angles.

Time-Series Approach

A discrete relative phase DRP angle (ϕ) is determined at a discrete event during a movement cycle using the time-series of two joint or segment angles. For example, many researchers have investigated the relative timing of two oscillators such as knee flexion/extension and subtalar inversion/eversion during the support phase of the running stride (Hamill et al., 1992; McClay & Manal, 1997). The key events in this analysis would relate to the functionally important instances when the knee joint reaches maximum flexion and the subtalar joint maximum eversion. An initial point, generally foot touchdown, is established as time zero, and the time to these key events is determined from the initial point. The DRP angle (ϕ) is then calculated as follows:

$$\varphi = \frac{\mathbf{t_1 - t_2}}{\mathbf{T}} \times \mathbf{360^o}$$

where t_1 is the time to maximum knee flexion, t_2 is the time to maximum subtalar eversion, and T is the support period. DRP can range from 0° to 360°, where $\phi = 360^o$ implies that the timing of the events are perfectly in-phase. Angles between 0° and 359° indicate that the timing of the events are not in-phase. In calculating the mean DRP over a number of strides and hence the variability, circular statistics should be used. Circular statistics methods will be discussed in a later section of this paper.

Return Maps

Another tool used to study the dynamics of a system at discrete intervals is a return map. Numerically, a return map is an iteration of an equation that plots one point against another with a specific time lag. May (1976) used return maps to model the complex behavior that a seasonally breeding population might display over the course of generations. Depending on the tuning of the parameters (that represent some biological or economic or sociological aspect), equilibrium values and their stability are identified from one generation to the next by the intersection of the function and the line of identity on the return map.

In an experimental sense, probing periodic data with discrete time lags is also known as a Poincare section (named after the famous French mathematician, Henri Poincare) and is the same as strobing a continuous time series at discrete intervals and studying the same plot. By systematically varying the interval, the periodicity of the system as well as attraction points can be found. Kelso and Jeka (1992) identified several frequency ratios and

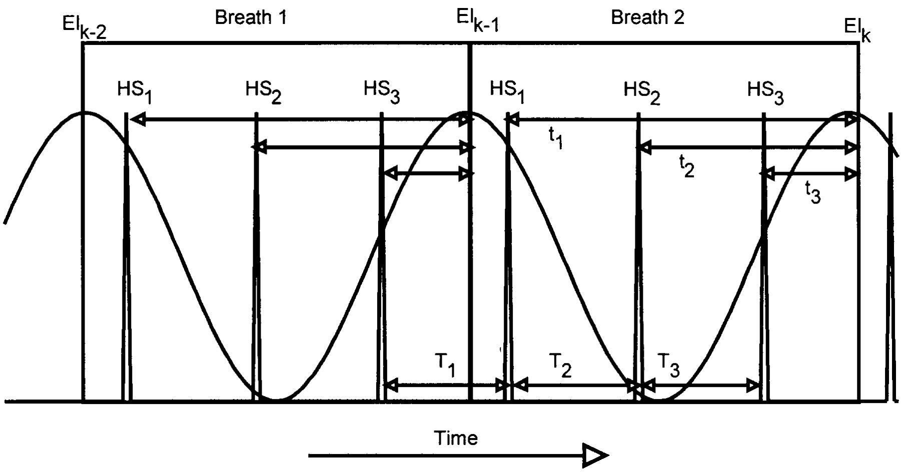

Figure 1 — A schematic of the coordination between breathing and stride frequency. EI is the end of inspiration; HS is heel strike; T is the time between heel strikes; and t is the time from heel strike to end of inspiration.

phase attraction points in human multi-limb coordination by calculating the discrete relative phase between the limbs and plotting it in return maps with different lags.

To illustrate the use of return maps for quantifying variability in frequency coupling, the coordination between stride and respiration during locomotion will be presented (McDermott et al., 2000). In this method, the DRP phase angle (ϕ) is calculated according to the following equation:

$$\varphi = \frac{\mathbf{T_n}}{\mathbf{t_n}} \times \mathbf{360^o}$$

where T_n is the time between consecutive heel contacts of the each foot and t_n is the time from heel strike to the end of inspiration. In this calculation, ϕ is the phase at which the next end-inspiration occurs with respect to the stride period (Figure 1).

Example data are presented from an experienced runner that was collected during 2.5 min of treadmill running at his preferred running speed. The ϕ data are plotted in Figure 2 as a return map with a phase shift of lag two. Frequency ratios are assessed by the periodicity or lag at which the points fall on the line of identity (indicating two consecutive breath cycles with the same number of strides) and the ranges of ϕ within which these points fall (indicated by boxes). The ranges are important for discriminating between different frequency ratios that have the same periodicity (i.e., 3:1 and 3:2). Points that fall outside the range and lag criteria in the maps are defined as non-couplings (NC) and represent those breath cycles that do not occur with the same frequency ratio in consecutive cycles. Given the relatively tight distribution around the line of identity in the lag 2 return map, it is clear from Figure 2 that the predominant frequency ratio is two strides to one breath.

The behavior of biologically coupled oscillators is based on the difference in their natural frequencies as well as the strength of coupling. Small differences in natural frequency and sufficient coupling result in absolute coordination—a state of frequency and phase locking. Greater differences in natural frequency and/or weaker coupling produce various states of relative coordination characterized by greater variability in phase relations and observed frequency ratios.

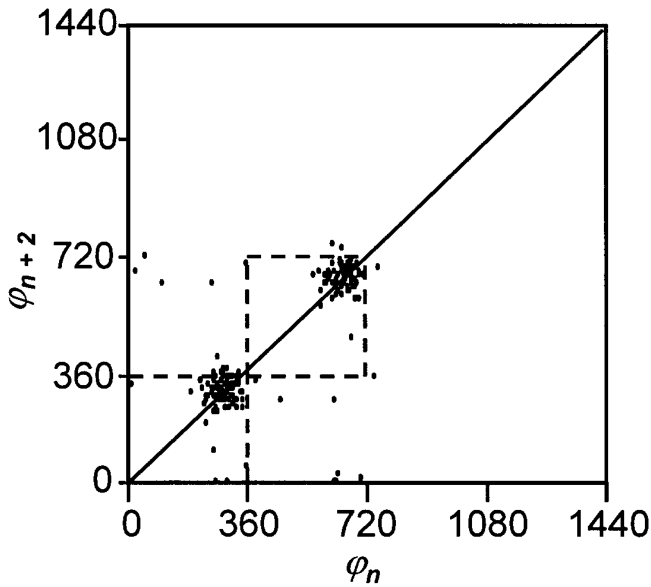

Figure 2 — An example of a return map with a lag 2.

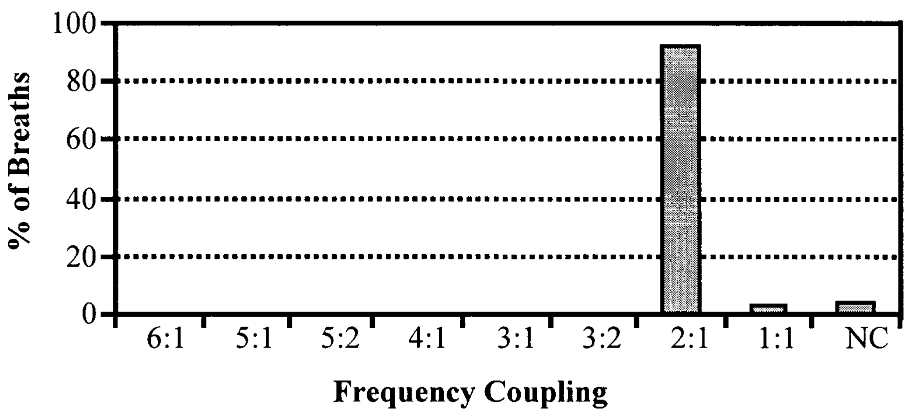

Figure 3 — Variability in the frequency coupling assessed by the percentage of breath cycles including different frequency ratios.

In this method, frequency coupling is assessed by the percentage of breaths using the dominant ratio. Variability in the frequency coupling is assessed by the distribution of breath cycles using ratios other than the dominant (Figure 3). The dominant ratio can be interpreted as the preferred state of the system, and the variability in coupling can be used to make inferences as to the next most preferred state and how close the system is to a transition to a new preferred state.

The sensitivity to multiple frequency ratios makes this method useful for studying systems such as locomotory-respiratory coupling where frequency ratios other than 1:1 are present. More generally, it is useful for systems where there is a driver signal (stride) that is very regular and a dependant signal (respiration) that varies based on the frequency of the driver. Furthermore, the phase attraction and variability can be calculated from the dispersion of points around the line of identity in the maps.

Continuous Methods

The continuous methods that we will discuss involve the evaluation of movement coordination over a period of time usually referred to as a cycle. These methods are different from the discrete methods in that they can be a spatial/temporal evaluation of coordination. Continuous methods have been used to assess the coordination or coupling between two oscillators. In movement science, the segment or joint angles are typically thought of as oscillators. Two methods will be presented. In the first case, angles for each oscillator of concern over the entire cycle and the difference between these angles constitute the relative motion. In the second method, phase angles for the relevant oscillators are calculated and the difference between the phase angles is the continuous relative phase. The advantage of these approaches is that coordination can be evaluated over the entire cycle.

Relative Motion

Relative motion as a measure of coordination can be assessed using angle-angle plots. The quantification of relative motion then is determined using vector coding techniques (see Sparrow et al., 1987). The orientation of the vector between two adjacent points on the angle-angle plot relative to the right horizontal is the coupling angle (Figure 4). The resulting values for the coupling angle range between 0 and 360°. Thus, values of 0°, 90°, 180°, and 270° indicate movement of one joint or segment. When the distal oscillator is fixed and the proximal oscillator is moving, the coupling angle is 0° or 180°, while 90° and 270° indicate the opposite. Vector orientations of 45°, 135°, 225°, and 315° indicate equal relative movement between the two segments. The two segments are moving in the same direction with values of 45° and 225°, while 135° and 315° indicate equal movement but in opposite directions.

Because the relative angle is directional, it is classified as a circular variable, and the mean and standard deviation of multiple trials must be determined using circular statistics (Batschelet, 1981). The mean direction is determined by calculating the mean cosine and sine of each direction of the relative motion angle (γ) over a number of trials (n):

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n}\cos\gamma_i$$

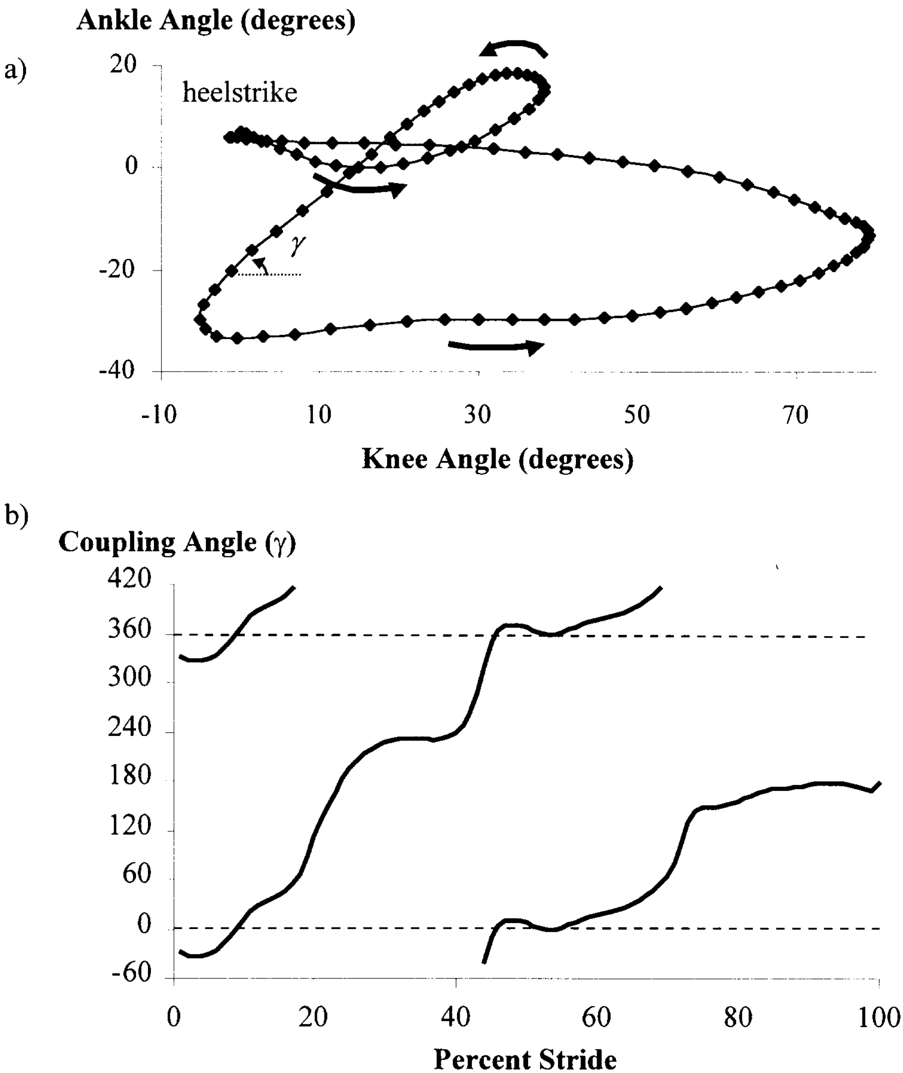

Figure 4 — Example of a continuous method based on an angle-angle plot: a) an illustration of the coupling angle as the orientation between two points; b) coupling angle over a complete stride.

and

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} \sin \gamma_i$$

The mean direction (γ) is then calculated by:

$$\bar{\gamma} = \tan^{-1}\left(\frac{\bar{y}}{\bar{x}}\right)$$

if $\overline{y} > 0$ and

$$\bar{\gamma} = 180 + \tan^{-1}\left(\frac{\bar{y}}{\bar{x}}\right)$$

if $\overline{y} < 0$.

The length of mean vector (r) provides an estimate of the variability of the data, as it reflects the directional concentration of the data:

$$r = \left(\bar{x}^2 + \bar{y}^2\right)^{1/2}$$

The calculation of the relative motion of two oscillators from angle-angle plots is advantageous because no normalization procedures are required. However, this method has the disadvantage of presenting only spatial information with no regard to temporal information.

Continuous Relative Phase (CRP)

CRP and CRP variability have been used as dependent variables in studies examining the coordination of finger oscillations (Kelso, 1995), interlimb leg oscillations (Clark & Phillips, 1993), and thorax and pelvic rotations (van Emmerik & Wagenaar, 1996). CRP is obtained by calculating the four-quadrant arctangent phase angle from a parametric phase plot of the oscillators of interest. Parametric phase plots illustrate velocity as a function of position (Figure 5). Once the phase angles are calculated for each oscillator, the time history is normalized to a fixed number of data points. The continuous relative phase is found by simply subtracting the phase angle of one oscillator from that of the other at each point in time over the entire stride. For example, the formula to calculate the CRP angle of the thigh/leg coupling is:

$$\mathrm{CRP(t)} = \phi_{\mathrm{thigh}}(\mathrm{t}) - \phi_{\mathrm{leg}}(\mathrm{t})$$

where $\phi_{\mathrm{thigh}}(\mathrm{t})$ and $\phi_{\mathrm{leg}}(\mathrm{t})$ are the normalized phase angles of the thigh and leg, respectively, at each instant in time of the movement cycle. CRP(t) values can range from 0° to 360°. However, there is a redundancy in angles (i.e., 0° and 360° mean the same thing) and, therefore, the scale is generally presented from 0° to 180°. Given the assumption that the motions are closely sinusoidal and a one-to-one frequency ratio, we can make statements about the phasing of the oscillators. Thus, based on this method of calculation, when the CRP(t) is 0°, the respective oscillators are perfectly in-phase, while a CRP(t) of 180° indicates that the oscillators are perfectly anti-phase. Any angle between 0° and 180° indicates a relative amount of in-phase or anti-phase.

If we are concerned with inter-limb coordination (i.e., right thigh-left thigh or right knee-left knee coordination), then it may not be necessary to undertake a normalizing procedure. Simply centering the parametric phase plots about an origin may be sufficient. However, if we are concerned with intra-limb coordination (i.e., right knee-right ankle coordination), the results can be questioned if no normalizing procedures are undertaken. Normalizing adjusts for amplitude differences in the range of motion of the oscillator and centers the phase plot about an origin.

For intra-limb coordination, prior to the calculation of phase angles, the phase plots of each oscillator must be normalized. In conducting this normalization, the raw data can

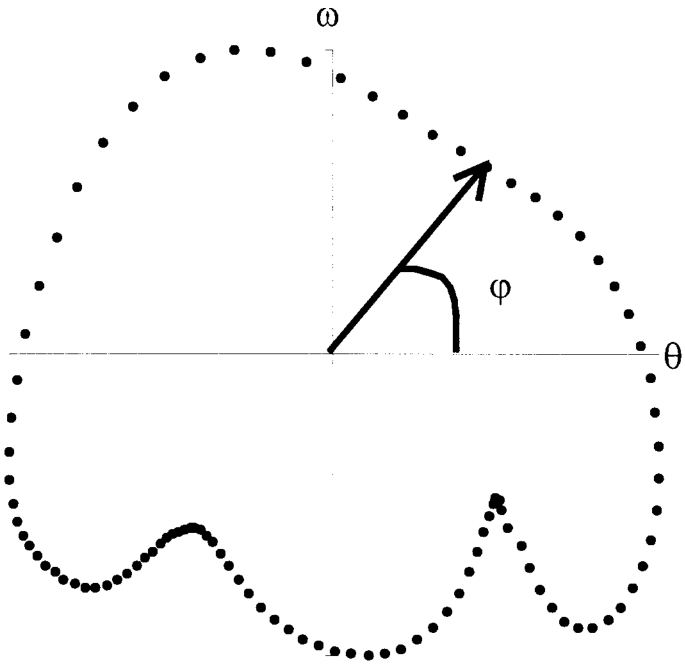

Figure 5 — A parametric phase plot. The phase angle (φ) is calculated at each instant in time using the arctangent of the ratio of ω to θ.

be distorted and the information of interest may be lost. As an example, we will use a data set in which subjects were required to walk at their preferred walking speed. Phase plots were then constructed for various lower body segments. The phase plots were normalized in each of two ways. First, angular position and velocity data were normalized to a unit circle using the maximum and minimum values (van Emmerik & Wagenaar, 1996). Second, angular position and velocity data were normalized to ±1 along the position axis, and to +1 or –1 along the velocity axis, depending on where the maximum absolute velocity occurred (Burgess-Limerick et al., 1993). Each of the above techniques were employed, with the normalization occurring on a stride-per-stride basis or with normalization occurring over the maximum values of multiple strides (Haddad et al., 1999). The effects of these normalization procedures on the phase plots of one of the oscillators in the coupling are illustrated in Figure 6.

Generally, differences in the normalization procedures based on either the individual strides or the maximum values of multiple strides were qualitatively minimal. However, normalizing to the maximum value of multiple strides will better maintain the true spatial properties among strides. In Figure 6a and 6b, with the individual stride method, there is an artificial merging at the reference points (i.e., ±1 along both axes in the unit circle method and ±1 along the position axis and +1 or –1 along the velocity axis in the maximum velocity method) that were used in the normalization procedure. In Figure 6c and 6d, when normalized to the maximum value of multiple strides, we see no merging at the reference points. The spatial layout is better maintained because each stride is normalized to a reference stride in lieu of each stride normalized individually. Normalizing over

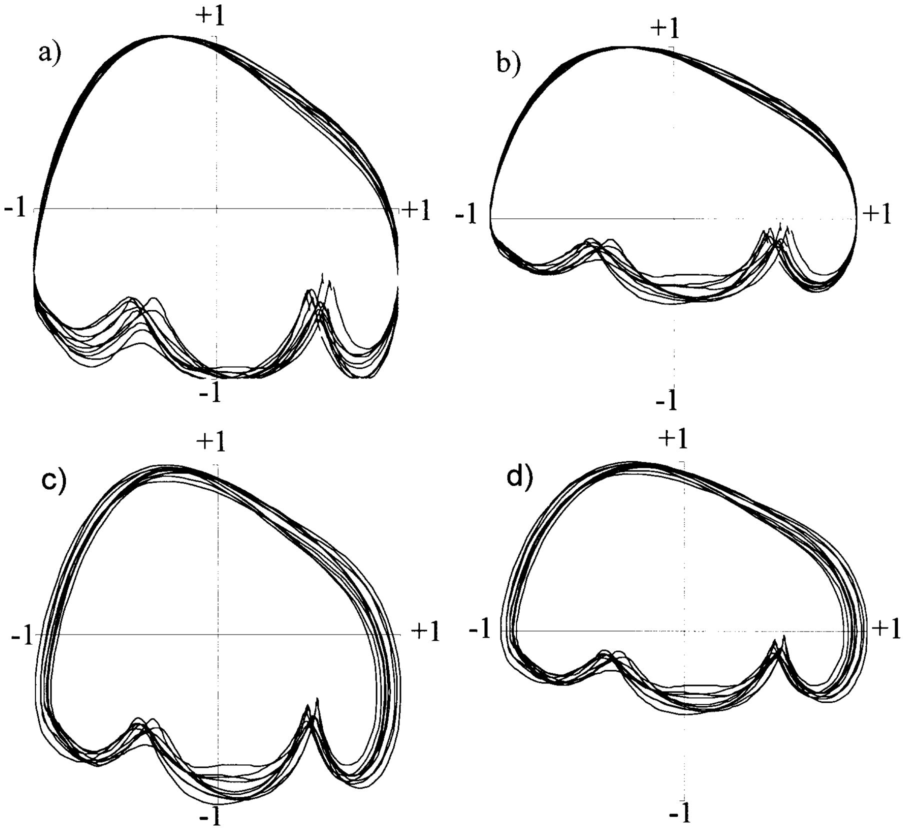

Figure 6 — Results of different normalization procedures of a parametric phase plot: a) phase plot normalized to the unit circle and to each individual stride; b) phase plot normalized to the maximum velocity and to each individual stride; c) phase plot normalized to the unit circle and to the maximum of multiple strides; and d) phase plot normalized to the maximum velocity and to the maximum of multiple strides.

strides (or to a reference stride) may preserve the original data to a greater degree than individual stride normalization. However, when using the maximum of multiple strides to normalize, extreme cases or outliers must be identified and a decision as to including or excluding them must be made. In this method, outliers will typically become the reference stride in normalization algorithms, causing a distortion in all of the strides. If the individual stride normalization technique is employed, the outlying stride will not affect the other strides. There was a difference between the two normalization techniques only when outliers were present in the multiple stride procedure.

Actual values of CRP were found to be affected based on the unit circle and maximum velocity normalization procedures. In the unit circle method, information regarding zero velocity is lost (Figure 6a and 6c). That is, zero on the y-axis of the phase plane does not correspond to an actual zero velocity as seen on the raw velocity data. In the maximum velocity method, the normalization procedure allows for the velocity trace to “float” below either the +1 or –1 axis (Figure 6b and 6d). The differences in phase angles result from

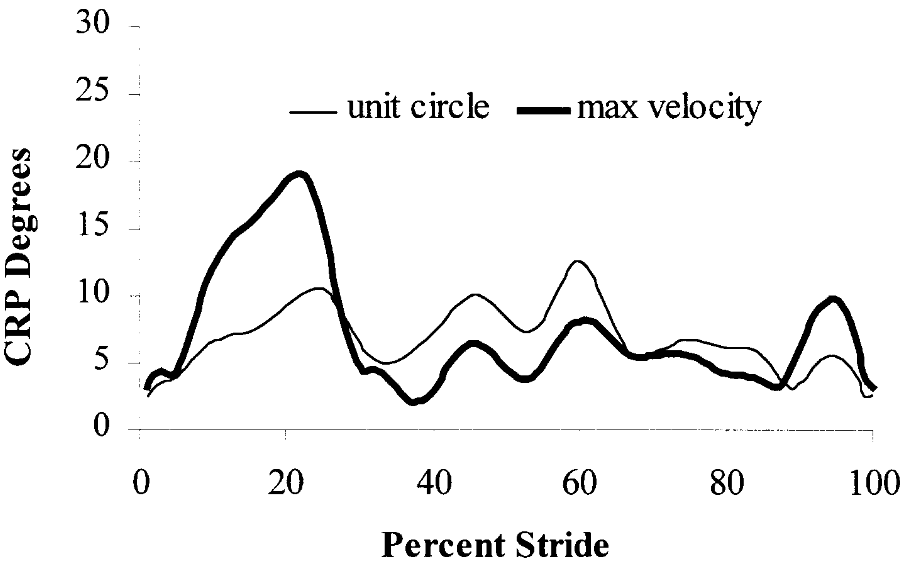

Figure 7 — Ensemble CRP over a complete stride cycle comparing the unit circle and maximum velocity methods.

discrepancies in the velocity traces. The calculations are the same for the position data irrespective of the normalization method chosen. An ensemble CRP history can be calculated by averaging on a point-by-point basis across multiple cycles. Figure 7 illustrates ensemble CRP strides using the unit circle and maximum velocity normalization techniques and scaled using maximum value of multiple strides.

CRP variability is calculated as the standard deviation on a point-by-point basis over the complete cycle. The normalization procedures discussed previously can also affect the CRP variability (Figure 8). Since any normalization based on strides does not alter the CRP, this normalization also does not affect the variability. However, there are differences in the variability between the unit circle and the maximum velocity normalization procedures.

Several methods of normalizing phase plot angles have been presented that will ultimately lead to the calculation of the continuous relative phase. There are many other methods that could have been employed. Ultimately, the choice of normalization procedure is likely to be dependent upon the specific aspects of the research question. There are limitations to calculating CRP, however. Diedrich and Warren (1995) suggested that CRP should be used only when the time series histories of the joint motions are sinusoidal.

Summary

We have presented several methods of evaluating oscillator coordination in human movement studies. In addition, each of these methods imply techniques that will allow the researcher to investigate the issue of variability of movement coordination. Ultimately, the methods that the researcher employs should be based on the question that is asked con-

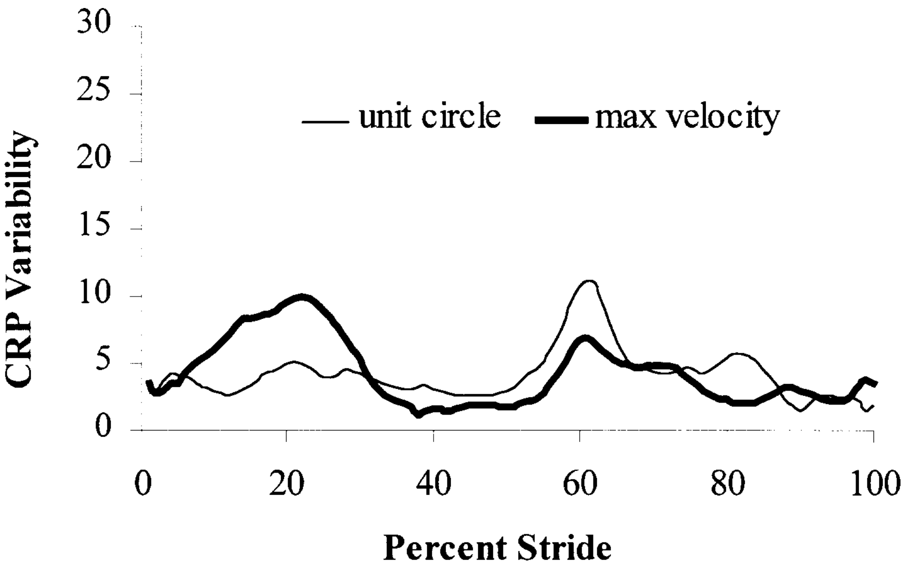

Figure 8 — CRP variability over a complete stride calculated from the unit circle and maximum velocity.

cerning movement coordination. The question should dictate the method to be used. The researcher should also be familiar with the advantages and drawbacks in using the types of analyses that have been presented.

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