

American Mathematical Society

Colloquium Publications

Volume 52

J-holomorphic Curves and Symplectic Topology

Dusa McDuff
Dietmar Salamon



American Mathematical Society
Providence, Rhode Island

Contents

Preface	ix
Chapter 1. Introduction	1
1.1. Symplectic manifolds	1
1.2. Moduli spaces: regularity and compactness	4
1.3. Evaluation maps and pseudocycles	7
1.4. The Gromov–Witten invariants	10
Chapter 2. J -holomorphic Curves	17
2.1. Almost complex structures	17
2.2. The nonlinear Cauchy-Riemann equations	19
2.3. Unique continuation	21
2.4. Critical points	26
2.5. Somewhere injective curves	30
2.6. The adjunction inequality	34
Chapter 3. Moduli Spaces and Transversality	37
3.1. Moduli spaces of simple curves	37
3.2. Transversality	46
3.3. A regularity criterion	54
3.4. Curves with pointwise constraints	58
3.5. Implicit function theorem	65
Chapter 4. Compactness	71
4.1. Energy	72
4.2. The bubbling phenomenon	75
4.3. The mean value inequality	80
4.4. The isoperimetric inequality	85
4.5. Removal of singularities	89
4.6. Convergence modulo bubbling	92
4.7. Bubbles connect	98
Chapter 5. Stable Maps	107
5.1. Stable maps	107
5.2. Gromov convergence	114
5.3. Gromov compactness	117
5.4. Uniqueness of the limit	125
5.5. Gromov compactness for stable maps	129
5.6. The Gromov topology	137
Chapter 6. Moduli Spaces of Stable Maps	143

6.1.	Simple stable maps	145
6.2.	Transversality for simple stable maps	148
6.3.	Transversality for evaluation maps	153
6.4.	Semipositivity	156
6.5.	Pseudocycles	159
6.6.	Gromov–Witten pseudocycles	165
6.7.	The pseudocycle of graphs	169
Chapter 7.	Gromov–Witten Invariants	187
7.1.	Counting pseudoholomorphic spheres	189
7.2.	Variations on the definition	195
7.3.	Counting pseudoholomorphic graphs	203
7.4.	Rational curves in projective spaces	208
7.5.	Axioms for Gromov–Witten invariants	222
Chapter 8.	Hamiltonian Perturbations	239
8.1.	Trivial bundles	240
8.2.	Locally Hamiltonian fibrations	246
8.3.	Pseudoholomorphic sections	252
8.4.	Pseudoholomorphic spheres in the fiber	259
8.5.	The pseudocycle of sections	261
8.6.	Counting pseudoholomorphic sections	266
Chapter 9.	Applications in Symplectic Topology	275
9.1.	Periodic orbits of Hamiltonian systems	276
9.2.	Obstructions to Lagrangian embeddings	290
9.3.	The nonsqueezing theorem	301
9.4.	Symplectic 4-manifolds	307
9.5.	The group of symplectomorphisms	320
9.6.	Hofer geometry	328
9.7.	Distinguishing symplectic structures	334
Chapter 10.	Gluing	343
10.1.	The gluing theorem	344
10.2.	Connected sums of J -holomorphic curves	347
10.3.	Weighted norms	349
10.4.	Cutoff functions	353
10.5.	Construction of the gluing map	356
10.6.	The derivative of the gluing map	365
10.7.	Surjectivity of the gluing map	373
10.8.	Proof of the splitting axiom	379
Chapter 11.	Quantum Cohomology	387
11.1.	The small quantum cohomology ring	388
11.2.	The Gromov–Witten potential	403
11.3.	Four examples	409
11.4.	The Seidel representation	432
11.5.	Frobenius manifolds	443
Chapter 12.	Floer Homology	451

12.1. Floer's cochain complex	452
12.2. Ring structure	463
12.3. Poincaré duality	467
12.4. Spectral invariants	469
12.5. The Seidel representation	478
12.6. Donaldson's quantum category	483
12.7. The symplectic vortex equations	487
Appendix A. Fredholm Theory	493
A.1. Fredholm theory	493
A.2. Determinant line bundles	495
A.3. The implicit function theorem	500
A.4. Finite dimensional reduction	506
A.5. The Sard–Smale theorem	508
Appendix B. Elliptic Regularity	511
B.1. Sobolev spaces	511
B.2. The Calderon–Zygmund inequality	523
B.3. Regularity for the Laplace operator	530
B.4. Elliptic bootstrapping	533
Appendix C. The Riemann–Roch Theorem	541
C.1. Cauchy–Riemann operators	541
C.2. Elliptic estimates	546
C.3. The boundary Maslov index (by Joel Robbin)	552
C.4. Proof of the Riemann–Roch theorem	558
C.5. The Riemann mapping theorem	562
C.6. Nonsmooth bundles	564
C.7. Almost complex structures	565
Appendix D. Stable Curves of Genus Zero	569
D.1. Möbius transformations and cross ratios	569
D.2. Trees, labels, and splittings	572
D.3. Stable curves	579
D.4. An embedding	582
D.5. The Grothendieck–Knudsen compactification	587
D.6. The GK topology	592
D.7. Examples	595
Appendix E. Singularities and Intersections (written with Laurent Lazzarini)	601
E.1. The main results	602
E.2. Positivity of intersections	606
E.3. Integrability	612
E.4. The Hartman–Wintner theorem	616
E.5. Local behaviour	621
E.6. Contact between branches	626
E.7. Singularities of J -holomorphic curves	634
Bibliography	643

List of Symbols

655

Index

659