

## Iterative IDMA Receivers with Random and Tree Based Interleavers

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### Abstract

In recent days, on the horizon of wireless world, newly proposed multiple access scheme known as Interleave-Division Multiple-Access (IDMA) has made its remarkable impact. Researchers all over world, are making hard marks to establish the scheme to establish its claim as potential candidate for 4<sup>th</sup> generation wireless communication systems. This paper is concerned with the performance enhancement of iterative IDMA systems under coded & uncoded environment. The performance of an interleave division multiple access (IDMA) system can be improved by the optimized power allocation techniques. Based on the optimized power allocation technique we compare the performance of coded & uncoded IDMA system with random interleaver & tree based interleaver. During the simulation, it has been observed that tree based interleaver demonstrate the similar bit error rate (BER) performance to that of random interleaver however on other fronts including bandwidth and memory requirement at transmitter and receiver ends, it outperforms the random interleavers.

**Keywords:** Tree Based Interleaver, Random Interleaver, IDMA, linear programming, power allocation, BER.

### 1. Introduction

The well established Direct-Sequence code-division multiple-access (DS-CDMA) which is also known as simply CDMA has been adopted in second and third generation cellular mobile standards. The CDMA scheme possesses many attractive features such as dynamic channel sharing, mitigation of cross-cell interference, asynchronous transmission, ease of cell planning, and robustness against fading.

In CDMA system, many users share the same transmission media so that signals from different users are superimposed, causing the multiple access interference (MAI) problems. At the receiver side, it is necessary to separate the mixed signals. Multi-user detection (MUD) is a technique to improve performance by jointly processing the signals forms all of the users.

Recently, it has been shown that multiuser detection together with unequal power control can enhance the performance of code division multiple access (CDMA) systems. The complexity and computational cost is the major problem of CDMA systems.

Interleave Division Multiple Access (IDMA) scheme in which interleavers are employed as the only means of user separation. Interleave division multiple access (IDMA) is a special case of random waveform CDMA, and the accompanying chip-by-chip (CBC) estimation algorithm is essentially a low cost iterative soft cancellation technique [Ping L. & Lihai L. (2004)].

The chip-by-chip (CBC) MUD algorithm for IDMA is an iterative soft cancellation technique for treating multiple access interference (MAI). Its computational cost is very low, being independent of the total number of users when normalized to each user [Wang P., Ping L. & Liu L. (2006)].

With equal power control, it has been shown in [Liu L., Tong J. & Ping L. (2006)] that IDMA together with CBC algorithm can achieve performance close to the theoretical limits. Unequal power allocation is a technique to further enhance system performance. In [3], a linear programming technique is employed for

power optimization of IDMA systems over additive white Gaussian noise (AWGN) multiple access channels (MACs).

In this paper, we use the unequal optimized power allocation technique to the uncoded-IDMA system with random interleavers & tree based interleavers. After that we compare their performance and reach to the conclusion.

Section 2 contains an introduction of the IDMA communication system. In section 3, we discussed about tree based interleaver. In section 4 we discussed the power allocation techniques in brief. Section 5 presents the simulation results of uncoded-IDMA with random interleaver and tree based interleaver. Section 6 concludes the paper.

## 2. Iterative IDMA System

### 2.1. System Model

Consider an IDMA system with  $k$  users. At the transmitter side, the information bits for user  $k$  are first encoded by an encoder (ENC) based on a FEC code and then interleaved and transmitted over a Gaussian multiple access channel (MAC). The received signal can be written as;

$$r(j) = \sum_{k=1}^K h_k x_k(j) + n(j), \quad j = 1, 2, \dots, J \quad (1)$$

Where  $x_k(j) \in \{+1, -1\}$  is the  $j$ th chip transmitted by user- $k$ ,  $h_k$  the coefficient for user- $k$  representing the combined effect of power control and channel loss, and  $\{n(j)\}$  are samples of an AWGN process with zero-mean and variance  $\sigma^2 = N_0 / 2$  [4].

The receiver consists of an elementary signal estimator (ESE) and  $k$  a posteriori probability (APP) decoders (DECs), operating iteratively.

For each user  $k$ , we rewrite (1) as;

$$r(j) = h_k x_k(j) + \xi_k(j) \quad (2)$$

where,

$$\xi_k(j) = \sum_{k' \neq k} h_{k'} x_{k'}(j) + n(j) \quad (3)$$

### 2.2 The CBC Algorithm

$$E(x_k(j)) = \tanh(e_{DEC}(x_k(j))/2), \forall k, j \quad (4)$$

$$\text{Var}(x_k(j)) = 1 - (E(x_k(j)))^2, \forall k, j \quad (5)$$

$$E(\xi_k(j)) = \sum h_{k'} x_{k'}(j), \forall k, j \quad (6)$$

$$\text{Var}(\xi_k(j)) = \sum |h_{k'}|^2 \text{Var}(x_{k'}(j)) + \sigma^2, \forall k, j \quad (7)$$

$$e_{ESE} = \{2h_k / \text{Var}(\xi_k(j))\} (r(j) - E(\xi_k(j))), \forall k, j \quad (8)$$

After (8), the APP decoding in the DECs is performed to generate the LLRs  $\{e_{DEC}(x_k(j)), \forall k, j\}$ . Then go back to (4) for the next iteration.

## 3. Tree Based Interleaving Mechanism

In iterative IDMA system each user has a specific interleaver  $\{\pi_k\}$  having length equal to chip length 'J'.

A considerable amount of memory will be required to store the indexes for these interleavers. The tree based interleaver is basically aimed to minimize the computational complexity and memory requirement

that occurs in power interleaver & random interleavers respectively [Shukla M., Srivastava V.K. & Tiwari S. (2009)].

In a tree based interleaver generation, two randomly generated interleaver are chosen. Let  $\pi_1$  and  $\pi_2$  be the two random interleavers. The combinations of these two interleavers in a particular fashion as shown in the figure 2 are used as interleaving masks for the users [Shukla M., Srivastava V.K. & Tiwari S. (2009)].

The allocations of the interleaving masks follow the binary tree format. The interleaving masking diagram is shown upon 14 users for simplicity. It is clearly shown through the figure that, for obtaining the interleaving sequence of the 14<sup>th</sup> user, it need only 2 cycles, as in [Shukla M., Srivastava V.K. & Tiwari S. (2009)].

$$\pi_{14} = \pi_2 (\pi_2 (\pi_2))$$

#### 4. Power Allocation Strategy for Users

This section is concerned with the power optimization technique for IDMA systems over AWGN channels.

##### 4.1 Problem Formulation:

For simplicity, we assume that, for all users, the same FEC code is used and the same BER performance is required. Let  $\{\gamma_k^{(l)}\}$  be the average SNIR for the outputs of the ESE after the l-th iteration &  $f_k(\gamma_k^{(l)})$  be the average variance of the outputs of  $DEC_k$  driven by an input sequence with SNIR  $\gamma_k^{(l)}$  [Shukla M., Srivastava V.K. & Tiwari S. (2006), Shukla M., Srivastava V.K. & Tiwari S. (2009)].

We can approximately track  $\gamma_k^{(l)}$  using the following recursion;

$$\gamma_k^{(l+1)} = \frac{P_k |h_k|^2}{\sum_{i \neq k} P_i |h_i|^2 f_i(\gamma_i^{(l)}) + \sigma^2}, \forall k, l = 0, 1, \dots, (L-1) \quad (10)$$

where L is the maximum number of iterations.

For AWGN channels, we have,

$$|h_k|^2 = 1, \forall k$$

Then recursion (10) reduces to,

$$\gamma_k^{(l+1)} = \frac{P_k}{\sum_{i \neq k} P_i f_i(\gamma_i^{(l)}) + \sigma^2}, \forall k, l = 0, 1, \dots, (L-1) \quad (11)$$

Our objective now is to minimize total power  $\sum_k P_k$  while achieving the required performance  $\gamma_k^{(l)} \geq \Gamma, \forall k$ , for specified  $\Gamma$  [2]. This power optimization problem can be formulated as follows.

##### 4.2 Power optimization over AWGN channels:

Find the distribution  $\{P_k\}$  that minimizes,

$$\Phi = \sum_k P_k \quad (12)$$

subject to,

$$\gamma_k^{(l)} \geq \Gamma, \forall k \quad (13)$$

where  $\{\gamma_k^{(l)}\}$  are obtained through the following SNIR evolution process with initialization  $\gamma_k^{(0)} = 0$ , for all k.

$$\gamma_k^{(l+1)} = \frac{P_k}{\sum_{i \neq k} P_i f_i(\gamma_i^{(l)}) + \sigma^2}, \forall k, l = 0, 1, \dots, (L-1) \quad (14)$$

Problem (14) is generally nonlinear and non-convex. The problem considered above is to find the minimum sum-power solution within the feasible region [Shukla M., Srivastava V.K. & Tiwari S. (2009)]. We define the total interference after the  $l$ -th iteration of recursion (11) as,

$$I^{(l)} = \sum_{k=1} P_k f_k(\gamma_k^{(l)}) + \sigma^2 \quad (15)$$

Then (11) can be rewritten as

$$\gamma_k^{(l+1)} = \frac{P_k}{I^{(l)} - P_k f_k(\gamma_k^{(l)})}, \forall k, l = 0, 1, \dots, (L-1) \quad (16)$$

### B. Linear Programming Approach:

In problem (12), the constraints (13) & (14) are nonlinear with respect to the optimization variables  $\{P_k\}$ , which makes the problem complicated. We therefore consider a linearization technique to solve this problem.

We quantize the power levels  $\{P_k\}$  into  $(M+1)$  discrete levels :  $\{P(m), m=0, 1, \dots, M\}$  with  $P(m-1) < P(m), m=1, 2, \dots, M$ , and partition the  $k$  users into  $(M+1)$  groups according to their power levels. Letting  $\lambda(m)$  denote the number of users assigned with power level  $P(m)$  [Shukla M., Srivastava V.K. & Tiwari S. (2009)].

Then total power (12) is rewritten as

$$\phi = \sum_m \lambda(m) P(m) \quad (17)$$

and the number of users  $k$  can be represented as,

$$k = \sum_m \lambda(m) \quad (18)$$

$\gamma^{(l)}(m)$  be the SNIR of the ESE outputs for the users in the group with power  $P(m)$  after the  $l$ -th iteration and assume that user- $k$  has power  $P(m)$ . Then (15) & (16) can be rewritten as,

$$I^{(l)} = \sum_m \lambda(m) P(m) f(\gamma^{(l)}(m)) + \sigma^2 \quad (19)$$

$$\gamma^{(l+1)}(m) = \frac{P(m)}{I^{(l)} - P(m) f(\gamma^{(l)}(m))}, \forall k, l = 0, 1, \dots, (L-1) \quad (20)$$

Since  $\{I^{(l)}\}$  is a monotonically decreasing sequence, i.e.,

$$I^{(l+1)} \leq I^{(l)}, \quad l=0, 1, \dots, (L-1) \quad (21)$$

Substituting (21) into (20) and introducing a delay factor  $\delta (0 < \delta < 1)$  to control the convergence speed of the iterative detection, we get,

$$\sum_m \lambda(m) P(m) f(\gamma^{(l)}(m)) + \sigma^2 \leq (1 - \delta) I^{(l)} \quad (22)$$

Now we take  $\{\lambda(m)\}_{l=0, 1, \dots, (L-1)}$  as optimization variables. In this case both the target function (17) and the constraints (18) & (22) appear linear with respect to  $\{\lambda(m)\}$ . However  $\gamma^{(l+1)}(m)$  in (20) is dependent on  $\{\lambda(m)\}$ . Now,  $\gamma^{(l+1)}(m)$  can be treated as pre-calculated parameter when linear programming is applied.

Using an approximation,

$$\gamma^{(l+1)}(m) \approx \gamma^*(m) = \frac{P(m)}{I^{(l)}} \quad (23)$$

\* denotes approximation.

We can rewrite (22) as

$$\sum_m \lambda(m)P(m)f\left(\frac{P(m)}{I}\right) + \sigma^2 \leq (1-\delta)I, I_{\min} \leq I \leq I_{\max} \quad (24)$$

where  $I_{\max}$  &  $I_{\min}$  specify the total interference at the beginning and end of the iterative detection and the iteration index  $l$  for  $I^{(l)}$  is omitted. We evaluate (24) at a set of quantized values of  $I: \{I(n), n=0,1,..N\}$ . By pre-calculating  $\{f_{m,n} = f(P(m)/I(n))\}$  and using  $\{f_{m,n}\}$  as constants, we obtain a group of linear constraints with respect to  $\{\lambda(m)\}$  [2].

In summary, we have

$$\phi = \sum_m \lambda(m)P(m) \quad (25)$$

subject to

$$\sum_m \lambda(m) = k \quad (26)$$

$$\lambda(m) \geq 0, m = 0, 1, \dots, M \quad (27)$$

$$\sum_m \lambda(m)P(m)f\left(\frac{P(m)}{I}\right) + \sigma^2 \leq (1-\delta)I, I_{\min} \leq I \leq I_{\max} \quad (28)$$

Let  $y(m)$  denotes the total power of these  $\lambda(m)$  users. As such,

$$y(m) = \lambda(m)P(m) \quad (29)$$

Substituting (29) into (25),(27) & (28) we get

$$\phi = \sum_m y(m) \quad (30)$$

subject to,

$$\sum_m y(m)f\left(\frac{P(m)}{I}\right) + \sigma^2 \leq (1-\delta)I, I_{\min} \leq I \leq I_{\max} \quad (31)$$

$$y(m) \geq 0, m = 0, 1, \dots, M. \quad (32)$$

In practice in [1], both  $P$  and  $I$  should be quantized. We need to determine the searching ranges  $P_{\min}, P_{\max}, I_{\min}$  &  $I_{\max}$ . Let us quantize  $P$  and  $I$  as

$$P(m) = \alpha^m P_{\min}, m = 0, 1, \dots, M \quad (33)$$

$$I(n) = \beta^n I_{\min}, n = 0, 1, \dots, N \quad (34)$$

with  $P(0) = P_{\min} > 0, P(M) = P_{\max}, I(0) = I_{\min} > 0, I(N) = I_{\max}, \alpha > 1$  and  $\beta > 1$ .

Then (31) becomes,

$$\sum_m y(m)f\left(\alpha^m \beta^{-n} \gamma\right) + \sigma^2 \leq (1-\delta)\beta^n I_{\min} \quad (35)$$

$$\text{Since } \gamma = P_{\min} / I_{\min} \quad (36)$$

The minimum SNR is obtained at the end of the iterative decoding process. Here  $\gamma$  can be determined by the desired BER. For more details about the relation between  $\gamma$  and BER see in [1]. Since

$$k = \sum_m \lambda(m) = \sum_m \frac{y(m)}{P(m)} = \sum_m \frac{y(m)\alpha^{-m}}{\gamma I_{\min}} \quad (37)$$

We get,

$$\sum_m y(m)\alpha^{-m} - k\gamma I_{\min} = 0 \quad (38)$$

We can replace (31) by (35). We can also include (38) as part of the LP constraints. In this way  $I_{\min}$  becomes an optimization variable and  $P_{\min}$  can be determined from  $I_{\min}$  by (36) [1].

We still need to determine M and N in (33) & (34). In general, we just use a sufficient large M, since the LP will automatically determine the maximum power level used. We start with a relatively small N.

If after the LP,

$$I_{\max} = \sum_m y(m) \times 1 + \sigma^2 > I(N) \quad (39)$$

then the quantization range in (34) is not sufficient and we increase N and repeat the LP until  $I_{\max} \leq I(N)$ .

## 5. Simulation Results

The power profiles for 32, 48, 64 users are given below in the Table 1.

As we see that the performance improves with increased data lengths. We can also see that a data length of 512 is sufficient to achieve relatively good performance. Performance enhancement is marginal with information block longer than 512.

If user count is  $k \leq 32$ , then fewer than 20 iterations are sufficient (measured at  $10^{-4}$ ) and more iterations do not bring about significant performance improvement. However, when user count is  $K \geq 48$ , more iteration can be beneficial [Shukla M., Srivastava V.K. & Tiwari S. (2008), Shukla M., Srivastava V.K. & Tiwari S. (2006)].

## 6. Conclusion

As we see from the simulation results that in unequal power allocation technique with Tree based interleaver based uncoded and coded IDMA system gives the better results in comparison to random interleaver based IDMA system.

So using Tree based interleaver in IDMA system with unequal optimized power allocation technique will enhance the performance in comparison to random interleaver and also reduce the memory cost & computational complexity.

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Table 1  
 Power Profile For 32, 48 & 64 Users

	(power level(dB)) $\times$ (user number)
K=32	0 $\times$ 25, 5.3811 $\times$ 7
K=48	0 $\times$ 26, 7.4509 $\times$ 8, 10.3484 $\times$ 8, 10.7623 $\times$ 6
K=64	0 $\times$ 25, 7.8645 $\times$ 7, 8.2784 $\times$ 7, 13.2455 $\times$ 5 13.6594 $\times$ 7, 18.6266 $\times$ 13

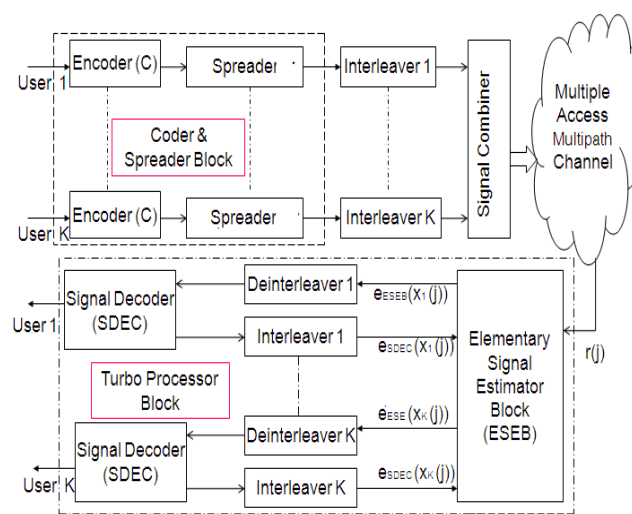


Figure 1. (a) The transmitter for user-k. (b) A part of the receiver related to user-k.

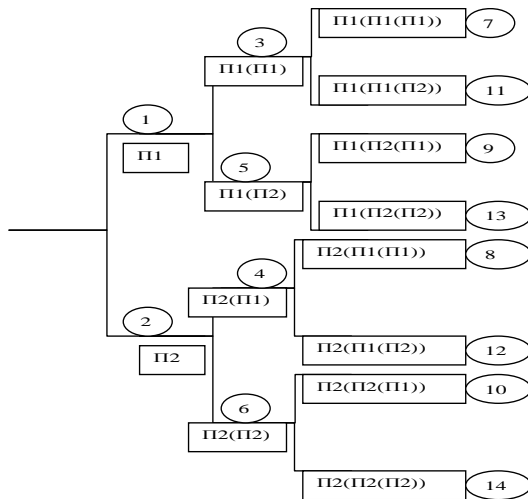


Figure 2. Interleaving mask allocation for the proposed Tree based interleaving scheme

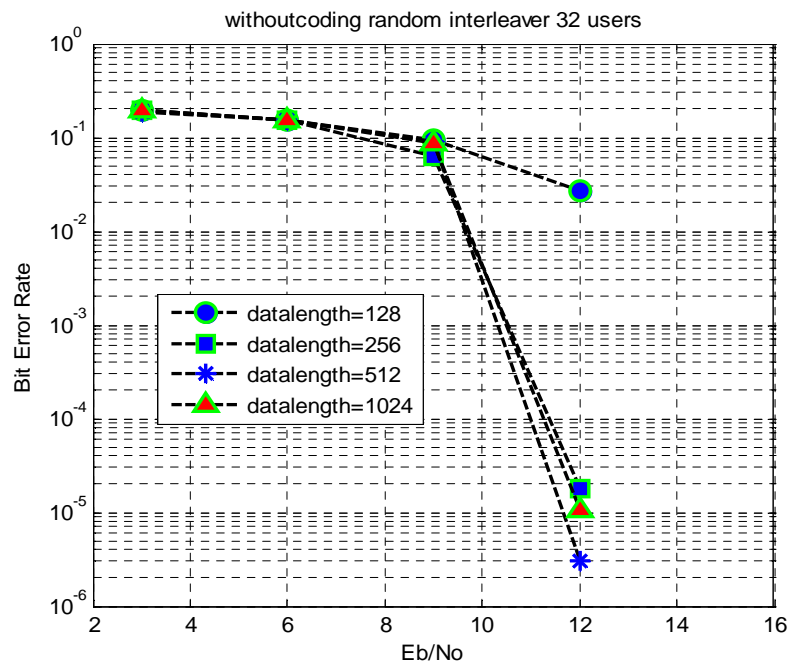


Figure 3 (a). 32 users Random Interleaver



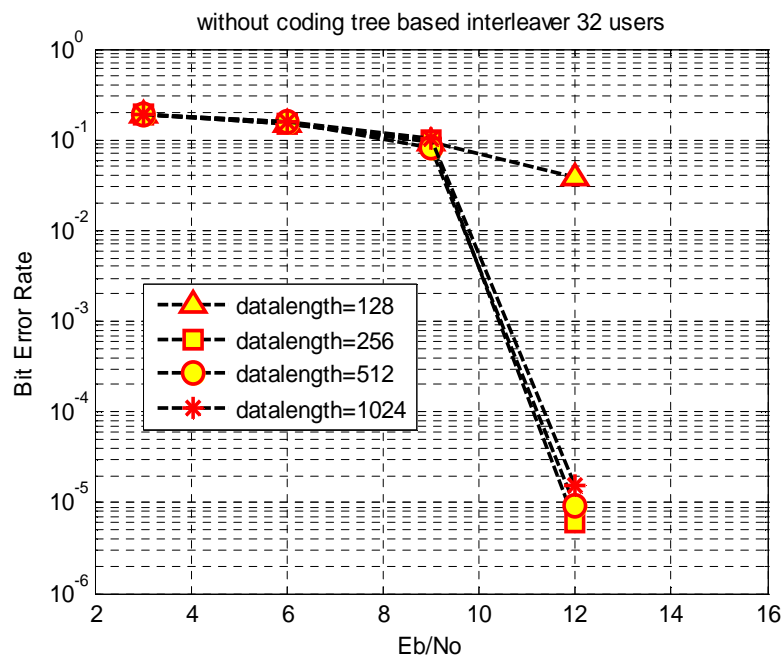


Figure 3 (b). 32 users Tree Based Interleaver

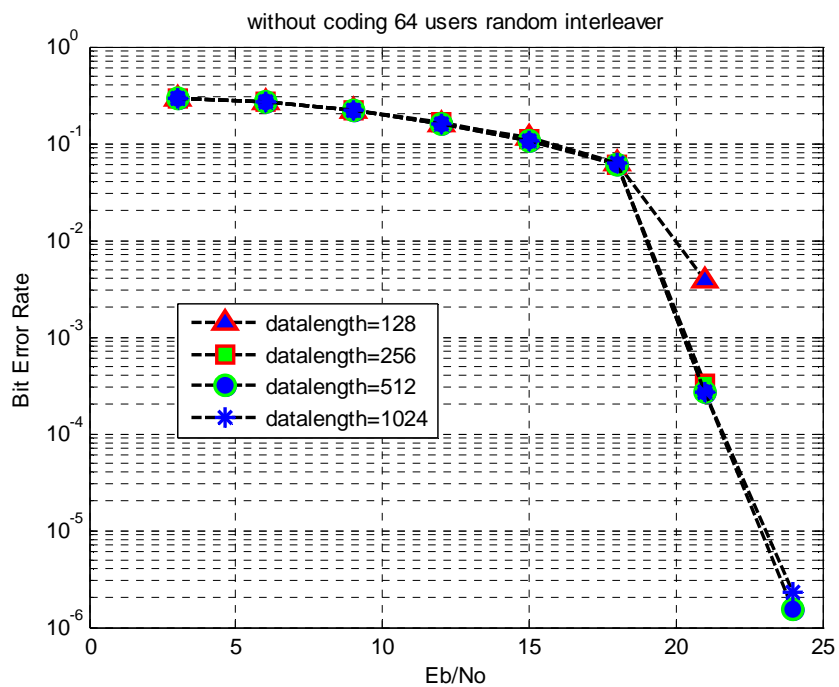


Figure 4 (a). 64 users Random Interleaver

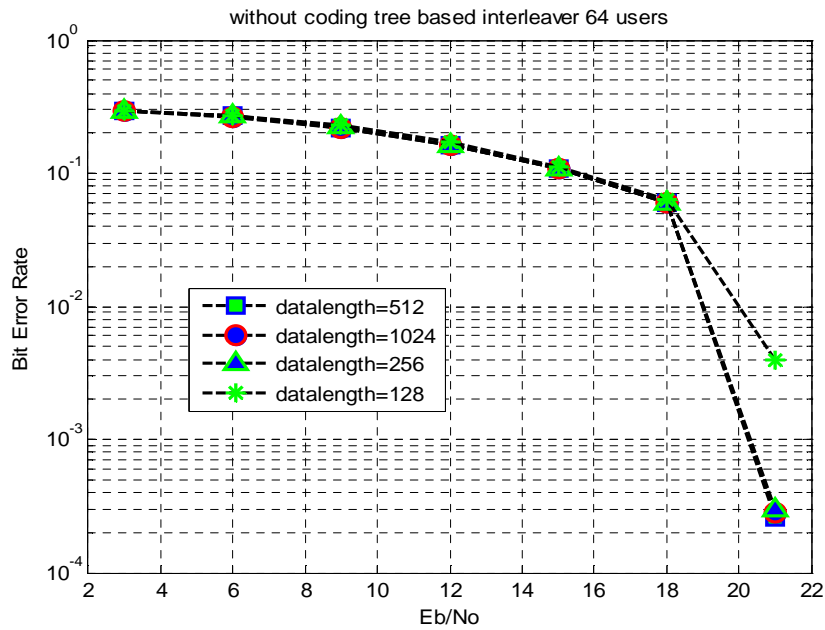


Figure 4 (b). 64 users Tree Based Interleaver

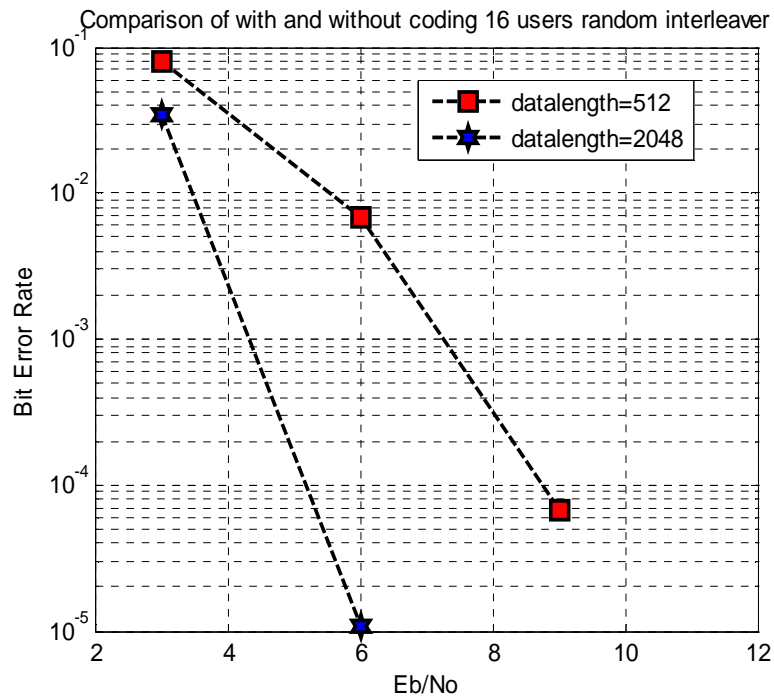


Figure 5 (a). 16 users Random Interleaver with and without coding

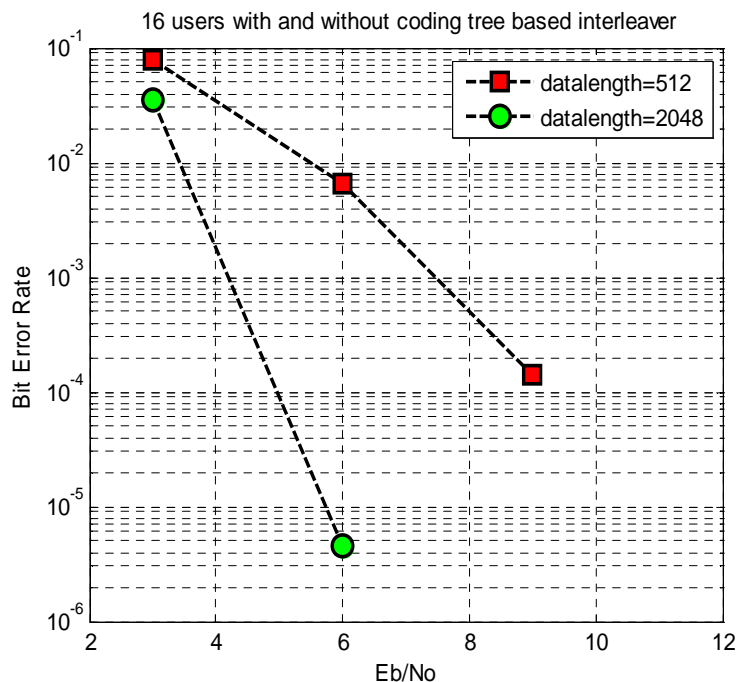


Figure 5 (b). 16 users Tree-Based Interleaver with and without Coding

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