$J_C(B, T, \varepsilon)$ Parameterization for the ITER Nb_3Sn Production

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***Abstract*—A number of models for the critical surface of Nb_3Sn, and in general A15 superconductors, have been developed in the past years. This paper compares the most common parameterizations using consistent notation. Although the parameterizations appear dissimilar at first sight, they are in reality all based on a fit of the normalized pinning force vs. the reduced field, and have similar scalings for the critical field and critical temperature based on a Unified Scaling Law. In this paper we take the various parameterizations as a basis for a generic scaling proposed for the characterization and production follow-up of the ITER Nb_3Sn strands. The accuracy of the scaling is estimated using the fitting residuals on various sets of $I_C(B, T, \varepsilon)$ data available in literature. We discuss the results, and give our view of the work towards a unified, practical parameterization.**

***Index Terms*—Critical current, critical surface, Nb_3Sn superconducting material, pinning force.**

I. Introduction

PARAMETRIZATIONS for the critical current I_C or current density J_C in Nb_3Sn have a large degree of complexity necessary to describe the field, temperature and strain dependence observed experimentally. Several attempts at finding a generic scaling of the critical surface have led to a number of, apparently, very different parameterizations [1]–[6]. These apparent dissimilarities can be reduced by adopting the idea of a *separable* parameterization [1], [6], [7] of the Unified Scaling Law proposed in [1], which also provides a framework for comparison. Our objective is to review the parameterizations developed in the past years, re-writing them in a uniform and consistent notation, and use this work as a basis to select a parameterization suitable to the characterization of the ITER Nb_3Sn production.

For later use, we define the critical field and temperature:

- critical field: $B^*_{C2}(T, \varepsilon)$;
- maximum critical field: $B^*_{C20\max} = B^*_{C2}(0, 0)$;
- critical temperature: $T^*_C(B, \varepsilon)$;
- maximum critical temperature: $T^*_{C0\max} = T^*_{C0}(0, 0)$;

functions of field B, temperature T and intrinsic longitudinal strain ε, i.e. the difference $\varepsilon = \varepsilon_{applied} - \varepsilon_{ma}$ of the applied strain $\varepsilon_{applied}$, and the strain $\varepsilon_{\max}$ at which the critical properties are maximum. The critical field and temperature are intended as *effective* values (i.e. obtained extrapolating I_C data). The parameterizations are best written using reduced variables:

- reduced magnetic field: $b = B/B^*_{C2}(T, \varepsilon)$.
- reduced temperature: $t = T/T^*_C(0, \varepsilon)$.

Most parameterizations require the knowledge of the temperature and strain dependent Ginzburg-Landau (GL) parameter $\kappa(T, \varepsilon)$. In practice the GL parameter is normalized to the value at zero temperature and strain, i.e. $\mathrm{k}(T, \varepsilon) = \kappa(T, \varepsilon)/\kappa(0, 0)$.

II. Unified Scaling Law in Separable Form

One of the most important results in the attempt to model the dependency of the critical current density of Nb_3Sn on field, temperature and strain was to recognize that J_C *scales* with these three variables [1], [6]–[8]. In simpler terms, this means that the shape of the dependency of J_C on one of the variables (B, T or ε) is maintained when the other variables are changed. The most useful form of scaling law for J_C is obtained making the additional assumption that the dependencies can be separated as follows [1], [6], [7]:

$$F_P = J_C(B, T, \varepsilon) \times B = Cg(\varepsilon)h(t)f_P(b) \tag{1}$$

where C is a constant and the three functions $f_P(b)$, $h(t)$ and $g(\varepsilon)$ describe the dependencies on reduced field, reduced temperature and intrinsic strain. This separable form is indeed practical, but it is just a mathematical statement to be supported by results. However, if substantiated, separation of functions allows the determination of the three dependencies with a reduced number of measurements, and can be used to design an optimal measurement plan.

III. J_C Parameterizations

Several attempts at providing a generic J_C parameterization have been documented in the references listed, spanning a period of over 20 years. For our work we have selected those that have been most widely used, or most significant for the understanding of the scaling properties. A compilation of the parameterizations is reported in Table I, while each sub-section below reports references and comments to the specific properties of each.

A. Parameterization of Ekin

Documented in [1], [6], [7], this parameterization is based on simple expressions for the three functions $f_P(b)$, $h(t)$ and $g(\varepsilon)$. The functions are determined empirically, parametrically incorporating the collected contributions from the upper critical field, the Ginzburg-Landau parameter κ, and the density and strength of pinning centers in the material. The resulting parameterization is flexible and can provide very good interpolation. In its original form [1], the strain function is the simplest and most consistent description of the moderate intrinsic strain

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TABLE I
SUMMARY OF SEPARABLE FUNCTIONS FOR THE PARAMETERIZATION OF J_C IN Nb_3Sn SURVEYED

	$g(\varepsilon)$	$h(t)$	$f_P(b)$	$s(\varepsilon)$
Ekin	$[s(\varepsilon)]^{\sigma\ (\dagger)}$	$(1-t^{v})^{\eta}$	$b^{p}(1-b)^{q}$	$1-a\lvert\varepsilon\rvert^{1.7}$
Summers	$s(\varepsilon)$	$[1-0.31\,t^{2}(1-1.77\ln(t))]^{0.5}(1-t^{2})^{25}$	$b^{0.5}(1-b)^{2}$	$1-a\lvert\varepsilon\rvert^{1.7}$
Durham	$[s(\varepsilon)]^{\frac{w(n-2)+2+u}{w}}$	$(1-t^{v})^{n-2}(1-t^{2})^{2}$	$b^{p}(1-b)^{q}$	$1+c_2\varepsilon^2+c_3\varepsilon^3+c_4\varepsilon^4$
Twente	$s(\varepsilon)$	$(1-t^{1.52})(1-t^{2})$	$b^{0.5}(1-b)^{2}$	$1+\dfrac{C_{a1}\left(\sqrt{\varepsilon_{sh}^2+\varepsilon_{0,a}^2}-\sqrt{(\varepsilon-\varepsilon_{sh})^2+\varepsilon_{0,a}^2}\right)-C_{a2}\varepsilon}{1-C_{a1}\varepsilon_{0,a}}$ $\varepsilon_{sh}=\dfrac{C_{a2}\varepsilon_{0,a}}{\sqrt{C_{a1}^2-C_{a2}^2}}$
Markiewicz	-	-		$\dfrac{1}{1+c_2\varepsilon^2+c_3\varepsilon^3+c_4\varepsilon^4}$
Oh and Kim	$[s_B(\varepsilon)]^{2.5}[k(T,\varepsilon)]^{0.5}(1-t^{2.17})^{2.5(\ddagger)}$		$b^{0.5}(1-b)^{2}$	$1-\beta\lvert\varepsilon\rvert^{1.7\,(*)}$
ITER-2008	$s(\varepsilon)$	$(1-t^{1.52})(1-t^{2})$	$b^{p}(1-b)^{q}$	$1+\dfrac{C_{a1}\left(\sqrt{\varepsilon_{sh}^2+\varepsilon_{0,a}^2}-\sqrt{(\varepsilon-\varepsilon_{sh})^2+\varepsilon_{0,a}^2}\right)-C_{a2}\varepsilon}{1-C_{a1}\varepsilon_{0,a}}$ $\varepsilon_{sh}=\dfrac{C_{a2}\varepsilon_{0,a}}{\sqrt{C_{a1}^2-C_{a2}^2}}$

$^{(\dagger)}$ modified to cover the high compressive regime to $g(\varepsilon)=(1-a|\varepsilon|^{u})^{\sigma}+H(\varepsilon_0-\varepsilon)g_1|\varepsilon_0-\varepsilon|^{g_2}$, where $\varepsilon_0=-0.5\%$ and $H(x)$ is the heavyside function $H(x)=0$ for $x<0$ and $H(x)=1$ for $x\geq 1$.
$^{(\ddagger)}$ the product $g(\varepsilon)h(t)$ cannot be separated for this parametrization
$^{(*)}$different $s(\varepsilon)$ functions are used for the field, temperature and critical current scaling

range $(-0.5\% < \varepsilon < 0.5\%)$. To accurately include high compressive strains (below -0.5%), it has been modified with the addition of a term to form the *extended power law* [6]. In its most general form, the number of free fitting parameters is 10 for the moderate strain regime and 14 for extended compressive strains. Parameter values can be built up from separate strain and temperature measurements, a great advantage from an engineering standpoint.

B. Parameterization of Summers, Guinan, Miller and Hahn

Proposed in [2], this parameterization is the fruit of the initial work on the material database for ITER Nb_3Sn. The parameterization has few free parameters (a total of 4), and is hence relatively easy to handle. The main drawback is that the pinning force fit has many pre-determined exponents that are suitable only for relatively pure Nb_3Sn and fails to describe many of the present Nb_3Sn wires for ITER that have considerable composition gradients in the filaments. In addition, the strain function, identical to the moderate-strain range power-law proposed by Ekin [1], is too simple to describe properly the high compressive strains unique to the ITER CICC's (i.e., below -0.5%).

C. Parameterization of Durham University

This parameterization has been proposed in the present form in [3], and has been derived using a combination of microscopic theory and empirically determined parameter values. It has been tested extensively in its full form, which is very flexible. This parameterization, along with the extended power law of Ekin, is possibly the best parameterization for interpolation over a very wide range of different strands. The flexibility is reflected in the number of free parameters, which is relatively large (13 parameters for the interpolative scheme and 17 for the full general form). Some of the parameters, in particular the exponents of the strain function, have competing effects, which can make the fitting procedure delicate. The strain function proposed is a 4th degree polynomial, with good interpolation properties. The drawback is that it has several coefficients whose values depend on the fit range, and is not suitable for extrapolation outside the measurement data range. A simplified form, with a reduced parameter set, has been proposed recently [9]. The results are interesting, and the simplified parameterization achieves good accuracy with 6 fitting parameters (the other parameters in the general form are set to a given recommended value). Further testing will be necessary to establish the universality of this choice; see the later discussion.

D. Parameterization of Twente University

Godeke, *et al.* [4], have derived this parameterization using a combination of microscopic theory and empirical fits, i.e. an approach similar to the one of Durham University. The general form of the parameterization obtained has been greatly simplified (fixing fit exponents and dependencies) using results obtained on a number of ITER strands. Specifically, the pinning force and temperature dependence fits have fixed exponents. In particular the temperature dependence includes a $1/\kappa$ term rather than the $1/\kappa^2$ commonly expected. The model for the strain function has been derived to match the observation that the deviatoric strain has a dominant effect in tapes, and adjusted to fit wire data still maintaining the asymptotic behavior measured in tapes [10], [11]. The resulting number of fitting parameters is relatively small (7 parameters) for the accuracy reached. The main drawback is that the range of validity of the simplified form is restricted to reduced fields from ≈0.1 to ≈0.9 and for compressive strain down to -0.8%. Outside this range the parameterization is not sufficiently accurate to reproduce the features observed in measured data.

E. Strain Function of Markiewicz

All the strain functions used in the parameterizations discussed above are empirical fits of $J_C(\varepsilon)$, $B^*_{C2}(\varepsilon)$ or $T^*_C(\varepsilon)$. In contrast to this approach, Markiewicz has numerically derived the dependence of the critical temperature on strain [12]. Based on these numerical results, he has suggested an empirical *invariant strain function* [13] representing contributions from all 3D strain components either through the hydrostatic strain, or the deviatoric strain invariants. From the 3D function, Markiewicz has derived a specific form of the invariant strain function that applies to the case of uniaxial strain which is used for testing strands. The suggested approximation for the normalized dependence of the critical temperature on strain is a rational function [13] that could be used directly as an alternative to the functions used in other parameterizations. Even more interesting, including the full form of the invariant strain function, with all strain components, would offer a very powerful generalization of the scalings discussed in this paper to longitudinal as well as transverse strain. This work in progress deserves priority, especially in the light of the recent findings on the importance of the 3-D strain state in the ITER and HEP $\mathrm{Nb_3Sn}$ strands.

F. Other Parameterizations

Oh and Kim [5] have proposed a parameterization in which a part of the dependencies are determined through a theoretical analysis of pinning in $\mathrm{Nb_3Sn}$. Although this is interesting, fundamental work, this parameterization is not very practical as it cannot be cast in a separable form. In addition, the resulting number of fitting parameters is still relatively large (9 to 12), for a moderate fitting accuracy.

G. Parameterization for the ITER Nb_3Sn Production

Following the review of the various parameterizations described above, a specific set of expressions has been tentatively selected to describe the critical surface of the $\mathrm{Nb_3Sn}$ strands to be produced for ITER. This is essentially the same as the parameterizations used by the group at Twente University, but taking a flexible pinning force fit. The *ITER-2008* parameterization of the critical surface is:

$$J_C = \frac{C}{B} s(\varepsilon)(1-t^{1.52})(1-t^2)b^p(1-b)^q \quad (2)$$

$$B^*_{C2}(T,\varepsilon) = B^*_{C20\max} s(\varepsilon)(1-t^{1.52}) \quad (3)$$

$$T^*_C(B,\varepsilon) = T^*_{C0\max}[s(\varepsilon)]^{\frac{1}{3}}\left(1-\frac{B}{B^*_{C2}(0,\varepsilon)}\right)^{\frac{1}{1.52}} \quad (4)$$

$$s(\varepsilon) = 1 + \frac{C_{a1}\left(\sqrt{\varepsilon_{sh}^2+\varepsilon_{0,a}^2}-\sqrt{(\varepsilon-\varepsilon_{sh})^2+\varepsilon_{0,a}^2}\right)-C_{a2}\varepsilon}{1-C_{a1}\varepsilon_{0,a}} \quad (5)$$

$$\varepsilon_{sh} = \frac{C_{a2}\varepsilon_{0,a}}{\sqrt{C_{a1}^2-C_{a2}^2}} \quad (6)$$

The model requires the following 9 parameters determined by a data fitting procedure:

C	scaling constant
$B^*_{C20\max}$	upper critical field at zero temperature and strain
$T^*_{C0\max}$	critical temperature at zero field and strain
p	low field exponent of the pinning force ($p \approx 0.5$)
q	high field exponent of the pinning force ($q \approx 2$)
C_{a1}	strain fitting constant
C_{a2}	strain fitting constant
$\varepsilon_{0,a}$	residual strain component
$\varepsilon_{\max}$	tensile strain at which the maximum critical properties are reached

TABLE II
SOURCES AND REFERENCES FOR THE DATASETS USED TO TEST THE ITER-2008 PARAMETERIZATION

Dataset	Points	Description	Ref.
1	161	Furukawa Billet 10-SG96071-34U	[3]
2	118	OST Billet 7567-2	[3]
3	362	OKSC Billet NT6801	[3]
4	264	EM-LMI Billet 285-17	[3]
5	531	VAC Billet 21	[3]
6	182	OST RRP Spool 8712-2	[14]
7	123	EFDA dipole OCSI strand	[15]
8	93	Luvata billet 8401	[16]-[18]
9	96	Luvata billet 8404	[16]-[18]
10	93	OST billet 10019 T2	[18]-[21]
11	99	ITER OST strand	[22]
12	148	ITER Luvata strand	[22]
13-15		ITER OST, OKSC, OCSI	[9]
16		ITER CSMC stand	[23]
18-20		JA ITER strands	[23]

IV. ACCURACY TESTS

Data from several sources have been collected to test the accuracy of the interpolation of the ITER-2008 parameterization. A summary of the datasets taken for the tests is given in Table II, and covers strands from approximately 10 years of $\mathrm{Nb_3Sn}$ R&D and production. The datasets have typically 100 to at most 500 data points, each obtained for a different range of strain, field and temperature. The whole envelope of data covers an intrinsic strain range from -1.5% to 0.4%, a temperature range from 2.35 to 16 K and a field range from 0.5 to 19 T. The first 12 datasets have been fitted with the proposed parameterization. For the other datasets, the results of the comparison are quoted from [9] (datasets 13 through 15) and [23] (datasets 16 through 20). The quality of the fit has been judged based on the r.m.s. error on the measured vs. computed values of $\mathrm{I_C}$.

A summary of the r.m.s. error of the $\mathrm{I_C}$ fit is reported in Fig. 1. Overall, the ITER-2008 parameterization achieves an average accuracy of 3.8 A, best value of 1.5 A and worst value of 7.5 A. As shown in the scatter plot of Fig. 2, when compared to the highly accurate full parameterization of Durham University, the ITER-2008 parameterization yields r.m.s. errors that are on average 1.5 times larger, which is significant, but not dramatic.

An additional result of interest is that, examining the fitting parameters, the values of C_{a1} and C_{a2} appear to be strongly correlated. In particular we found that:

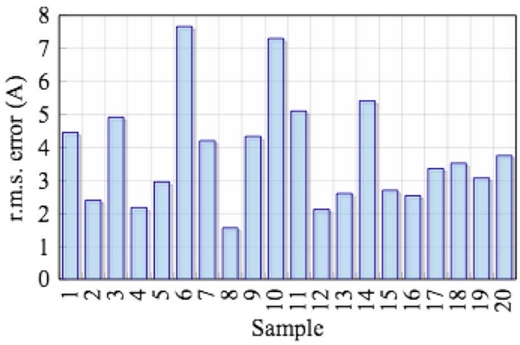

Fig. 1. Summary of r.m.s. error between measured I_C and I_C fit using the ITER-2008 parameterization.

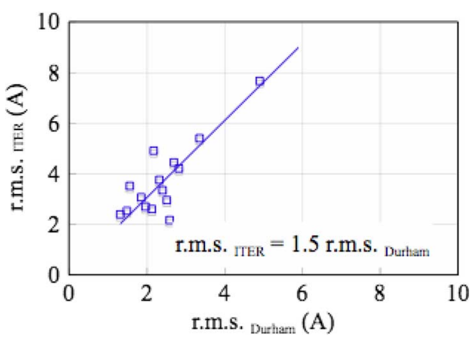

Fig. 2. Comparison of r.m.s. error achieved with the ITER-2008 parameterization, and the r.m.s. error obtained with the parameterization of Durham University.

$$C_{a2} \approx 0.36 C_{a1} - 12 \tag{7}$$

which can be used to eliminate one of the fitting parameters.

The fitting accuracy, however, is not the only quality indicator for the parameterization. Studying the single scaling functions we have identified two main reasons for the modest accuracy achieved, namely:

- the strain function is only appropriate in the moderate strain region, down to -0.8%. Beyond this value the measured behavior of the strain function exhibits an inflection and a change in curvature, which is not included in (5) and (6). In fact, already a direct fit of strain data beyond an intrinsic compressive strain of -0.5% affects the regime of moderate strain and reduces the accuracy of the fits;
- the temperature dependence may lack some degree of freedom necessary to describe accurately the scaling for some of the strands in the data sample analysed.

We are presently working to find suitable modifications of the parameterization to solve these drawbacks.

V. Conclusions and Perspectives

Most parameterizations for the critical surface of Nb_3Sn can be cast in the form of a separable Unified Scaling Law, as defined in [1], [6]. A parameterization for the ITER Nb_3Sn production (ITER-2008) was chosen among those presented based on criteria of simplicity and stability. Indeed, the main advantage of the expression selected is that it can be used for robust extrapolation outside the domain of fitting. Extensive data modeling tests show that ITER-2008 reaches modest but satisfactory accuracy for the majority of cases analysed, with typical r.m.s. errors below 4 A. It was recognized, however, that adaptations may be necessary to extend the domain of validity beyond compressive strains of -0.8%, and to properly describe the temperature dependencies of specific strands, which is the path upon which we are proceeding.

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