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J/ψ-Nucleon Scattering Length and In-Medium Mass Shift of J/ψ in QCD Sum Rule Analysis

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We calculate the spin-averaged J/ψ-nucleon scattering length $a_{J/\psi}$ by directly applying the QCD sum rule to the J/ψ-N forward scattering amplitude. Our result, $a_{J/\psi} = -0.10 \pm 0.02$ fm, implies the possibility of bound states with nuclei, though the force is weaker than that of the light vector mesons (ρ, ω, ϕ)-N cases. Up to dimension-4 gluonic operators, we evaluate the scattering length with a twist-2 contribution. This increases the absolute value of the scattering length by about 30%. If we apply $a_{J/\psi}$ to the effective mass of J/ψ in nuclear matter on the basis of the linear density approximation, it exhibits very slight decrease (4–7 MeV) at normal matter density.

§1. Introduction

Theoretical analysis on the in-medium properties of hadrons is increasingly necessary for various on-going and forthcoming heavy-ion experiments (such as SPS, LHC (CERN) and AGS, RHIC (BNL)).[1] In particular, experimentally it is important to observe vector mesons, because they decay into lepton pairs and carry the information inside matter without the disturbance of the strong interaction. The properties of light vector mesons in nuclear matter have been studied extensively in various theoretical approaches, including effective hadronic models[2] and QCD sum rules (QSR's).[3)-6)] Vacuum properties of the vector mesons have been successfully studied using the QSR's.[7),8)] The method enables us to express physical quantities such as mass and decay width in terms of the parameters of the QCD Lagrangian and vacuum condensates. Extending the vacuum QSR to finite density, we can consistently incorporate the effects of nuclear matter into the form of in-medium condensates. There are two methodologically different ways for applying in-medium QSR. First, Hatsuda and Lee developed the in-medium QSR formalism for light vector mesons.[3] They found a $10-20\%$ decrease of the masses of the ρ and ω mesons at normal matter density. Second, for light vector mesons, we formulated in-medium QSR[4),6)] based on the relation between the scattering length and the mass shift.[9] In this approach with the Fermi gas model, the in-medium correlation function is divided into a vacuum part and a one nucleon part. This one nucleon part corresponds to the forward vector meson-nucleon scattering amplitude. The QSR analysis on the forward scattering amplitude enables us to obtain information concerning the vector meson-nucleon interaction. Moreover, from this information we can estimate the change of spectra for vector mesons in nuclear matter. The difference between

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these two approaches has been discussed in Refs. 10) and 6). Eventually we derived in Ref. 6) the result that both of them are based on almost the same idea and can lead to results consistent with those of the effective models.

In this paper we apply the QSR analysis established in Ref. 6) to a heavy quark system with equal masses for quarks and antiquarks. As a concrete system we focus on J/ψ, which is a low-lying charmonium state ($^3S_1$). The study of medium modification of J/ψ has the following motivations:

1. We have detailed experimental information for charmonium. In particular, the spectrum of J/ψ is extremely narrow for leptonic decay ($\Gamma_{l^+l^-} \simeq 5$ keV). Thus it would be a good tool to observe the change of the spectra (e.g., mass shift) in nuclear matter.
2. Since charmonium and nucleons consist of quarks with different kinds of flavors, the J/ψ-N interaction is purely gluonic without quark exchange to first order in elastic scattering. This simplification reduces our practical calculation.
3. Theoretical studies for J/ψ in QSR are successful only in the description of the free state.[8),11)]
4. In order to utilize J/ψ suppression[12),13)] as a direct signal of the quark-gluon plasma phase, we need to understand the effect of nuclear absorption (*L-scaling*[14)]) theoretically. In particular, recent experimental data[15)] suggest a drastic deviation from the *L-scaling* in lead-lead collisions. We should estimate the elementary J/ψ-nucleon interaction to investigate the origin of such additional suppression. For this purpose, it is reasonable as a first step to study J/ψ-N elastic scattering at low energy.

Motivated by these points, we calculate the J/ψ-N scattering length and the mass shift of J/ψ in nuclear matter obtained by the scattering length. That is, the first aim is to estimate the essential features of the interaction between J/ψ and N through the scattering length. In practice, by applying QSR to the J/ψ-N forward scattering amplitude we calculate the scattering length. The scattering length is a physically very important quantity in free space, because it is the unique observable in J/ψ-N elastic scattering at low energy. If it is negative, then we could predict an attractive interaction capable of binding J/ψ to a nucleus, so that J/ψ could lead to a bound state with a nucleus. The prediction of such an exotic state would open exciting new directions in nuclear physics. As is well known, since the meson-nucleon interaction is repulsive for isovector mesons, a π meson forms a π-nucleus bound state by a Coulomb attractive force. On the other hand, J/ψ is expected to be bounded only by the attractive interaction from the isoscalar property. As was pointed out in Ref. 16), this interaction should be sufficiently attractive to allow a bound state. The probability of such exotic states has been recently discussed for η, ω and D cases.[17)]

The second aim is to determine the manner in which the superposition of elementary J/ψ-N scattering at low energy affects the effective mass of J/ψ in nuclear matter. When we work in a dilute nucleon gas, we find that the mass shift is linearly dependent on the density (linear density approximation).

This paper is organized as follows. In §2 we summarize the relation between the scattering length and the mass shift in the linear density approximation.[6)] In the actual calculation we adopt a moment sum rule method to the forward scattering amplitude. In §3 the Wilson coefficients on the OPE side are explicitly given for a twist-2 operator. In §4, in order to obtain unknown hadronic parameters for the forward scattering amplitude, we apply the moment sum rule to the vacuum correlation function. In §5 the numerical results of the scattering length and the mass shift of J/ψ are shown. Finally concluding remarks are given.

§2. The relation between scattering length and mass shift

Let us first review the relation between the scattering length and the mass shift on the basis of the QSR method.[4),6)] The starting point of this approach is the following vector current correlation function in the ground state of nuclear matter with nucleon density ρ_N:

$$\Pi^{\rm NM}_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle {\rm T}J_\mu(x)J^\dagger_\nu(0)\rangle_{{\rm NM}(\rho_N)}. \qquad (2\cdot1)$$

Here $q^\mu = (\omega, \boldsymbol{q})$ is the four-momentum carried by the J/ψ vector meson current $J_\mu(x) = \bar{c}\gamma_\mu c(x)$. Following the QSR method, when we apply an operator product expansion (OPE) to this correlator in the deep Euclidean region ($Q^2 = -q^2 > 0$), it is supposed that the ρ_N-dependence of this correlator is contained entirely in the ρ_N-dependence of various condensates. Moreover, we assume the Fermi gas model taking account of the Pauli principle among uncorrelated nucleons in nuclear matter. In this approximation, the in-medium correlation function reads

$$\Pi^{\rm NM}_{\mu\nu}(q) = \Pi^0_{\mu\nu}(q) + \sum_{\rm spin,isospin}\int^{p_F}\frac{d^3p}{(2\pi)^3 2p_0}T_{\mu\nu}(q), \qquad (2\cdot2)$$

where $\Pi^0_{\mu\nu}(q)$ is the in-vacuum correlation function and $\sum_{\rm spin,isospin}$ denotes the sum of spin and isospin states for nucleons in nuclear matter. $T_{\mu\nu}(q)$ is the vector current-nucleon forward scattering amplitude, defined as

$$T_{\mu\nu}(\omega,\boldsymbol{q}) = i\int d^4x e^{iq\cdot x}\langle N(ps)|{\rm T}J_\mu(x)J^\dagger_\nu(0)|N(ps)\rangle. \qquad (2\cdot3)$$

Here $|N(ps)\rangle$ denotes the nucleon state with four momentum $p = (p_0, \boldsymbol{p})$ and spin s normalized covariantly as $\langle N(\boldsymbol{p})|N(\boldsymbol{p}')\rangle = (2\pi)^3 2p^0\delta^3(\boldsymbol{p}-\boldsymbol{p}')$. The quantity $\Pi^0_{\mu\nu}(q)$ gives the main contribution to $\Pi^{\rm NM}_{\mu\nu}(q)$ due to the perturbative contribution. On the other hand, $T_{\mu\nu}(q)$ leads to a small contribution for $\Pi^{\rm NM}_{\mu\nu}(q)$, but the effect is vital.

If we consider a normal matter density ($\rho_N \sim 0.17$ fm^{-3}), the integral of the last term in Eq. (2·2) can be approximated up to the first order of nucleon density ρ_N reasonably well. The linear density term corresponds to matter with static nucleons ($\boldsymbol{p} = \boldsymbol{0}$), and the higher order correction terms correspond to the velocity-dependent terms involving the effect of Fermi motion ($\boldsymbol{p} \neq \boldsymbol{0}$) and the complex interaction among nucleons. The linear expression can be calculated model-independently. On the other hand, the higher order corrections depend on the model calculation, but in a few effective theories[18)] it is known that the effect for the linear result is fairly small ($\sim 10\%$) at the nuclear matter saturation density. Hatsuda et al. [10] also insist that the Fermi momentum correction is fairly small ($\sim 10\%$), up to twist-4 operators. Thus we can safely neglect the effect at the saturation density. Therefore we can set $p = (M_N, \mathbf{0})$ for $T_{\mu\nu}(p, q)$, so that we proceed to discussions based on the assumption that all nucleons are at rest in nuclear matter.

In Eq. (2·2) the second term implies a slight deviation from the properties in the free state determined by $\Pi^0_{\mu\nu}$. By applying the QSR method to $T_{\mu\nu}$ directly, we can relate the scattering length extracted from the QSR for $T_{\mu\nu}$ with the mass shift in the framework of QSR. Near the pole position of J/ψ, $T_{\mu\nu}$ can be associated with the T matrix for the forward J/ψ-N helicity amplitude $\mathcal{T}_{hH,h'H'}(\omega, \boldsymbol{q})$, where h (h') and H (H') are the helicity of the initial (final) J/ψ and the initial (final) nucleon, respectively. The relation between $T_{\mu\nu}$ and $\mathcal{T}_{hH,h'H'}$ is given by the relation

$$\epsilon^{\mu*}_{(h')}(q) T_{\mu\nu}(\omega, \boldsymbol{q}) \epsilon^{\nu}_{(h)}(q) \simeq \frac{-f^2_{J/\psi} m^4_{J/\psi}}{(q^2 - m^2_{J/\psi} + i\varepsilon)^2} \mathcal{T}_{hH,h'H'}(\omega, \boldsymbol{q}). \qquad (2·4)$$

Here we introduce the coupling $f_{J/\psi}$ and the J/ψ mass $m_{J/\psi}$ by the relation $\langle 0|J_\mu |J/\psi^{(h)}(q)\rangle = f_{J/\psi} m^2_{J/\psi} \epsilon^{(h)}_\mu(q)$ with the polarization vector $\epsilon^{(h)}_\mu$ normalized as $\sum_h \epsilon^{(h)*}_\mu(q) \epsilon^{(h)}_\nu(q) = -g^{\mu\nu} + q^\mu q^\nu / q^2$. Taking the spin average on both sides of Eq. (2·4), $T_{\mu\nu}(\omega, \boldsymbol{q})$ is projected onto $T(\omega, \boldsymbol{q}) = T^{\ \mu}_{\mu}/(-3)$, and $\mathcal{T}_{hH,h'H'}(\omega, \boldsymbol{q})$ is projected onto the spin averaged J/ψ-N T-matrix, $\mathcal{T}(\omega, \boldsymbol{q})$. At low energy, $q = (m_{J/\psi}, \mathbf{0})$ and $p = (M_N, \mathbf{0})$. $\mathcal{T}$ is reduced to the spin averaged J/ψ-N scattering length $a_{J/\psi} = 1/3(2a_{3/2} + a_{1/2})$ ($a_{1/2}$ and $a_{3/2}$ are the scattering lengths in the spin-1/2 and spin-3/2 channels, respectively) as $\mathcal{T}(m_{J/\psi}, \boldsymbol{q} = \mathbf{0}) = 8\pi(M_N + m_{J/\psi}) a_{J/\psi}$. We note that the negative value of $a_{J/\psi}$ corresponds to attraction in our convention.

We relate the parameters of the QCD Lagrangian with the hadronic mass and coupling using the dispersion relation. If one utilizes the retarded correlation function as a useful quantity for dispersion analysis, we obtain the following dispersion relation for $T(\omega, \boldsymbol{q})$:

$$T(\omega, \mathbf{0}) = \frac{1}{\pi} \int_{-\infty}^{\infty} du \frac{\rho(u, \mathbf{0})}{u - \omega - i\varepsilon} = \frac{1}{\pi} \int_0^{\infty} du^2 \frac{\rho(u, \mathbf{0})}{u^2 - \omega^2}. \qquad (2·5)$$

Here the spectral function $\rho(u, \boldsymbol{q} = \mathbf{0})$ is given with three unknown phenomenological parameters, a, b and c, in terms of the spin-averaged J/ψ-N forward T-matrix $\mathcal{T}$ such as

$$\rho(u, \boldsymbol{q} = \mathbf{0}) = \frac{1}{\pi} \mathrm{Im} \left[\frac{-f^2_{J/\psi} m^4_{J/\psi}}{(u^2 - m^2_{J/\psi} + i\varepsilon)^2} \mathcal{T}(u, \mathbf{0}) \right] + \cdots \qquad (2·6)$$

$$= a\, \delta'(u^2 - m^2_{J/\psi}) + b\, \delta(u^2 - m^2_{J/\psi}) + c\, \delta(u^2 - s_0). \qquad (2·7)$$

The terms denoted by $\cdots$ in Eq. (2·6) represent the continuum contribution, and δ' in Eq. (2·7) is the first derivative of the δ function with respect to u^2. The first a-term is the double-pole term corresponding to the on-shell effect of the T matrix, and the coefficient is associated with the scattering length $a_{J/\psi}$ as $a = 8\pi f^2_{J/\psi} m^4_{J/\psi}(M_N +$

$m_{J/\psi})a_{J/\psi}$. The second b-term is the simple-pole term corresponding to the off-shell effect of the T matrix. The third c-term is the continuum term corresponding to other remaining effects, where s_0 is regarded as the continuum threshold in vacuum. Now the contribution from the inelastic channels is not included in the ansatz of Eq. (2·7). In this system the OZI rule restricts the inelastic channels of the J/ψ-N interaction to those containing charmed quarks, for example, $J/\psi+N \to D+\bar{D}+N$ and $J/\psi + N \to \Lambda_c + \bar{D}$. But the contribution from all these processes is near the continuum threshold ($s_0 \simeq 13$ GeV^2) or included completely in the continuum. Here such inelastic channels are isolated from the contribution of the a- and b-terms in Eq. (2·7). This situation does not change even when one takes into account the effect of the Fermi motion of the nucleons. The parametrization Eq. (2·7) for the spectral function, however, may be quite simplified, compared to the behavior of a realistic spectral function. This point leaves us further discussion in the future.

The parameters a, b and c in Eq. (2·7) are not completely independent. That is, among these parameters we introduce a constraint relation, which is imposed by a low energy theorem for the J/ψ current-nucleon forward scattering amplitude. In the low energy limit $\omega \to 0$, $T(\omega, \mathbf{0})$ becomes equivalent to the Born term $T^{\mathrm{Born}}(\omega, \mathbf{0})$, which is zero in the J/ψ-N system for lack of intrinsic charmed quarks inside a nucleon. Now we obtain the constraint relation

$$\frac{a}{m_{J/\psi}^4} + \frac{b}{m_{J/\psi}^2} + \frac{c}{s_0} = 0 \tag{2·8}$$

from the low energy theorem. Therefore, the spectral function is parametrized with the two unknown phenomenological parameters a and b, by removing c from Eq. (2·8). The phenomenological (PH) side for $\Pi_{\mu\nu}^{\mathrm{NM}}$ can be expressed as the combination of the pole position for $\Pi_{\mu\nu}^0$ and $T_{\mu\nu}$ such as

$$\begin{aligned}\Pi_{\mu\nu}^{\mathrm{NM}} &= \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\left[\frac{F}{m_{J/\psi}^2 - q^2} + \frac{\rho_N}{2M_N}\left\{\frac{a}{(m_{J/\psi}^2 - q^2)^2} + \frac{b}{m_{J/\psi}^2 - q^2}\right\} + \cdots\right] \\ &\propto \frac{F + \Delta F}{(m_{J/\psi}^2 + \Delta m_{J/\psi}^2 - q^2)} + \cdots, \end{aligned} \tag{2·9}$$

where the pole residue F in $\Pi_{\mu\nu}^0$ is equivalent to $f_{J/\psi}^2 m_{J/\psi}^4$ and the deviation ΔF is $\rho_N b/2M_N$. The quantity expressed as the shift of the squared J/ψ mass in nuclear matter,

$$\Delta m_{J/\psi}^2 = 2m_{J/\psi}\delta m_{J/\psi} = \frac{\rho_N}{2M_N}\frac{a}{f_{J/\psi}^2 m_{J/\psi}^4} = \frac{\rho_N}{2M_N}\, 8\pi(M_N + m_{J/\psi})a_{J/\psi}, \tag{2·10}$$

is proportional to the scattering length $a_{J/\psi}$ through the double pole term in $T_{\mu\nu}$. Thus we can calculate the mass shift $\delta m_{J/\psi}$ in Eq. (2·10) from $a_{J/\psi}$ obtained by QSR for $T_{\mu\nu}$.

We explicitly write down the PH side with the unknown parameters a and b for $T(q^2)$ using Eq. (2·5), (2·7) and (2·8). We take the n-th derivative with respect to

q^2 after dividing $T^{\rm ph}$ by q^2 as follows and define it as $\widehat{T}^{(n)}$:

$$\begin{aligned}
\frac{1}{n!}\left(\frac{d}{dq^2}\right)^n \frac{T^{\rm ph}(q^2)}{q^2} &\equiv \widehat{T}^{(n)\,{\rm ph}}(q^2\,;\,a,b) \\
&= \frac{a}{m_{J/\psi}^4}\left[\frac{(n+1)m_{J/\psi}^2}{(m_{J/\psi}^2-q^2)^{n+1}}+\frac{1}{(m_{J/\psi}^2-q^2)^{n+1}}-\frac{1}{(s_0-q^2)^{n+1}}\right] \\
&\quad +\frac{b}{m_{J/\psi}^2}\left[\frac{1}{(m_{J/\psi}^2-q^2)^{n+1}}-\frac{1}{(s_0-q^2)^{n+1}}\right]. \qquad (2\cdot11)
\end{aligned}$$

In order to construct the QSR, we calculate the n-th derivative of the OPE side similarly in the next section.

§3. The calculation of the Wilson coefficients for $T_{\mu\nu}$

Now we give the OPE expression for $T_{\mu\nu}$. The main task on the OPE side is to calculate the Wilson coefficients based on perturbative QCD. In the case of J/ψ, the charmed quark mass is so heavy that the calculation of the Wilson coefficients must be carried out explicitly with the effect of heavy quark mass. We now expand local operators up to dimension-4 on the OPE side. Then pure gluonic contributions must be taken into account only for the local operators. Up to this order in the OPE, the nucleon matrix elements of two-gluon operators (GG) are most dominant. We note that, in contrast to the vacuum QCD sum rule, a new feature in $T_{\mu\nu}$ is that the nucleon matrix elements survive not only the Lorentz scalar operators but also nonscalar operators. That is, we must consider new contributions from twist-2 operators with two spins for the Wilson coefficients. In order to calculate the coefficient function, we adopt a well-known method for massive quarks propagating through coupling with soft gluons working as the external field, namely the fixed-point gauge method.[19] This gauge condition is expressed as $x^\mu A_\mu^a(x) = 0$. The nucleon matrix element of two-gluon operators can be decomposed into a scalar part and a twist-2 part with an additional four-vector u_μ ($u^2 = 1$), through simple tensor analysis:[20]

$$\begin{aligned}
\left\langle G^a_{\alpha\beta}G^b_{\gamma\delta}\right\rangle_N = \frac{\delta^{ab}}{96}\Bigg[&\left\langle G^2\right\rangle_N (g_{\alpha\gamma}g_{\beta\delta}-g_{\alpha\delta}g_{\beta\gamma}) - 4\left\langle (u\cdot G)^2-\frac{1}{4}G^2\right\rangle_N \\
&\times\left\{(g_{\alpha\gamma}g_{\beta\delta}-g_{\alpha\delta}g_{\beta\gamma}) - 2\left(g_{\alpha\gamma}u_\beta u_\delta - g_{\alpha\delta}u_\beta u_\gamma - g_{\beta\gamma}u_\alpha u_\delta + g_{\beta\delta}u_\alpha u_\gamma\right)\right\}\Bigg].
\end{aligned} \qquad (3\cdot1)$$

Here we have defined $(u\cdot G)^2 \equiv G^a_{\kappa\lambda}G^{a\ \lambda}_{\rho}\, u^\kappa u^\rho$. By introducing u_μ, one can imagine uniformly moving nucleons ($p_\mu = M_N u_\mu$) in nuclear matter, but in this case we set $u = (1, \mathbf{0})$. The OPE expression for $T_{\mu\nu}$ can be written as follows with the combination of Eq. (3·1) and the Wilson coefficients corresponding to each matrix element:

$$\frac{1}{n!}\left(\frac{d}{dq^2}\right)^n \frac{T^{\rm OPE}(q^2)}{q^2} \equiv \widehat{T}^{(n)\ \rm OPE}(q^2)$$
$$= \frac{1}{3}\left[C_G^{(n)}(\xi)\left\{ \left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_N - 4\left\langle \frac{\alpha_s}{\pi}\mathcal{ST}(G^a_{0\sigma}G^a_{0\sigma})\right\rangle_N \right\} + \{D_1^{(n)}(\xi) - D_2^{(n)}(\xi) - D_3^{(n)}(\xi)\}\left\langle \frac{\alpha_s}{\pi}\mathcal{ST}(G^a_{0\sigma}G^a_{0\sigma})\right\rangle_N \right]. \tag{3·2}$$

Here we define the dimensionless parameter as $\xi = -q^2/4m_c^2$ (m_c is the charmed quark mass) and $\rho = \xi/(1+\xi)$. The symbol $\mathcal{ST}$ makes the operators symmetric and traceless with respect to the Lorentz indices. In Eq. (3·2) each coefficient function is given using Gauss hypergeometric functions ${}_2F_1$ for arbitrary q^2 as follows:

$$C_G^{(n)}(\xi) = -\frac{2^n(n+1)(n+3)!}{(2n+5)!!}(4m_c^2)^{-(n+2)}(1+\xi)^{-(n+2)}\ {}_2F_1\left(n+2,-\frac{1}{2},n+\frac{7}{2};\rho\right), \tag{3·3}$$

$$D_1^{(n)}(\xi) = \frac{2^{n+3}(n+1)(n+1)!}{3(2n+3)!!}(4m_c^2)^{-(n+2)}(1+\xi)^{-(n+2)} \times\left[2\ {}_2F_1\left(n+2,\frac{1}{2},n+\frac{5}{2};\rho\right) - \frac{2(n+2)}{1+\xi}\ {}_2F_1\left(n+3,\frac{1}{2},n+\frac{5}{2};\rho\right) + \frac{3(n+2)^2}{(1+\xi)(2n+5)}\ {}_2F_1\left(n+3,\frac{1}{2},n+\frac{7}{2};\rho\right)\right], \tag{3·4}$$

$$D_2^{(n)}(\xi) = -\frac{2^{n+5}(n+1)(n+2)!}{3(2n+5)!!}(4m_c^2)^{-(n+2)}(1+\xi)^{-(n+2)} \times\left[{}_2F_1\left(n+2,\frac{1}{2},n+\frac{7}{2};\rho\right) - \frac{n+2}{2(1+\xi)}\ {}_2F_1\left(n+3,\frac{1}{2},n+\frac{7}{2};\rho\right)\right], \tag{3·5}$$

$$D_3^{(n)}(\xi) = \frac{2^{n+3}(n+1)(n+1)!}{3(2n+5)!!}(4m_c^2)^{-(n+2)}(1+\xi)^{-(n+2)} \times\left[(n+2)\ {}_2F_1\left(n+2,\frac{1}{2},n+\frac{7}{2};\rho\right) + 4(2n+5)\ {}_2F_1\left(n+2,\frac{1}{2},n+\frac{5}{2};\rho\right)\right]. \tag{3·6}$$

The Wilson coefficient of Eq. (3·3) for the scalar operator has already given in Ref. 11), and Eqs. (3·4)–(3·6) are new contributions for the twist-2 operator. Eventually, by equating Eqs. (2·11) and (3·2) we obtain the moment sum rule expressed in the form of the n-th derivative with respect to q^2:

$$\widehat{T}^{(n)\ \rm ph}(\xi\ ;\ a,b) = \widehat{T}^{(n)\ \rm OPE}(\xi). \tag{3·7}$$

The manipulation of the derivative ensures that the enhancement of the low energy part will not depend on the details of high energy part. The vacuum sum rules have been utilized for investigation of the free state of charmonium by Reinders et al.[11)] in the moment sum rule and by Bertlmann [21)] in the Borel sum rule. Furnstahl et al. [22)] have studied the spectra of J/ψ at finite temperature using both QCD sum rules. Here we summarize the well-known behavior of the moment sum rule for the variations of n and q^2.

- The convergence of the OPE side is worse for large n but is better for large q^2.
- In contrast to this behavior on the OPE side, the unwelcome contributions from the continuum on the PH side grow for large q^2, but decrease for large n.

We must choose a reliable stability region of the moment sum rule for investigation of the change of both n and q^2. The moment sum rule for $T_{\mu\nu}$ is a method to investigate the deviation from the properties in the vacuum obtained from $\Pi^0_{\mu\nu}$. Here we should adopt the same regions of n and q^2 as those used for $\Pi^0_{\mu\nu}$, which can reproduce the behavior of J/ψ in the moment sum rule reasonably well.

§4. Moment sum rule for $\Pi^0_{\mu\nu}$

In this section, we calculate the window of n for various values of ξ by applying the moment sum rule to the vacuum correlation function $\Pi_0(\omega^2) = \Pi^{0\,\mu}_{\mu}(\omega, \boldsymbol{q} = \boldsymbol{0})/(-3\omega^2)$ in Eq. (2·2).

On the OPE side, the n-th derivative for $\Pi_0^{(n)}(q^2)$ is expressed as [8), 11)]

$$\frac{1}{n!}\left(\frac{d}{dq^2}\right)^n \Pi_0^{\mathrm{OPE}}(q^2) \equiv \widehat{\Pi}_0^{(n)\,\mathrm{OPE}}(\xi) = \frac{1}{3}\left[C_0^{(n)}(\xi)\,\{1 + c_1^{(n)}(\xi)\,\alpha_s(\xi)\} + C_G^{(n)}(\xi)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_0\right], \tag{4·1}$$

where $C_I^{(n)}(\xi)$ and $c_1^{(n)}(\xi)$ are given by

$$C_0^{(n)}(\xi) = \frac{9}{4\pi^2}\frac{2^n(n+1)(n-1)!}{(2n+3)!!}(4m_c^2)^{-n}(1+\xi)^{-n}\ {}_2F_1\left(n, \frac{1}{2}, n+\frac{5}{2}; \rho\right), \tag{4·2}$$

$$\begin{aligned} c_1^{(n)}(\xi) = {} & \frac{(2n+1)!!}{3\cdot 2^{n-1}n!}\frac{2n+3}{2(n+1)}\frac{1}{{}_2F_1\left(n,\frac{1}{2},n+\frac{5}{2};\rho\right)} \\ & \times\left[\pi - \left\{\frac{\pi}{3}+\frac{1}{2}\left(\frac{\pi}{2}-\frac{3}{4\pi}\right)\right\}\frac{1}{n+1}\ {}_2F_1(n,1,n+2;\rho) \right. \\ & \left. + \frac{1}{3}\frac{1}{(n+1)(n+2)}\left(\frac{\pi}{2}-\frac{3}{4\pi}\right)\ {}_2F_1(n,2,n+3;\rho)\right] \\ & - \left(\frac{\pi}{2}-\frac{3}{4\pi}\right) - 2n\frac{\ln(2+\xi)}{\pi}\frac{2+\xi}{(1+\xi)^2}\frac{{}_2F_1\left(n+1,\frac{1}{2},n+\frac{5}{2};\rho\right)}{{}_2F_1\left(n,\frac{1}{2},n+\frac{5}{2};\rho\right)}. \end{aligned} \tag{4·3}$$

On the other hand, the relation between the J/ψ mass ($m_{J/\psi}$) of the lowest-lying resonance and $\widehat{\Pi}_0^{(n)}(q^2)$ on the PH side is given as

$$\frac{1}{n!}\left(\frac{d}{dq^2}\right)^n \Pi_0^{\rm ph}(q^2) \equiv \widehat{\Pi}_0^{(n)\,{\rm ph}}(q^2) = \frac{9}{4}\frac{m_{J/\psi}^2}{g_{J/\psi}^2}\frac{1}{(m_{J/\psi}^2-q^2)^{n+1}}[1+\delta_n], \qquad (4\cdot4)$$

where $g_{J/\psi}$ is the coupling parameter and δ_n represents the contribution from low resonances between J/ψ and the continuum.

We eliminate the parameter $g_{J/\psi}$ by calculating $\widehat{\Pi}_0^{(n)\,{\rm ph}}/\widehat{\Pi}_0^{(n-1)\,{\rm ph}}$. Then the ratio will be independent of δ_n or δ_{n-1} at sufficiently large n, because $\delta_n \simeq \delta_{n-1}$ for such n. Apart from these low resonances, we explicitly take account of the continuum contribution and include it on the OPE side. For simplicity the coefficient of the continuum term is assumed to be constant $(1/4\pi^2)$ without dependence on the charmed quark mass. Finally, the J/ψ mass is derived from the relation

$$m_{J/\psi} = \left[q^2 + \frac{\widehat{\Pi}_0^{(n-1)\,{\rm OPE}} - \frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\frac{1}{n-1}\frac{1}{(s_0-q^2)^{n-1}}}{\widehat{\Pi}_0^{(n)\,{\rm OPE}} - \frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\frac{1}{n}\frac{1}{(s_0-q^2)^{n}}}\right]^{1/2}. \qquad (4\cdot5)$$

We fix ξ ranging from 0.0 to 3.0 in 0.5 increments. This range corresponds to $0 \leq \sqrt{-q^2} \leq 4$ GeV. In Fig. 1 we show results for the J/ψ bare mass determined from Eq. (4·5) for the change of ξ. Here we have used $s_0 = 3.6^2$ GeV^2 and $\langle \frac{\alpha_s}{\pi}G^2\rangle_0 = 0.0126$ GeV^4.[11)] We must read off the range of n for each ξ from Fig. 1. The

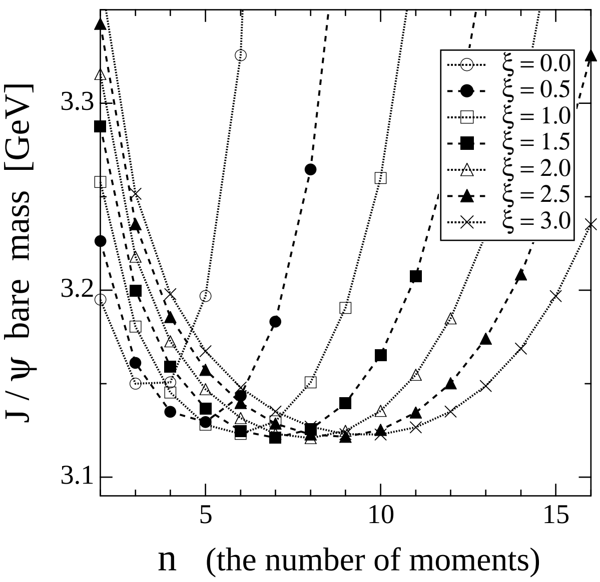

Fig. 1. The results of the moment sum rule analysis for $\Pi_0^{(n)}$ are shown for the determination of the J/ψ bare mass $m_{J/\psi}$. For each value of ξ fixed, we must read off the stability region of n that reproduces the experimental value $m_{J/\psi}^{\rm exp} = 3.096$ GeV.

window of n corresponds to finding the region stabilizing the J/ψ bare mass for the change of n. We obtain the window of n for each ξ as follows: $n_1 = 2, 3, 4$ for $\xi = 0.0$ ($Q^2 = 0.00$ GeV^2), $n_2 = 3, 4, 5$ for $\xi = 0.5$ ($Q^2 = 3.18$ GeV^2), $n_3 = 4, 5, 6, 7$ for $\xi = 1.0$ ($Q^2 = 6.25$ GeV^2), $n_4 = 5, 6, 7, 8$ for $\xi = 1.5$ ($Q^2 = 9.23$ GeV^2), $n_5 = 6, 7, 8, 9$ for $\xi = 2.0$ ($Q^2 = 12.1$ GeV^2), $n_4 = 7, 8, 9, 10$ for $\xi = 2.5$ ($Q^2 = 14.9$ GeV^2), $n_5 = 8, 9, 10, 11$ for $\xi = 3.0$ ($Q^2 = 17.6$ GeV^2). These points seem to reproduce the mass reasonably well for the experimental value $m_{J/\psi}^{\rm exp} = 3.096$ GeV.

§5. Numerical results

By inserting the sets of ξ and n obtained in §4 into Eq. (3·7), we can determine the unknown parameters a and b simultaneously by fitting the left-hand side to the right-hand side. The calculation proceeds as follows: First we arbitrarily choose two points in the window of n for fixed ξ and make a simultaneous equation for a and b by inserting the two chosen values of n. Next we consider all such combinations for each ξ and take the average of a solved for each combination. Eventually the scattering length is easily obtained from a.

To calculate the scattering length, we use the following values for other various parameters. On the PH side we adopt $m_{J/\psi} = 3.1$ GeV and $M_N = 0.94$ GeV. The coupling is determined from the experimental value of $\Gamma_{J/\psi}^{e^+e^-}$ as

$$f_{J/\psi}^2 = \frac{3\Gamma_{J/\psi}^{ee}}{4\pi e_q^2 \alpha^2 m_{J/\psi}} = 1.7 \times 10^{-2}, \tag{5·1}$$

where e_q is the electric charge of a quark ($e_c = 2/3$ for the charmed quark), and α is the fine structure constant ($= 1/137$). Indeed, the coupling $f_{J/\psi}$ ($= 1/e_c g_{J/\psi}$) can also be determined from the moment sum rule by oppositely substituting the experimental values of the J/ψ bare mass into Eq. (4·4). The value of the coupling obtained with this method agrees with the experimental value extremely well.[11] For QCD Lagrangian parameters we use the following functions dependent on ξ given in Ref. 11):

$$\alpha_s(\xi) = \frac{\alpha_s(4m_c^2)}{1 + \frac{25}{12\pi}\,\alpha_s(4m_c^2)\,\ln(1+\xi)}\ , \qquad \alpha_s(4m_c^2) \simeq 0.3\ , \tag{5·2}$$

$$m_c(\xi) = 1.28 \times \left[\, 1 - \frac{\alpha_s(\xi)}{\pi} \left\{ \frac{2+\xi}{1+\xi} \ln(2+\xi) - 2\ln 2 \right\} \right] \text{ GeV}\ . \tag{5·3}$$

On the OPE side, we determine the nucleon matrix elements as follows:

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_N = -(1.222 \pm 0.282)\ \text{ GeV}^2\ , \tag{5·4}$$

$$\left\langle \frac{\alpha_s}{\pi} \mathcal{ST}(G_{0\sigma}^a G_{0\sigma}^a) \right\rangle_N = -(0.094 \pm 0.010)\ \text{ GeV}^2\ . \tag{5·5}$$

The scalar part is evaluated from the trace anomaly.[20),23)] The twist-2 part is determined from the gluon distribution function of a nucleon, which is obtained through

Table I. J/ψ-N scattering length and the mass shift of J/ψ in the case of the scalar operator only and the scalar plus twist-2 operator at normal matter density $\rho_N = 0.17$ fm^{-3}.

ξ	0.0	0.5	1.0	1.5	2.0	2.5	3.0
$-\,a^{\rm scalar}_{J/\psi}$ [fm]	0.091	0.068	0.070	0.063	0.059	0.057	0.055
$\delta m^{\rm scalar}_{J/\psi}$ [MeV]	5.3	3.9	4.0	3.6	3.4	3.3	3.2
$-\,a^{\rm twist2}_{J/\psi}$ [fm]	0.120	0.090	0.092	0.083	0.078	0.075	0.073
$\delta m^{\rm twist2}_{J/\psi}$ [MeV]	6.9	5.2	5.3	4.8	4.5	4.3	4.2

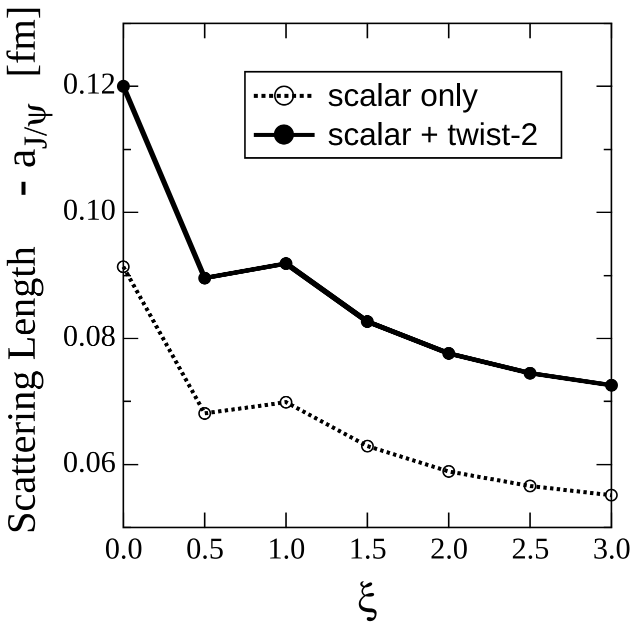

Fig. 2. We calculate the J/ψ-N scattering length for each set of ξ and n obtained from Fig. 1. The dotted line denotes the result calculated only by the contribution of the scalar operator. The solid line is that involving the twist-2 contribution.

leading order parametrization to the experimental data of deep-inelastic scattering. [20), 24)]

We list the results for the case of a scalar operator only and the case involving the twist-2 operator in Table I and display a graph of these results in Fig. 2. These values of the mass shift may be determined irrespective of the magnitude of the error ($\sim$ 20 MeV) between the theoretical values calculated with the vacuum sum rule and the experimental value for the J/ψ mass shown in Fig. 1, because the QSR for $T_{\mu\nu}$ is independent of the vacuum QSR for $\Pi^0_{\mu\nu}$. Therefore, the mass shift of J/ψ induced by the interaction with nuclear matter should be regarded as a shift from the experimental value for the J/ψ mass.

§6. Concluding remarks

The direct application of the moment sum rule to the forward J/ψ-N scattering amplitude gives an interesting result for the J/ψ-N interaction. That is, the J/ψ-N

scattering length $a_{J/\psi}$ is found to be a negative value (about -0.1 fm). This result suggests that the attractive J/ψ-N interaction is not sufficient to form a bound state with one nucleon, but it could make a bound state with nuclei. The absolute value is certainly smaller than the typical hadronic size, 1 fm, and the scattering length of light vector meson-N systems ($a_\rho \simeq -0.47$ fm, $a_\omega \simeq -0.41$ fm, $a_\phi \simeq -0.15$ fm),[6] but the experimental creation of J/ψ at the threshold would lead to the formation of a bound state inside a heavy nucleus. Our result is smaller than those obtained recently by Brodsky et al.[25] and de Téramond et al.[26] in the QCD sum rule approach.[27] Their method is based on an on-shell calculation to the charmed quark mass ($q^2 = 0$).

In this study we have given a new calculation of the Wilson coefficients for twist-2 gluon operators (dimension-4) in a form including the quark mass. The nucleon matrix elements of twist-2 gluon operators are about 1/10 times as large as those of the scalar part, but the total contribution with the Wilson coefficient makes the absolute value of the scalar part larger by about 30%. From $a_{J/\psi}$, we can estimate the total cross section ($\sigma_{J/\psi} = 4\pi a_{J/\psi}^2$). The result is about 1.26 mb at the threshold.

Next, in the linear density approximation we can calculate the J/ψ mass shift from $a_{J/\psi}$. The result indicates a very small decrease of the mass (about -4 to -7 MeV), about 0.1 to 0.2% at normal matter density. Since the slight mass shift is of the order of MeV, the change is sufficiently larger than the leptonic decay width of the order of several keV. Thus we conclude that J/ψ is a good probe for the observation of the medium effect.

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Note added: After the completion of this work, there appeared a preprint (nucl-th/9811070) by Klingl et al., who calculated the effective mass of J/ψ at normal matter density. They applied the QSR to the in-medium correlation function and obtained a mass shift of about -7 MeV (0.2%) in the second version of the preprint.