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Iterative Learning Based Accumulative Disturbance Observer for Repetitive Systems via a Virtual Linear Data Model

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ABSTRACT This work explores the problem of observing nonrepetitive disturbances under an almost data-driven framework. First, a linear data model among the inputs, states, and outputs is built for a repetitive system (linear or nonlinear) between two consecutive iterations where the nonrepetitive disturbances are accumulated along time axis as a total one. The accumulative disturbance contains all the influences on the system states or outputs caused by the disturbances from the initial time instant to the current time instant between two consecutive iterations. Furthermore, an iterative updating algorithm is designed to estimate the gradient matrix in the derived linear data model. Subsequently, iterative learning-based accumulative disturbance observer (ILADOB) is proposed employing the state information in the iteration domain when the system states are measurable; otherwise, when the states are immeasurable, an output-based ILADOB is presented as an alternative. The proposed two ILADOB methods are executed along the iteration direction all over the finite time interval pointwisely using the system data from preceding trials. The convergence and stability are proved mathematically. The simulation study confirms the validity of the state-based and output-based ILADOB methods.

INDEX TERMS Accumulate disturbance observer, iterative learning, linear data dynamic relationship, repetitive systems, almost data-driven design and analysis.

I. INTRODUCTION

In industrial applications, the existence of disturbances is inevitable. System disturbances include not only nonlinear disturbances, time delays, sensor measurement noises, but also external disturbances and unknown disturbance inputs [1]–[3]. The disturbances in the system may affect the control performance seriously. To solve this problem with unmatched uncertain systems, one can design a disturbance observer (DOB) for the estimation of uncertainties which are then incorporated into the controller to compensate their influences on the control performance [4]. In [5], a DOB-based control method for phase compensation is proposed to reduce the influence of inertial variation on motion control. In [6], a communication disturbance observer is used for compensation of influences caused by time-varying delays in network-based control systems. In [7], a sliding mode DOB is added to enhance control performance of the aircraft attitude control and to decrease the influence from disturbances such as wind and collision on the aircraft.

Currently, linear disturbance observers have been well developed with many theoretical results and practical applications. In [8], a frequency domain disturbance observer is designed by using an inverse model. In [9], a reduced order DOB is proposed where the control system is not

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required to be fully observable. Aiming at the time-delay problem, an uncertainty and disturbance estimator [10] is proposed to estimate the disturbances and uncertainties. In [11], an unknown input observer is used in the disturbance accommodation control to deal with external disturbance. Generalized proportional integral observer is proposed to estimate time-varying disturbance and uncertainties [12] where more disturbance information is used to increase estimation accuracy. On the other hand, nonlinear systems are more popular than the linear ones in practical applications. Therefore, some DOB methods have also been explored in recent years [13]–[16]. In [15], a high order DOB is proposed to deal with high order disturbances. In [16], an extended high-gain state observer is developed to estimate disturbances in nonlinear systems.

It is worth pointing out that most of the DOB methods [8]–[16], no matter linear or nonlinear, require the known model information as a priori. On the other words, these methods [8]–[16] are model-based and depend on an explicit model of the practical systems. However, it is actually too difficult to model a control plant using physical-chemical principle or identification methods because the practical processes are becoming more and more complex with increasing large scales. Therefore, the above model-based observers [8]–[16] may encounter challenges and difficulties when applied into practical problems, and thus data-driven approaches in modeling, control, optimization has become much more popular [17]–[19].

On the other hand, many practical systems are repetitive operating with a fixed time length. For example, many industrial robots repetitively execute an identical operation task [20], and high-speed trains travel repeatedly along a fixed pathway within a finite time interval [21]. Other examples include traffic systems [22], batch processes [23], [24], multi-agent systems [25], and so on. For such a repetitive system, iterative learning control [20]–[25] is most effective in perfect tracking using the control information in the preceding trials. ILC method is proposed for nonlinear systems originally where only the bound of the direct transmission term is needed for the controller design and implementation. And thus, ILC requires very little model information and is also called a ‘‘model-free’’ or ‘‘data-driven’’ method sometimes [26].

So, can we introduce the idea of ILC into the repetitive systems to solve the design problem of DOB bypassing the modeling steps? Recently, several works about state observer/estimation have been done for repetitive systems. In [27], a finite time observer is designed using the known nonlinear model. In [28], a Luenberger observer is designed for both state and output of the linear repetitive systems. In [29], an iterative learning observer is proposed along the iteration direction for linear time-invariant systems. In [30]–[32] it is assumed that the bounds of the disturbances exist and then a robust analysis of the ILC problem is provided. To the best of the authors knowledge, there is no existing work in designing a disturbance observer based on iterative learning for repetitive systems.

In this work, an iterative learning based accumulative disturbance observer (ILADOB) is proposed for repetitive systems. The design and analysis start from a linear discrete-time system where almost all the coefficient matrices are unknown, to a nonlinear discrete-time system in which both the nonlinear function and the system order are completely unknown. At first, a linear data dynamic model is developed among the inputs, states and outputs for a repetitive system. The linear data model includes an unknown Jacobian/gradient matrix as well as a total accumulative disturbance containing all the influences on system states and outputs caused by the disturbances from the initial time instant to the current time instant between two consecutive iterations.

Aiming to estimate the expanded disturbance in the derived linear data model, two ILADOB methods are presented in the iteration domain using the measurable system states and system outputs respectively where a parameter iterative updating algorithm is included for the estimation of the gradient matrix. The two proposed ILADOB methods are conducted in iteration domain pointwisely using the data of the system states, outputs and control inputs from preceding trials. The convergence and stability of the developed ILADOBs are addressed with rigorous analysis. Simulations further demonstrate the validity of the two ILADOB methods.

The rest of this paper is organized bellow. Section 2 designs an iterative learning-based disturbance observer for a linear repetitive system with analysis. Section 3 extends the results to nonlinear repetitive systems. The simulation study is proposed in Section 4. Finally, Section 5 concludes this paper.

II. ILADOB FOR LINEAR TIME-VARYING SYSTEMS

A. PROBLEM FORMULATION AND LINEAR DATA MODEL

A repetitive linear time-varying system is considered as follows,

$$\begin{cases} \boldsymbol{x}_k(t+1) = \boldsymbol{A}(t)\boldsymbol{x}_k(t) + \boldsymbol{B}(t)\boldsymbol{u}_k(t) + \boldsymbol{d}_k(t) \\ \boldsymbol{y}_k(t) = \boldsymbol{C}(t)\boldsymbol{x}_k(t) \end{cases} \tag{1}$$

where $\boldsymbol{x}_k(t) \in \boldsymbol{R}^n$, $\boldsymbol{u}_k(t) \in \boldsymbol{R}^l$, and $\boldsymbol{y}_k(t) \in \boldsymbol{R}^m$ are the system state, input and output respectively; $\boldsymbol{d}_k(t) \in \boldsymbol{R}^n$ is the system disturbance; $t \in \{0, \ldots, N\}$, and N denotes the terminal time instant of the repetitive system; k denotes the iteration number. The system matrix $\boldsymbol{A}(t) \in \boldsymbol{R}^{n\times n}$, $\boldsymbol{B}(t) \in \boldsymbol{R}^{n\times l}$ are unknown but bounded, and $\boldsymbol{C}(t) \in \boldsymbol{R}^{m\times n}$ is known and bounded, that is, $\|\boldsymbol{A}(t)\| \leq b_A$, $\|\boldsymbol{B}(t)\| \leq b_B$, $\|\boldsymbol{C}(t)\| \leq b_C$, where b_A, b_B, b_C are positive constants.

Two assumptions about the system (1) are made, shown as follows.

Assumption 1: The system initial state $\boldsymbol{x}_k(0)$ is invariant, that is, $\boldsymbol{x}_k(0) = \boldsymbol{x}_0$, where $\boldsymbol{x}_0$ is a constant matrix.

Assumption 2: The disturbance $\boldsymbol{d}_k(t)$ is bounded, i.e., $\|\boldsymbol{d}_k(t)\| \leq b_d$, where b_d is a constant.

According to [26], system (1) is transformed as,

$$\begin{cases} \boldsymbol{x}_k(t+1) = \prod_{i=0}^{t} \boldsymbol{A}(i)\boldsymbol{x}_k(0) \\ \qquad + \sum_{i=1}^{t+1}\prod_{j=1}^{i-1} \boldsymbol{A}(t+1-j)\boldsymbol{B}(t+1-i)\boldsymbol{u}_k(t+1-i) \\ \qquad + \sum_{i=1}^{t+1}\prod_{j=1}^{i-1} \boldsymbol{A}(t+1-j)\boldsymbol{d}_k(t+1-i) \\ \boldsymbol{y}_k(t) = \boldsymbol{C}(t)\boldsymbol{x}_k(t) \end{cases} \tag{2}$$

Define an iteration-difference operator Δ, i.e., $\Delta \boldsymbol{x}_k(t) = \boldsymbol{x}_k(t) - \boldsymbol{x}_{k-1}(t)$. Then, according to Assumption 1, we have

$$\begin{cases} \Delta \boldsymbol{x}_k(t+1) \\ \quad = \sum_{i=1}^{t+1}\prod_{j=1}^{i-1} \boldsymbol{A}(t+1-j)\boldsymbol{B}(t+1-i)\Delta \boldsymbol{u}_k(t+1-i) + \boldsymbol{\delta}_k(t) \\ \boldsymbol{y}_k(t) = \boldsymbol{C}(t)\boldsymbol{x}_k(t) \end{cases} \tag{3}$$

where $\boldsymbol{\delta}_k(t)$ denotes the total disturbance to the system state,

$$\boldsymbol{\delta}_k(t) = \sum_{i=1}^{t+1}\prod_{j=1}^{i-1} \boldsymbol{A}(t+1-j)\Delta \boldsymbol{d}_k(t+1-i).$$

Define

$$\boldsymbol{\Phi}(t) = \left[\prod_{i=1}^{t} \boldsymbol{A}(i)\boldsymbol{B}(0) \; \prod_{i=2}^{t} \boldsymbol{A}(i)\boldsymbol{B}(1) \; \ldots \; \boldsymbol{A}(t)\boldsymbol{B}(t-1) \; \boldsymbol{B}(t) \right]$$

and $\boldsymbol{U}_k(t) = \left[\boldsymbol{u}_k^T(0) \; \boldsymbol{u}_k^T(1) \; \ldots \; \boldsymbol{u}_k^T(t) \right]^T$.

Then, an alternative linear data model of system (1) is built finally as

$$\begin{cases} \boldsymbol{x}_k(t+1) = \boldsymbol{x}_{k-1}(t+1) + \boldsymbol{\Phi}(t)\Delta \boldsymbol{U}_k(t) + \boldsymbol{\delta}_k(t) \\ \boldsymbol{y}_k(t) = \boldsymbol{C}(t)\boldsymbol{x}_k(t) \end{cases} \tag{4}$$

where $\boldsymbol{\Phi}(t)$ is bounded because $\boldsymbol{A}(t)$ and $\boldsymbol{B}(t)$ are bounded.

Remark 1: According to (3), the accumulate disturbance $\boldsymbol{\delta}_k(t)$ contains both the system uncertainties (e.g., the unknown matrix $\boldsymbol{A}$) and the external disturbances together. So, its estimation is important for the subsequent controller design as a compensation of the uncertainties.

Remark 2: The linear data model (4) does not have any special physical backgrounds but describes the data relationships and virtually exists in the computer, which is most different from the traditional mechanistic model.

B. STATE-BASED ILADOB WITH MEASURABLE SYSTEM STATES

1) STATE-BASED ILADOB DESIGN

Considering system (1) where system states are measurable and using the linear data model (4), a state-based ILADOB is developed as,

$$\hat{\boldsymbol{\delta}}_k(t) = \boldsymbol{K}x_{k-1}(t+1) - z_k(t+1) \tag{5a}$$

$$z_{k+1}(t+1) = z_k(t+1) + \boldsymbol{K}(\hat{\boldsymbol{\Phi}}_k(t)\Delta \boldsymbol{U}_k(t) + \hat{\boldsymbol{\delta}}_k(t)) \tag{5b}$$

$$\hat{\boldsymbol{\Phi}}_k(t) = \hat{\boldsymbol{\Phi}}_{k-1}(t) + \frac{\eta_1(\Delta \boldsymbol{x}_{k-1}(t+1) - \hat{\boldsymbol{\Phi}}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}(t))\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_1 + \left\| \Delta \boldsymbol{U}_{k-1}^T(t) \right\|^2} \tag{5c}$$

where $\hat{\boldsymbol{\delta}}_k(t)$ is the estimation of $\boldsymbol{\delta}_k(t)$, $z_k(t) \in \boldsymbol{R}^n$ is a state variable, $\hat{\boldsymbol{\Phi}}_k(t)$ is the estimation of $\boldsymbol{\Phi}(t)$, $\mu_1 > 0$ is a weighting factor and $\eta_1 \in (0, 2)$ is a step factor; $\boldsymbol{K} = (\boldsymbol{I}_n - \boldsymbol{\Gamma}) \in \boldsymbol{R}^{n\times n}$, where $\boldsymbol{\Gamma} = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$, and $|\gamma_i| < 1, i = 1, 2, \ldots, n$, is a proper constant.

Remark 3: Different from the traditional DOB in time domain, the proposed state based ILADOB (5a)-(5c) is executed in a batch over $\{1, 2, \ldots, N\}$ along iteration direction and aims to estimate the accumulative disturbance in the linear data model (4) as a total influence to the system states.

Remark 4: It is seen from (5a)-(5c) that no model information about system (1), such as $\boldsymbol{A}(t)$ and $\boldsymbol{B}(t)$, is used at all. Therefore, proposed state based ILADOB is a data-driven method even though it is derived from linear systems. Actually, the results can be easily extended to a general unknown nonlinear system, as shown subsequently.

2) STABILITY ANALYSIS

Define $\boldsymbol{e}_k(t) = \boldsymbol{\delta}_k(t) - \hat{\boldsymbol{\delta}}_k(t)$ as the disturbance estimation error and $\tilde{\boldsymbol{\Phi}}_k(t) = \boldsymbol{\Phi}(t) - \hat{\boldsymbol{\Phi}}_k(t)$ as the parameter estimate error.

The convergence of the state-based ILADOB (5a)-(5c) is guaranteed by the theorem below.

Theorem 1: For system (1) satisfying Assumption 1-2, by properly selecting the observer parameters such that $\eta_1 \in (0, 2)$, $\mu_1 > 0$, and $|\gamma_i| < 1, i = 1, 2, \ldots, n$, the proposed state-based ILADOB (5a)-(5c) guarantees that $\hat{\boldsymbol{\Phi}}_k(t)$ is bounded and the disturbance estimate error $\boldsymbol{e}_k(t)$ converges to a bound related to the parameters b_δ, b_K, and b_γ which are defined subsequently.

Proof: Subtracting $\boldsymbol{\Phi}(t)$ from both sides of (5c), in terms of (4), we have

$$\begin{aligned} \tilde{\boldsymbol{\Phi}}_k(t) &= \tilde{\boldsymbol{\Phi}}_{k-1}(t) - \frac{\eta_1(\boldsymbol{\Phi}(t)\Delta \boldsymbol{U}_{k-1}(t) + \boldsymbol{\delta}_{k-1}(t) - \hat{\boldsymbol{\Phi}}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}(t))\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_1 + \left\| \Delta \boldsymbol{U}_{k-1}^T(t) \right\|^2} \\ &= \tilde{\boldsymbol{\Phi}}_{k-1}(t)\left(\boldsymbol{I} - \frac{\eta_1 \Delta \boldsymbol{U}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_1 + \left\| \Delta \boldsymbol{U}_{k-1}^T(t) \right\|^2} \right) \\ &\quad - \frac{\eta_1 \boldsymbol{\delta}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_1 + \left\| \Delta \boldsymbol{U}_{k-1}^T(t) \right\|^2} \end{aligned} \tag{6}$$

Since both the system matrix $\boldsymbol{A}(t)$ and the disturbance $\boldsymbol{d}_k(t)$ are bounded and the system time interval is finite, one can find a constant b_δ such that the accumulative disturbance is also bounded, i.e., $\|\boldsymbol{\delta}_k(t)\| \leq b_\delta$. In addition, $\boldsymbol{U}_k(t)$ is given bounded for the practical system identification. Then, we have,

$$\left\| \frac{\eta_1 \boldsymbol{\delta}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_1 + \left\|\Delta \boldsymbol{U}_{k-1}^T(t)\right\|^2} \right\| \leq b_1$$

where b_1 is a positive constant.

Taking norm on both sides of (6), we have,

$$\left\|\tilde{\boldsymbol{\Phi}}_k(t)\right\| \leq \left\|\tilde{\boldsymbol{\Phi}}_{k-1}(t)\left(\boldsymbol{I} - \frac{\eta_1 \Delta \boldsymbol{U}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_1 + \left\|\Delta \boldsymbol{U}_{k-1}^T(t)\right\|^2}\right)\right\| + b_1 \tag{7}$$

Consider the first item on the right of (7),

$$\begin{aligned} &\left\|\tilde{\boldsymbol{\Phi}}_{k-1}(t)\left(\boldsymbol{I} - \frac{\eta_1 \Delta \boldsymbol{U}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_1 + \left\|\Delta \boldsymbol{U}_{k-1}^T(t)\right\|^2}\right)\right\|^2 \\ &= \left\|\tilde{\boldsymbol{\Phi}}_{k-1}(t)\right\|^2 - \left(2 - \frac{\eta_1 \left\|\Delta \boldsymbol{U}_{k-1}(t)\right\|^2}{\mu_1 + \left\|\Delta \boldsymbol{U}_{k-1}^T(t)\right\|^2}\right) \\ &\quad \times \frac{\eta_1 \left\|\tilde{\boldsymbol{\Phi}}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}(t)\right\|^2}{\mu_1 + \left\|\Delta \boldsymbol{U}_{k-1}^T(t)\right\|^2} \end{aligned}$$

Due to $\eta_1 \in (0, 2)$ and $\mu_1 > 0$,

$$2 - \frac{\eta_1 \left\|\Delta \boldsymbol{U}_{k-1}(t)\right\|^2}{\mu_1 + \left\|\Delta \boldsymbol{U}_{k-1}^T(t)\right\|^2} > 0$$

Therefore, there exists a positive constant $0 < b_2 < 1$ such that

$$\left\|\tilde{\boldsymbol{\Phi}}_{k-1}(t)\left(\boldsymbol{I} - \frac{\eta_1 \Delta \boldsymbol{U}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_1 + \left\|\Delta \boldsymbol{U}_{k-1}^T(t)\right\|^2}\right)\right\| \leq b_2 \left\|\tilde{\boldsymbol{\Phi}}_{k-1}(t)\right\| \tag{8}$$

According to (7) and (8), we can obtain that

$$\left\|\tilde{\boldsymbol{\Phi}}_k(t)\right\| \leq b_2 \left\|\tilde{\boldsymbol{\Phi}}_{k-1}(t)\right\| + b_1 \leq \cdots \leq b_2^k \left\|\tilde{\boldsymbol{\Phi}}_0(t)\right\| + \frac{b_1}{1 - b_2} \tag{9}$$

from which we have that $\tilde{\boldsymbol{\Phi}}_k(t)$ is bounded because $\boldsymbol{\Phi}(t)$ is bounded and $\hat{\boldsymbol{\Phi}}_0(t)$ is also given bounded. Consequently, $\hat{\boldsymbol{\Phi}}_0(t)$ is bounded too.

Now, we proceed to the proof of the stability of the disturbance observer. According to (5a), the dynamics of the accumulative disturbance estimation error is

$$\boldsymbol{e}_{k+1}(t) = \boldsymbol{\delta}_{k+1}(t) - \boldsymbol{K} x_k(t+1) + z_{k+1}(t+1) \tag{10}$$

Then from (4) and (5b), equation (10) becomes,

$$\begin{aligned} \boldsymbol{e}_{k+1}(t) &= \boldsymbol{\delta}_{k+1}(t) - \boldsymbol{K}(\boldsymbol{x}_{k-1}(t+1) + \boldsymbol{\Phi}(t)\Delta \boldsymbol{U}_k(t) + \boldsymbol{\delta}_k(t)) \\ &\quad + z_k(t+1) + \boldsymbol{K}(\hat{\boldsymbol{\Phi}}_k(t)\Delta \boldsymbol{U}_k(t) + \hat{\boldsymbol{\delta}}_k(t)) \\ &= \boldsymbol{\delta}_{k+1}(t) - \boldsymbol{\delta}_k(t) + \boldsymbol{\delta}_k(t) - (\boldsymbol{K} x_{k-1}(t+1) - z_k(t+1)) \\ &\quad - \boldsymbol{K} e_k(t) - \boldsymbol{K}\tilde{\Phi}_k(t)\Delta \boldsymbol{U}_k(t) \end{aligned} \tag{11}$$

By virtue of (5a), one can obtain from (11) that

$$\boldsymbol{e}_{k+1}(t) = \Delta \boldsymbol{\delta}_{k+1}(t) + (\boldsymbol{I}_n - \boldsymbol{K})\boldsymbol{e}_k(t) - \boldsymbol{K}\tilde{\Phi}_k(t)\Delta \boldsymbol{U}_k(t) \tag{12}$$

Since $\boldsymbol{K} = (\boldsymbol{I}_n - \boldsymbol{\Gamma})$, one can rewrite (12) as

$$\boldsymbol{e}_{k+1}(t) = \Delta \boldsymbol{\delta}_{k+1}(t) + \boldsymbol{\Gamma} e_k(t) - \boldsymbol{K}\tilde{\Phi}_k(t)\Delta \boldsymbol{U}_k(t) \tag{13}$$

Taking the norm on the both sides of (13), yields,

$$\begin{aligned} \|\boldsymbol{e}_{k+1}(t)\| &\leq \|\boldsymbol{\Gamma}\| \, \|\boldsymbol{e}_k(t)\| + \|\Delta \boldsymbol{\delta}_{k+1}(t)\| \\ &\quad + \|\boldsymbol{K}\| \left\|\tilde{\boldsymbol{\Phi}}_k(t)\right\| \|\Delta \boldsymbol{U}_k(t)\| \end{aligned} \tag{14}$$

Since $\boldsymbol{u}_k(t)$ is bounded, then $\Delta \boldsymbol{U}_k(t)$ is bounded too. Further, the boundedness of $\tilde{\boldsymbol{\Phi}}_k(t)$ has been proved and $\boldsymbol{K}$ is also bounded, thereafter, $\|\boldsymbol{K}\| \left\|\tilde{\boldsymbol{\Phi}}_k(t)\right\| \|\Delta \boldsymbol{U}_k(t)\| \leq b_K$ is bounded where b_K is a constant. As for the diagonal matrix $\boldsymbol{\Gamma}$, we have $\|\boldsymbol{\Gamma}\| \leq b_\gamma$, where b_γ denotes its maximum eigenvalue and $0 < b_\gamma < 1$ because $\gamma_i < 1$(i = 1,2,…,n).

Therefore, one can get from (14) that

$$\begin{aligned} \|\boldsymbol{e}_{k+1}(t)\| &\leq b_\gamma \|\boldsymbol{e}_k(t)\| + 2b_\delta \\ &\quad + b_K \leq \cdots \leq b_\gamma^{k+1} \|\boldsymbol{e}_0(t)\| + \frac{2b_\delta + b_K}{1 - b_\gamma} \end{aligned}$$

It is obvious that the disturbance estimation error convergence boundedly with the increasing iterations, i.e.,

$$\lim_{k\to\infty} \|\boldsymbol{e}_k(t)\| \leq \frac{2b_\delta + b_K}{1 - b_\gamma}.$$

Remark 5: It is apparent that Theorem 1 guarantees the convergence of the state-based ILADOB along the iteration axis. By using the information in previous trials, the observing performance of the proposed state based ILADOB can be improved iteratively. Instead, the traditional DOB in the time domain achieves convergence only when the time instants approach to infinity. So, the traditional DOB is not suitable for observing the disturbances of a repetitive system due to the finite time interval.

C. OUTPUT-BASED ILADOB WITH UNMEASURABLE SYSTEM STATES

1) OUTPUT-BASED ILADOB DESIGN

By considering the case that the system state is unmeasurable and using the linear data model (4), an output based ILADOB method is proposed as follows,

$$\begin{aligned} \hat{\boldsymbol{\delta}}_k(t) &= \boldsymbol{K}\boldsymbol{C}^+(t+1)\boldsymbol{y}_{k-1}(t+1) - z_k(t+1) \\ &\quad + \boldsymbol{K}\boldsymbol{N}(t+1)\boldsymbol{x}_{k-1}(t+1) \end{aligned} \tag{15a}$$

$$\begin{aligned} &z_{k+1}(t+1) \\ &\quad = z_k(t+1) + \boldsymbol{K}(\hat{\boldsymbol{\Phi}}_k(t)\Delta \boldsymbol{U}_k(t) + \hat{\boldsymbol{\delta}}_k(t)) \end{aligned} \tag{15b}$$

$$\begin{aligned} \hat{\boldsymbol{\Phi}}_k(t) &= \hat{\boldsymbol{\Phi}}_{k-1}(t) - \frac{\eta_1 \hat{\boldsymbol{\Phi}}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_1 + \left\|\Delta \boldsymbol{U}_{k-1}^T(t)\right\|^2} \\ &\quad + \frac{\eta_1 (\boldsymbol{M}\boldsymbol{C}(t+1))^{L+}\boldsymbol{M}\Delta \boldsymbol{y}_{k-1}(t+1)\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_1 + \left\|\Delta \boldsymbol{U}_{k-1}^T(t)\right\|^2} \end{aligned} \tag{15c}$$

where C^+ denotes a pseudo-inverse of the matrix C, $(MC)^{L+}$ denotes a left inverse of MC, which satisfies $(MC)^{L+}MC = I$, and I is the identity matrix, $N(t+1) = I_n - C^+(t+1)C(t+1)$, and $KN(t+1)x_{k-1}(t+1)$ is added in (15a) as a compensation.

According to [9], there exist matrix $H(t+1) \in R^{n\times h}$ and $V \in R^{h\times n}$ satisfying $H(t+1)V = KN(t+1)$, where $h = \mathrm{rank}(KN(t+1))$.

Since $KN(t+1)x_{k-1}(t+1)$ cannot be measured, we define a state function vector,

$$\beta_k(t) = Vx_{k-1}(t+1) \in R^h \tag{16}$$

where $\beta_k(t)$ denotes an unknown state, and an estimator for $\beta_k(t)$ is designed further.

So, the final output based ILADOB is designed as

$$\hat{\delta}_k(t) = KC^+(t+1)y_{k-1}(t+1) - z_k(t+1) + H(t+1)\hat{\beta}_k(t) \tag{17a}$$

$$z_{k+1}(t+1) = z_k(t+1) + K(\hat{\Phi}_k(t)\Delta U_k(t) + \hat{\delta}_k(t)) \tag{17b}$$

$$\hat{\Phi}_k(t) = \hat{\Phi}_{k-1}(t) - \frac{\eta_1\hat{\Phi}_{k-1}(t)\Delta U_{k-1}(t)\Delta U_{k-1}^T(t)}{\mu_1 + \left\|\Delta U_{k-1}^T(t)\right\|^2} + \frac{\eta_1(MC(t+1))^{L+}M\Delta y_{k-1}(t+1)\Delta U_{k-1}^T(t)}{\mu_1 + \left\|\Delta U_{k-1}^T(t)\right\|^2} \tag{17c}$$

$$\hat{\beta}_k(t) = \xi_k(t) + Q(t+1)y_{k-1}(t+1) \tag{17d}$$

$$\xi_{k+1}(t) = R(t+1)\xi_k(t) + S(t+1)y_{k-1}(t+1) + W_{u,k}(t+1)\Delta U_k(t) + W_\delta(t+1)\hat{\delta}_k(t) \tag{17e}$$

where $\hat{\beta}_k(t) \in R^h$ is to estimate $\beta_k(t)$, and $W_{u,k}(t+1) = (V - Q(t+1)C(t+1))\hat{\Phi}_k(t)$, $\xi_k(t) \in R^h$, $W_\delta(t+1) = V - Q(t+1)C(t+1)$; for the matrices $S(t) \in R^{h\times m}$, $Q(t) \in R^{h\times m}$, and $R(t) \in R^{h\times h}$, they satisfy that

$$(I_h - R(t))(V - Q(t)C(t)) - S(t)C(t) = 0 \tag{18}$$

Remark 6: Different from the state based ILADOB (5a)-(5c), the proposed output based ILADOB utilizes the system output and the pseudo-inverse of output matrix to denote the system state. Further, a state observer (17d) is employed to denote the comprehensive state as a compensation.

Remark 7: The solvability of the condition (18) has been shown in [9]. In addition, M can be set as $M = I_n$, but M can also be set as other values, and the choice of M is relatively free [9].

2) STABILITY ANALYSIS

Theorem 2: Consider a repetitive linear time-varying system (1) satisfying assumptions 1-2. By choosing $Q(t)$, $R(t)$, and $S(t)$ properly such that condition (18) is satisfied, and $\eta_1 \in (0,2)$, $\mu_1 > 0$, and $|\gamma_i| < 1, i = 1,2,\ldots,n$, properly, the proposed output based ILADOB (17a)-(17e) guarantees that: (i) $\hat{\Phi}_k(t)$ is bounded for all iterations over $\{1,2,\ldots,N\}$; (ii) both the observer error of the accumulative disturbance, $e_k(t)$, and the observer error of the extended state, $\varepsilon_k(t) = \beta_k(t) - \hat{\beta}_k(t)$, over $\{1,2,\ldots,N\}$ convergence boundedly with increasing iterations.

Proof: Subtracting $\Phi(t)$ from both sides of (17c), in terms of (4), we have

$$\begin{aligned}\tilde{\Phi}_k(t) &= \tilde{\Phi}_{k-1}(t) + \frac{\eta_1\hat{\Phi}_{k-1}(t)\Delta U_{k-1}(t)\Delta U_{k-1}^T(t)}{\mu_1 + \left\|\Delta U_{k-1}^T(t)\right\|^2} - \frac{\eta_1(MC(t+1))^{L+}MC(t+1)\Delta x_{k-1}(t+1)\Delta U_{k-1}^T(t)}{\mu_1 + \left\|\Delta U_{k-1}^T(t)\right\|^2}\\ &= \tilde{\Phi}_{k-1}(t)\left(I - \frac{\eta_1\Delta U_{k-1}(t)\Delta U_{k-1}^T(t)}{\mu_1 + \left\|\Delta U_{k-1}^T(t)\right\|^2}\right) - \frac{\eta_1\delta_{k-1}(t)\Delta U_{k-1}^T(t)}{\mu_1 + \left\|\Delta U_{k-1}^T(t)\right\|^2}\end{aligned} \tag{19}$$

After that, the proof of the stability of $\hat{\Phi}_k(t)$ is similar to those in Theorem 1. So, we omit the detail proof for simplicity. Now, we proceed to the stability analysis of estimation error $e_k(t)$.

Define $\varepsilon_k(t) = \beta_k(t) - \hat{\beta}_k(t)$ as the estimation error of $\beta_k(t)$. By virtue of (16) and (17d), one has

$$\begin{aligned}\xi_k(t) &= \beta_k(t) - \varepsilon_k(t) - Q(t+1)C(t+1)x_{k-1}(t+1)\\ &= (V - Q(t+1)C(t+1))x_{k-1}(t+1) - \varepsilon_k(t)\end{aligned} \tag{20}$$

Then, we can get that,

$$\varepsilon_{k+1}(t) = (V - Q(t+1)C(t+1))x_k(t+1) - \xi_{k+1}(t) \tag{21}$$

Substituting (4) and (17e) into (21), yields

$$\begin{aligned}\varepsilon_{k+1}(t) = {}&(V - Q(t+1)C(t+1))(x_{k-1}(t+1) + \Phi(t)\Delta U_k(t) + \delta_k(t))\\ &- (R(t+1)\xi_k(t) + S(t+1)C(t+1)x_{k-1}(t+1) + W_{u,k}(t+1)\Delta U_k(t) + W_\delta(t+1)\hat{\delta}_k(t))\end{aligned} \tag{22}$$

In view of (20) and (22), one can get

$$\begin{aligned}\varepsilon_{k+1}(t) = {}&((V - Q(t+1)C(t+1)) - R(t+1)(V - Q(t+1)C(t+1)) - S(t+1)C(t+1))x_{k-1}(t+1) - W_{u,k}(t+1)\Delta U_k(t)\\ &+ ((V - Q(t+1)C(t+1))\Phi(t)\Delta U_k(t) - W_\delta(t+1)\hat{\delta}_k(t)\\ &+ ((V - Q(t+1)C(t+1))\delta_k(t) + W_\delta(t+1)e_k(t)\\ &+ R(t+1)\varepsilon_k(t)\end{aligned} \tag{23}$$

By virtue of equation (18) and the definitions of $W_{u,k}(t+1)$ and $W_\delta(t+1)$, (23) is rewritten as

$$\begin{aligned}\varepsilon_{k+1}(t) = {}&(V - Q(t+1)C(t+1))e_k(t) + R(t+1)\varepsilon_k(t)\\ &+ (V - Q(t+1)C(t+1))\tilde{\Phi}_k(t)\Delta U_k(t)\end{aligned} \tag{24}$$

Since $\boldsymbol{\beta}_k(t) = \boldsymbol{V}x_{k-1}(t+1)$ and $\hat{\boldsymbol{\beta}}_k(t) = \boldsymbol{\beta}_k(t) - \boldsymbol{\varepsilon}_k(t)$, we have

$$\begin{aligned}\hat{\boldsymbol{\delta}}_k(t) &= \boldsymbol{K}\boldsymbol{C}^{+}(t+1)\boldsymbol{y}_{k-1}(t+1) - z_k(t+1)\\ &\quad + \boldsymbol{H}(t+1)(\boldsymbol{V}x_{k-1}(t+1) - \boldsymbol{\varepsilon}_k(t))\\ &= \boldsymbol{K}\boldsymbol{C}^{+}(t+1)\boldsymbol{y}_{k-1}(t+1) - z_k(t+1)\\ &\quad + \boldsymbol{K}\boldsymbol{N}(t+1)\boldsymbol{x}_{k-1}(t+1) - \boldsymbol{H}(t+1)\boldsymbol{\varepsilon}_k(t)\\ &= \boldsymbol{K}x_{k-1}(t+1) - z_k(t+1) - \boldsymbol{H}(t+1)\boldsymbol{\varepsilon}_k(t)\end{aligned} \tag{25}$$

Then,

$$\begin{aligned}\boldsymbol{e}_{k+1}(t) &= \boldsymbol{\delta}_{k+1}(t) - \hat{\boldsymbol{\delta}}_{k+1}(t)\\ &= \boldsymbol{\delta}_{k+1}(t) - \boldsymbol{K}\boldsymbol{x}_k(t+1) + z_{k+1}(t+1)\\ &\quad + \boldsymbol{H}(t+1)\boldsymbol{\varepsilon}_{k+1}(t)\end{aligned} \tag{26}$$

According to (4) and (17b), one can obtain from (26) that

$$\begin{aligned}\boldsymbol{e}_{k+1}(t) &= \boldsymbol{\delta}_{k+1}(t) - \boldsymbol{K}(\boldsymbol{x}_{k-1}(t+1) + \boldsymbol{\Phi}(t)\Delta\boldsymbol{U}_k(t) + \boldsymbol{\delta}_k(t))\\ &\quad + z_k(t+1) + \boldsymbol{K}(\hat{\boldsymbol{\Phi}}_k(t)\Delta\boldsymbol{U}_k(t) + \hat{\boldsymbol{\delta}}_k(t))\\ &\quad + \boldsymbol{H}(t+1)\boldsymbol{\varepsilon}_{k+1}(t)\\ &= \boldsymbol{\delta}_{k+1}(t) - (\boldsymbol{K}x_{k-1}(t+1) - z_k(t+1))\\ &\quad - \boldsymbol{K}(\boldsymbol{\delta}_k(t) - \hat{\boldsymbol{\delta}}_k(t))\\ &\quad - \boldsymbol{K}\tilde{\Phi}_k(t)\Delta\boldsymbol{U}_k(t) + \boldsymbol{H}(t+1)\boldsymbol{\varepsilon}_{k+1}(t)\end{aligned} \tag{27}$$

Substituting (25) into (27), one has

$$\begin{aligned}\boldsymbol{e}_{k+1}(t) &= \boldsymbol{\delta}_{k+1}(t) - \boldsymbol{\delta}_k(t) + \boldsymbol{\delta}_k(t) - \hat{\boldsymbol{\delta}}_k(t) - \boldsymbol{H}(t+1)\boldsymbol{\varepsilon}_k(t)\\ &\quad - \boldsymbol{K}\boldsymbol{e}_k(t) - \boldsymbol{K}\tilde{\Phi}_k(t)\Delta\boldsymbol{U}_k(t) + \boldsymbol{H}(t+1)\boldsymbol{\varepsilon}_{k+1}(t)\\ &= \Delta\boldsymbol{\delta}_{k+1}(t) + (\boldsymbol{I}_n - \boldsymbol{K})\boldsymbol{e}_k(t) - \boldsymbol{K}\tilde{\Phi}_k(t)\Delta\boldsymbol{U}_k(t)\\ &\quad - \boldsymbol{H}(t+1)\boldsymbol{\varepsilon}_k(t) + \boldsymbol{H}(t+1)\boldsymbol{\varepsilon}_{k+1}(t)\end{aligned} \tag{28}$$

Again, substituting (24) into (28), yields,

$$\begin{aligned}\boldsymbol{e}_{k+1}(t) &= (\boldsymbol{I}_n - \boldsymbol{K} + \boldsymbol{H}(t+1)(\boldsymbol{V} - \boldsymbol{Q}(t+1)\boldsymbol{C}(t+1)))\boldsymbol{e}_k(t)\\ &\quad + (\boldsymbol{H}(t+1)\boldsymbol{R}(t+1) - \boldsymbol{H}(t+1))\boldsymbol{\varepsilon}_k(t) + \Delta\boldsymbol{\delta}_{k+1}(t)\\ &\quad + (\boldsymbol{H}(t+1)(\boldsymbol{V} - \boldsymbol{Q}(t+1)\boldsymbol{C}(t+1))\\ &\quad - \boldsymbol{K})\tilde{\boldsymbol{\Phi}}_k(t)\Delta\boldsymbol{U}_k(t)\end{aligned} \tag{29}$$

Combining (29) and (24), the dynamics of $\boldsymbol{e}_k(t)$ and $\boldsymbol{\varepsilon}_k(t)$ can be formulated as

$$\begin{pmatrix}\boldsymbol{e}_{k+1}(t)\\ \boldsymbol{\varepsilon}_{k+1}(t)\end{pmatrix} = \boldsymbol{E}_1(t+1)\begin{pmatrix}\boldsymbol{e}_k(t)\\ \boldsymbol{\varepsilon}_k(t)\end{pmatrix} + \boldsymbol{E}_2(t+1) \tag{30}$$

where

$$\begin{aligned}&\boldsymbol{E}_1(t+1)\\ &= \begin{bmatrix}\boldsymbol{E}_{1,1}(t+1) & \boldsymbol{H}(t+1)\boldsymbol{R}(t+1) - \boldsymbol{H}(t+1)\\ \boldsymbol{V} - \boldsymbol{Q}(t+1)\boldsymbol{C}(t+1) & \boldsymbol{R}(t+1)\end{bmatrix},\\ &\boldsymbol{E}_{1,1}(t+1)\\ &= \boldsymbol{I}_n - \boldsymbol{K} + \boldsymbol{H}(t+1)(\boldsymbol{V} - \boldsymbol{Q}(t+1)\boldsymbol{C}(t+1)),\\ &\boldsymbol{E}_2(t+1)\\ &= \begin{pmatrix}\boldsymbol{E}_{2,1}(t+1)\\ (\boldsymbol{V} - \boldsymbol{Q}(t+1)\boldsymbol{C}(t+1))\tilde{\boldsymbol{\Phi}}_k(t)\Delta\boldsymbol{U}_k(t)\end{pmatrix},\\ &\boldsymbol{E}_{2,1}(t+1)\\ &= \Delta\boldsymbol{\delta}_{k+1}(t)\\ &\quad + (\boldsymbol{H}(t+1)(\boldsymbol{V} - \boldsymbol{Q}(t+1)\boldsymbol{C}(t+1)) - \boldsymbol{K})\tilde{\boldsymbol{\Phi}}_k(t)\Delta\boldsymbol{U}_k(t).\end{aligned}$$

Since the elements of $\boldsymbol{E}_2(t+1)$ are bounded, $\|\boldsymbol{E}_2(t+1)\|$ is bounded too. Since condition (18) is satisfied, it is clear that $\boldsymbol{E}_1(t+1)$ is stable, then one can guarantee that both $\boldsymbol{e}_k(t)$ and $\boldsymbol{\varepsilon}_k(t)$ are bounded as the iteration number increases.

III. EXTENSION TO MIMO NONLINEAR NONAFFINE SYSTEMS

A. FORMULATION AND LINEAR DATA MODEL

A MIMO nonlinear discrete-time repetitive system is considered,

$$\begin{cases}\boldsymbol{x}_k(t+1) = \boldsymbol{f}(\boldsymbol{x}_k(t), \boldsymbol{u}_k(t)) + \boldsymbol{d}_k(t)\\ \boldsymbol{y}_k(t) = \boldsymbol{C}(t)\boldsymbol{x}_k(t)\end{cases} \tag{31}$$

where $\boldsymbol{x}_k(t) \in \boldsymbol{R}^n$, $\boldsymbol{u}_k(t) \in \boldsymbol{R}^l$, and $\boldsymbol{y}_k(t) \in \boldsymbol{R}^m$ are the system state, input and output; $\boldsymbol{d}_k(t) \in \boldsymbol{R}^n$ is the system disturbance; $t \in \{0, \ldots, N\}$ and N denotes the terminal time instant of the repetitive system; $k \in \{0, 1, \ldots, \infty\}$ denotes the iteration number; $\boldsymbol{f}(\ldots)$ is an unknown vector-valued function. The system matrix $\boldsymbol{C}(t) \in \boldsymbol{R}^{m\times n}$ is a known and bounded matrix, that is

$$\|\boldsymbol{C}(t)\| \le b_C$$

where b_C is a positive constant.

Besides assumptions 1-2, the nonlinear nonaffine system (31) also satisfies the following assumption.

Assumption 3: The partial gradients $\boldsymbol{f}(\ldots)$ with respect to the input vectors $\boldsymbol{u}_k(t)$ and the disturbance $\boldsymbol{d}_k(t)$ are both continuous and bounded.

Then, according to [24], the nonlinear state equation of system (31) becomes

$$\begin{aligned}\boldsymbol{x}_k(1) &= \boldsymbol{f}(\boldsymbol{x}_k(0), \boldsymbol{u}_k(0)) + \boldsymbol{d}_k(0) = \boldsymbol{g}^0(\boldsymbol{x}_k(0), \boldsymbol{u}_k(0), \boldsymbol{d}_k(0))\\ \boldsymbol{x}_k(2) &= \boldsymbol{f}(\boldsymbol{x}_k(1), \boldsymbol{u}_k(1)) + \boldsymbol{d}_k(1)\\ &= \boldsymbol{f}(\boldsymbol{g}^0(\boldsymbol{x}_k(0), \boldsymbol{u}_k(0), \boldsymbol{d}_k(0)), \boldsymbol{u}_k(1)) + \boldsymbol{d}_k(1)\\ &= \boldsymbol{g}^1(\boldsymbol{x}_k(0), \boldsymbol{u}_k(0), \boldsymbol{u}_k(1), \boldsymbol{d}_k(0), \boldsymbol{d}_k(1))\\ &\ \ \vdots\\ \boldsymbol{x}_k(t+1) &= \boldsymbol{g}^t(\boldsymbol{x}_k(0), \boldsymbol{u}_k(0), \ldots, \boldsymbol{u}_k(t-1), \boldsymbol{u}_k(t),\\ &\qquad \boldsymbol{d}_k(0), \ldots, \boldsymbol{d}_k(t-1), \boldsymbol{d}_k(t))\end{aligned}$$

where $\boldsymbol{g}^t(\cdots) \in \boldsymbol{R}^n$ is a corresponding nonlinear vector-valued functions and is a compound function of $\boldsymbol{f}(\ldots)$ and thus its partial derivative of $\boldsymbol{u}_k(t)$ and $\boldsymbol{d}_k(t)$ is also continuous and bounded according to Assumption 3.

Define $\boldsymbol{U}_k(t) = \left[\boldsymbol{u}_k^T(0)\ \boldsymbol{u}_k^T(1)\ \ldots\ \boldsymbol{u}_k^T(t)\right]^T$ and $\boldsymbol{D}_k(t) = \left[\boldsymbol{d}_k^T(0)\ \boldsymbol{d}_k^T(1)\ \ldots\ \boldsymbol{d}_k^T(t)\right]^T$.

Making a difference of system outputs between two iterations, there exist some functions $\boldsymbol{\zeta}_{1,k} \in (\boldsymbol{x}_k(0) - \Delta\boldsymbol{x}_k(0), \boldsymbol{x}_k(0))$, $\boldsymbol{\zeta}_{2,k}(t) \in (\boldsymbol{U}_k(t) - \Delta\boldsymbol{U}_k(t), \boldsymbol{U}_k(t))$, and $\boldsymbol{\zeta}_{3,k}(t) \in (\boldsymbol{D}_k(t) - \Delta\boldsymbol{D}_k(t), \boldsymbol{D}_k(t))$ such that

$$\begin{aligned}\Delta\boldsymbol{x}_k(t+1) &= \boldsymbol{g}_x^t(\boldsymbol{\zeta}_{1,k})\Delta\boldsymbol{x}_k(0) + \boldsymbol{g}_U^t(\boldsymbol{\zeta}_{2,k}(t))\Delta\boldsymbol{U}_k(t)\\ &\quad + \boldsymbol{g}_D^t(\boldsymbol{\zeta}_{3,k}(t))\Delta\boldsymbol{D}_k(t)\end{aligned} \tag{32}$$

where $\boldsymbol{g}_x^t = \partial\boldsymbol{g}^t/\partial\boldsymbol{x}$, $\boldsymbol{g}_U^t = \partial\boldsymbol{g}^t/\partial\boldsymbol{U}$, $\boldsymbol{g}_D^t = \partial\boldsymbol{g}^t/\partial\boldsymbol{D}$.

According to Assumption 3, it is obvious that $\boldsymbol{g}_x^t(\boldsymbol{\zeta}_{1,k})$, $\boldsymbol{g}_U^t(\boldsymbol{\zeta}_{2,k}(t))$, and $\boldsymbol{g}_D^t(\boldsymbol{\zeta}_{3,k}(t))$ are bounded. By virtue of Assumption 1, i.e., $\Delta \boldsymbol{x}_k(0) = 0$, therefore (32) becomes

$$\Delta \boldsymbol{x}_k(t+1)=\boldsymbol{g}_U^t(\boldsymbol{\zeta}_{2,k}(t))\Delta \boldsymbol{U}_k(t)+\boldsymbol{g}_D^t(\boldsymbol{\zeta}_{3,k}(t))\Delta \boldsymbol{D}_k(t) \quad (33)$$

So, a linear data model of nonlinear system (31) is derived as,

$$\begin{cases} \boldsymbol{x}_k(t+1) = \boldsymbol{x}_{k-1}(t+1) + \Xi_k(t)\Delta \boldsymbol{U}_k(t) + \boldsymbol{\delta}_k(t) \\ \boldsymbol{y}_k(t) = \boldsymbol{C}(t)\boldsymbol{x}_k(t) \end{cases} \quad (34)$$

where $\Xi_k(t) = \boldsymbol{g}_U^t(\boldsymbol{\zeta}_{2,k}(t))$ is the gradient parameter vector and $\boldsymbol{\delta}_k(t) = \boldsymbol{g}_D^t(\boldsymbol{\zeta}_{3,k}(t))\Delta \boldsymbol{D}_k(t)$ denotes the accumulative disturbances of system (31), and both of them are bounded.

Remark 8: One can see that linear data model (34) for a nonlinear nonaffine system (31) is similar to the one for linear system (1) where the only difference is that the gradient parameter matrix $\Xi_k(t) = \boldsymbol{g}_U^t(\boldsymbol{\zeta}_{2,k}(t))$ in (34) is iteration-dependent, while $\boldsymbol{\Phi}(t)$ in (4) is iteration-independent.

B. ILADOB FOR NONLINEAR NONAFFINE SYSTEMS

Comparing linear data model (34) with (4), the previous designed ILADOB methods (5a)-(5c) and (17a)-(17e) can also be applicable to observe the accumulative disturbance in (34). For the convenience of the readers, we rewrite them here by replacing $\hat{\boldsymbol{\Phi}}_k(t)$ with $\hat{\Xi}_k(t)$.

1) STATE-BASED ILADOB FOR NONLINEAR NONAFFINE SYSTEM (31)

$$\hat{\boldsymbol{\delta}}_k(t) = \boldsymbol{K}x_{k-1}(t+1) - z_k(t+1) \quad (35a)$$

$$z_{k+1}(t+1) = z_k(t+1) + \boldsymbol{K}(\hat{\Xi}_k(t)\Delta \boldsymbol{U}_k(t) + \hat{\boldsymbol{\delta}}_k(t)) \quad (35b)$$

$$\hat{\Xi}_k(t) = \hat{\Xi}_{k-1}(t) + \frac{\eta_2(\Delta \boldsymbol{x}_{k-1}(t+1) - \hat{\Xi}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}(t))\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_2 + \left\| \Delta \boldsymbol{U}_{k-1}^T(t) \right\|^2} \quad (35c)$$

where $\hat{\boldsymbol{\delta}}_k(t)$ is to estimate $\boldsymbol{\delta}_k(t)$, $z_k(t) \in \boldsymbol{R}^n$ is a state variable, $\hat{\Xi}_k(t)$ is the estimation of $\Xi(t)$, $\mu_2 > 0$ is a weighted factor and $\eta_2 \in (0, 2)$ is a step-size factor; $\boldsymbol{K} = (\boldsymbol{I}_n - \boldsymbol{\Gamma}) \in \boldsymbol{R}^{n\times n}$, where $\boldsymbol{\Gamma} = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_n\}$, and $|\gamma_i| < 1, i = 1, 2, \ldots, n$ is a proper constant.

2) OUTPUT-BASED ILADOB FOR NONLINEAR NONAFFINE SYSTEM (31)

$$\hat{\boldsymbol{\delta}}_k(t) = \boldsymbol{K}\boldsymbol{C}^{+}(t+1)\boldsymbol{y}_{k-1}(t+1) - z_k(t+1) + \boldsymbol{H}(t+1)\hat{\boldsymbol{\beta}}_k(t) \quad (36a)$$

$$z_{k+1}(t+1) = z_k(t+1) + \boldsymbol{K}(\hat{\Xi}_k(t)\Delta \boldsymbol{U}_k(t) + \hat{\boldsymbol{\delta}}_k(t)) \quad (36b)$$

$$\hat{\Xi}_k(t) = \hat{\Xi}_{k-1}(t) - \frac{\eta_2 \hat{\Xi}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}(t)\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_2 + \left\| \Delta \boldsymbol{U}_{k-1}^T(t) \right\|^2} + \frac{\eta_2(\boldsymbol{M}\boldsymbol{C}(t+1))^{L+}\boldsymbol{M}\Delta \boldsymbol{y}_{k-1}(t+1)\Delta \boldsymbol{U}_{k-1}^T(t)}{\mu_2 + \left\| \Delta \boldsymbol{U}_{k-1}^T(t) \right\|^2} \quad (36c)$$

$$\hat{\boldsymbol{\beta}}_k(t) = \boldsymbol{\xi}_k(t) + \boldsymbol{Q}(t+1)\boldsymbol{y}_{k-1}(t+1) \quad (36d)$$

$$\boldsymbol{\xi}_{k+1}(t) = \boldsymbol{R}(t+1)\boldsymbol{\xi}_k(t) + \boldsymbol{S}(t+1)\boldsymbol{y}_{k-1}(t+1) + \boldsymbol{W}_{u,k}(t+1)\Delta \boldsymbol{U}_k(t) + \boldsymbol{W}_\delta(t+1)\hat{\boldsymbol{\delta}}_k(t) \quad (36e)$$

where $\hat{\boldsymbol{\beta}}_k(t) \in \boldsymbol{R}^h$ is to estimate $\boldsymbol{\beta}_k(t)$; $\boldsymbol{\xi}_k(t) \in \boldsymbol{R}^h$, $\boldsymbol{W}_{u,k}(t+1) = (\boldsymbol{V} - \boldsymbol{Q}(t+1)\boldsymbol{C}(t+1))\hat{\Xi}_k(t)$, $\boldsymbol{W}_\delta(t+1) = \boldsymbol{V} - \boldsymbol{Q}(t+1)\boldsymbol{C}(t+1)$, the matrices $\boldsymbol{S}(t) \in \boldsymbol{R}^{h\times m}$, $\boldsymbol{Q}(t) \in \boldsymbol{R}^{h\times m}$, and $\boldsymbol{R}(t) \in \boldsymbol{R}^{h\times h}$ satisfy that

$$(\boldsymbol{I}_h - \boldsymbol{R}(t))(\boldsymbol{V} - \boldsymbol{Q}(t)\boldsymbol{C}(t)) - \boldsymbol{S}(t)\boldsymbol{C}(t) = 0.$$

3) STABILITY ANALYSIS

Assumption 4: Assume the gradient parameter $\Xi_k(t)$ is slowly iteration-varying or iteration-invariant for a strong repetitive system.

Remark 9: Actually, $\Xi_k(t)$ denotes differential signal of a control system and is not sensitive to iteration-varying factors such as iteration-varying parameters, structures, etc. So, it can be regarded as a slow-iteration-varying parameter matrix especially when the difference of $\boldsymbol{U}_k(t)$ is not too large. Consequently, one can regard $\Xi_k(t)$ as iteration-invariant in the convergence analysis, similar to the most popular methods in the well-known adaptive control theory in dealing with slowly time-varying parameters which are regarded as constants directly.

The convergence theorems of the state-based and output-based ILADOB methods for the nonlinear nonaffine system (31) are given respectively as follows.

Theorem 3: For the repetitive nonlinear nonaffine system (31) satisfying assumptions 1-3, by properly selecting the observer parameters such that $\eta_2 \in (0, 2)$, $\mu_2 > 0$, and $|\gamma_i| < 1, i = 1, 2, \ldots, n$, the proposed state based ILADOB (35a)-(35c) guarantees that $\hat{\Xi}_k(t)$ is bounded and the accumulative disturbance estimate error $\boldsymbol{e}_k(t)$ converges to a bound iteratively.

Theorem 4: Consider a repetitive nonlinear nonaffine system (31) satisfying assumptions 1-3. By choosing $\boldsymbol{Q}(t)$, $\boldsymbol{R}(t)$, and $\boldsymbol{S}(t)$ properly such that condition (18) is satisfied, and $\eta_2 \in (0, 2)$, $\mu_2 > 0$, and $|\gamma_i| < 1, i = 1, 2, \ldots, n$, properly, the proposed output based ILADOB (36a)-(36e) guarantees that: (i) $\hat{\Xi}_k(t)$ is bounded over $\{1, 2, \ldots, N\}$ for all iterations; (ii) both the observer error of the accumulative disturbance $\boldsymbol{e}_k(t)$ and the observer error of the extended state $\boldsymbol{\varepsilon}_k(t) = \boldsymbol{\beta}_k(t) - \hat{\boldsymbol{\beta}}_k(t)$ over $\{1,2,\ldots, N\}$ convergence boundedly with increasing iterations.

Proof: The proofs of theorems 3 and 4 are similar to that of theorems 1 and 2, respectively. So, the detail proofs are excluded for simplicity.

IV. SIMULITION STUDY

Two examples are provided to show the validity of the ILADOB methods. One is a MIMO linear time-varying system and another is a MIMO nonlinear time-varying system. The given mathematical models are used to represent a practical plant in the simulations. They only serve in generating I/O data and are assumed not available for the observer design. The performance of the designed ILADOB can be evaluated by using the estimation errors of the system states and outputs, respectively.

Example 1: Consider a MIMO linear time-varying system,

$$\begin{cases} \boldsymbol{x}_k(t+1) = \boldsymbol{A}(t)\boldsymbol{x}_k(t) + \boldsymbol{B}(t)\boldsymbol{u}_k(t) + \boldsymbol{d}_k(t) \\ \boldsymbol{y}_k(t) = \boldsymbol{C}(t)\boldsymbol{x}_k(t) \end{cases}$$

where $t \in \{0, \ldots, N\}$, $N = 80$, and the system matrices are given as

$$\boldsymbol{A}(t) = \begin{bmatrix} 0.9630 + 0.01\sin(0.1t) & 0.0181 \\ 0.1808 & 0.8195 + 0.01\cos(0.1t) \end{bmatrix},$$

$$\boldsymbol{B}(t) = \begin{bmatrix} 1 + 0.1\sin(2\pi t/80) \\ 1 + 0.1\cos(2\pi t/80) \end{bmatrix},$$

$$\boldsymbol{C}(t) = \begin{bmatrix} 0.6 & 0.3 + 0.1\sin(2\pi t/50) \\ 0.8 + 0.2\cos(2\pi t/50) & -0.1 \end{bmatrix}.$$

The disturbance $\boldsymbol{d}_k(t) = \begin{bmatrix} \sin(t/40) + \cos(k/40) \\ \cos(t/40) + \cos(k/40) \end{bmatrix}$.

The estimate model of system states and outputs is taken as the linear data model with estimated gradient parameter vector and observed accumulative disturbances, shown as follows,

$$\begin{cases} \hat{\boldsymbol{x}}_k(t+1) = \boldsymbol{x}_{k-1}(t+1) + \hat{\boldsymbol{\Phi}}_k(t)\Delta\boldsymbol{U}_k(t) + \hat{\boldsymbol{\delta}}_k(t) \\ \hat{\boldsymbol{y}}_k(t) = \boldsymbol{C}(t)\hat{\boldsymbol{x}}_k(t) \end{cases}$$

where $\hat{\boldsymbol{x}}_k(t)$ is the estimation of $\boldsymbol{x}_k(t)$, $\hat{\boldsymbol{y}}_k(t)$ is the estimation of $\boldsymbol{y}_k(t)$. In the simulation, the system input is given as $u_k(t) = 0.5\sin(t/k)$.

Define that $e^i_{x,k}(t) = x^i_k(t) - \hat{x}^i_k(t)(i = 1, 2, \ldots, n)$ is the estimation error of the system state, $e^i_{y,k}(t) = y^i_k(t) - \hat{y}^i_k(t)$ is the estimation error of the system output, and $\bar{e}^i_{x,k} = \sup\left|e^i_{x,k}(t)\right|$, $\bar{e}^i_{y,k} = \sup\left|e^i_{y,k}(t)\right|$ $(t \in \{1, \ldots, N\})$ are the maximum estimation error of system state and output for each iteration respectively.

When the system states are measurable, we apply the proposed state-based ILADOB (5a)-(5c) by selecting $\eta_1 = 1$, $\mu_1 = 1$, $\boldsymbol{\Gamma} = \mathrm{diag}\{0.1, 0.3\}$, $\boldsymbol{K} = \mathrm{diag}\{0.9, 0.7\}$, $\boldsymbol{M} = \boldsymbol{I}_n$. The system state estimation performance and the state estimation error convergence are shown in Figs.1-2, respectively.

One can see that the proposed state-based ILADOB achieves a well performance in estimating the total disturbance with an iterative convergence of the state estimation error.

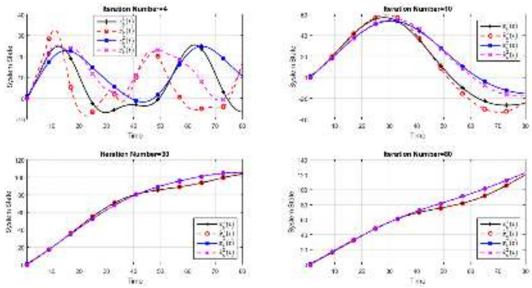

FIGURE 1. The estimation performance of system states in Example 1.

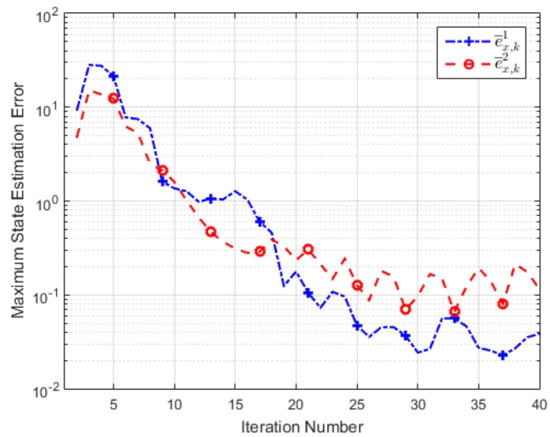

FIGURE 2. The convergence of state estimation error in Example 1.

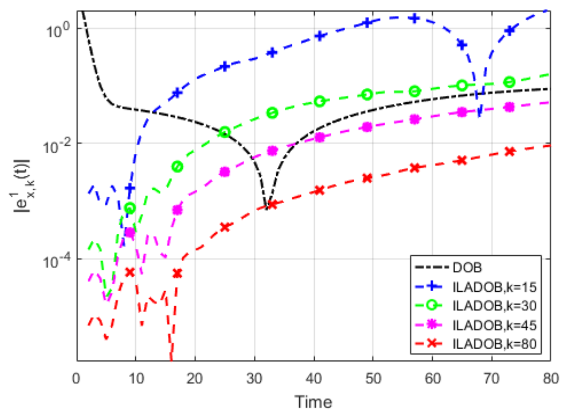

FIGURE 3. The profile of state estimation error e^1_x in Example 1.

For comparison, the traditional state-based DOB for non-repetitive linear systems [9] is also applied,

$$\hat{\boldsymbol{d}}(t) = \boldsymbol{K}x(t) - z(t)$$
$$z(t+1) = z(t) + \boldsymbol{K}((\boldsymbol{A}(t) - \boldsymbol{I}_n)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) + \hat{\boldsymbol{d}}(t))$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are system state matrix, $\boldsymbol{K} = \mathrm{diag}\{K_1, \ldots, K_n\}$, and $|K_i| < 1$, $i = 1, 2, \ldots, n$ is a proper constant.

By setting the parameter as $\boldsymbol{K} = \mathrm{diag}\{0.9, 0.7\}$, which is the same as the state-based ILADOB, the absolute value of state estimation error along time axis is shown in Figs.3-4.

It is seen that the traditional DOB has no learning ability from iterations, and the profile of state estimation error holds

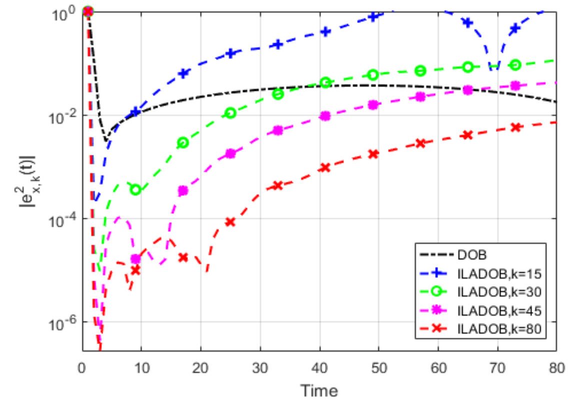

FIGURE 4. The profile of state estimation error e_x^2 in Example 1.

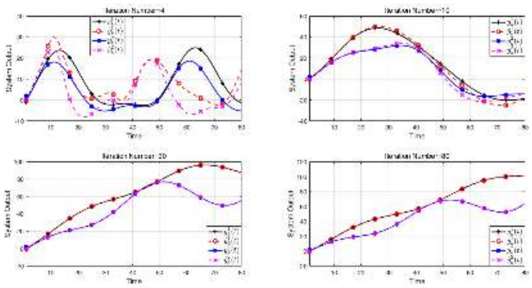

FIGURE 5. The estimation performance of system outputs in Example 1.

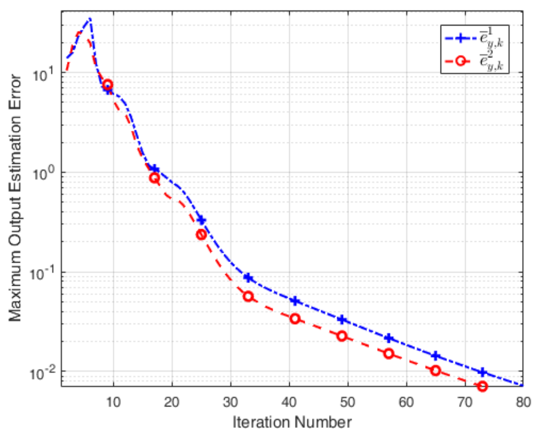

FIGURE 6. The convergence of output estimation error in Example 1.

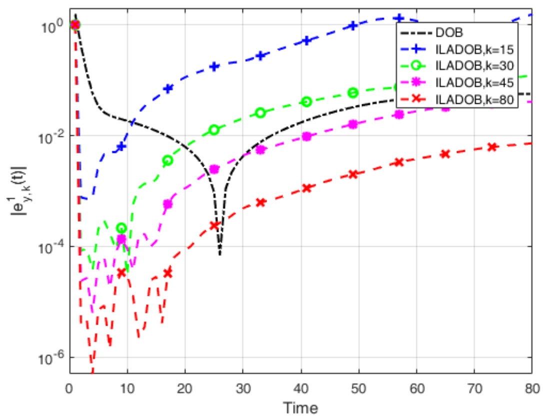

FIGURE 7. The profile of output estimation error e_y^1 in Example 1.

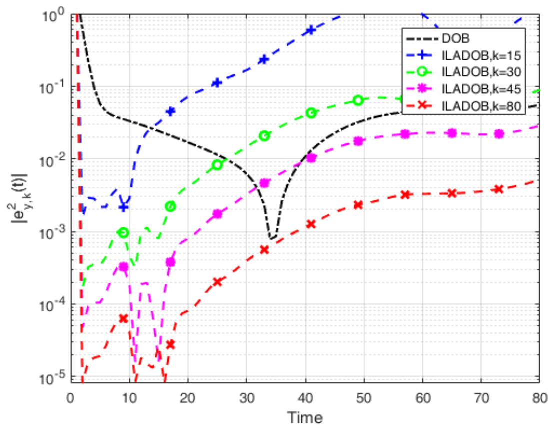

FIGURE 8. The profile of output estimation error e_y^2 in Example 1.

unchanged from iteration to iteration. Instead, the proposed state-based ILADOB method can learn from the I/O data in previous iterations and the state estimation errors are reduced with increasing iterations.

When the system state is not measurable, the proposed output-based ILADOB (17a)-(17e) is applied as an alternative. Selecting $\eta_1 = 1$, $\mu_1 = 1$, $\boldsymbol{\Gamma} = \text{diag}\{0.1, 0.3\}$, $\boldsymbol{V} = [\,1\ -1\,]$, $\boldsymbol{K} = \text{diag}\{0.9, 0.7\}$, $\boldsymbol{M} = \boldsymbol{I}_n$. Applying the output-based ILADOB (17a)-(17e), the estimation performance of system outputs and the maximum output estimation errors are shown in Figs. 5-6, respectively.

It is clear that the output-based ILADOB method (17a)-(17e) is capable of achieving a good output estimation performance which reflects the effectiveness of the ILADOB in observing the accumulative disturbance.

Further the traditional output-based DOB for nonrepetitive linear systems [9] is also applied for the purpose of comparison,

$$\begin{aligned}
\hat{\boldsymbol{d}}(t) &= \boldsymbol{K}\boldsymbol{C}^{+}(t)\boldsymbol{y}(t) - z(t) + \boldsymbol{H}_1(t)\hat{\boldsymbol{\beta}}(t)\\
z(t+1) &= z(t) + \boldsymbol{H}_2(t)\hat{\boldsymbol{\beta}}(t)\\
&\quad + \boldsymbol{K}((\boldsymbol{A}(t) - \boldsymbol{I}_n)\boldsymbol{C}^{+}(t)\boldsymbol{y}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) + \hat{\boldsymbol{d}}(t))\\
\hat{\boldsymbol{\beta}}(t) &= \boldsymbol{\xi}(t) + \boldsymbol{Q}(t)\boldsymbol{y}(t)\\
\boldsymbol{\xi}(t+1) &= \boldsymbol{R}(t)\boldsymbol{\xi}(t) + \boldsymbol{S}(t)\boldsymbol{y}(t) + \boldsymbol{W}_u(t)\boldsymbol{u}(t) + \boldsymbol{W}_d(t)\hat{\boldsymbol{d}}(t)
\end{aligned}$$

where $\boldsymbol{H}_1(t)\boldsymbol{V} = \boldsymbol{K}\boldsymbol{N}(t), \boldsymbol{H}_2(t)\boldsymbol{V} = \boldsymbol{K}(\boldsymbol{A}-\boldsymbol{I}_n)\boldsymbol{N}(t), \boldsymbol{W}_u(t) = (\boldsymbol{V} - \boldsymbol{Q}(t)\boldsymbol{C}(t))\boldsymbol{B}(t)$, $\boldsymbol{W}_d(t) = \boldsymbol{V} - \boldsymbol{Q}(t)\boldsymbol{C}(t)$, the definition of matrix $\boldsymbol{V}$ and $\boldsymbol{N}(t)$ are similar to those in Section 2; for the matrices $\boldsymbol{R}(t)$, $\boldsymbol{Q}(t)$, and $\boldsymbol{S}(t)$, they satisfy that

$$(\boldsymbol{V} - \boldsymbol{Q}(t)\boldsymbol{C}(t))\boldsymbol{B}(t) - \boldsymbol{R}(t)(\boldsymbol{V} - \boldsymbol{Q}(t)\boldsymbol{C}(t)) - \boldsymbol{S}(t)\boldsymbol{C}(t) = 0$$

By setting the parameter as $\boldsymbol{K} = \text{diag}\{0.9, 0.7\}$, $\boldsymbol{V} = [\,1\ -1\,]$, which are the same as the output-based ILADOB, the absolute value of output estimation error along time axis is shown in Fig.7-8.

One can see that the proposed output-based ILADOB method can reduce the output estimation error with the increasing iterations. However, the traditional output-based DOB has no learnability from the repetitive dynamics and the output estimation error is repeated without any improvement though the system runs repetitively.

To show the effectiveness of the proposed ILADOB for the accumulative disturbance estimation, we assume that the linear system in Example is known exactly, i.e., $\boldsymbol{\Phi}(t)$ is known a priori without need of estimation.

Let $E_{x,k}^{i}(t)=\delta_{k}^{i}(t)-\hat{\delta}_{k}^{i}(t),(i=1,2,\ldots,n)$ be the estimation error of the total disturbance with state-based ILADOB and $E_{y,k}^{i}(t)=\delta_{k}^{i}(t)-\hat{\delta}_{k}^{i}(t)$ be the estimation error of the total disturbance with output-based ILADOB, and $\bar{E}_{x,k}^{i}=\sup\left|E_{x,k}^{i}(t)\right|$, $\bar{E}_{y,k}^{i}=\sup\left|E_{y,k}^{i}(t)\right|$ $(t\in\{1,\ldots,N\})$ be the maximum estimation error of the total disturbance for each iteration.

When the system states are measurable, we apply the proposed state-based ILADOB (5a)-(5c) by selecting $\boldsymbol{\Gamma}=\mathrm{diag}\{0.1,0.3\}$, $\boldsymbol{K}=\mathrm{diag}\{0.9,0.7\}$, $\boldsymbol{M}=\boldsymbol{I}_n$. The accumulative disturbance estimation performance and estimation error are shown in Figs.9-10, respectively.

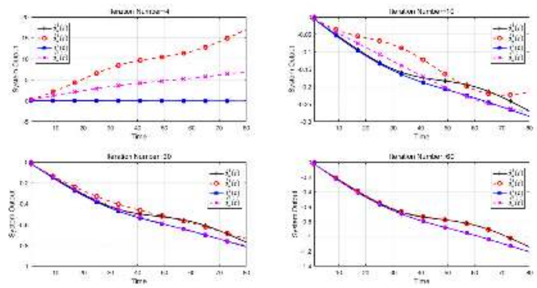

FIGURE 9. The estimation performance of total disturbance by using state-based ILADOB in Example 1.

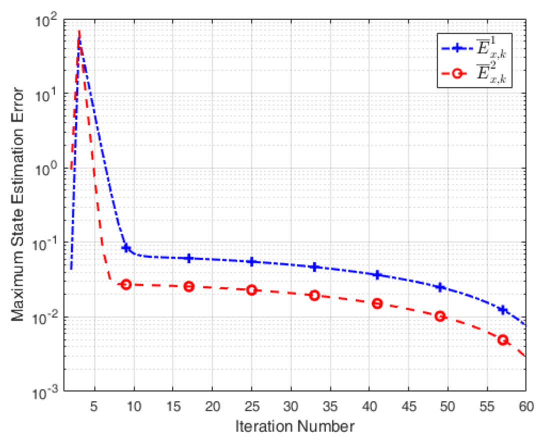

FIGURE 10. The convergence of total disturbance estimation error by using state-based ILADOB in Example 1.

One can see that the proposed state-based ILADOB is effective for the estimation of the accumulative disturbance.

When the system state is not measurable, the proposed output-based ILADOB (17a)-(17e) is applied. Selecting $\boldsymbol{\Gamma}=\mathrm{diag}\{0.1,0.3\}$, $\boldsymbol{V}=[\,1\ -1\,]$, $\boldsymbol{K}=\mathrm{diag}\{0.9,0.7\}$, $\boldsymbol{M}=\boldsymbol{I}_n$,

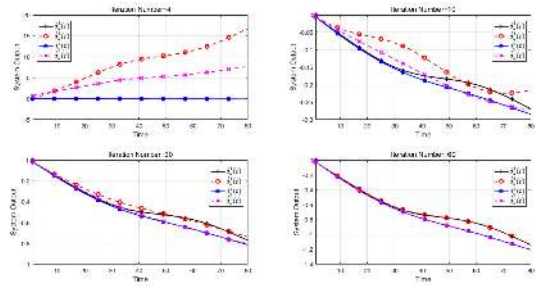

FIGURE 11. The estimation performance of total disturbance by using output-based ILADOB in Example 1.

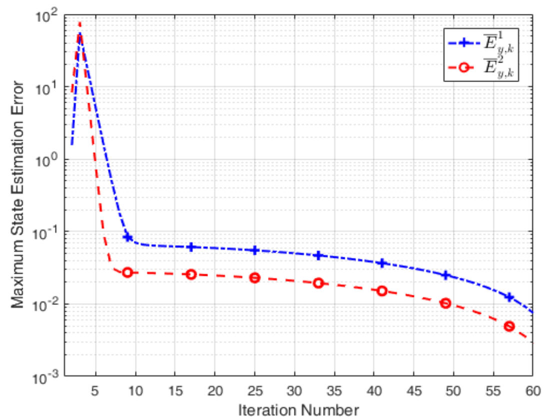

FIGURE 12. The convergence of total disturbance estimation error by using output-based ILADOB in Example 1.

the simulation results are shown in Figs. 11-12. Clearly, the output-based ILADOB also achieves a good estimation of the accumulative disturbance.

Example 2: Consider a MIMO nonlinear system [33],

$$\begin{cases} x_k^1(t+1) \\ \quad = \dfrac{2.5x_k^1(t)x_k^1(t-1)+0.09u_k^1(t)u_k^1(t-1)}{1+x_k^1(t)^2+x_k^1(t-1)^2}+d_k^1(t) \\ \quad\quad +1.2u_k^1(t)+1.6u_k^1(t-2)+0.09u_k^1(t)u_k^2(t)+0.5u_k^2(t) \\ \quad\quad +0.7\sin(0.5(x_k^1(t)+x_k^1(t-1)))\cos(0.5(x_k^1(t)+x_k^1(t-1))) \\ x_k^2(t+1)=\dfrac{5x_k^2(t)x_k^2(t-1)}{1+x_k^2(t)^2+x_k^2(t-1)^2+x_k^2(t-2)^2}+u_k^2(t) \\ \quad\quad +1.1u_k^2(t-1)+1.4u_k^2(t)+0.5u_k^1(t) \\ y_k^1(t)=(1+0.5\sin(2\pi t/100))x_k^1(t)-x_k^2(t) \\ y_k^2(t)=x_k^1(t)+(1+0.5\cos(2\pi t/100))x_k^2(t) \end{cases}$$

where the disturbance $\boldsymbol{d}_k(t)$ is

$$\boldsymbol{d}_k(t)=\begin{bmatrix} 0.02\cos(0.3t)+0.01\cos(0.1k) \\ 0.02\sin(0.2t)+0.01\cos(0.1k) \end{bmatrix}$$

In the simulation, the system input is given as $u_k^1(t)=0.01(\sin(t/100)-\sin(k/100))$, $u_k^2(t)=-0.01(\cos(t/100)+\sin(k/100))$.

Consider the case that the system state is measurable. Applying the state-based ILADOB for nonlinear system

(34a)-(34c) with $\eta_2 = 0.5$, $\mu_2 = 1$, $\boldsymbol{\Gamma} = \text{diag}\{0.3, 0.3\}$, $\boldsymbol{M} = \boldsymbol{I}_n$, the results are shown in Figs. 13-14. One sees that the proposed state-based ILADOB method is applicable to general nonlinear systems without depending any model information and a perfect estimation performance can be achieved with increasing iterations.

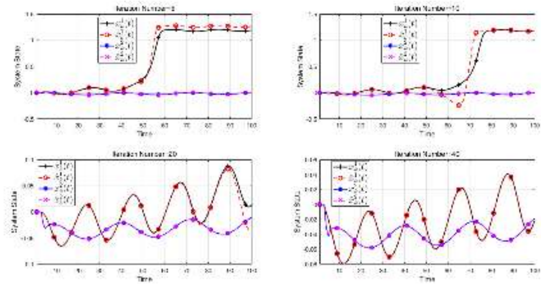

FIGURE 13. The estimation performance of system states in Example 2.

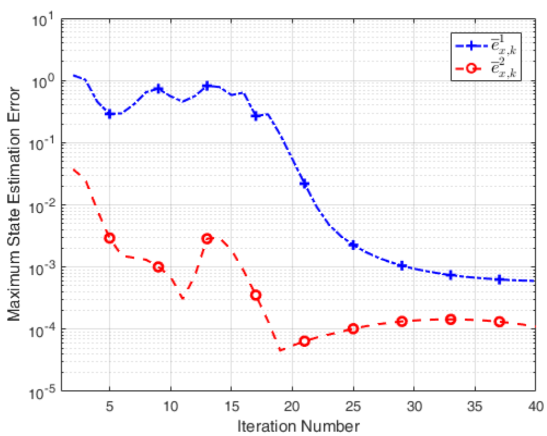

FIGURE 14. The convergence of state estimation error in Example 2.

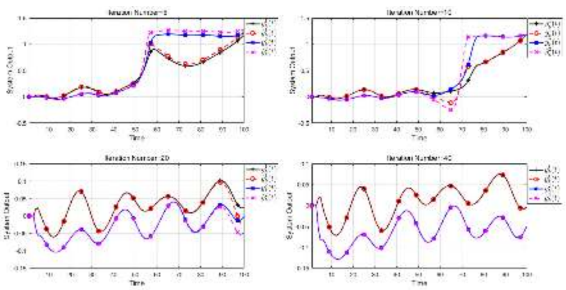

FIGURE 15. The estimation performance of system output in Example 2.

Further, consider the case that the system state is unmeasurable. Select $\eta_2 = 0.5$, $\mu_2 = 1$, $\boldsymbol{\Gamma} = \text{diag}\{0.3, 0.3\}$, $\boldsymbol{V} = \begin{bmatrix} 1 & -1 \end{bmatrix}$, $\boldsymbol{K} = \text{diag}\{0.7, 0.7\}$, $\boldsymbol{M} = \boldsymbol{I}_n$. Applying the proposed output-based ILADOB (35a)-(35e), the results are shown in figures 15 and 16, respectively.

Apparently, the proposed output-based ILADOB is also a data-driven approach, which only uses I/O data and very little model information, and is able to estimate the accumulative disturbance of nonlinear systems and attains an iteratively convergent performance of the output estimation error.

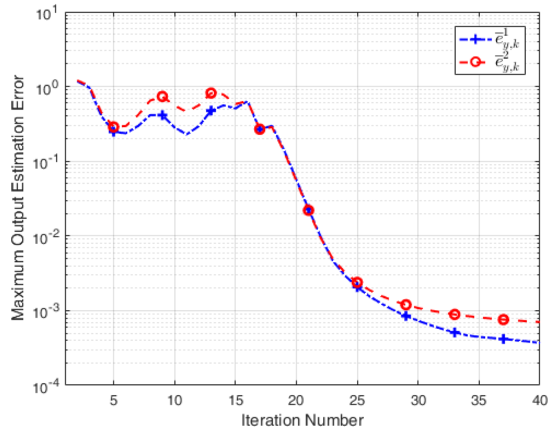

FIGURE 16. The convergence of output estimation error in Example 2.

V. CONCLUSION

Two novel ILADOB methods are proposed to observe the nonrepetitive disturbances in a repetitive system. By using a virtual linear data model, the dynamic relationship of the repetitive system is built between two consecutive iterations where the nonrepetitive disturbances are also represented as an accumulative one. Both state-based and output-based ILADOB methods are proposed by incorporating a gradient parameter estimation law to update the linear data model using I/O data. The two ILADOB methods are executed along the iteration axis using little information of mechanistic model.

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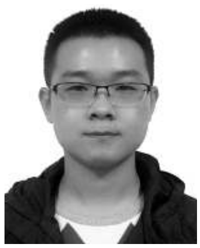

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