Iterative Based Time Domain Equalization Method for DFTS-OFDM in Highly Mobile Environments

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ABSTRACT

In highly mobile environments, signal quality of discrete Fourier transform spreading-orthogonal frequency division multiplexing (DFTS-OFDM) with a frequency domain equalization method would be degraded due to the occurrence of inter-channel interference (ICI). To solve this problem, this paper proposes an iterative based time domain equalization (TDE) method with a time domain channel impulse response (CIR) estimation for DFTS-OFDM signals. The salient features of the proposed method are the use of a time domain training sequence (TS) in the estimation of CIR instead the conventional pilot subcarriers and to employ the TDE method with a maximum likelihood (ML) estimation instead of the conventional one-tap minimum mean square error frequency domain equalization (One-Tap MMSE- FDE) method. This paper also proposes a low complexity iterative based TDE method using the properties of a symmetric banded CIR transfer matrix for solving the simultaneous equations instead of direct calculation of an inverse matrix. It also presents various simulation results in highly mobile environments to demonstrate the effectiveness of the proposed iterative based TDE with CIR estimation for the TS inserted DFTS-OFDM signals compared to the conventional One-Tap MMSE-FDE method.

Keywords: DFTS-OFDM, Time Domain Equalization (TDE), Channel Impulse Response (CIR) Estimation, Time Domain Training Sequence (TS), Maximum Likelihood (ML) Estimation, PCGS Algorithm, Banded Matrix

1. INTRODUCTION

Discrete Fourier transform spreading-orthogonal frequency division multiplexing (DFTS-OFDM) has

received much attention as an alternative technique to OFDM due to its lower peak to average power ratio (PAPR) and robustness to multipath fading using a simple one-tap minimum mean square error frequency domain equalization (One-Tap MMSE-FDE) method [1-4]. Owing to these advantages, the DFTS-OFDM with a One-Tap MMSE-FDE method has been adopted as the standard transmission technique for the uplink from a user terminal to a base station in the 4th generation mobile communication system (LTE: Long Term Evolution) [5-6].

When the DFTS-OFDM is employed in highly mobile environments such as high speed vehicles or high speed trains, the transmitted DFTS-OFDM signal experiences severe inter-channel interference (ICI) due to the Doppler frequency spread. Under these conditions, the time domain channel impulse response (CIR) is no longer constant even during one the DFTS-OFDM symbol period. Accordingly, the channel frequency response (CFR), which is used in the One-Tap MMSE-FDE method at the receiver is also changed during the one DFTS-OFDM symbol period. In light of this, it is impossible to mitigate the ICI using the One-Tap MMSE-FDE method in highly mobile environments. This leads fatal degradation of the bit error rate (BER) performance [7].

Until now, many FDE methods of using the CFR transfer matrix were proposed for the OFDM signal to mitigate ICI in highly mobile environments [8-10]. The authors proposed an iterative based FDE method for the OFDM signal which can solve the inverse of CFR matrix iteratively with less computation complexity than using an inverse matrix calculation [10]. By using the iterative based FDE method, the order of computation complexity $O(N^3)$ required in the inverse matrix calculation can be reduced to $O(N_{Aver} \cdot N^2)$, where N is the number of FFT points and N_{Aver} is the average number of required iterations. However, the order of complexity for the iterative based FDE method is still higher. This method is conducted in the frequency domain which requires higher computation complexity, on the order of $O(N^2 \cdot \log_2 N)$, in the construction of the CFR transfer matrix. To solve the problem of the FDE method, the authors proposed a time domain equalization (TDE) method for the DFTS-OFDM signal

[11]. Although this method can mitigate the ICI precisely with lower complexity in the construction of a time domain CIR transfer matrix, it is required to calculate the inverse matrix at every data symbol, which leads extremely higher computational complexity. It is unsuitable in the practical implementation of DFTS-OFDM receivers.

To solve the above problems, this paper proposes an iterative based TDE method for the time domain training sequence (TS) inserted DFTS-OFDM signal in highly mobile environments [12]. The salient features of proposed TDE method are its use of a CIR transfer matrix in the mitigation of ICI and to employ a preconditioned conjugate gradient squared (PCGS) algorithm [13] in the calculation of an inverse matrix instead of using a direct inverse matrix calculation.

Using the proposed TDE method for the TS inserted DFTS-OFDM signal, the orders of complexity required in the construction of a time domain CIR transfer matrix and in the calculation of the resulting inverse matrix are $O(S^2 \cdot N)$ and $O(2S \cdot N \cdot N_{Aver})$, respectively. These are much lower than the $O(N^2 \cdot N_{Aver})$ and $O(N^2 \cdot \log_2 N)$ values for the conventional iterative based FDE method for OFDM signals [10]. S is the length of the time domain of the TS signal that corresponds to the guard interval (GI) of a OFDM signal. The proposed iterative based TDE method can achieve much better BER performance than the conventional One-Tap MMSE-FDE method and almost the same BER performance as the TDE method of using a direct inverse matrix calculation with much lower computational complexity [11] in highly mobile environments.

The remainder of this paper is organized as follows. Section 2 introduces a problem of a conventional One-Tap MMSE-FDE method for a DFTS-OFDM signal in highly mobile environments. Section 3 proposes a low complexity iterative based TDE with a time domain CIR estimation method for the DFTS-OFDM signal. Section 4 presents various computer simulation results to verify the effectiveness of the proposed iterative based TDE method compared to the conventional One-Tap MMSE-FDE method, and Section 5 draws some conclusions.

2. THE PROBLEM OF A CONVENTIONAL ONE-TAP MMSE-FDE FOR DFTS-OFDM SIGNALS

When assuming the quasi-static or lower time-varying and fading channels, the channel impulse response (CIR) can be considered a constant during a one DFTS-OFDM symbol period. Fig.1 shows a schematic diagram for the relationships between the CIR in the time domain and the channel frequency response (CFR) in the frequency domain, where the CFR can be obtained by performing the DFT on the CIR at a certain sampling time during a one DFTS-OFDM symbol period. Fig. 1(a) shows the relation-

ships between the CIR and CFR in the quasi-static channels. From the figure, it can be seen that the CIRs at any sampling time are almost constant.
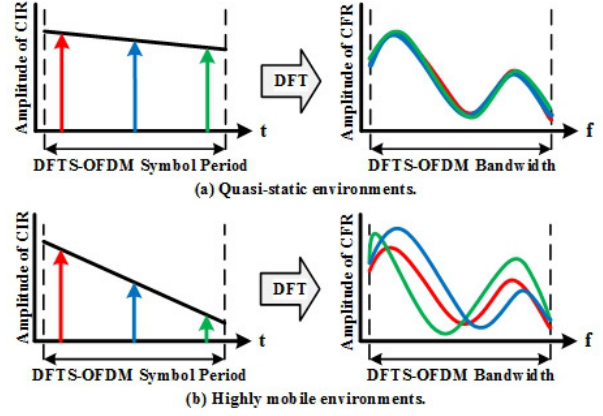

Fig.1: *Relationships between CIR and CFR in quasi-static and highly time-varying fading channels.*

Accordingly, the CFRs converted from the CIRs at any sampling time during a one symbol period are also nearly constant as shown in Fig. 1(a). From this, the received DFTS-OFDM signal caused by lower time-varying fading distortion can be precisely equalized using the One-Tap MMSE-FDE method with the CFR converted from the CIR at any sampling time during a one symbol period. However, the CIR would no longer be constant even during a one DFTS-OFDM symbol period with highly time-varying fading channels [11]. Fig. 1(b) shows the relationships between the CIRs and CFRs in highly time-varying fading channels. The CIRs are changing during a one symbol period and the CFRs are converted from the CIRs at the different sampling times. They are also changing as shown in Fig.1(b). From this, it is difficult to mitigate the time-varying fading distortions by the One-Tap MMSE-FDE method of using a fixed CFR during one symbol period. This leads to fatal degradation of BER performance.

3. PROPOSAL OF ITERATIVE BASED TIME DOMAIN EQUALIZATION METHOD

To solve the above problem of the conventional One-Tap MMSE-FDE method in highly mobile environments, this paper proposes an iterative TDE method for the TS inserted DFTS-OFDM signal using an estimated CIR at every sampling time.

3.1 TS inserted DFTS-OFDM system

Fig. 2(a) shows the frame format for a TS inserted DFTS-OFDM signal that can be used in the CIR estimation at every sampling time. The authors used a CIR estimation method at every sampling time for the TS inserted OFDM signal with a maximum likelihood (ML) estimation [11]. The time domain

training sequences, TS1 and TS2, with a length of S samples were added at both ends of each data symbol used in the estimation of CIR. They also were used in the role of GI to remove the inter-symbol interference (ISI). The employment of a time domain TS signal in the estimation of CIR in highly mobile environments can achieve higher estimation accuracy than the conventional pilot subcarriers in the frequency domain. This is the reason that the time duration of the TS signal is much shorter than the DFTS-OFDM symbol and the fluctuation of CIR due to Doppler spread can be considered a constant during the period of a TS signal. The pilot subcarriers were inserted into the data subcarriers with a certain interval in the frequency domain in which the assumption of constant CIR over one DFTS-OFDM symbol time period was no longer satisfied [11].

Fig. 3 shows a structure of a transceiver for the TS inserted DFTS-OFDM system with the proposed iterative based TDE method. At the transmitter, the data is encoded by a forward error correction (FEC) code [14] and the encoded data is modulated in the time domain. Then, the M modulated data is converted to M data subcarriers using an M-points discrete Fourier transform (DFT) which is given by:

$$X_D(m,k) = \sum_{n=0}^{M-1} x_D(m,n) \cdot e^{-j\frac{2\pi kn}{M}} \qquad (1)$$

where $x_D(m,n)$ is the time domain data signal at the n-th sampling time of the m-th symbol and $X_D(m,k)$ is the frequency domain data signal at the k-th sub- carrier. Then, the M data subcarriers of $X_D(m,k)$ were mapped into the allocated f requency bandwidth with N subcarriers. This is called subcarrier mapping. In subcarrier mapping, M data subcarriers are continuously mapped into a certain frequency band from the data subcarrier numbered N_{Z1} to N_{Z2} ($N_{Z2} - N_{Z1} = M$) within N subcarriers in which the zero padding (null subcarrier) with a length of $(N-M)/2$ subcarriers added at the both ends of M data subcarriers. Here, it should be noted that DFT and inverse DFT (IDFT) processing are required both at the transmitter and receiver in the proposed DFTS-OFDM system. Although the required processing loads for the DFT and IDFT are higher than those for the fast Fourier transform (FFT) and inverse FFT (IFFT), the DFTS-OFDM technique is already employed in the uplink of LTE systems as the standard transmission technique [5-6]. From this, the proposed system using the DFTS- OFDM technique can also be used to implement practical systems.

The frequency domain signal over N subcarriers after subcarrier mapping is given by:
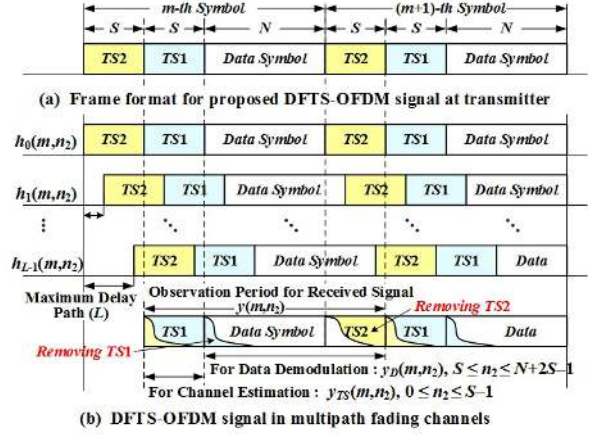

Fig.2: *Frame format of DFTS-OFDM at transmitter (a) and receiver (b) in multipath fading channels.*

$$X(m,k_1) = \begin{cases} 0 \ (zero\ padding), & 0 \le k_1 \le N_{Z1}-1 \\ X_D(m,k_1-N_{Z1}), & N_{Z1} \le k_1 \le N_{Z2} \\ 0 \ (zero\ padding), & N_{Z2}+1 \le k_1 \le N-1 \end{cases} \qquad (2)$$

where N_{Z1} and N_{Z2} in (2) are the starting and ending data subcarrier numbers for M data subcarriers in the frequency domain. The starting and ending data subcarrier numbers are given by $N_{Z1} = (N-M)/2$ and $N_{Z2} = M+N_{Z1}-1$, respectively. Here, it should be noted that zero padding is employed to avoid aliasing at the output of the D/A converter. Using zero padding, a simple analogue band-pass filter located after the D/A converter can be used to reject aliasing, which leads to an easy implementation of a transmitter. After subcarrier mapping, $X(m,k_1)$ including the zero padding is converted into a time domain signal similar to the conventional OFDM signal by an N-point IFFT which is given by,

$$x(m,n_1) = \frac{1}{N} \sum_{k_1=N_{Z1}}^{N_{Z2}} X(m,k_1) \cdot e^{j\frac{2\pi n_1 k_1}{N}},$$
$$0 \le n_1 \le N-1 \qquad (3)$$

where $x(m,n_1)$ is the transmitted time domain DFTS-OFDM signal at the n_1-th sampling time of the m-th symbol. Here, it should be noted that since the computational complexity both for the FFT and IFFT are decided by N, including the zero padding. From this, zero padding would not affect the computational complexity. In the proposed method, the data pattern of the TS signal was generated using some part of time domain of the DFTS-OFDM signalto achieve a higher CIR e stimation accuracy. The optimum data pattern of the TS signal that can achieve a higher CIR estimation accuracy in the multipath fading channels was selected from computer
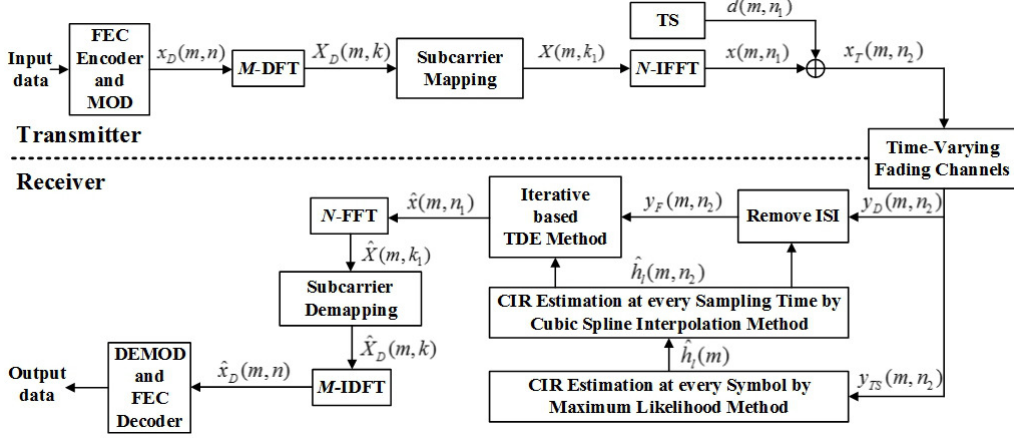
Fig.3: *Structure of transceiver for DFTS-OFDM signal with proposed iterative based TDE method.*

simulation results by changing the r andomly generated data patterns of the DFTS-OFDM symbols.

For simplicity, this paper assumes that the data patterns both for TS1 and TS2 are the same as $d(m, n_1)$ at the m-th symbol. The length of S should be longer than the length of delay paths (L) has the same as the role of GI to remove the ISI. After adding the TS1 and TS2 signals at the both ends of the data symbol as shown in Fig. 2(a), the transmitted time domain signal including TS1 and TS2 signals can be expressed as:

$$x_T(m, n_2) =$$
$$\begin{cases} d(m, n_2), & 0 \leq n_2 \leq S-1 \\ x(m, n_2 - S), & S \leq n_2 \leq N+S-1 \\ d_2(m, n_2 - N - S), & N+S \leq n_2 \leq N+2S-1 \end{cases}$$
$$(4)$$

where $x_T(m, n_2)$ is the transmitted time domain signal including the TS1 and TS2 signals with a length of $N + 2S$ sampling time, d_1 and d_2 are the time domain of TS1 and TS2 with a length of S sampling time ($0 \leq n_2 \leq S - 1$) of which data patterns are known at the receiver.

As shown in Fig. 2(b), the received DFTS-OFDM signal $y(m, n_2)$ can be divided into two parts that consist of the observation period for the CIR estimation $y_{TS}(m, n_2)$ with a length of S sampling time and for the data demodulation period $y_D(m, n_2)$ with the length of $N + S$ sampling time, respectively.

3.2 CIR Estimation method for TS inserted DFTS-OFDM signal [11]

Consider the observation period for CIR estimation, the received time domain TS1 signal $y_{TS}(m, n_2)$ during the observation period for CIR estimation from 0 to $S - 1$ as shown in Fig. 2(b) can be expressed as,

$$y_{TS}(m, n_2) = \sum_{l=0}^{L-1} h_1(m, n_2) \cdot d_1(m, n_2 - l) + z(m, n_2),$$
$$0 \leq n_2 \leq S-1$$
$$(5)$$

where $d_1(m, n_2)$ is the TS1 signal given in (4), hl(m, n2) is the complex amplitude of the CIR for the l-th delay path at the n_2-th sampling time of m-th symbol and $z(m, n_2)$ is additive white Gaussian noise (AWGN). Here, it was assumed that $h_l(m, n_2)$ was constant during the short period of TS1 even in highly mobile environments. Under the above assumption, the expected received TS1 signal passed through the multipath channels can be expressed as:

$$\hat{y}_{TS}(m, n_2) = \sum_{l=0}^{S-1} \hat{h}_1(m) \cdot d(m, m_2 - l)$$
$$0 \leq n_2 \leq S - 1$$
$$(6)$$

The unknown parameters of complex amplitude of $\hat{h}l(m)$ at the m-th symbol can be estimated by using the maximum likelihood (ML) estimation method under the constraint minimizing the difference between the actual received time domain TS signal in (5) and the expected received time domain signal in (6). Here the estimated CIR $\hat{h}_l(m)$ corresponds to the CIR in the middle sampling time of TS1. It should be noted that the fluctuation of CIR even in highly mobile environments can be considered constant because of the use of a short time period of the TS signal. From this, the CIR estimated using the TS signal in highly mobile environments can achieve greater estimation accuracy than using pilot subcarriers assigned over one DFTS-OFDM symbol in the frequency domain as discussed in Section 3.1. Then, the time domain CIR $\hat{h}_l(m, n_2)$ at every sampling time can be estimated

by applying a cubic spline interpolation for CIR estimation at every symbol over one frame. Then, the received data symbol can be equalized in the time domain to mitigate the Doppler spread. In other words, the proposed TDE method can mitigate the Doppler spread at every time sampling basis in the time domain. This is completely different from the conventional frequency domain equalization using the estimated CFR for each subcarrier [11].

3.3 Proposed Time Domain Equalization Method

In the proposed time domain equalization (TDE) method, the estimated CIRs at every sampling time are employed in the construction of a CIR transfer matrix that enables compensation for Doppler spread at every sampling time basis. This feature of the proposed TDE method is completely different from the conventional One-Tap MMSE-FDE method in which the CFR is converted from a fixed CIR during a one symbol period which leads fatal degradation of BER performance. By assuming the actual channel impulse response $h_l(m, n_2)$ at e very sampling time, the received time domain data signal $y_D(m, n_2)$ during the observation period for the data demodulation from S to $N + 2S - 1$, as shown in Fig. 2(b), can be expressed by:

$$y_D(m, n_2) = \sum_{l=0}^{L-1} h_l(m, n_2) \cdot x_T(m, n_2 - l) + z(m, n_2),$$
$$S \leq n_2 \leq N + 2S - 1 \tag{7}$$

where $x_T(m, n_2 - l)$ corresponds to the transmitted time domain signal given in (4) and the following relationships are employed in the derivation of (7).

$$n_2 - l \leq S - 1, \quad x_T(m, n_2 - l) = d(m, n_2 - l) \tag{8}$$

$$n_2 - l \leq N + S, \quad x_T(m, n_2 - l) = d(m, n_2 - N - S - l) \tag{9}$$

The received data signal in (7) includes the ISI which are added at the start and end of the data symbol from the TS1 and TS2 signals, respectively, as shown in Fig. 2(b). Since the data patterns of the TS signal are known at the receiver, the ISI caused from the TS signals in the multipath fading channel can be removed using the estimated CIRs at every sampling time and the known data pattern of TS signal, which is given by:

$$y_F(m, n_2) =$$
$$\begin{cases} y_D(m, n_2) - \displaystyle\sum_{l=n_2-S+1}^{S-1} \hat{h}_l(m, n_2) \cdot d(m, n_2 - l), \\ \qquad\qquad (S \leq n_2 \leq 2S - 2) \\ y_D(m, n_2), (2S - 1 \leq n_2 \leq N + S - 1) \\ y_D(m, n_2), \displaystyle\sum_{l=0}^{n_2-N-S} \hat{h}_l(m, n_2) \cdot d(m, n_2 - N - S - l), \\ \qquad\qquad (N + S \leq n_2 \leq N + 2S - 2) \end{cases}$$
$$\tag{10}$$

where $y_F(m, n_2)$ is the received time domain signal after removing the ISI from the actual received DFTS-OFDM signal $y_D(m, n_2)$ in (7). When the transmitted time domain data signal $x(m, n_1)$ given in (3) is assumed to be an unknown parameter, the expected time domain received data signal $\hat{y}_E(m, n_2)$ without the ISI, which corresponds to (10), can be given by:

$$\hat{y}_E(m, n_2) = \begin{cases} \displaystyle\sum_{l=0}^{n_2-S} \hat{h}_l(m, n_2) \cdot \hat{x}(m, n_2 - S - l), \\ \qquad\qquad (S \leq n_2 \leq 2S - 2) \\ \displaystyle\sum_{l=0}^{S-1} \hat{h}_l(m, n_2) \cdot \hat{x}(m, n_2 - S - l), \\ \qquad\qquad (2S - 1 \leq n_2 \leq N + S - 1) \\ \displaystyle\sum_{l=n_2-N-S+1}^{S-1} \hat{h}_l(m, n_2) \cdot \hat{x}(m, n_2 - S - l), \\ \qquad\qquad (N + S \leq n_2 \leq N + 2S - 2) \end{cases}$$
$$\tag{11}$$

The unknown parameter of the time domain data signal $\hat{x}(m, n_1)$ can be estimated by solving the following maximum likelihood (ML) equation under the constraint that minimizes the difference between the actual received data signal $y_F(m, n_2)$ in (10) and the expected received data signal $\hat{y}_E(m, n_2)$ in (11). This can be expressed as:

$$\Upsilon = \underset{\hat{x}(m,s)}{\arg\min} \left[\sum_{n_2=S}^{N+2S-2} |y_F(m, n_2) - \hat{y}_E(m, n_2)|^2 \right]$$
$$, 0 \leq n_2 \leq N + 2S - 2 \tag{12}$$

The ML equation in (12) can be solved by taking the partial differential for unknown parameters of $\hat{x}^*(m, n_3)$ which can be expressed as,

$$\frac{\partial \Upsilon}{\partial \hat{x}^*(m,n_3)} = \frac{\partial \left(\sum_{n_2=n_3}^{N+2S-2} |y_F(m,n_2) - \hat{y}_E(m,n_2)|^2 \right)}{\partial \hat{x}^*(m,n_3)} = 0$$
$$, 0 \le N_3 \le N-1$$

(13)

where $(\cdot)^*$ represents the conjugate complex. Using (13), the ML equation (12) can be expressed by the following simultaneous equations with N unknown parameters of $\hat{x}(m,n_1)$.

$$[b(m,n_3)]_{N\times 1} = [A_m(n_3,n_1)]_{N\times N} \cdot [\hat{x}(m,n_1)]_{N\times 1}$$

(14)

where $b(m,n_3)$ and $A_m(n_3,n_1)$ can be expressed as,

$$b(m,n_3) = \sum_{n_2=S}^{N+2S-2} y_F(m,n_2) \frac{\partial \hat{y}_E^*(m,n_2)}{\partial \hat{x}^*(m,n_3)}$$
$$= \underbrace{\left[\hat{h}_l^H(m,n_3) \right]}_{N\times(N+S-1)} \cdot \underbrace{[y_F(m,n_1)]}_{(N+S-1)\times 1}, \; 0 \le n_3 \le N-1$$

(15)

$$A_m(n_3,n_1) = \sum_{n_2=S}^{N+2S-2} \hat{y}_E(m,n_2) \frac{\partial \hat{y}_E^*(m,n_2)}{\partial \hat{x}^*(m,n_3)}$$
$$= \underbrace{\left[\hat{h}_l^H(m,n_3) \right]}_{N\times(N+S-1)} \cdot \underbrace{\left[\hat{h}_l(m,n_1) \right]}_{(N+S-1)\times 1}, \; 0 \le n_1 \le N-1$$

(16)

where $(\cdot)^H$ represents the Hermitian transpose operation and the following relationships are used in the derivation of (15) and (16).

$$n_3 = n_2 - S - l \text{ in } \partial \hat{y}_E^*(m,n_2)/\partial \hat{x}^*(m,n_3) \quad (17)$$

$$n_1 = n_2 - S - l \text{ in } \hat{y}_E^*(m,n_2) \quad (18)$$

Here the CIR matrix $[\hat{h}_l(m,n_1)]$ in (16) is given by:

$[\hat{h}_l(m,n_1)]$

$$= \begin{bmatrix} \hat{h}_0(m,S) & 0 & \cdots & 0 \\ \hat{h}_1(m,S+1) & \hat{h}_0(m,S+1) & \ddots & \vdots \\ \vdots & \hat{h}_1(m,S+2) & \ddots & 0 \\ \hat{h}_{S-1}(m,2S-1) & \vdots & \ddots & \hat{h}_0(m,N+S-1) \\ 0 & \hat{h}_{S-1}(m,2S-1) & \vdots & \hat{h}_1(m,N+S) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \hat{h}_{S-1}(m,N+2S-2) \end{bmatrix}$$

(19)

From (19), the order of complexity required in the construction of CIR transfer matrix $A_m(n_3,n_1)$ in (16) which is obtained by the multiplication of $\hat{h}_l(m,n_1)$ and $\hat{h}_l^H(m,n_1)$, can be given by $O(S^2 \cdot N)$. The proposed TDE method using the CIR transfer matrix can achieve a lower, complexity than the conventional FDE method [10] which employs the full CFR transfer matrix with a complexity of $O(N^2 \cdot \log_2 N)$. The CIR transfer matrix $[A_m(n_3,n_1)]$ in (16) which is obtained after the partial differentiation can be represented by,

$$A_{i,j} = \begin{bmatrix} A_{0,0} & A_{0,1} & \cdots & A_{0,S-1} & 0 & \cdots & 0 \\ A_{0,1}^H & A_{1,01} & & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & & & \ddots & 0 \\ A_{0,S-1}^H & & & \ddots & & & A_{N-S,N-1} \\ 0 & \ddots & & & \ddots & & \vdots \\ \vdots & \ddots & \ddots & & & A_{N-2,N-2} & A_{N-2,N-1} \\ 0 & \cdots & 0 & A_{N-S,N-1}^H & \cdots & A_{N-2,N-1}^H & A_{N-1,N-1} \end{bmatrix}$$

(20)

In (20), $A_m(i,j)$ in (16) is represented by $A_{i,j}$ and the index of m-th symbol is omitted for brevity. From (20), it can be seen that the matrix is a banded matrix with an upper and lower bandwidth (S-1) whose non-zero entries are confined to a diagonal band. Also, the lower band matrix, with an index of (j,i) below the diagonal term, is the Hermitian transpose of the upper band matrix with indices of (i,j). From this, the CIR transfer matrix $[A_m(n_3,n_1)]$ in (16) is a symmetric banded matrix with a block size of $(N \times N)$. This paper employs the favourable property of symmetric banded matrices in the proposed iterative based TDE method for further reduction of computation complexity.

From (14), the unknown parameters $\hat{x}(m,n_1)$ can be simply solved by using the inverse matrix of $[A_m(n_3,n_1)]$ which is given as,

$$[\hat{x}(m, n_1)]_{N \times 1} = [A_m(n_3, n_1)]_{N \times N}^{-1} \cdot [b(m, n_3)]_{N \times 1} \tag{21}$$

where $[\cdot]^{-1}$ represents the inverse matrix. In the demodulation of the time domain data for the DFTS-OFDM signal, the estimated $\hat{x}(m, n_1)$ in (21) is converted to the frequency domain signal $\hat{X}(m, k)$ as given in (2) by an N-points FFT. After subcarrier demapping to $\hat{X}(m, k)$, M data subcarriers $\hat{X}_D(m, k)$ as given in (2) can be obtained in the frequency domain. Then $\hat{X}_D(m, k)$ is converted to the time domain as $\hat{x}_D(m, n)$ using an M-points IDFT as given in (1). The time domain data can be obtained after demodulation and FEC decoding for $\hat{x}_D(m, n)$ which are the reverse processing at the transmitter side as shown in Fig. 3.

In (21), the order of computational complexity for the calculation of an inverse matrix with size of $N \times N$ is $O(N^3)$ is required at every data symbol demodulation. To reduce the computational complexity in solving the simultaneous equations for the proposed TDE method, the next section proposes an iterative based TDE method that employs a precondition conjugate gradient squared (PCGS) algorithm for the symmetric banded CIR transfer matrix given in (20).

3.4 Proposed Iterative based TDE Method

The conjugate gradient squared (CGS) algorithm [13] is well known as an iterative method that can solve linear simultaneous equations for N unknown parameters with much smaller computational complexity compared to the inverse matrix calculation. Considering the simultaneous equations $A\hat{x} = b$, where A corresponds to $A_m(n_3, n_1)$ in (14). Its matrix is a symmetric banded matrix with a size of $N \times N$. The exact CGS solution can be obtained after at most N steps. Hence, stopping the iteration after Niter $N_{iter}(< N)$ steps would yield an approximate solution of the problem. In the equalization of every data symbol, the CGS algorithm iteratively minimizes the cost function in a reduced-rank Krylov subspace. When the spectral condition number of the matrix A is too high, a pre-conditioned matrix D is employed that is called the precondition CGS (PCGS) algorithm. The PCGS algorithm solves simultaneous equations as:

$$D^{-1}A\hat{x} = D^{-1}b \tag{22}$$

where the inverse of matrix D should be a computationally efficient operation. In the rest of our analysis, assuming the simplicity that the matrix $D(m)$ is constructed as a diagonal of matrix $A_m(n_3, n_1)$ and the initial solution of $\hat{x}(m, n_1)$ can be given by,

$$[\hat{x}(m, n_1)]_{N \times 1}^{(0)} = [D(m)]_{N \times N}^{-1} \cdot [b(m, n_3)]_{N \times 1} \tag{23}$$

In PCGS algorithm [13], the residual vector $r^{(i)}$ can be regarded as the product of $r^{(0)}$ and an i-th degree polynomial in matrix A which is expressed as,

$$r^{(i)} = P_i(A)r^{(0)} \tag{24}$$

where $P_i(A)$ is a polynomial of A at the i-th degree and $r^{(0)}$ is obtained from the initial solution given in (23) as $r^{(0)} = b - A\hat{x}^{(0)}$. The iteration co-efficients can be recovered from the i-th vectors $r^{(i)}$ and finding the corresponding approximations for $[\hat{x}(m, n_1)]$ is quite easy. From (24), multiplication of $P_i(A)r^{(0)}$ is required at every symbol. Since the CIR transfer matrix A given in (20) is a symmetric banded matrix, the order of complexity for this multiplication is only $O(2S \cdot N)$. This is lower than the conventional iterative based FDE method using all the elements of the CFR transfer matrix [10] which requires an order of complexity of $O(N^2 \cdot \log_2 N)$.

The iteration of the PCGS algorithm is stopped when the following normalized mean square error (NMSE) between the $(i - 1)$-th and i-th solutions of $[\hat{x}(m, n_1)]$ is smaller than a predetermined threshold level (TOL) [10].

$$NMSE = \frac{\sum\limits_{n_1=0}^{N-1} \left| [\hat{x}(m, n_1)]^{(i)} - [\hat{x}(m, n_1)]^{(i+1)} \right|^2}{\sum\limits_{n_1=0}^{N-1} \left| [\hat{x}(m, n_1)]^{(i)} \right|^2} \tag{25}$$

In the proposed iterative based TDE using the PCGS algorithm, the following procedures are repeated until either the value of $NMSE$ in (25) becomes less than the predetermined threshold level(TOL) or the number of iterations reaches to a predetermined maximum (N_{max}).

Step 1: The maximum iteration number is set to N_{max} (N_{max} = 5), the threshold level is set to TOL and the initial solution of $[\hat{x}(m, n_1)]^{(0)}$ is given by (23).

Step 2: Calculate the $i - th$ solution of $[\hat{x}(m, n_1)]^{(i)}$ using the PCGS algorithm and calculate the $NMSE$ by (25).

Step 3: Compare the $NMSE$ obtained at the $i - th$ iteration with the predetermined threshold level of TOL. If the $NMSE$ is less than TOL, the $[\hat{x}(m, n_1)]^{(i)}$ is output as the estimated data. If not, iterate the same procedures. If the number of iterations reaches to predetermined number, N_{max}, $[\hat{x}(m, n_1)]^{(N_{max})}$ is output as the estimated information.

The order of computation complexity for the proposed PCGS algorithm can be evaluated by $O(2S \cdot N \cdot N_{Aver})$ which is a lower complexity than $O(N^2 \cdot N_{Aver})$ when the PCGS algorithm is applied to the conventional FDE method [10] with the all elements

of transfer CFR matrix. The order of complexity ratio between TDE with the inverse matrix calculation $O(N^3)$ and proposed TDE with the iterative method is evaluated by the following equation.

$$R_c = \frac{2S \cdot N \cdot N_{Aver}}{N^3} = \frac{2S \cdot (N_{Aver})}{N^2} \qquad (26)$$

where N_{Aver} is the average number of required iterations which satisfies the predetermined threshold level of TOL. The average number of required iterations may depend on the predetermined threshold level TOL, mobile environments and operating carrier to noise power ratio (C/N), which are evaluated by computer simulations in the next section.

4. PERFORMANCE EVALUATIONS

Various computer simulations were conducted to evaluate the performance of the proposed iterative based TDE method and compare it with a conventional One-Tap MMSE-FDE method in highly time-varying fading channels. The simulation pa- rameters used in the following evaluations are listed in Table 1. The total numbers of subcarriers (FFT/IFFT points) is N=128, the number of data subcarriers (DFT/IDFT points) is M=96, and the number of null subcarriers at both ends of data sub-carriers (zero padding) is N M= 32. The communications channel was modelled as a Rician multipath f ading channel that is usually experienced by a user in high speed vehicles or trains [7]. In the following evaluations, the normalized Doppler frequency $R_D = f_{dmax}/\Delta f$ (%) was employed as a measure of mobile conditions where f_{dmax} is the maximum Doppler frequency and Δf is the subcarrier spacing of DFTS-OFDM signal.

The estimation accuracy for the time domain CIR at every sampling time was evaluated using a normalized mean square error (NMSE) which can be expressed as,

$$\Psi_{NMSE} = \frac{\sum_{l=0}^{L-1} \sum_{m=0}^{N_s-1} \sum_{n_2=0}^{N+2S-1} \left| \hat{h}_l(m, m_2) - h_l(m, m_2) \right|^2}{\sum_{l=0}^{L-1} \sum_{m=0}^{N_s-1} \sum_{n_2=0}^{N+2S-1} \left| h_l(m, n_2) \right|^2}$$

$$(27)$$

where N_S is the number of data symbols per one DFTS-OFDM frame.

Fig. 4 shows the time domain CIR estimation accuracy at every sampling time, which was evaluated using the normalized mean square error (NMSE) given in (27) for the proposed CIR estimation method of using the time domain TS when changing the normalized Doppler frequency R_D and operational C/N. From the figure, it can be observed that the proposed CIR estimation method using the time domain TS signal can keep higher estimation accuracy even at lower operating C/N values when the R_D is as

Table 1: *Simulation parameters.*

Parameters	Values
No. of FFT/IFFT points (N)	128
No. of DFT/IDFT points (M)	96
No. od data subcarriers (M)	96
No. of zero padding $(N-M)$	32
Length of GI for One-Tap MMSE-FDE	16
Length of training sequence (S)	16
Modulation for training sequence (TS)	16QAM
Modulation for data information	16QAM
No. of symbols per one frame (N_S)	33
Allocated frequency bandwidth	1MHz
Radio frequency	5.9GHz
Forward Error Correction (FEC) Codec [14]	
Encoding	Convolution
FEC rate	1/2
Constraint length	7
Decoding	Viterbi with hard decision
Interleaver	Matrix size with one frame
Rician multipath fading channel model	
Rice factor (K)	6dB
Delay profile for Rayleigh fading	Exponential
Decay constant	-1dB
No. of delay paths (L)	14
No. of scattered rays per one delay path	20

much as 20%, which corresponds to a vehicle speed of 381km/hr.

Fig. 5 shows the BER performance of the proposed TDE method using the direct inverse matrix calculation and the proposed iterative method with the PCGS algorithm when changing the threshold level (TOL) of $NMSE$ at the normalized Doppler frequency R_D=15%, which corresponds to a vehicle speed of 286 km/hrs.

Here, it should be noted that all the BER performance values in Fig. 5 were evaluated by performing 5 iterations regardless of TOL. From Fig. 5, it can be seen that the BER performance of the proposed iterative method can achieve almost the same level as that for the direct inverse matrix calculation when the TOL is less than 2×10^{-2}. From the figure, it can also be seen that the proposed iterative method can achieve almost the same BER as that for the direct inverse matrix calculation after performing 5 iterations even at $TOL = 10^{-3}$, which would r equire more iterations to satisfy system when the $TOL = 10^{-3}$ than $TOL = 2 \times 10^{-2}$. From the above results, this approach employed $N_{max} = 5$ and a threshold $TOL = 2 \times 10^{-2}$ in the following evaluations.

Fig. 6 shows the average number of iterations N_{Aver} when N_{max} and TOL were fixed at 5 and 2×10^{-2} based on the results in Fig. 5 and a changing normalized Doppler frequency (R_D). From the figure, it can be observed that the average number of iterations N_{Aver} became larger with an increasingly normalized Doppler frequency, R_D and decreasing the operational C/N.

From Fig. 6, it can also be observed that the average number of iterations N_{Aver} required to satisfy
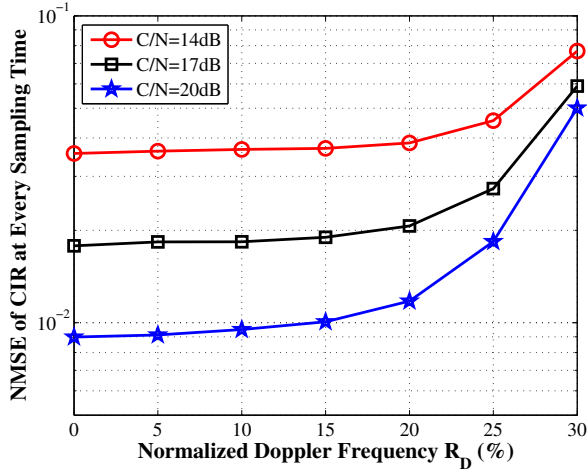
Fig.4: *CIR estimation accuracy when changing R_D and operation C/N.*
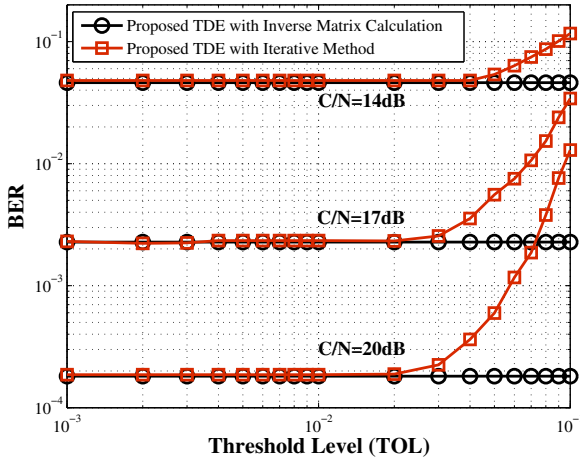

Fig.5: *BER performance for proposed iterative based TDE method when changing TOL and C/N at R_D=15%.*

$TOL = 2 \times 10^{-2}$ was always less than 3.5 even at lower C/N ratios and higher R_D=15%. From Figs. 5 and 6, it can be concluded that the proposed iterative method can achieve almost the same BER as that for an inverse matrix calculation while keeping N_{Aver} lower than N_{max}=5 which provided for a reduction in computational complexity.

Table 2 shows the order of complexity ratio R_C defined in (26). The order of computational complexity for the proposed iterative based TDE method and the inverse matrix calculation method were evaluated as $O(2S \cdot N \cdot N_{Aver})$ and $O(N^3)$, respectively. From this, the orders of complexities of both methods are highly depending on the size of parameter N. From the table, it can be observed that the average number of iterations for the proposed method decreased with increasing operational C/N. This is the reason that

a larger number of iterations is required to satisfy the predetermined small TOL at a lower C/N. However, the average number of iterations NAver was always less than 3.5 for all operational C/N ratios and R_D. From the results in Table 2, it can be concluded that the proposed iterative based TDE method can reduce computational complexity by about 150 times compared to the inverse matrix calculation. Here, it should be noted that the proposed iterative based TDE method can achieve much lower computational complexity, especially when employing larger values of N.
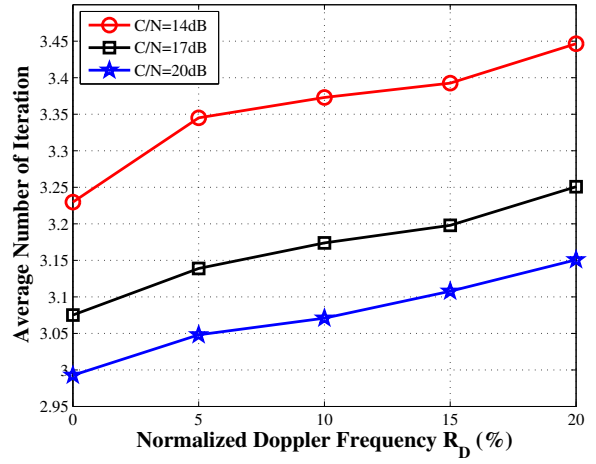

Fig.6: *Average number of required iterations for proposed iterative based TDE method when changing R_D and C/N at TOL= 0.02.*

Table 2: *Ratio of computation complexity for proposed iterative based TDE method.*

C/N	Proposed iterative based TDE method ($N = 128, S = 16, TOL = 0.02$)			
	$R_D(= f_{dmax}/\Delta f) = 5\%$		$R_D(= f_{dmax}/\Delta f) = 15\%$	
	N_{Aver}	Complexity ratio R_C in (26)	N_{Aver}	Complexity ratio R_C in (26)
14dB	3.34	0.0065	3.39	0.0066
17dB	3.13	0.0061	3.19	0.0062
20dB	3.04	0.0059	3.11	0.0061

Fig. 7 shows the BER performance when changing the normalized Doppler frequency R_D at C/N=20dB for a conventional One-Tap MMSE-FDE and the proposed TDE using both the inverse matrix calculation and proposed iterative methods. From the figure, it can be seen that the proposed iterative based TDE method shows a much better BER than the conventional One-Tap MMSE-FDE method, especially when the normalized Doppler frequency R_D is higher. The proposed iterative based TDE method shows almost the same BER as the inverse matrix calculation method with much lower complexity, even when the normalized Doppler frequency R_D is as high as 15%.

Fig. 8 shows the BER performance when chang-

ing C/N at R_D =15% for the conventional One-Tap MMSE-FDE method, proposed TDE with both the iterative method and the direct inverse matrix method. From the figure, it can be observed that the proposed TDE with the iterative method can achieve much better BER values than the conventional One-Tap MMSE-FDE method. It achieved almost the same BER as the proposed TDE with the inverse matrix calculation. From the results in Table 2 and Fig. 8, it can be concluded that the proposed iterative TDE can reduce computation complexity by 150 times while retaining the same BER performance as the inverse matrix calculation.
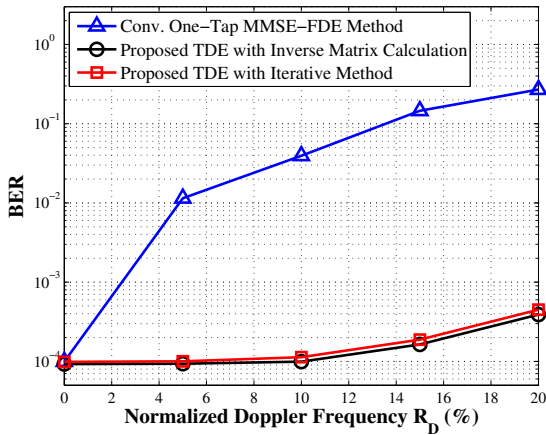
Fig.7: *BER performance for proposed iterative based TDE method when changing R_D at C/N=20dB.*
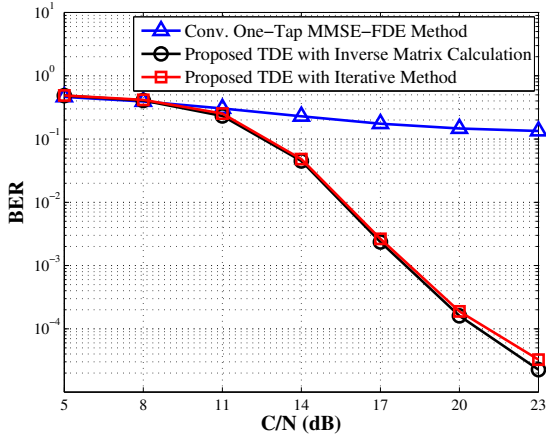
Fig.8: *BER performance for proposed iterative based TDE method when changing C/N at R_D =15%.*

From Figs. 7 and 8, it can be seen that the BER of the proposed method is not exactly equal to the conventional inverse matrix calculation. This is the reason that the proposed iterative method can reduce computation complexity while retaining almost the same BER as that for the inverse matrix calculation.

5. CONCLUSIONS

This paper proposes a low-complexity iterative based TDE method with a time domain CIR estimation for the a TS inserted DFTS-OFDM signal in highly mobile e nvironments. The proposed method employs partial differentiation in solving the ML equation in the form of a symmetric banded CIR transfer matrix. This allows the use of an iterative method for solving simultaneous equations iteratively with much lower computational complexity for the inverse matrix calculation. From the computer simulation, it can be concluded that the proposed iterative based TDE method can achieve much better BER performance than the conventional One-Tap MMSE-FDE method in highly mobile environments. It can be also concluded that the proposed method can achieve much lower computation complexity than the inverse matrix calculation while retaining almost the same BER.

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