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ITERATIVE APPROXIMATION OF SOLUTION OF GENERALIZED MIXED SET–VALUED VARIATIONAL INEQUALITY PROBLEM

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Abstract. In this paper, we consider a generalized mixed set-valued variational inequality problem which includes many important known variational inequality problems and equilibrium problem, and its related some auxiliary variational inequality problems. We prove the existence of solutions of the auxiliary variational inequality problems and suggest a two-step iterative algorithm and an inertial proximal iterative algorithm. Further, we discuss the convergence analysis of iterative algorithms. The theorems presented in this paper improve and generalize many known results for solving equilibrium problems, variational inequality and complementarity problems in the literature.

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Key words and phrases: equilibrium problem, variational inequality, auxiliary principle, two-step iterative algorithm, inertial proximal iterative algorithm, skew-symmetric function, mixed monotone, partially relaxed strongly mixed monotone, KKM mapping, fixed-point theorem.

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