J.V. Field *The Invention of Infinity: Mathematics and Art in the Renaissance Oxford University Press, 1997*

Reviewed by Mark Peterson

The 15th century mathematician Luca Pacioli, in recommending the crucial importance of mathematics (and by implication the importance of his own services) to Duke Ludovico Sforza of Milan, emphasized its martial utility, recalling the machines of Archimedes at the defense of Syracuse, the engineering feats of the Romans, and many other examples. Similarly Leonardo da Vinci, seeking a position at this same Ludovico's court, emphasized his ability to design defensive fortifications and offensive weapons of terrible ingenuity. But when these two met in Milan, their collaboration produced something more benign: a book about symmetrical polyhedra, with perspective woodcut illustrations by Leonardo.

As the anecdote suggests, mathematicians and artists had interesting things to say to each other in the Renaissance, and sometimes they actually said them. There are even figures like Piero della Francesca (1410?-1492) who combined artistic and mathematical accomplishment in one person. *The Invention of Infinity* surveys the interaction of mathematics and art in the period roughly 1300-1650, with particular attention to one fascinating development: the invention of perspective in drawing, and the invention of projective geometry in mathematics. Both of these have a right to be called "the invention of infinity," and projective geometry, one feels, might have arisen as a kind of abstraction from the practical perspective construction methods of artists. J.V. Field's investigations in this area since the 1980's have laid the groundwork for this highly readable overview of mathematics and the arts in the Renaissance. In particular, she traces the evolution of ideas about linear perspective until they reach what would be a mathematical high point in the 17th century in the geometry of Girard Desargues. (The real flowering of projective geometry, after 1800, which apparently owes nothing further to its roots in the arts, is beyond the scope of this book.)

In an idealized and sanitized account, this story might be presented as somehow logical and inevitable. The historical reality, however, is much messier, and more fun. Prof. Field shares her experience in tracking it down, in a voice that, without losing authority, is frequently chatty and witty. Chapter 3, for example, is a first person account of her investigation (with two colleagues) of Masaccio's Trinity fresco (c. 1426), involving close-up measurement and inspection of this early perspective masterpiece while scaffolding was in place in 1986-7. She

gives us real numerical data, in case we want to try drawing our own conclusions. I hope I would have come independently to her conclusion, because it feels right: Masaccio knew all he needed to know about making an accurate perspective construction, but he didn't always do it. He bent the rules, for very good reasons, as artists of ability will always do. As Field says, it was "a painting, not a theorem." This example highlights what a little reflection might also have suggested: that, in spite of logic, mathematics and art lie uneasily together. The art world by and large did not pay close attention to the mathematics of its procedures, but simply adopted methods which seemed to work, and which had supposedly been proved in the past. The mathematical world was very slow to appreciate that there was something of value here for mathematics: projective geometry lay dormant even after Desargues.

The complexity of the real story is an invitation to range freely over a wide body of material. What kind of mathematics did an artisan typically know? What mathematics did a patron know? What perspective methods were actually used, and how were they described in treatises? How did these descriptions evolve over time? How were these ideas realized in paintings, architecture, and theaters? Professor Field sticks close to the primary sources and quotes many representative passages (in English translation).

The book is lavishly illustrated with photographs, black and white reproductions, and diagrams from original treatises. One has the feeling of looking over the shoulder of an historian at work. Her frequent amusing asides may perhaps derive from her lecture series at Imperial College, London, out of which this book grew. She insists that the primary sources should be made to speak intelligibly: "Historians of science get used to meeting a nice class of intellect among the illustrious dead, and one of the rules of the game is accordingly never to be hasty in coming to the conclusion that the person one is reading is more of a fool than oneself." This generosity of spirit aims at understanding one part at least of the mathematics of the Renaissance in its own terms.

One is left wondering, however, how significant the developments described here really were for mathematics in the Renaissance. The Invention of Infinity explicitly restricts itself to mathematics in its relation to the arts. While it is an attractive notion that Renaissance mathematics had its roots in the arts, and while it is clear that to some extent it really did, one still senses that there is a larger context which is missing. Professor Field says this herself in her introduction, but it is worth returning to this point. If we are to understand the new beginnings of mathematics in the Renaissance, or, to put it more grandly, the beginnings of modern mathematical science, we must look at the whole picture. In this reviewer's opinion, that picture is still very puzzling. The mathematics of Piero della Francesca, for example, apart from his book on perspective, arguably has NOTHING to do with art, but springs from entirely different motives. Piero, who really is a central figure in this whole story, gets two chapters in The Invention of Infinity, but his peculiar double nature as artist and mathematician is only described, not really addressed. To take another example, Galileo, who might have figured near the end of the story, does not appear, for the simple reason that (as far as I know) he never even mentions perspective. But this is very strange: Galileo was passionate about geometry; his friends included patrons of the arts; he was expert in practical triangulation methods; he was a protégé of Guidobaldo del Monte, a pivotal figure in the perspective story; and he championed the application of mathematics to physical reality. Why does he seem totally uninterested in perspective?

None of these comments is intended as a criticism of *The Invention of Infinity*, which takes on a problem of manageable size and deals with it in a thoroughgoing and satisfying way. Rather, the illumination of one part of the rather neglected subject of Renaissance mathematics, which Professor Field has unquestionably achieved, calls attention to the unilluminated cultural context surrounding it. It is hard to assess the significance of the book because we know so little about the larger context, what mathematics really meant in the Renaissance. That, in a way, gives the book a different and additional significance. We may hope that in the future more of Renaissance mathematics will be explored and elucidated in its cultural context, in the way that Professor Field has done here for the mathematics of perspective.

The Reviewer

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