Iterative Multiuser Detection for CDMA with FEC: Near-Single-User Performance

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***Abstract*—This paper introduces an iterative multiuser receiver for direct sequence code-division multiple access (DS-CDMA) with forward error control (FEC) coding. The receiver is derived from the maximum *a posteriori* (MAP) criterion for the joint received signal, but uses only single-user decoders. Iterations of the system are used to improve performance, with dramatic effects. Single-user turbo code decoders are utilized as the FEC system and a complexity study is presented. Simulation results show that the performance approaches single-user performance even for moderate signal-to-noise ratios.**

***Index Terms*— Code-division multiaccess, decoding, multiuser channels, random codes, turbo codes.**

I. Introduction

WITH THE standardization of direct spread code-division multiple access (DS-CDMA) for mobile communications [1], a number of vendors have introduced their products onto the world market. This has raised a lot of interest on the potential capabilities and capacity of this multiple-access technology [2]–[7]. In this paper we study the uplink, or the base-station (BS) receiver. In the design of these systems most are currently symbol-synchronous or quasisymbol-synchronous so that orthogonal codes can be utilized. When orthogonal codes are used the BS linear filter receivers perform well in detecting the signal sent by taking advantage of this orthogonality, which gives performance equal to single-user performance.

In a true mobile wireless system, synchronization is difficult to maintain and needs tight closed-loop timing control between the BS and the mobile station (MS). If this timing control is not maintained then the orthogonal properties are lost and performance degrades severely. Multipath effects, common in mobile radio channels, also destroy this orthogonal property. If the codes are randomly selected, however, the performance of a synchronous system is on average the same as that of an asynchronous system. Work produced by Grant *et al.* [8] shows that the capacity penalty vanishes, for a large number of users, using randomly selected spreading codes, as the ratio between number of users and spreading length becomes large. Jana *et al.* [9] have shown a slightly different result; they showed that the upper bound of the normalized minimum distance for a trellis-coded multiuser CDMA system with nonorthogonal spreading is identical to that of the single-user case. This means that asymptotically, using nonorthogonal codes, or random codes, single-user performance should be possible. With such a receiver the performance under asynchronous conditions will be the same as that under synchronous conditions. We are therefore motivated to look for new multiuser receiver structures that use random codes to achieve near-single-user performance.

A paper by Giallorenzi *et al.* [7] formulated the optimal multiuser sequence estimator for an asynchronous DS-CDMA system where each user employs convolutional error control coding. Giallorenzi *et al.* found that the complexity per information bit using the MLSE solution depends exponentially on the number of users in the system and the number of states in each user's encoder. We propose to partition the receiver to reduce the complexity, without sacrificing performance. This paper therefore describes a partitioned trellis-based receiver with separate equalization and decoding. We develop a multiuser receiver (or equalizer) from the maximum-*a posteriori* (MAP) criterion. The MAP criterion maximizes the probability of a correct bit decision and, hence, minimizes the probability of error [10, p. 245].

Recently, a new coding method, called turbo codes, was introduced [11]. This technique achieves reliable transmission while operating close to the Shannon limit. Turbo codes combine the concept of soft-in/soft-out decoding, iterative decoding, nonuniform random interleaving, and parallel concatenated convolutional codes (PCCC). Further to this, published results by Benedetto and Montorsi [12] discuss serial concatenated convolutional codes (SCCC).

Several authors have proposed using turbo codes for DS-CDMA systems [4], [5]. These papers discuss system implementations but show no performance results. The authors would like to note that since the submission of this paper several independent publications have shown similar results

Paper approved by R. Kohno, the EDitor for Spread Spectrum Theory and Applications of the IEEE Communications Society. Manuscript received April 28, 1997; revised December 17, 1997 and April 28, 1998. This work was supported in part by Telstra Australia under Contract 700136 and by the Commonwealth of Australia under International S & T Grant 56. This paper was presented in part at the International Symposium on Personal, Indoor and Mobile Radio Communications, Helsinki, Finland, September 1997.

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Publisher Item Identifier S 0090-6778(98)09380-5.

(albeit without the use of turbo codes for the channel code), namely, work by Moher [14] and Tarköy [15].

In this paper we concatenate the MAP-based multiuser receiver and soft-in/soft-out single-user trellis decoders to produce a type of serial concatenated convolutional code. The proposed structure differs from successive interference cancellation [6], because no signal cancellation takes place. Our system can be viewed as a soft parallel interference canceler or more correctly as an SCCC. A paper by Douillart *et al.* [13] discusses the use of an SCCC to cancel intersymbol interference (ISI). The technique described in [13] has similarities to this work but the end application is very different. The channel coding we use is turbo codes (PCCC) which gives very low error rates and soft-in/soft-out decoding can be implemented. A complexity analysis is presented and complexity reduction techniques are discussed as a way of minimizing the computational load.

The paper is organized as follows. In Section II we describe the coded bit synchronous channel model. In Section III we introduce the iterative multiuser receiver and derive the metric for the single-user decoders. We also describe the iterative process and detail assumptions made. In Section IV we describe the integration of the single-user turbo code decoders before detailing a complexity analysis in Section V. Section VI shows simulation results including near/far tests, and Section VII contains a concluding discussion.

Throughout this paper scalars are lower case, vectors are underlined lower case, and matrices are underlined upper case. The symbols $(\cdot)^\top$, $(\cdot)^{-1}$, and $|\cdot|$ are the transposition, inversion, and determinant operators, respectively. Variables have subscripts that refer to the time increment and superscripts that refer to the user.

II. System Model

We model the uplink of a DS/CDMA communication system, as a coded, discrete-time system using perfect square pulses (no pulse shaping and no ISI), perfect sample timing (no synchronization errors), and no multipath. The channel adds zero-mean complex white Gaussian noise with variance $\sigma^2 = N_0/2$, where N_0 is the single-sided noise power spectral density. The channel model is coded bit- and chip-synchronous with the samples taken at the chip rate. The coding method we use is limited to trellis codes [16] and FEC is provided by convolutional codes. K users each transmit L coded bits $d_t^{(k)} \in \{+1, -1\}$, where $k \in \{1, \cdots, K\}$ is the user number, and $t \in \{0, \cdots, L-1\}$ identifies the coded bit interval. The spreading code employed by user k at coded bit interval t consists of N chips and is denoted

$$\underline{s}_t^{(k)} \in \{-1/\sqrt{N}, +1/\sqrt{N}\}^N.$$

The chips of the spreading codes are selected independently for every user k for every coded bit interval t, this is statistically equivalent to using a pseudorandom sequence of period much greater than the spreading length N. Fig. 1 shows the channel model. The channel output $\underline{e}_t$ at time t can be expressed as

$$\underline{e}_t = \underline{A}_t \underline{d}_t + \underline{n}_t \in \mathbb{C}^N$$

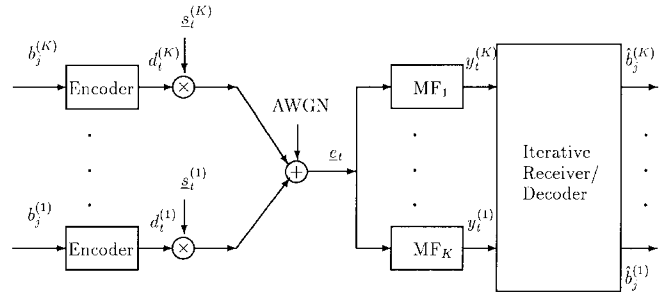

Fig. 1. Channel model.

where

$$\underline{A}_t = (\underline{s}_t^1, \cdots, \underline{s}_t^K) \in \left\{-1/\sqrt{N}, +1/\sqrt{N}\right\}^{N \times K}$$

is the bank of spreading codes, one spreading code for each user. The matched filter (MF) output $\underline{y}_t$ at time t then is

$$\begin{aligned} \underline{y}_t &= \underline{A}_t^\top \underline{A}_t \underline{d}_t + \underline{A}_t^\top \underline{n}_t \in \mathbb{C}^K \\ &= \underline{H}_t \underline{d}_t + \underline{z}_t \in \mathbb{C}^K \end{aligned} \tag{1}$$

where

$$\underline{d}_t = (d_t^{(1)}, \cdots, d_t^{(K)})^\top \in \{+1, -1\}^K$$

is the data vector, $\underline{H}_t$ is the crosscorrelation matrix of the spreading sequences, where $\underline{H}_t = \underline{A}_t^\top \underline{A}_t \in \mathbb{R}^{K \times K}$, and $\underline{z}_t$ and $\underline{n}_t$ are the correlated and uncorrelated noise vectors, respectively.

The noise samples $\underline{z}_t$ are Gaussian distributed where $E\{\underline{z}_t \underline{z}_t^\top\} = \underline{H}_t \sigma^2$, while the noise samples $\underline{n}_t$ are Gaussian distributed and have correlation $E\{\underline{n}_t \underline{n}_t^\top\} = \sigma^2$.

III. The Iterative Mulituser Receiver

In this section we derive the iterative multiuser receiver in terms of the MAP criterion. We then describe the channel decoder before discussing iterations of the receiver/decoder.

A. Deriving the Iterative Multiuser Receiver

The problem faced when designing a partitioned multiuser receiver/decoder with an iterative structure is that of generating the correct single-user input probability information (marginal probabilities) for the soft-in/soft-out FEC decoders and of supplying appropriate *a priori* information to the multiuser receiver on each iteration. Fortunately, both these problems can be solved simultaneously.

Fig. 2 shows the iterative multiuser receiver system. The receiver takes the matched filter channel output as described in (1) and generates the conditional channel probabilities $p(\underline{y}_t|\underline{d}_t)$, which are multivariate Gaussian conditional probabilities [10]. The metric generator then calculates the marginal probabilities $p(\underline{y}_t|d_t^{(k)})$ for the kth decoder. Single-user soft in/soft out FEC decoders generate the *a posteriori* coded bit probabilities $\Pr\{d_t^{(k)} = d|\underline{y}^{(k)}\}$, for user k for coded block size 0 to $L-1$. In the FEC decoder *a posteriori* coded bit probabilities are then used as *a priori* information for the metric generator on the next iteration. When the required

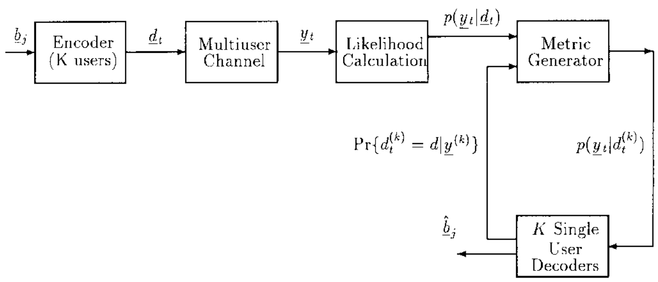

Fig. 2. Iterative multiuser model.

number of iterations have been completed a decision $(\hat{\underline{b}}_j)$ is output by the single-user FEC decoders. The distribution, $p(\underline{(y)}_t|\underline{(d)}_t$, of the matched filter output (1) conditioned on the information is multivariate Gaussian [10, p. 49]. The MAP decision rule [10] for the metric generator sets

$$\begin{aligned}\hat{\underline{d}}_t &= \arg\max_{\underline{d}_t} \frac{p(\underline{d}_t, \underline{y}_t)}{p(\underline{y}_t)} \\ &= \arg\max_{\underline{d}_t} p(\underline{y}_t|\underline{d}_t) \Pr\{\underline{d}_t\}. \end{aligned} \tag{2}$$

The Bayesian detector (2) is based on the coded bits and does not take into account the FEC code. This lowers the complexity from $\mathcal{O}(2^{K\nu})$ to $\mathcal{O}(2^{K+\nu})$ where ν is the memory length of the convolutional code. The above MAP criterion (2) was also used in [17] to compute suboptimal MAP metrics for single-user decoders in multiuser CDMA. Also note that (2) can be realized by a tree search with 2^{K-1} nodes, this is discussed further in Section V-A.

B. Single-User Input and Output Metrics

We now study generation of the input and output probabilities required by the K single-user decoders. The K single-user decoders calculate the *a posteriori* probabilities (APP) for the multiuser receiver using the algorithm due to Bahl *et al.* [18]. Assuming a rate $R = 1/n$ convolutional code, there are n coded bits (channel bits) $d_t^{(k)}$ for every uncoded information bit b_j, or encoder state transition at time $j = \lfloor t/n \rfloor$. Where $\lfloor x \rfloor$ indicates the largest integer not greater than x. The channel bits are denoted by $(d_t^{(k)}, \cdots, d_{t+n-1}^{(k)}) = \underline{d}_j^{(k)}$. We then calculate

$$\Pr\{d_{t'}^{(k)} = d|\underline{y}^{(k)}\} = \sum_{m'} \sum_{\substack{\underline{d}_j^{(k)} \\ (d_{t'}^{(k)}=d)}} \Pr\{S_{j-1} = m'; \underline{d}_j^{(k)}|\underline{y}^{(k)}\} \tag{3}$$

where S_j is the state at time j and m' ranges over all code states. Vector $\underline{d}_j^{(k)}$ is the hypothesized channel bit vector for a particular user k for a particular FEC code trellis transition at time j, where $t \le t' \le t + n - 1$.

The APP (3) for the kth-decoder output can be expressed [18] using three probability variables. They are the forward state probability, the reverse state probability, and the transition probability. While the first two are internal variables of the algorithm the transition probability accepts the input information $p(y_{t'}^{(k)}|d_{t'}^{(k)} = d)$ and is given by

$$\begin{aligned}&\gamma_j(m', m) \\ &\quad = \Pr\{S_j = m|S_{j-1} = m'\} \prod_{t'=t}^{t+n-1} p(y_{t'}^{(k)}|d_{t'}^{(k)} = d)\end{aligned} \tag{4}$$

if the transition $m' \to m$ exists in the code trellis, and its *a priori* probability equals $\Pr\{S_j = m|S_{j-1} = m'\}$. We will slightly modify this equation in Section IV-A so that we can accept the metric generator output.

C. Computation of Single-User Decoder Metrics

Now that we have described the input and output probabilistic requirements of the single-user decoders, we address the task of calculating $p(y_t^{(k)}, d_t^{(k)})$ which is a precursor to the conditional input probability as required by the single-user decoders. This metric is generated by manipulating the conditional probability $p(\underline{y}_t|\underline{d}_t)$ of $\underline{y}_t$ to obtain the joint probability in terms of the $d_t^{(k)}$. Using

$$p(\underline{y}_t, d_t^{(k)} = d) = \sum_{\substack{\underline{d}_t \\ (d_t^{(k)}=d)}} p(\underline{y}_t|\underline{d}_t) \Pr\{\underline{d}_t\}$$

the metric generator of Fig. 2 therefore calculates

$$\begin{aligned}p(\underline{y}_t|d_t^{(k)} = d) &= \frac{p(\underline{y}_t, d_t^{(k)})}{\Pr\{d_t^{(k)} = d\}} \\ &= \sum_{\substack{\underline{d}_t \\ (d_t^{(k)}=d)}} p(\underline{y}_t|\underline{d}_t) \prod_{\substack{i=1 \\ (i\neq k)}}^{K} \Pr\{d_t^{(i)}\}.\end{aligned} \tag{5}$$

We have assumed that the coded bits $d_t^{(i)}$ are independent among the different users (i.e., there is no transmission cooperation).

D. Iterating the Receiver

Multiuser systems describe receivers where users share information. If this is performed correctly a joint detection process results with improved performance over systems without joint detection. In turbo code decoding [11], the output probability of the first MAP decoder, $\Pr\{b_t = b|\underline{y}\}$, is used as *a priori* information for a second MAP decoder. In a similar fashion we assign the single-user decoder output probabilities from iteration i to the *a priori* input probabilities to the metric generator for iteration $i + 1$ in (5), i.e., we set

$$\Pr\{d_t^{(k)}\} = \Pr\{d_t^{(k)}|\underline{y}^{(k)}\}.$$

Analogous to [11] this is justified due to no correlation between the single-user convolutional codes and the spreading codes. On the first iteration we set the multiuser receiver *a priori* information to $\Pr\{d_t^{(k)}\} = 1/2$, i.e., all coded bit sequences are assumed to be equiprobable.

Due to the fact that the *a posteriori* information cannot be factored from the output of the metric generator, the *a priori* probability influence cannot be removed as in [11] where the extrinsic information is determined. The K single-user decoders also do not subtract *a priori* information as shown in [12] for the same reasons.

The iteration step discussed here highlights the fact that the receiver design is not a successive interference canceler, like for example in [6], as no hard decisions are made. Instead, soft APP's are passed between the soft-in/soft-out decoders and the metric generator. Interleaving between the multiuser receiver metric calculation and the single-user decoders is not required because the data is coded bit-synchronous where the multiuser receiver detects across users and the single-user decoders decode over time. If, however, the system were asynchronous, interleaving would be necessary to reduce correlations between the metric generator and the single-user decoders.

IV. Single-User Turbo Code Decoder

The use of turbo codes for error correction gives us a very powerful decoding stage. As with single-user turbo code results we expect and obtain better error performance than using a convolutional code as our channel code. The turbo code decoder structure needs to be modified slightly to accept soft input probabilities and produce suitable soft output probabilities for the *a priori* input to the multiuser metric generator. In this section we describe the use of turbo codes [11] as the channel code in the structure shown in Fig. 2. We describe just the kth user's decoder, which consists of two MAP decoders. We label the MAP decoders as MAP1 for the first decoder and MAP2 for the second decoder. There are K single-user turbo code encoders and decoders, as shown in Fig. 2, implemented in a similar way to those used in [11].

A. Turbo Code Decoder Soft Input

At the receiver a turbo code decoder is required for each user. These turbo code decoders receive likelihood values $p(\underline{y}_t|d_t^{(k)})$ from the metric generator. This data is used directly in MAP1's branch metric calculation [18] which is modified from (4) to

$$\gamma_j(m', m) = \Pr\{S_j = m|S_{j-1} = m'\} \prod_{t'=t}^{t+n-1} p(\underline{y}_{t'}|d_{t'}^{(k)})$$

where the product is over all the $d_t^{(k)}$ values that produce the transition of the MAP decoder from state m to state m'. Like the SCCC solution [12] MAP1 does not take any *a priori* information on the first iteration, that is, $\Pr\{S_j = m|S_{j-1} = m'\} = 1/2$ if the transition exists. On subsequent iterations of the turbo code decoder, *a priori* information for the systematic bits in MAP1 is generated by MAP2. There is no *a priori* information for the redundant coded bits.

B. Turbo Code Decoder Soft Output Generation

As in [11], after the desired number of iterations of the turbo code decoder are performed a hard decision $(\hat{\underline{b}}_j)$ on the information bits is output as the final result. We, however, want to produce a soft *a posteriori* probability output which can be used as *a priori* information for the metric generator. This *a posteriori* probability output must be generated for all bits that were input to the turbo code decoder (not just the information bits as in [11]).

The turbo code decoder is modified to produce systematic (uncoded) and redundant (coded) conditional bit probabilities as *a priori* information for the metric calculation $(\Pr\{d_t^{(k)}|\underline{y}^{(k)}\})$. The output probabilities are calculated using (3). The redundant coded bit probabilities are taken from the output of MAP1 and MAP2. As the turbo code is punctured, the redundant coded bit probabilities are taken alternately from MAP1 and MAP2 according to the puncturing scheme as described in [11]. The uncoded systematic bit probabilities are taken from the output of MAP2. Each turbo code decoder outputs a block containing L values of $\Pr\{d_t^{(k)}|\underline{y}^{(k)}\}$ in the same order as the input vector that it received from the metric generator on the previous iteration $p(\underline{y}_t|d_t^{(k)})$.

TABLE I
Complexity of Iterative Multiuser Receiver Components

Procedure	*Complexity(FLOP per Information Symbol)*
Likelihood Calc.	$\frac{(2^K(7K^3/6+4K^2+6K+4))}{(RK)}$
Metric Gen.(1 Iter.)	$(2^K(K-1))/(RK)$
Turbo Decoder(1 Iter.)	$2(PS(R^{-1}/2+10)-2S+16)$

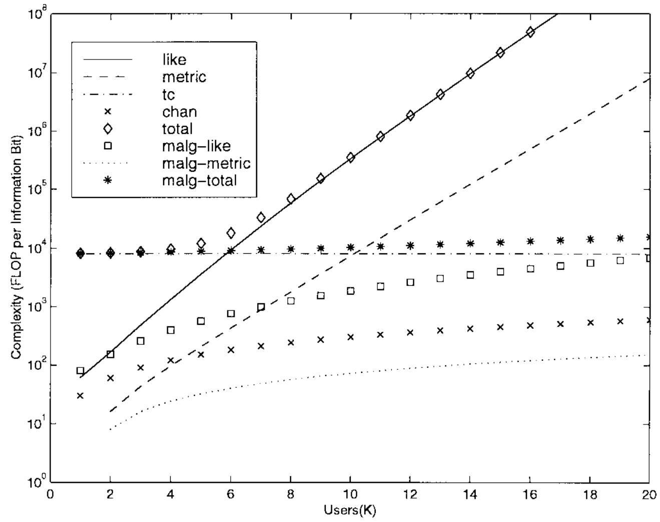

Fig. 3. Complexity-reduction techniques.

V. Complexity Analysis

In this section we study the complexity of the iterative multiuser receiver/decoder. This is important so that we can determine which parts of the system will consume most computational resources, and, therefore, where to concentrate efforts for complexity reduction.

We determine the number of floating-point operations required per information bit transmitted for the likelihood calculation, metric generator, and turbo decoder. We assign the variable S to be the number of states in the decoder, the number of users to equal K, the number of paths out of each state to equal P, and the rate of the FEC code to equal R.

Table I shows the complexity in terms of those variables. Fig. 3 shows the complexity of the three components of the

TABLE II
REDUCED COMPLEXITY OF ITERATIVE MULTIUSER RECEIVER COMPONENTS

Procedure	*Complexity(FLOP per Information Symbol)*
Likelihood Calc.(1 Iter.)	$(2K^2+3K+5)/R$
Metric Gen.(1 Iter.)	$(K^2-K)/(RK)$

system for a varying number of users. For more than six users (the case of interest for practical implementation) the 2^K term in the likelihood generation and metric generation dominates complexity. Fig. 3 shows the complexity of computing the likelihood function $p(\underline{y}_t|\underline{d}_t)$ by the curve labeled "like," the complexity of the metric generation (5) by the curve labeled "metric," and the complexity of the turbo code decoder by the curve labeled "tc," against users. The curve "total" is the sum of these three operations. For this curve the number of turbo code iterations was set to four, the number of multiuser iterations was set to three, the number of states was set to $S=16$, the paths out of each state was set to $P=2$, and the rate of the code $R=0.5$; the same as used in the final performance tests.

A. Complexity Reduction Techniques

From the complexity analysis it is clear that our solution is exponentially large with the number of users, like the optimal decoder [19]. In this section we propose techniques to reduce this complexity. The computation of $p(\underline{y}_t|\underline{d}_t)$ contains a number of complex linear algebraic tasks. As the first term of the exponential is independent of the conditioning variable $\underline{d}$ we can simplify to

$$p(\underline{y}_t|\underline{d}_t) = C_h \exp\left\{\frac{1}{2\sigma^2}\left(2\underline{y}_t^\top \underline{d}_t - \underline{d}_t^\top \underline{H}_t \underline{d}_t\right)\right\} \tag{6}$$

where C_h is a constant and does not have to be computed. This is a significant reduction as we have removed the need to compute the inverse of the crosscorrelation matrix $(\underline{H}_t)$. Even with this scheme the dominant 2^K term still exists in the likelihood calculation and the metric generation.

Another technique, suggested independently by Hoeher [20] and Nasiri-Kenari and Rushforth [21], is to calculate likelihoods $p(\underline{y}_t|\underline{d}_t)$ based on a one-bit difference from a previous likelihood. This means that the likelihood calculation to compute $p(\underline{y}_t|\underline{d}_t)$ only has to be computed once. Thereafter, a step-wise difference calculation is required to determine all the possible likelihood values. The technique of [21] still requires the computation of 2^K likelihoods. However, we now propose to use an M algorithm [16] and setting $M=K$ we compute only K likelihoods which are used to approximate (5), this reduces complexity from $\mathcal{O}(2^{K+\nu})$ to $\mathcal{O}(K2^\nu)$. Using the two reduction techniques discussed the complexity of the likelihood calculator and the metric generator is now shown in Table II.

The M-algorithm complexity of the likelihood calculation is shown in Fig. 3 as "malg-like," and the M-algorithm complexity of the metric calculation is "malg-metric" when $M=K$. The total sum complexity (including "tc") is now "malg-total" which meets our linear complexity requirement.

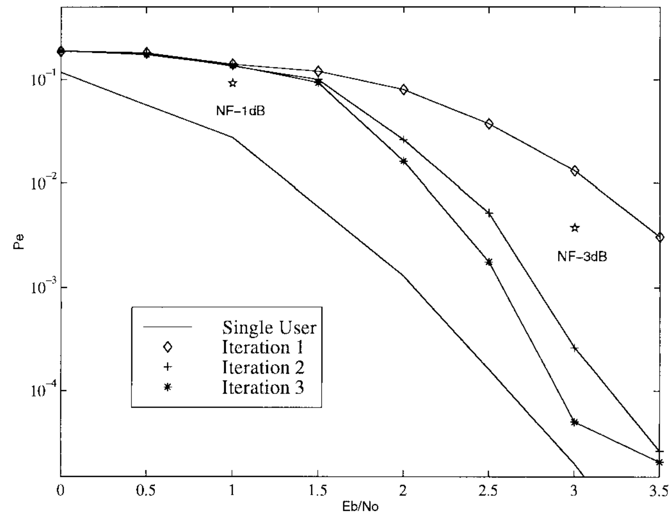

Fig. 4. Receiver performance for $k=5$, $N=7$ synchronous CDMA channel.

If the likelihood difference calculation is used to calculate only a list of M outputs we no longer have all the possible likelihoods available. To try to maximize performance under these conditions a number of techniques are used. The M-algorithm likelihood calculation is run with a different hypothesized starting vector $(\underline{d}_t)$ on the second iteration generating a second likelihood list. This starting vector is selected based on making a hard decision on the likelihood output from the single-user decoders. The likelihood lists from previous iterations are combined to increase the likelihood data available for metric generation.

VI. SIMULATIONS

The simulation result in Figs. 4 and 5 shows the average performance over all users. For each simulation a turbo code encoder of rate $R=1/2$, consisting of two parallel recursive systematic encoders, generators G (37, 21), memory $\nu=4$, separated by random interleaving, was used as the error control code. The block size was set to $RL=200$ information bits as is typical in mobile applications. We assume the receiver and decoder know the noise variance (σ^2) and the spreading codes exactly.

Fig. 4 shows performance for one to three multiuser iterations. In this paper we make no effort to determine the best combination of turbo code decoder iterations to multiuser iterations, we leave this as an implementation problem; instead, we highlight the principle of iterative multiuser detection. We choose what we believe is a likely combination of four turbo decoder iterations and one to three multiuser iterations. The label "single user" indicates single-user performance, that is, no multiple-access interference is present and the turbo code decoder completes four iterations. The simulation was performed with random spreading codes and a processing gain of $N=7$, with $K=5$ users. All of the possible 2^K likelihood values $p(\underline{y}_t|\underline{d}_t)$ were calculated. We see that most of the improvement occurs in the second iteration. It is also apparent that after three iterations in this highly loaded random

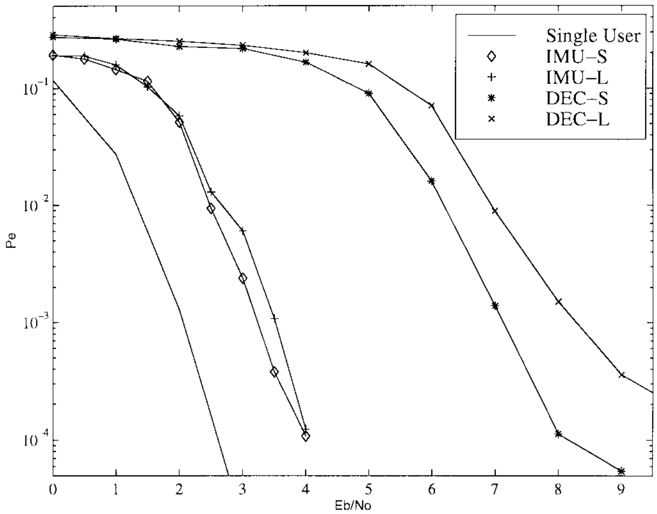

Fig. 5. Reduced complexity receiver performance.

code case, a result within 0.3 dB of single-user performance is obtained at a probability of error of 10^{-4}.

The near–far performance of the proposed receiver was also investigated. Three users were set to 1 dB and two users to 3 dB, resulting in a 2-dB difference. All other parameters were the same as previously described. The average performance of the 1-dB users along with the average performance of the 3-dB set of users is shown in Fig. 4 by the points labeled "NF-1 dB" and "NF-3 dB," respectively. The ideal result is where performance is independent of the received power of the other users. We see that the strong power users are degraded by approximately 0.7 dB while the weak power users are improved by 0.5 dB. The near–far effect in this receiver is therefore not severe.

Fig. 5 shows the simulation results for an M algorithm reduced-complexity solution as discussed in Section V-A. Four turbo code iterations and three multiuser iterations were used to produce these results. The performance of a $K = 5$ user system, random spreading codes of length $N = 7$, with $M = K$ is shown by the curve labeled "IMU-S" and a larger system with $N = 15$, $K = 10$, and $M = K$ is shown by the curve labeled "IMU-L." From this result we can see that even with a large, highly loaded, suboptimal, $\mathcal{O}(K2^{\nu})$ system the performance is approximately 1.5 dB away from single-user performance. We compare this result with the decorrelator performance [22] using turbo code decoding as the FEC. We see that this performance is 5.1 dB away from single-user performance for $K = 5, N = 7$, plotted as "DEC-S" and 6.2 dB away from single-user performance for the $K = 10, N = 15$ system, plotted as "DEC-L" all measured at a probability of error of 10^{-3}. Therefore, even under suboptimal operation, to reduce the complexity, the iterative multiuser receiver performs at least 3 dB better than the decorrelator receiver with turbo code decoding. Note also that the decorrelator receiver has less multiple-access interference power due to the constraint that $\underline{H}_t$ must not be singular. This is required to invert $\underline{H}_t$ for decorrelator operation.

VII. Conclusions

The main contribution of this paper is the derivation of the maximum *a posteriori* synchronous multiuser receiver which approaches single-user performance even for large system loads, i.e., spectrally efficient scenarios. We described the implementation of the iterative multiuser receiver and the modifications necessary to apply single-user turbo code decoders.

Simulation results show the performance of the system, which indicate that the iterative multiuser receiver design, combined with turbo code decoding, approaches turbo code single-user performance. This is the case even with random spreading codes and a large number of users relative to the spreading factor. The complexity of the optimal joint receiver is well known to be exponential with the number of users and the memory of the FEC code. We apply the M-algorithm to the likelihood calculation and set $M = K$ to obtain $\mathcal{O}(K2^{\nu})$ complexity, where ν is the memory of the FEC code. The performance loss under these conditions is approximately 1.5 dB with respect to single-user performance for $K = 10$ users and a spreading gain of $N = 15$.

Acknowledgment

The authors wish to thank the anonymous reviewers and system administrator B. Cooper who has been invaluable in providing a reliable computer network to produce these results.

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