# Jet properties from dihadron correlations in $p+p$ collisions at $\sqrt{s}=200 \mathbf{G e V}$ 

S.S. Adler,,${ }^{5}$ S. Afanasiev, ${ }^{20}$ C. Aidala, ${ }^{10}$ N. N. Ajitanand, ${ }^{44}$ Y. Akiba, ${ }^{21,40}$ A. Al-Jamel,,${ }^{35}$ J. Alexander,${ }^{44}$ K. Aoki, ${ }^{25}$ L. Aphecetche, ${ }^{46}$ R. Armendariz, ${ }^{35}$ S. H. Aronson, ${ }^{5}$ R. Averbeck, ${ }^{45}$ T. C. Awes, ${ }^{36}$ V. Babintsev, ${ }^{17}$ A. Baldisseri, ${ }^{11}$ K. N. Barish, ${ }^{6}$ P.D. Barnes, ${ }^{28}$ B. Bassalleck, ${ }^{34}$ S. Bathe,,${ }^{6,31}$ S. Batsouli, ${ }^{10}$ V. Baublis, ${ }^{39}$ F. Bauer, ${ }^{6}$ A. Bazilevsky, ${ }^{5}{ }^{5,41}$ S. Belikov, ${ }^{19,17}$ M. T. Bjorndal, ${ }^{10}$ J. G. Boissevain, ${ }^{28}$ H. Borel, ${ }^{11}$ M. L. Brooks, ${ }^{28}$ D. S. Brown, ${ }^{35}$ N. Bruner, ${ }^{34}$ D. Bucher, ${ }^{31}$ H. Buesching,,${ }^{5,31}$ V. Bumazhnov, ${ }^{17}$ G. Bunce,,${ }^{5,41}$ J. M. Burward-Hoy, ${ }^{28,27}$ S. Butsyk, ${ }^{45}$ X. Camard, ${ }^{46}$ P. Chand, ${ }^{4}$ W. C. Chang, ${ }^{2}$ S. Chernichenko, ${ }^{17}$ C. Y. Chi, ${ }^{10}$ J. Chiba, ${ }^{21}$ M. Chiu,,${ }^{10}$ I. J. Choi, ${ }^{53}$ R. K. Choudhury, ${ }^{4}$ T. Chujo, ${ }^{5}$ V. Cianciolo, ${ }^{36}$ Y. Cobigo, ${ }^{11}$ B. A. Cole, ${ }^{10}$ M. P. Comets, ${ }^{37}$ P. Constantin, ${ }^{19}$ M. Csanád, ${ }^{13}$ T. Csörgő, ${ }^{22}$ J. P. Cussonneau, ${ }^{46}$ D. d'Enterria, ${ }^{10}$ K. Das, ${ }^{14}$ G. David, ${ }^{5}$ F. Deák,,${ }^{13}$ H. Delagrange,,${ }^{46}$ A. Denisov, ${ }^{17}$ A. Deshpande, ${ }^{41}$ E. J. Desmond,,${ }^{5}$ A. Devismes, ${ }^{45}$ O. Dietzsch,,${ }^{42}$ J. L. Drachenberg, ${ }^{1}$ O. Drapier, ${ }^{26}$ A. Drees, ${ }^{45}$ A. Durum, ${ }^{17}$ D. Dutta, ${ }^{4}$ V. Dzhordzhadze, ${ }^{47}$ Y. V. Efremenko, ${ }^{36}$ H. En'yo, ${ }^{40,41}$ B. Espagnon, ${ }^{37}$ S. Esumi, ${ }^{49}$ D. E. Fields, ${ }^{34,41}$ C. Finck, ${ }^{46}$ F. Fleuret, ${ }^{26}$ S. L. Fokin, ${ }^{24}$ B. D. Fox, ${ }^{41}$ Z. Fraenkel, ${ }^{52}$ J. E. Frantz, ${ }^{10}$ A. Franz, ${ }^{5}$ A. D. Frawley, ${ }^{14}$ Y. Fukao, ${ }^{25,40,41}$ S.-Y. Fung, ${ }^{6}$ S. Gadrat, ${ }^{29}$ M. Germain, ${ }^{46}$ A. Glenn, ${ }^{47}$ M. Gonin, ${ }^{26}$ J. Gosset, ${ }^{11}$ Y. Goto, ${ }^{40,41}$ R. Granier de Cassagnac, ${ }^{26}$ N. Grau, ${ }^{19}$ S. V. Greene, ${ }^{50}$ M. Grosse Perdekamp,,${ }^{18,41}$ H.-Å. Gustafsson, ${ }^{30}$ T. Hachiya, ${ }^{16}$ J. S. Haggerty, ${ }^{5}$ H. Hamagaki, ${ }^{8}$ A. G. Hansen, ${ }^{28}$ E. P. Hartouni, ${ }^{27}$ M. Harvey, ${ }^{5}$ K. Hasuko, ${ }^{40}$ R. Hayano, ${ }^{8}$ X. He, ${ }^{15}$ M. Heffner, ${ }^{27}$ T. K. Hemmick, ${ }^{45}$ J. M. Heuser, ${ }^{40}$
P. Hidas, ${ }^{22}$ H. Hiejima, ${ }^{18}$ J. C. Hill, ${ }^{19}$ R. Hobbs, ${ }^{34}$ W. Holzmann, ${ }^{44}$ K. Homma, ${ }^{16}$ B. Hong, ${ }^{23}$ A. Hoover, ${ }^{35}$ T. Horaguchi, ${ }^{40,41,48}$ T. Ichihara, ${ }^{40,41}$ V. V. Ikonnikov, ${ }^{24}$ K. Imai, ${ }^{25,40} \mathrm{M}$. Inaba, ${ }^{49}$ M. Inuzuka, ${ }^{8}$ D. Isenhower, ${ }^{1}$
L. Isenhower, ${ }^{1}$ M. Ishihara, ${ }^{40}$ M. Issah, ${ }^{44}$ A. Isupov, ${ }^{20}$ B. V. Jacak, ${ }^{45}$ J. Jia, ${ }^{45}$ O. Jinnouchi, ${ }^{40,41}$ B. M. Johnson, ${ }^{5}$
S. C. Johnson, ${ }^{27}$ K. S. Joo, ${ }^{32}$ D. Jouan, ${ }^{37}$ F. Kajihara, ${ }^{8}$ S. Kametani, ${ }^{8,51}$ N. Kamihara, ${ }^{40,48}$ M. Kaneta, ${ }^{41}$ J. H. Kang, ${ }^{53}$ K. Katou, ${ }^{51}$ T. Kawabata, ${ }^{8}$ A. V. Kazantsev, ${ }^{24}$ S. Kelly, ${ }^{9}{ }^{910}$ B. Khachaturov, ${ }^{52}$ A. Khanzadeev, ${ }^{39}$ J. Kikuchi, ${ }^{51}$ D. J. Kim, ${ }^{53}$ E. Kim, ${ }^{43}$ G.-B. Kim,,${ }^{26}$ H. J. Kim, ${ }^{53}$ E. Kinney, ${ }^{9}$ A. Kiss, ${ }^{13}$ E. Kistenev, ${ }^{5}$ A. Kiyomichi, ${ }^{40}$ C. Klein-Boesing, ${ }^{31}$ H. Kobayashi, ${ }^{41}$ L. Kochenda, ${ }^{39}$ V. Kochetkov, ${ }^{17}$ R. Kohara, ${ }^{16}$ B. Komkov, ${ }^{39}$ M. Konno, ${ }^{49}$ D. Kotchetkov, ${ }^{6}$ A. Kozlov, ${ }^{52}$ P. J. Kroon, ${ }^{5}$ C. H. Kuberg, ${ }^{1, *}$ G. J. Kunde, ${ }^{28}$ K. Kurita, ${ }^{40}$ M. J. Kweon, ${ }^{23}$ Y. Kwon, ${ }^{53}$ G. S. Kyle, ${ }^{35}$ R. Lacey, ${ }^{44}$ J. G. Lajoie, ${ }^{19}$ Y. Le Bornec, ${ }^{37}$ A. Lebedev, ${ }^{19,24}$ S. Leckey, ${ }^{45}$ D. M. Lee, ${ }^{28}$ M. J. Leitch, ${ }^{28}$ M. A. L. Leite,,${ }^{42}$ X. H. Li, ${ }^{6}$ H. Lim, ${ }^{43}$ A. Litvinenko, ${ }^{20}$ M. X. Liu, ${ }^{28}$ C.F. Maguire, ${ }^{50}$ Y. I. Makdisi, ${ }^{5}$ A. Malakhov, ${ }^{20}$ V. I. Manko, ${ }^{24}$ Y. Mao, ${ }^{38,40}$ G. Martinez ${ }^{46}$ H. Masui, ${ }^{49}$ F. Matathias, ${ }^{45}$ T. Matsumoto, ${ }^{8,51}$ M. C. McCain, ${ }^{1}$ P. L. McGaughey,${ }^{28}$ Y. Miake, ${ }^{49}$ T.E. Miller, ${ }^{50}$ A. Milov, ${ }^{45}$ S. Mioduszewski, ${ }^{5}$ G. C. Mishra, ${ }^{15}$ J. T. Mitchell,,${ }^{5}$ A. K. Mohanty, ${ }^{4}$ D. P. Morrison, ${ }^{5}$ J. M. Moss, ${ }^{28}$ D. Mukhopadhyay, ${ }^{52}$ M. Muniruzzaman, ${ }^{6}$ S. Nagamiya, ${ }^{21}$ J.L. Nagle,,${ }^{9,10}$ T. Nakamura, ${ }^{16}$ J. Newby, ${ }^{47}$ A. S. Nyanin, ${ }^{24}$ J. Nystrand, ${ }^{30}$ E. O'Brien, ${ }^{5}$ C. A. Ogilvie, ${ }^{19}$ H. Ohnishi, ${ }^{40}$ I. D. Ojha, ${ }^{3,50}$ H. Okada, ${ }^{25,40}$ K. Okada, ${ }^{40,41}$ A. Oskarsson, ${ }^{30}$ I. Otterlund, ${ }^{30}$ K. Oyama, ${ }^{8}$ K. Ozawa, ${ }^{8}$ D. Pal, ${ }^{52}$ A. P. T. Palounek, ${ }^{28}$ V. Pantuev, ${ }^{45}$ V. Papavassiliou ${ }^{35}$
J. Park, ${ }^{43}$ W. J. Park, ${ }^{23}$ S. F. Pate, ${ }^{35}$ H. Pei,,${ }^{19}$ V. Penev, ${ }^{20}$ J.-C. Peng, ${ }^{18}$ H. Pereira,,${ }^{11}$ V. Peresedov, ${ }^{20}$ A. Pierson, ${ }^{34}$ C. Pinkenburg, ${ }^{5}$ R.P. Pisani, ${ }^{5}$ M. L. Purschke, ${ }^{5}$ A. K. Purwar, ${ }^{45}$ J. M. Qualls, ${ }^{1}$ J. Rak, ${ }^{19}$ I. Ravinovich, ${ }^{52}$ K. F. Read, ${ }^{36,47}$ M. Reuter, ${ }^{45}$ K. Reygers, ${ }^{31}$ V. Riabov, ${ }^{39}$ Y. Riabov, ${ }^{39}$ G. Roche, ${ }^{29}$ A. Romana, ${ }^{26, *}$ M. Rosati, ${ }^{19}$ S. S. E. Rosendahl, ${ }^{30}$ P. Rosnet, ${ }^{29}$ V. L. Rykov, ${ }^{40}$ S. S. Ryu, ${ }^{53}$ N. Saito, ${ }^{25,40,41}$ T. Sakaguchi, ${ }^{8,51}$ S. Sakai, ${ }^{49}$ V. Samsonov, ${ }^{39}$ L. Sanfratello, ${ }^{34}$ R. Santo, ${ }^{31}$ H. D. Sato, ${ }^{25,40}$ S. Sato,,${ }^{5,49}$ S. Sawada, ${ }^{21}$ Y. Schutz, ${ }^{46}$ V. Semenov, ${ }^{17}$ R. Seto, ${ }^{6}$ T. K. Shea, ${ }^{5}$ I. Shein, ${ }^{17}$ T.-A. Shibata, ${ }^{40,48}$ K. Shigaki, ${ }^{16}$ M. Shimomura, ${ }^{49}$ A. Sickles, ${ }^{45}$ C. L. Silva, ${ }^{42}$ D. Silvermyr, ${ }^{28}$ K. S. Sim, ${ }^{23}$ A. Soldatov, ${ }^{17}$ R. A. Soltz, ${ }^{27}$ W. E. Sondheim, ${ }^{28}$ S. P. Sorensen, ${ }^{47}$ I. V. Sourikova, ${ }^{5}$ F. Staley, ${ }^{11}$ P. W. Stankus, ${ }^{36}$ E. Stenlund, ${ }^{30}$ M. Stepanov,,${ }^{35}$ A. Ster, ${ }^{22}$ S. P. Stoll, ${ }^{5}$ T. Sugitate, ${ }^{16}$ J. P. Sullivan, ${ }^{28}$ S. Takagi, ${ }^{49}$ E. M. Takagui, ${ }^{42}$ A. Taketani, ${ }^{40,41}$ K. H. Tanaka, ${ }^{21}$ Y. Tanaka, ${ }^{33}$ K. Tanida, ${ }^{40}$ M. J. Tannenbaum, ${ }^{5}$ A. Taranenko, ${ }^{44}$ P. Tarján, ${ }^{12}$ T. L. Thomas, ${ }^{34}$ M. Togawa, ${ }^{25,40}$ J. Tojo, ${ }^{40}$ H. Torii, ${ }^{25,41}$ R.S. Towell, ${ }^{1}$ V-N. Tram, ${ }^{26}$ I. Tserruya, ${ }^{52}$ Y. Tsuchimoto, ${ }^{16}$ H. Tydesjö, ${ }^{30}$ N. Tyurin,,${ }^{17}$ T. J. Uam, ${ }^{32}$ H. W. van Hecke, ${ }^{28}$ J. Velkovska, ${ }^{5}$ M. Velkovsky, ${ }^{45}$ V. Veszprémi, ${ }^{12}$ A. A. Vinogradov, ${ }^{24}$ M. A. Volkov, ${ }^{24}$ E. Vznuzdaev, ${ }^{39}$ X.R. Wang, ${ }^{15}$ Y. Watanabe, ${ }^{40,41}$ S. N. White, ${ }^{5}$ N. Willis, ${ }^{37}$ F. K. Wohn, ${ }^{19}$ C. L. Woody, ${ }^{5}$ W. Xie, ${ }^{6}$ A. Yanovich, ${ }^{17}$ S. Yokkaichi, ${ }^{40,41}$ G.R. Young, ${ }^{36}$ I. E. Yushmanov, ${ }^{24}$ W. A. Zajc, ${ }^{10, \dagger}$ C. Zhang, ${ }^{10}$ S. Zhou, ${ }^{7}$ J. Zimányi, ${ }^{22}$ L. Zolin, ${ }^{20}$ and X. Zong ${ }^{19}$

## (PHENIX Collaboration)

[^0]
# ${ }^{5}$ Brookhaven National Laboratory, Upton, New York 11973-5000, USA 

${ }^{6}$ University of California - Riverside, Riverside, California 92521, USA
${ }^{7}$ China Institute of Atomic Energy (CIAE), Beijing, People's Republic of China
${ }^{8}$ Center for Nuclear Study, Graduate School of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan
${ }^{9}$ University of Colorado, Boulder, Colorado 80309, USA
${ }^{10}$ Columbia University, New York, New York 10027 and Nevis Laboratories, Irvington, New York 10533, USA
${ }^{11}$ Dapnia, CEA Saclay, F-91191, Gif-sur-Yvette, France
${ }^{12}$ Debrecen University, H-4010 Debrecen, Egyetem tér 1, Hungary
${ }^{13}$ ELTE, Eötvös Loránd University, H-1117 Budapest, Pázmány P. s. 1/A, Hungary
${ }^{14}$ Florida State University, Tallahassee, Florida 32306, USA
${ }^{15}$ Georgia State University, Atlanta, Georgia 30303, USA
${ }^{16}$ Hiroshima University, Kagamiyama, Higashi-Hiroshima 739-8526, Japan
${ }^{17}$ IHEP Protvino, State Research Center of Russian Federation, Institute for High Energy Physics, Protvino, 142281, Russia
${ }^{18}$ University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA
${ }^{19}$ Iowa State University, Ames, Iowa 50011, USA
${ }^{20}$ Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia
${ }^{21}$ KEK, High Energy Accelerator Research Organization, Tsukuba, Ibaraki 305-0801, Japan
${ }^{22}$ KFKI Research Institute for Particle and Nuclear Physics of the Hungarian Academy of Sciences (MTA KFKI RMKI),
H-1525 Budapest 114, P.O. Box 49, Budapest, Hungary
${ }^{23}$ Korea University, Seoul, 136-701, Korea
${ }^{24}$ Russian Research Center "Kurchatov Institute", Moscow, Russia
${ }^{25}$ Kyoto University, Kyoto 606-8502, Japan
${ }^{26}$ Laboratoire Leprince-Ringuet, Ecole Polytechnique, CNRS-IN2P3, Route de Saclay, F-91128, Palaiseau, France
${ }^{27}$ Lawrence Livermore National Laboratory, Livermore, California 94550, USA
${ }^{28}$ Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
${ }^{29}$ LPC, Université Blaise Pascal, CNRS-IN2P3, Clermont-Fd, 63177 Aubiere Cedex, France
${ }^{30}$ Department of Physics, Lund University, Box 118, SE-221 00 Lund, Sweden
${ }^{31}$ Institut für Kernphysik, University of Muenster, D-48149 Muenster, Germany
${ }^{32}$ Myongji University, Yongin, Kyonggido 449-728, Korea
${ }^{33}$ Nagasaki Institute of Applied Science, Nagasaki-shi, Nagasaki 851-0193, Japan
${ }^{34}$ University of New Mexico, Albuquerque, New Mexico 87131, USA
${ }^{35}$ New Mexico State University, Las Cruces, New Mexico 88003, USA
${ }^{36}$ Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA
${ }^{37}$ IPN-Orsay, Universite Paris Sud, CNRS-IN2P3, BP1, F-91406, Orsay, France
${ }^{38}$ Peking University, Beijing, People's Republic of China
${ }^{39}$ PNPI, Petersburg Nuclear Physics Institute, Gatchina, Leningrad region, 188300, Russia
${ }^{40}$ RIKEN (The Institute of Physical and Chemical Research), Wako, Saitama 351-0198, JAPAN
${ }^{41}$ RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973-5000, USA
${ }^{42}$ Universidade de São Paulo, Instituto de Física, Caixa Postal 66318, São Paulo CEP05315-970, Brazil
${ }^{43}$ System Electronics Laboratory, Seoul National University, Seoul, South Korea
${ }^{44}$ Chemistry Department, Stony Brook University, SUNY, Stony Brook, New York 11794-3400, USA
${ }^{45}$ Department of Physics and Astronomy, Stony Brook University, SUNY, Stony Brook, New York 11794, USA
${ }^{46}$ SUBATECH (Ecole des Mines de Nantes, CNRS-IN2P3, Université de Nantes) BP 20722-44307, Nantes, France
${ }^{47}$ University of Tennessee, Knoxville, Tennessee 37996, USA
${ }^{48}$ Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro, Tokyo 152-8551, Japan
${ }^{49}$ Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan
${ }^{50}$ Vanderbilt University, Nashville, Tennessee 37235, USA
${ }^{51}$ Waseda University, Advanced Research Institute for Science and Engineering, 17 Kikui-cho, Shinjuku-ku, Tokyo 162-0044, Japan
${ }^{52}$ Weizmann Institute, Rehovot 76100, Israel
${ }^{53}$ Yonsei University, IPAP, Seoul 120-749, Korea
(Received 12 May 2006; published 5 October 2006)
The properties of jets produced in $p+p$ collisions at $\sqrt{s}=200 \mathrm{GeV}$ are measured using the method of two-particle correlations. The trigger particle is a leading particle from a large transverse momentum jet while the associated particle comes from either the same jet or the away-side jet. Analysis of the angular width of the near-side peak in the correlation function determines the jet-fragmentation transverse momentum $j_{\mathrm{T}}$. The extracted value, $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}=585 \pm 6($ stat $) \pm 15(\mathrm{sys}) \mathrm{MeV} / c$, is constant with respect to the trigger particle transverse momentum, and comparable to the previous lower $\sqrt{s}$ measurements. The

[^1]
#### Abstract

width of the away-side peak is shown to be a convolution of $j_{\mathrm{T}}$ with the fragmentation variable, $z$, and the partonic transverse momentum, $k_{\mathrm{T}}$. The $\langle z\rangle$ is determined through a combined analysis of the measured $\pi^{0}$ inclusive and associated spectra using jet-fragmentation functions measured in $e^{+} e^{-}$collisions. The final extracted values of $k_{\mathrm{T}}$ are then determined to also be independent of the trigger particle transverse momentum, over the range measured, with value of $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}=2.68 \pm 0.07$ (stat) $\pm 0.34$ (sys) $\mathrm{GeV} / c$.


DOI: 10.1103/PhysRevD.74.072002
PACS numbers: 13.85.-t, 25.75.Dw

## I. INTRODUCTION

The goal of this paper is to explore the systematics of jet production and fragmentation in $p+p$ collisions at $\sqrt{s}=$ 200 GeV by the method of two-particle azimuthal correlations. Knowledge of the jet-fragmentation process is useful not only as a reference measurement for a similar analysis in $\mathrm{Au}+\mathrm{Au}$ collisions, but can be used as a stringent test of perturbative QCD (pQCD) calculations beyond leading order.

The two-particle azimuthal correlations method worked well at ISR energies $(\sqrt{s}=63 \mathrm{GeV})$ and below [1-3], where it is difficult to directly reconstruct jets, but has not been attempted at higher values of $\sqrt{s}$. This method is also suitable for jet-analysis in heavy-ion data where the large particle multiplicity severely interferes with direct jet reconstruction.

With the beginning of RHIC operation, heavy-ion physics entered a new regime, where pQCD phenomena can be fully explored. High-energy partons materializing into hadronic jets can be used as sensitive probes of the early stage of heavy-ion collisions. Measurements carried out during the first three years of RHIC operation at $\sqrt{s_{\mathrm{NN}}}=$ 130 and 200 GeV exhibit many new and interesting features. The high- $p_{\mathrm{T}}$ particle yield was found to be strongly suppressed in $\mathrm{Au}+\mathrm{Au}$ central collisions [4-6]. Furthermore, the nonsuppression of the high- $p_{\mathrm{T}}$ particle yield in $d+\mathrm{Au}$ induced collisions [7-10] confirmed that the suppression can be fully attributed to the final state interaction of high-energy partons with an extremely opaque nuclear medium formed in $\mathrm{Au}+\mathrm{Au}$ collisions at RHIC.

Other striking features found in RHIC data are the large asymmetry of particle azimuthal distributions which is attributed to sizable elliptic flow $[11,12]$ and the observation of the apparent disappearance of the back-to-back jet correlation in central $\mathrm{Au}+\mathrm{Au}$ collisions [13].

Many of the above mentioned observations can be explained by a large opacity of the medium produced in central $\mathrm{Au}+\mathrm{Au}$ collisions which causes the scattered partons to lose energy via coherent (Landau-Pomeranchuk-Migdal [14]) gluon bremsstrahlung [1518]. It is expected that the medium effect will cause the apparent modification of fundamental properties of hardscattering like broadening of intrinsic parton transverse momentum $k_{\mathrm{T}}[19,20]$ and modification of jet fragmentation [21]. Thus the measurement of jet fragmentation properties and intrinsic parton transverse momentum $k_{\mathrm{T}}$
for $p+p$ collisions presented here provides a baseline for comparison to the results in heavy-ion collisions, helping to disentangle the complex processes of propagation and possible fragmentation of partons within the excited nuclear medium.

This paper is organized as follows: Sec. II discusses the method of two-particle correlations and the relations between jet properties and the angular correlation between parton fragments. The details of the PHENIX experiment relevant to this analysis are outlined in Section III. Section IV deals with the analysis of the correlation functions extracted from the $p+p$ data and an evaluation of the $\left\langle j_{\mathrm{T}}\right\rangle$ and $\left\langle k_{\mathrm{T}}\right\rangle$ quantities. The combined analysis of the inclusive and associated $p_{\mathrm{T}}$-distributions is discussed in Sec. V and the sensitivity of the associated $p_{\mathrm{T}}$-distributions to the fragmentation function is discussed in Sec. VI. Section VII presents the resulting values of the partonic transverse momenta $k_{\mathrm{T}}$ corrected for the mean momentum fraction $\left\langle z_{\mathrm{t}}\right\rangle$. Section VIII summarizes the results from this paper.

## II. JET ANGULAR CORRELATIONS

Jets are produced in the hard scattering of two partons [22-25]. The overall $p+p$ hard-scattering cross section in "leading logarithm" pQCD is the sum over parton reactions $a+b \rightarrow c+d$ (e.g. $g+q \rightarrow g+q$ ) at partonparton center-of-mass (c.m.) energy $\sqrt{\hat{s}}$,

$$
\begin{equation*}
\frac{d^{3} \sigma}{d x_{1} d x_{2} d \cos \theta^{*}}=\frac{1}{s} \sum_{a b} f_{a}\left(x_{1}\right) f_{b}\left(x_{2}\right) \frac{\pi \alpha_{s}^{2}\left(Q^{2}\right)}{2 x_{1} x_{2}} \Sigma^{a b}\left(\cos \theta^{*}\right) \tag{1}
\end{equation*}
$$

where $f_{a}\left(x_{1}\right), f_{b}\left(x_{2}\right)$, are parton distribution functions, the differential probabilities for partons $a$ and $b$ to carry momentum fractions $x_{1}$ and $x_{2}$ of their respective protons (e.g. $\left.u\left(x_{2}\right)\right)$, and where $\theta^{*}$ is the scattering angle in the partonparton c.m. system. The parton-parton c.m. energy squared is $\hat{s}=x_{1} x_{2} s$, where $\sqrt{s}$ is the c.m. energy of the $p+p$ collision. The parton-parton c.m. system moves with rapidity $y=(1 / 2) \ln \left(x_{1} / x_{2}\right)$ in the $p+p$ c.m. system.

Equation (1) gives the $p_{T}$ spectrum of outgoing parton $c$ (emitted at $\theta^{*}$ ), which then fragments into hadrons, e.g. a $\pi^{0}$. The fragmentation function $D_{c}^{\pi^{0}}\left(z, \mu^{2}\right)$ is the probability for a $\pi^{0}$ to carry a fraction $z=p^{\pi^{0}} / p^{c}$ of the momentum of outgoing parton $c$. Equation (1) must be summed over all subprocesses leading to a $\pi^{0}$ in the final state. The parameter $\mu^{2}$ is an unphysical "factorization" scale intro-
duced to account for collinear singularities in the structure and fragmentation functions [26,27], which will be ignored for the purposes of this paper.

In this formulation, $f_{a}\left(x_{1}\right), f_{b}\left(x_{2}\right)$, and $D_{c}^{\pi^{0}}(z)$ represent the "long-distance phenomena" to be determined by experiment; while the characteristic subprocess angular distributions, $\Sigma^{a b}\left(\cos \theta^{*}\right)$, and the coupling constant, $\alpha_{s}\left(Q^{2}\right)=\frac{12 \pi}{25 \ln \left(Q^{2} / \Lambda^{2}\right)}$, are fundamental predictions of QCD [28-30] for the short-distance, large- $Q^{2}$, phenomena. The momentum scale $Q^{2} \sim p_{\mathrm{T}}^{2}$ for the scattering subprocess, while $Q^{2} \sim \hat{s}$ for a Compton or annihilation subprocess, but the exact meaning of $Q^{2}$ tends to be treated as a parameter rather than a dynamical quantity.

Figure 1 shows a schematic view of a hard-scattering event. The transverse momentum of the outgoing scattered parton is

$$
\begin{equation*}
p_{\mathrm{T}}=p_{\mathrm{T}}^{*}=\frac{\sqrt{\hat{s}}}{2} \sin \theta^{*} \tag{2}
\end{equation*}
$$

The two scattered partons propagate nearly back-to-back in azimuth from the collision point and fragment into the jetlike spray of final state particles [see Fig. 1(a) where only one fragment of each parton is shown).

It was originally thought that parton collisions were collinear with the $p+p$ collision axis so that the two emerging partons would have the same magnitude of transverse momenta pointing opposite in azimuth.


FIG. 1 (color online). (a) Schematic view of a hard-scattering event in the plane perpendicular to the beam. Two scattered partons with transverse momenta $\hat{p}_{\text {T }}$ in the partons' center-ofmass frame are seen in the laboratory frame to have a momenta $\hat{p}_{\mathrm{Tt}}$ and $\hat{p}_{\mathrm{Ta}}$. The net pair transverse momentum $\hat{p}_{\mathrm{T}, \text { pair }}$ pair corresponds to the sum of the $\vec{k}_{\mathrm{T}}$-vectors of the two colliding partons. The trigger and associated jet fragments producing high- $p_{\mathrm{T}}$ particles are labeled as $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$. The projection of $\vec{k}_{T}$ perpendicular to $\hat{p}_{\mathrm{Tt}}$ is labeled as $k_{\mathrm{Ty}}$. The transverse momentum component of the away-side particle $\vec{p}_{\mathrm{Ta}}$ perpendicular to trigger particle $\vec{p}_{\text {Tt }}$ is labeled as $p_{\text {out }}$. (b) The same schematics as in (a), but the jet-fragmentation transverse momentum component $j_{\text {Ty }}$ of the trigger jet is also shown.

However, it was found [3] that each of the partons carries initial transverse momentum $\vec{k}_{\mathrm{T}}$, originally described as "intrinsic" [31]. This results in a momentum imbalance (the partons' $p_{\mathrm{T}}$ are not equal) and an acoplanarity (the transverse momentum of one jet does not lie in the plane determined by the transverse momentum of the second jet and the beam axes). The jets are noncollinear having a net transverse momentum $\left\langle p_{\mathrm{T}}^{2}\right\rangle_{\text {pair }}=2 \cdot\left\langle k_{\mathrm{T}}^{2}\right\rangle$.

It is important to emphasize that the $\left\langle k_{\mathrm{T}}\right\rangle$ denotes the effective magnitude of the apparent transverse momentum of each colliding parton. The net transverse momentum of the outgoing parton-pair is $\sqrt{2} \cdot\left\langle k_{\mathrm{T}}\right\rangle$. The naive expectation for the pure intrinsic parton transverse momentum based on nucleon constituent quark mass is about $\approx$ $300 \mathrm{MeV} / c$ [31,32]. However, the measurement of net transverse momenta of diphotons, dileptons, or dijets over a wide range of center-of-mass energies gives $\left\langle k_{\mathrm{T}}\right\rangle$ as large as $5 \mathrm{GeV} / c$ [33]. It is common to think of the net transverse momentum of a dilepton or dijet pair as composed of 3 components:

$$
\begin{equation*}
\frac{\left\langle p_{\mathrm{T}}^{2}\right\rangle_{\mathrm{pair}}}{2}=\left\langle k_{\mathrm{T}}^{2}\right\rangle=\left\langle k_{\mathrm{T}}^{2}\right\rangle_{\mathrm{intrinsic}}+\left\langle k_{\mathrm{T}}^{2}\right\rangle_{\mathrm{soft}}+\left\langle k_{\mathrm{T}}^{2}\right\rangle_{\mathrm{NLO}} \tag{3}
\end{equation*}
$$

where the intrinsic part refers to the possible "fermi motion" of the confined quarks or gluons inside a proton, the NLO part refers to the power law tail at large values of $p_{\mathrm{T}_{\text {pair }}}$ due to the radiation of an initial state or final state hard gluon, which is divergent as the momentum of the radiated gluon goes to zero, and the soft part refers to the actual Gaussian-like distribution observed as $p_{\mathrm{T}_{\text {pair }}} \rightarrow 0$, which is explained by resummation [34].

In the discussion below we will assume that the two components of the vector $\vec{k}_{\mathrm{T}}, k_{\mathrm{Tx}}$ and $k_{\mathrm{Ty}}$ are Gaussian distributed with equal standard deviations $\sigma_{1 \text { parton,1D }}$, in which case $k_{\mathrm{T}}^{2}=k_{\mathrm{Tx}}^{2}+k_{\mathrm{Ty}}^{2}$ is distributed according to a 2-dimensional (2D) Gaussian [33]. For the net transverse momentum of the jet pair, $\left\langle p_{\mathrm{T}}^{2}\right\rangle_{\text {pair }}=\sigma_{2 \text { partons,2d }}^{2}=$ $2 \sigma_{1 \text { parton, } 2 \mathrm{~d}}^{2}$. Note that the principal difference between the 1 and 2 dimensional Gaussians is that $\left\langle k_{\mathrm{Tx}}\right\rangle=\left\langle k_{\mathrm{Ty}}\right\rangle=0$, while $\left\langle k_{\mathrm{T}}\right\rangle \neq 0$ since $\vec{k}_{\mathrm{T}}$ is a 2 D radius vector.

The two components of $k_{T}$ result in different experimentally measurable effects. $k_{T y}$ leads to the acoplanarity of the dijet pair while $k_{\mathrm{Tx}}$ makes the momenta of the jets unequal which results in the smearing of the steeply falling $p_{\mathrm{T}}$ spectrum. This causes the measured inclusive jet or single particle cross section to be larger than the pQCD value given by Eq. (1). This was observed in the original discovery of high $p_{\text {T }}$ particle production at the CERN ISR in 1972 [35] and led to much confusion until the existence and effects of $k_{T}$ were understood.

Before the advent of QCD, the invariant cross section for the hard-scattering of the electrically charged partons of deeply inelastic scattering was predicted for $p+p$ collisions to follow a general scaling form [22,36]:

$$
\begin{equation*}
E \frac{d^{3} \sigma}{d^{3} p}=\frac{1}{p_{\mathrm{T}}^{n}} F\left(x_{T}\right)=\frac{1}{\sqrt{s}^{n}} G\left(x_{T}\right) \tag{4}
\end{equation*}
$$

where $x_{T}=2 p_{\mathrm{T}} / \sqrt{s}$. The cross section has two factors, a function $F\left(x_{T}\right)\left(G\left(x_{T}\right)\right)$ which "scales," i.e. depends only on the ratio of momenta, and a dimensioned factor, $1 / p_{\mathrm{T}}^{n}$ $\left(1 / \sqrt{s}^{n}\right)$, where $n$ equals 4 for QED, and for LO-QCD [Eq. (4)], analogous to the $1 / q^{4}$ form of Rutherford scattering. The structure and fragmentation functions are all in the $F\left(x_{T}\right)\left(G\left(x_{T}\right)\right)$ term. The original high $p_{T}$ measurements at CERN [35] and Fermilab [37], showed beautiful $x_{T}$ scaling, but with a value of $n=8$ instead of $n=4$, for values of $3 \leq p_{\mathrm{T}} \leq 7 \mathrm{GeV} / c$. Later measurements at larger $p_{\mathrm{T}}$ showed the correct scaling in agreement with pQCD and it was realized that the value $n=8$ at lower values of $p_{\mathrm{T}}$ and $\sqrt{s}$ was produced by the $k_{T \mathrm{x}}$ smearing [23,24]. More recently, the deviation of $\pi^{0}$ and direct photon inclusive cross sections measurements from pQCD predictions has been used to derive the values of $k_{T}$ required to bring the measured and smeared pQCD predictions into agreement [33].

A more direct method to determine $k_{\mathrm{Ty}}$ is to measure the acoplanarity of the dijet pair. Such measurements were originally performed at the CERN-ISR using two-particle correlations [1-3,31]. The same method will be used in the present work.

Hard-scattering in $p+p$ collisions at $\sqrt{s}=200 \mathrm{GeV}$ is detected by triggering on a $\pi^{0}$ with transverse momentum $p_{T_{\mathrm{t}}} \geq 3 \mathrm{GeV} / c$; and the properties of jets are measured using the method of two-particle correlations. The trigger $\pi^{0}$ is a leading particle from a large transverse momentum jet while the associated particle comes from either the same jet or the away-side jet. We will analyze an outgoing dijet pair, with trigger jet transverse momentum magnitude $\hat{p}_{\mathrm{Tt}}$ which fragments to a trigger particle with transverse momentum $\vec{p}_{\mathrm{Tt}}$, and an away-side jet transverse momentum magnitude of $\hat{p}_{T a}$ which fragments to a particle with transverse momentum $\vec{p}_{\text {Ta }}$. The average transverse momentum component of the away-side particle $\vec{p}_{\text {Ta }}$ perpendicular to trigger particle $\vec{p}_{\mathrm{Tt}}$ in the azimuthal plane is labeled as $p_{\text {out }}$. If the magnitude of the jet transverse fragmentation momentum $j_{\mathrm{T}}$ [Fig. 1(a)] is neglected, the magnitude of $\sqrt{2} k_{\text {Ty }}$ can be related to $p_{\text {out }}: \sqrt{2} k_{\mathrm{Ty}}=$ $p_{\text {out }} \hat{p}_{\mathrm{Ta}} / p_{\mathrm{Ta}} \equiv p_{\text {out }} / z_{\text {a }}$. Thus the measurement of $p_{\text {out }}$ and the knowledge of the fragmentation variable $\left(z_{\mathrm{a}}\right)$ determines the magnitude of the parton's transverse momentum $k_{\mathrm{T}}$.

The smearing of the steeply falling parton $\hat{p}_{\mathrm{T}}$ spectrum by the $k_{\mathrm{Tx}}$ distribution tends to make the trigger jet transverse momentum $\hat{p}_{\mathrm{Tt}}$ larger than the away-jet transverse momentum $\hat{p}_{\text {Ta }}$. The component of the net transverse momentum of the parton pair along the trigger direction is smeared by $\sqrt{2} k_{T \mathrm{x}}$ such that

$$
\begin{equation*}
\left\langle\left(\hat{p}_{\mathrm{Tt}}-\hat{p}_{\mathrm{Tax}}\right)^{2}\right\rangle=2\left\langle k_{\mathrm{Tx}}^{2}\right\rangle=\left\langle k_{\mathrm{T}}^{2}\right\rangle \tag{5}
\end{equation*}
$$

For a flat $\hat{p}_{T}$ spectrum, the smearing would average to zero so that there would be no net shift in the transverse momentum spectrum:

$$
\begin{equation*}
\left\langle\hat{p}_{\mathrm{Tt}}-\hat{p}_{\mathrm{T}}\right\rangle=\left\langle\hat{p}_{\mathrm{T}}-\hat{p}_{\mathrm{Tax}}\right\rangle=0 \tag{6}
\end{equation*}
$$

However, due to the steeply falling $\hat{p}_{\mathrm{T}}$ spectrum, the $k_{\mathrm{Tx}}$ smearing results in a net imbalance of the jet-pair towards the trigger direction. In the limit when $k_{\mathrm{T}}$ is collinear with the trigger jet and with the requirement of the Lorentz invariance of $\hat{s}\left(\hat{p}_{\mathrm{T}}^{2}=\hat{p}_{\mathrm{Tt}} \hat{p}_{\mathrm{Ta}}\right)$ it is easy to see that

$$
\begin{equation*}
\left\langle\hat{p}_{\mathrm{Tt}}-\hat{p}_{\mathrm{T}}\right\rangle=\left\langle\frac{\hat{p}_{\mathrm{Tt}}}{\hat{p}_{\mathrm{T}}}\left(\hat{p}_{\mathrm{T}}-\hat{p}_{\mathrm{Ta}}\right)\right\rangle \simeq \frac{1}{2}\left\langle\hat{p}_{\mathrm{Tt}}-\hat{p}_{\mathrm{Ta}}\right\rangle>0 . \tag{7}
\end{equation*}
$$

We denote the imbalance of $\hat{p}_{\mathrm{Ta}}$ and $\hat{p}_{\mathrm{Tt}}$ by the quantity

$$
\begin{equation*}
\hat{x}_{\mathrm{h}}=\left\langle\hat{p}_{\mathrm{Ta}}\right\rangle /\left\langle\hat{p}_{\mathrm{Tt}}\right\rangle . \tag{8}
\end{equation*}
$$

Jet fragments have a momentum $\vec{j}_{\text {T }}$ perpendicular to the partonic transverse momentum [Fig. 1(b)]. This vector is again a two-dimensional vector with one component perpendicular to the jet transverse axis, $\overrightarrow{\hat{p}}_{\mathrm{T}}$, in the transverse plane and the other component perpendicular to the jet transverse axis in the longitudinal plane (defined by the beam and jet axes). The component of $\vec{j}_{\mathrm{T}}$ projected onto the azimuthal plane is labeled as $j_{\mathrm{Ty}}$. The magnitude of $\left\langle j_{\mathrm{Ty}}\right\rangle$, the mean value of $j_{\mathrm{T}}$ projected into the plane perpendicular to the jet thrust (see Appendix A), measured at lower energies [1] has been found to be $p_{\mathrm{T}}$ independent and $\approx$ $400 \mathrm{MeV} / c$, consistent with measurements in $e^{+} e^{-}$collisions [38,39].

This analysis uses two-particle azimuthal correlation functions to measure the average relative angles between a trigger $\pi^{0}$ and an associated charged hadron. The angular width of the near- and away-side peak in the correlation function is used to extract the value of $\left\langle j_{\mathrm{T}}^{2}\right\rangle$ and $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \times$ $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$. An analysis of the associated yields is used to confirm the fragmentation function which provides the $\left\langle z_{\mathrm{t}}\right\rangle$ and $\left\langle z_{\mathrm{a}}\right\rangle$ values used for $\left\langle k_{\mathrm{T}}^{2}\right\rangle$ extraction. The details on the PHENIX experiment relevant to this analysis follow.

## III. EXPERIMENTAL DETAILS

The PHENIX experiment consists of four spectrometer arms - two around mid-rapidity (the central arms) and two at forward rapidity (the muon arms) - along with a set of global detectors. The layout of the PHENIX experiment during the 2003 RHIC run is shown in Fig. 2.

Each central arm covers the pseudorapidity range $|\eta|<$ 0.35 and 90 degrees in azimuthal angle $\phi$. In each of the central arms, charged particles are tracked by a drift chamber (DC) positioned from 2.0 to 2.4 m radially outward from the beam axis and 2 or 3 layers of pixel pad chambers ( $\mathrm{PC} 1, \mathrm{PC} 2, \mathrm{PC} 3$ located at $2.4 \mathrm{~m}, 4.2 \mathrm{~m}, 5 \mathrm{~m}$ in the radial direction, respectively). Particle identification is provided


FIG. 2 (color online). The PHENIX experimental layout for the $2003 \mathrm{Au}+\mathrm{Au}$ run. The top panel shows the PHENIX central arm spectrometers viewed along the beam axis. The bottom panel shows a side view of the PHENIX muon arm spectrometers and the position of the global detectors (BBC and ZDC).
by ring imaging Čerenkov counters (RICH), a time of flight scintillator wall (TOF), and two types of electromagnetic calorimeters (EMCal), lead scintillator ( PbSc ) and lead glass ( PbGl ). The magnetic field for the central arm spectrometers is axially symmetric around the beam axis. Its component parallel to the beam axis has an approximately Gaussian dependence on the radial distance from the beam axis, dropping from 0.48 T at the center to 0.096 T $(0.048 \mathrm{~T})$ at the inner (outer) radius of the DC. A pair of zero-degree calorimeters (ZDC) and a pair of beam-beam counters (BBC) were used for global event characterization. Further details about the design and performance of PHENIX can be found in [40-42].

A $p+p$ data sample corresponding to an integrated luminosity $0.35 \mathrm{pb}^{-1}$ at $\sqrt{s}=200 \mathrm{GeV}$ has been used for the present analysis. It contains a minimum bias (MB) sample of 121 M events and a high- $p_{\mathrm{T}}$ triggered sample of 50 M events. The MB trigger is obtained from the charge multiplicity in the two BBCs situated at large pseudorapidity ( $\eta \approx \pm(3.0-3.9)$ ). The BBCs were also used to determine the collision vertex, which is limited to a $\pm 30 \mathrm{~cm}$ range in this analysis. The high- $p_{\mathrm{T}}$ trigger requests an additional discrimination on sums of the analog signals from nonoverlapping, $2 \times 2$ groups of adjacent

EMCal towers situated at mid-rapidity $(|\eta|<0.35)$ equivalent to an energy deposition of 750 MeV [43]. The analysis has been performed separately on the two data sets and no trigger selection bias was found within the quoted errors.

Neutral pions, which are used as trigger particles, are detected by the reconstruction of their $\gamma \gamma$ decay channel. Photons are detected in the EMCal, which has a timing resolution of $\approx 100 \mathrm{ps}(\mathrm{PbSc})$ and $\approx 300 \mathrm{ps}(\mathrm{PbGl})$ and energy resolution of $\sigma_{\mathrm{E}} / \mathrm{E}=1.9 \% \oplus 8.2 \% / \sqrt{\mathrm{E}(\mathrm{GeV})}$ $(\mathrm{PbSc})$ and $\sigma_{\mathrm{E}} / \mathrm{E}=0.8 \% \oplus 8.4 \% / \sqrt{\mathrm{E}(\mathrm{GeV})}(\mathrm{PbGl})$. In order to improve the signal/background ratio we require the minimum hit energy $>0.3 \mathrm{GeV}$, a shower profile cut as described in [41], and no accompanying hit in the RICH detector, which serves as a veto for conversion electrons. A sample of the invariant mass distribution of photon pairs detected in the EMCal is shown in Fig. 3.

Charged particles are reconstructed in each PHENIX central arm using a drift chamber, followed by two layers of multiwire proportional chambers with pad readout [40]. Particle momenta are measured with a resolution $\delta p / p=$ $0.7 \% \oplus 1.1 \% p(\mathrm{GeV} / c)$. A confirmation hit is required in PC3. We also require that no signal in the RICH detector is associated with these tracks. These requirements eliminate charged particles which do not originate from the event vertex, such as beam albedo and weak decays, as well as conversion electrons.

High momentum charged pions (above the RICH Čerenkov threshold) are identified using the RICH and EMCal detectors. Candidate tracks must be associated


FIG. 3. The measured $\gamma \gamma$ invariant mass distribution for pair $p_{\mathrm{T}}$ in $4<p_{\mathrm{T} \gamma \gamma}<5 \mathrm{GeV} / c$. The peak is fitted with a Gaussian. The signal/background ratio within $2 \sigma$ of the mean ranges from $\approx 6$ at $p_{\mathrm{T}}$ of $3 \mathrm{GeV} / c$ up to $\approx 15$ at $8 \mathrm{GeV} / c$.
with a hit in the RICH [44], which corresponds to a minimum momentum of $18 \mathrm{MeV} / c$ for electrons, $3.5 \mathrm{GeV} / c$ for muons, and $4.9 \mathrm{GeV} / c$ for charged pions. In a previous PHENIX publication [45], we have shown that charged particles with reconstructed $p_{T}$ above $4.9 \mathrm{GeV} / c$, which have an associated hit in the RICH, are dominantly charged pions and background electrons from photon conversions albedo. The efficiency for detecting charged pions rises quickly past $4.9 \mathrm{GeV} / c$, reaching an efficiency of $>90 \%$ at $p_{\mathrm{T}}>6 \mathrm{GeV} / c$.

To reject the electron background in the charged pion candidates, the shower information at the EMCal is used. Since most of the background electrons are genuine low $p_{T}$ particles that were misreconstructed as high $p_{\mathrm{T}}$ particles, simply requiring a large deposition of shower energy in the EMCal is effective in suppressing the electron background. In this analysis, a momentum- dependent energy cut on the EMCal is applied

$$
\begin{equation*}
\mathrm{E}(\mathrm{GeV})>0.3+0.15 p_{\mathrm{T}} \tag{9}
\end{equation*}
$$

In addition to this energy cut, the shower shape information [41] is used to further separate the broad hadronic showers from the narrow electromagnetic showers and hence reduce the conversion backgrounds. The difference of the EM shower and hadronic shower is typically characterized by a $\chi^{2}$ variable [41],

$$
\begin{equation*}
\chi^{2}=\sum_{i} \frac{\left(\mathrm{E}_{i}^{\text {meas }}-\mathrm{E}_{i}^{\mathrm{pred}}\right)^{2}}{\sigma_{i}^{2}} \tag{10}
\end{equation*}
$$

where $\mathrm{E}_{i}^{\text {meas }}$ is the energy measured at tower $i$ and $\mathrm{E}_{i}^{\text {pred }}$ is the predicted energy for an electromagnetic particle of total energy $\sum_{i} \mathrm{E}_{i}^{\text {meas }}$.

In this analysis we use the probability calculated from this $\chi^{2}$ value for an EM shower, ranging from 0 to 1 with a flat distribution expected for an EM shower, and a peak around 0 for an hadronic shower.

Figure 4 shows the probability distribution for pion and electron candidates, each normalized to one. The pion candidates were required to pass the energy cut and the electrons were selected using particle ID cuts similar to that used in [46]. The electron distribution is relatively flat, while the charged pion distribution peaks at 0 . A cut of shower shape probability $<0.2$ selects pions above the energy cut with an efficiency of $\gtrsim 80 \%$. Detailed knowledge of the pion efficiency is not necessary, since we present in this paper the per trigger pion conditional-yield distributions, for which this efficiency cancels out.

Since the energy and shower shape cuts are independent of each other, we can fix one cut and then vary the second to check the remaining background level from conversions. The energy cut in Eq. (9) is chosen such that the raw pion yield is found to be insensitive to the variation in the shower shape probability. Figure 5 shows the raw pion spectra for EMCal-RICH triggered events as a function of $p_{\mathrm{T}}$, with the above cuts applied. The pion turn on from


FIG. 4 (color online). The probability distribution for charged pion candidates and electrons derived from the EM shower shape using identified electrons and pions. The integrals have been normalized to one.


FIG. 5 (color online). The raw charged pion transverse momentum spectrum, with the final cuts applied. The level of the remaining background is estimated from an extrapolation from low- $p_{\mathrm{T}}$ and is shown as a black line.
$4.9-7 \mathrm{GeV} / c$ is clearly visible. Below $p_{\mathrm{T}}$ of $5 \mathrm{GeV} / c$, the remaining background comes mainly from the random association of charged particles with hits in the RICH detector. The background level is less than 5\% from $5-16 \mathrm{GeV} / c$, which is the $p_{T}$ range for the charged pion data presented in this paper.

## IV. CORRELATION FUNCTION

The analysis uses two-particle azimuthal correlation functions between a neutral pion and an associated charged hadron to measure the distribution of the azimuthal angle difference $\Delta \phi=\phi_{t}-\phi_{a}$ (see Fig. 6). Whenever a $\pi^{0}$ was found in an event, the real, $d N_{\text {uncorr }} / d \Delta \phi$, and mixed, $d N_{\text {mix }} / d \Delta \phi$, distributions for given $p_{\mathrm{Tt}}\left(\pi^{0}\right)$ and $p_{\mathrm{Ta}}$ (charged hadron) were accumulated (upper panel of Fig. 6). Mixed events were obtained by randomly selecting


FIG. 6 (color online). An example of the correlation functions for $3<p_{\mathrm{Tt}}<3.5 \mathrm{GeV} / c$ and associated particles in $1.4<$ $p_{\mathrm{Ta}}<5 \mathrm{GeV} / c$. (upper) Unnormalized pair-yield distribution plotted with the fit function which is two Gaussians modulated by the pair detection efficiency derived from the mixed distribution (dashed line). (lower) Per $\pi^{0}$ trigger yield distribution corrected for the pair detection efficiency. Dashed line represents the constant term in the fit.
each member of a particle pair from different events having similar vertex position. Then the mixed event distribution was used to correct the correlation function for effects of the limited PHENIX azimuthal acceptance and for the detection efficiency, to the extent that it remains constant over the data sample.

We fit the raw $d N_{\text {uncorr }} / d \Delta \phi$ distribution with the product

$$
\begin{align*}
\frac{d N_{\mathrm{uncorr}}}{d \Delta \phi}= & \frac{1}{\mathcal{N}} \frac{d N_{\mathrm{mix}}}{d \Delta \phi} \cdot\left(C_{0}+C_{1} \cdot f_{\text {near }}(\Delta \phi)+C_{2}\right. \\
& \left.\cdot f_{\text {away }}(\Delta \phi)\right), \tag{11}
\end{align*}
$$

where the mixed event distribution is normalized to $2 \pi$ ( $\mathcal{N}=\sum d N_{\text {mix }}^{i} / d \Delta \phi$ see dashed line on the upper panel
of Fig. 6), $C_{0-2}$ are constant factors to be determined from the fit, $f_{\text {near }}(\Delta \phi)$ and $f_{\text {away }}(\Delta \phi)$ are the near- and awayside peak fit functions, respectively. Traditionally, the Gaussian functions, around $\Delta \phi=0$ and around $\Delta \phi=$ $\pi$, are used for $f_{\text {near }}(\Delta \phi)$ and $f_{\text {away }}(\Delta \phi)$. This leaves a total of five free parameters to be determined - the areas and widths of the above two Gaussians: $Y_{\mathrm{N}}, \sigma_{\mathrm{N}}$ for the near-angle component and $Y_{F}, \sigma_{\mathrm{A}}$ for the away-angle component and the constant term describing an uncorrelated distribution of particle pairs which are not associated with jets. However, the assumption of the Gaussian shape of the angular correlation induced by jet fragmentation is justified only in the high- $p_{\mathrm{T}}$ region where the relative angles are small. In order to access also a lower $p_{\mathrm{T}}$ region we used an alternative parameterization of $f_{\text {near }}(\Delta \phi)$ and $f_{\text {away }}(\Delta \phi)$ which will be discussed later in the text.


FIG. 7. (upper) Inclusive charged hadron efficiency correction function. (lower) $\eta$ acceptance correction factor for loss of jet pairs outside the limited $\eta$-acceptance of the PHENIX experiment.

The normalized correlation function was constructed as a ratio of real and mixed distributions multiplied by $\eta$-acceptance correction factor $R_{\Delta \eta}$, divided by $p_{\mathrm{T}}$-dependent efficiency correction $\epsilon\left(p_{\mathrm{T}}\right)$ (see upper panel of Fig. 7) and divided by the number of $\pi^{0}$ triggers.

$$
\begin{equation*}
\frac{1}{N_{\text {trigg }}} \frac{d N}{d \Delta \phi}=\frac{R_{\Delta \eta}}{N_{\text {trigg }} \epsilon\left(p_{\mathrm{T}}\right)} \frac{d N_{\text {uncorr }}(\Delta \phi) / d \Delta \phi}{d N_{\text {mix }}(\Delta \phi) / d \Delta \phi} \cdot \mathcal{N} . \tag{12}
\end{equation*}
$$

The $R_{\Delta \eta}$ correction factor which accounts for limited $\eta$ acceptance of the PHENIX experiment (see right panel of Fig. 7) for the near-side yield, with an assumption that the angular jet width is the same in $\Delta \eta$ and in $\Delta \phi$, can be written as

$$
\begin{equation*}
R_{\Delta \eta}=\frac{1}{\frac{1}{\sqrt{2 \pi \sigma_{\mathrm{N}}^{2}}} \int_{-0.7}^{0.7} \exp \left(-\frac{\Delta \eta}{2 \sigma_{\mathrm{N}}^{2}}\right) \operatorname{acc}(\Delta \eta) d \Delta \eta} \tag{13}
\end{equation*}
$$

where $\operatorname{acc}(\Delta \eta)$ represent the PHENIX pair acceptance function in $|\Delta \eta|$. It can be obtained by convolving two
flat distributions in $|\Delta \eta|<0.35$, so $\operatorname{acc}(\Delta \eta)$ has a simple triangular shape: $\operatorname{acc}(\Delta \eta)=(0.7-|\Delta \eta|) / 0.7$. For the away-side yield the corresponding $R_{\Delta \eta}$ is

$$
\begin{equation*}
\mathrm{R}_{\Delta \eta}=\frac{2(0.7)}{\int_{-0.7}^{0.7} \operatorname{acc}(\Delta \eta) d \Delta \eta}=2 \tag{14}
\end{equation*}
$$

$R_{\Delta \eta}$ equals 2, because the pair efficiency has a triangular shape in $|\Delta \eta|<0.7$, which results in $50 \%$ average efficiency when the real jet-pair distribution is flat in $|\Delta \eta|<$ 0.7. Figure 8 and Table I show the normalized correlation functions for various $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$.

For two particles with transverse momenta $p_{\mathrm{Tt}}, p_{\mathrm{Ta}}$ from the same jet, the width of the near-side correlation distribution can be related to the RMS value of the $j_{\mathrm{T}}$ vector component, $j_{\text {Ty }}$, perpendicular to the parton momentum as

$$
\begin{equation*}
\sigma_{\mathrm{N}}^{2}=\left\langle\Delta \phi^{2}\right\rangle=\left\langle\left(\frac{j_{\mathrm{Ty}}}{p_{\mathrm{Ta}}}\right)^{2}+\left(\frac{j_{\mathrm{Ty}}}{p_{\mathrm{Tt}}}\right)^{2}\right\rangle \tag{15}
\end{equation*}
$$

where we assume $\left\langle j_{\mathrm{Ty}}^{2}\right\rangle \ll p_{\mathrm{Tt}}^{2}$ and $p_{\mathrm{Ta}}^{2}$ and thus the arcsine function can be approximated by its argument and we


FIG. 8. (left) Measured yield of charged hadrons with away-side transverse momentum $1.4<p_{\mathrm{Ta}}<5.0 \mathrm{GeV} / \mathrm{c}$ associated with a trigger $\pi^{0}$ of transverse momenta given in in Table I. (right) Measured yield of charged hadrons associated with a trigger $\pi^{0}$ of fixed transverse momentum $3.0<p_{\mathrm{Tt}}<10.0 \mathrm{GeV} / \mathrm{c}$ and the away-side transverse momenta given in Table I. The dashed lines corresponds to the fit of two Gaussian functions representing the trigger ( t ) jet and away-side (a) jet correlation. The $\chi^{2}$ (d.o.f.) $\sigma_{\mathrm{N}}$ and $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ values extracted from these fits are tabulated in Table I.

TABLE I. The $\chi^{2}$ (d.o.f.) $\sigma_{N}$ and $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ values extracted for the correlation function shown in Fig. 8. All units in rad and $\mathrm{GeV} / c$. Only the statistical errors are shown.

|  | $1.4<p_{\text {Ta }}<5.0 \mathrm{GeV} / c$ |  |  |  | $3.0<p_{\mathrm{Tt}}<10.0 \mathrm{GeV} / c$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $p_{\mathrm{Tt}}$ | $\chi^{2}$ (d.o.f. $=34$ ) | $\sigma_{\mathrm{N}}$ | $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ | $p_{\text {Ta }}$ | $\chi^{2}($ d.o.f. $=34)$ | $\sigma_{\mathrm{N}}$ |

can solve for [47]

$$
\begin{equation*}
\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}=\sqrt{2\left\langle j_{\mathrm{Ty}}^{2}\right\rangle} \simeq \sqrt{2} \frac{p_{\mathrm{Tt}} p_{\mathrm{Ta}}}{\sqrt{p_{\mathrm{Tt}}^{2}+p_{\mathrm{Ta}}^{2}}} \sigma_{\mathrm{N}} \tag{16}
\end{equation*}
$$

In order to extract $\langle | k_{\mathrm{Ty}}| \rangle$, or $\left\langle k_{\mathrm{T}}^{2}\right\rangle$, we start with the relation $[1,31]$ between the magnitude of $p_{\text {out }}$ (see Fig. 1),

$$
\begin{equation*}
p_{\text {out }}=p_{\mathrm{Ta}} \sin \Delta \phi, \tag{17}
\end{equation*}
$$

which is the transverse momentum component of the away-side particle $\vec{p}_{\text {Ta }}$ perpendicular to trigger particle $\vec{p}_{\mathrm{Tt}}$ in the azimuthal plane (see Fig. 1), and $k_{\mathrm{Ty}}$ :

$$
\begin{equation*}
\langle | p_{\text {out }}| \rangle^{2}=x_{\mathrm{E}}^{2}\left[2\langle | k_{\mathrm{Ty}}| \rangle^{2}+\langle | j_{\mathrm{Ty}}| \rangle^{2}\right]+\langle | j_{\mathrm{Ty}}| \rangle^{2}, \tag{18}
\end{equation*}
$$

where

$$
\begin{equation*}
x_{\mathrm{E}}=-\frac{\vec{p}_{\mathrm{Tt}} \cdot \vec{p}_{\mathrm{Ta}}}{p_{\mathrm{Tt}}^{2}}=-\frac{p_{\mathrm{Ta}} \cos \Delta \phi}{p_{\mathrm{Tt}}} \simeq \frac{z_{\mathrm{a}} \hat{p}_{\mathrm{Ta}}}{z_{\mathrm{t}} \hat{p}_{\mathrm{Tt}}} \tag{19}
\end{equation*}
$$

represents the fragmentation variable of the away-side jet. [2,3] We note however, that [31] explicitly neglected $\left\langle z_{\mathrm{t}}\right\rangle=$ $\left\langle p_{\mathrm{Tt}} / \hat{p}_{\mathrm{Tt}}\right\rangle$ in the formula at ISR energies, where $\left\langle z_{\mathrm{t}}\right\rangle \simeq 0.85$, while it is not negligible at $\sqrt{s}=200 \mathrm{GeV}$. Furthermore, as mentioned earlier, the average values of trigger and associated jet momenta are generally not the same. There is a systematic momentum imbalance due to $k_{\mathrm{T}}$-smearing of the steeply falling parton momentum distribution. The event sample with a condition of $p_{\mathrm{Tt}}>p_{\mathrm{Ta}}$ is dominated by configurations where the $k_{\mathrm{T}}$-vector is parallel to the trigger jet and antiparallel to the associated jet and $\left\langle\hat{p}_{T t}-\right.$ $\left.\hat{p}_{\mathrm{Ta}}\right\rangle \neq 0$. Here we introduce the hadronic variable $x_{\mathrm{h}}$ in analogy to the partonic variable $\hat{x}_{\mathrm{h}}$

$$
\begin{equation*}
x_{\mathrm{h}} \equiv \frac{p_{\mathrm{Ta}}}{p_{\mathrm{Tt}}}, \quad \hat{x}_{\mathrm{h}}=\hat{x}_{\mathrm{h}}\left(\left\langle k_{\mathrm{T}}^{2}\right\rangle, x_{\mathrm{h}}\right) \equiv \frac{\left\langle\hat{p}_{\mathrm{Ta}}\right\rangle}{\left\langle\hat{p}_{\mathrm{Tt}}\right\rangle} \tag{20}
\end{equation*}
$$

The detailed discussion on the magnitude of this imbalance is given later. In order to derive the relation between the magnitude of $p_{\text {out }}$ and $k_{\mathrm{T}}$ let us first consider the simple case where we have neglected both trigger and associated $\left\langle j_{\mathrm{T}}\right\rangle$ [see panel (a) on Fig. 1]. In this case one can see that

$$
\begin{aligned}
\left.\langle | p_{\text {out }}| \rangle\right|_{j_{\mathrm{T}}=j_{\mathrm{Ta}}=0} & \equiv\langle | p_{\text {out }}| \rangle_{00}=\sqrt{2}\langle | k_{\mathrm{Ty}}| \rangle \frac{p_{\mathrm{Ta}}}{\left\langle\hat{p}_{\mathrm{Ta}}\right\rangle} \\
& =\sqrt{2}\langle | k_{\mathrm{Ty}}| \rangle\left\langle z_{\mathrm{t}}\right\rangle \frac{x_{\mathrm{h}}}{\hat{x}_{\mathrm{h}}} .
\end{aligned}
$$

Rewriting the formula for $p_{\text {out }}$ in terms of RMS we get

$$
\sqrt{\left\langle p_{\mathrm{out}}^{2}\right\rangle_{00}}=\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle} \frac{x_{\mathrm{h}}}{\hat{x}_{\mathrm{h}}}
$$

where we have taken $\left\langle k_{\mathrm{T}}^{2}\right\rangle=\left\langle 2 k_{\text {Ty }}^{2}\right\rangle$.
However, the jet fragments are produced with finite jet transverse momentum $j_{\mathrm{T}}$. The situation when the trigger particle is produced with $j_{\text {Tty }}>0 \mathrm{GeV} / c$ and the associated particle with $j_{\text {Tay }}=0 \mathrm{GeV} / c$ is shown in Fig. 1(b). The $p_{\text {out }}$ vector picks up an additional component

$$
\begin{aligned}
\left.\left\langle p_{\text {out }}^{2}\right\rangle\right|_{j_{\mathrm{Tt}}>0, j_{\mathrm{Ta}}=0}= & {\left[\left\langle p_{\text {out }}^{2}\right\rangle_{00}+\frac{\left\langle j_{\mathrm{Tty}}^{2}\right\rangle}{p_{\mathrm{Tt}}^{2}}\left(p_{\mathrm{Ta}}^{2}-\left\langle p_{\text {out }}^{2}\right\rangle_{00}\right)\right] } \\
& \times \frac{p_{\mathrm{Tt}}^{2}-\left\langle j_{\mathrm{Tty}}^{2}\right\rangle}{p_{\mathrm{Tt}}^{2}} .
\end{aligned}
$$

With an assumption of $j_{\text {Tty }} \ll p_{\mathrm{Tt}}$ we found that


FIG. 9 (color online). (top) The width of the near-side peak $\sigma_{\mathrm{N}}$ with $p_{\mathrm{Ta}}$ for various values of $p_{\mathrm{Tt}}$ as indicated in legend. (bottom) The width of the far-side peak $\sigma_{\mathrm{A}}$ with $p_{\mathrm{Ta}}$ for the same $p_{\mathrm{Tt}}$ selection. The data values are given in Table II.


FIG. 10 (color online). The near-side (squares) and away-side (circles) width as a function or trigger $-\pi^{0} p_{\mathrm{Tt}}$. The associated charged particle momenta are in the $1.4<p_{\mathrm{Ta}}<5.0 \mathrm{GeV} / c$ region. The curves are from a PYTHIA calculation with the values of $k_{\mathrm{T}}$ indicated. The data values are given in Table III.

$$
\left.\left\langle p_{\text {out }}^{2}\right\rangle\right|_{j_{\mathrm{Tt}}>0, j_{\mathrm{T}_{\mathrm{a}}}=0}=x_{\mathrm{h}}^{2}\left[\left\langle z_{\mathrm{t}}\right\rangle^{2}\left\langle k_{\mathrm{T}}^{2}\right\rangle \frac{1}{\hat{x}_{\mathrm{h}}^{2}}+\left\langle j_{\mathrm{T}_{\mathrm{ty}}}^{2}\right\rangle\right] .
$$

We include $j_{\text {Ta }}$ in the same approximation, $j_{\text {Tay }} \ll p_{\text {Ta }}$, i.e. collinearity of $j_{\mathrm{Ta}}$ and $p_{\text {out }}$ with result

$$
\begin{equation*}
\left\langle p_{\text {out }}^{2}\right\rangle=x_{\mathrm{h}}^{2}\left[\left\langle z_{\mathrm{t}}\right\rangle^{2}\left\langle k_{\mathrm{T}}^{2}\right\rangle \frac{1}{\hat{x}_{\mathrm{h}}^{2}}+\left\langle j_{\text {Tty }}^{2}\right\rangle\right]+\left\langle j_{\text {Tay }}^{2}\right\rangle \tag{21}
\end{equation*}
$$

and we solve for $\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle} / \hat{x}_{h}$

$$
\frac{\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}}{\hat{x}_{\mathrm{h}}}=\frac{1}{x_{\mathrm{h}}} \sqrt{\left\langle p_{\text {out }}^{2}\right\rangle-\left\langle j_{\text {Tay }}^{2}\right\rangle-x_{\mathrm{h}}^{2}\left\langle j_{\text {Tty }}^{2}\right\rangle}
$$

If we assume no difference between $j_{\mathrm{Tt}}$ and $j_{\mathrm{Ta}}$ then we have

TABLE III. The $\sigma_{\mathrm{N}}$ and $\sigma_{\mathrm{A}}$ values shown in Fig. 10. All units in rad and $\mathrm{GeV} / c$. Only the statistical errors are shown.

| $p_{\mathrm{Tt}}$ | $\sigma_{\mathrm{N}}$ | $\sigma_{\mathrm{A}}$ |
| :--- | :---: | :---: |
| 2.23 | $0.247 \pm 0.002$ | $0.565 \pm 0.013$ |
| 2.72 | $0.227 \pm 0.003$ | $0.548 \pm 0.014$ |
| 3.22 | $0.235 \pm 0.004$ | $0.521 \pm 0.016$ |
| 3.89 | $0.215 \pm 0.004$ | $0.464 \pm 0.014$ |
| 4.90 | $0.210 \pm 0.006$ | $0.431 \pm 0.020$ |
| 5.91 | $0.197 \pm 0.009$ | $0.396 \pm 0.025$ |
| 7.23 | $0.185 \pm 0.012$ | $0.350 \pm 0.028$ |

$$
\begin{equation*}
\frac{\left\langle z_{\mathrm{t}}\left(k_{\mathrm{T}}, x_{\mathrm{h}}\right)\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}}{\hat{x}_{\mathrm{h}}\left(k_{\mathrm{T}}, x_{\mathrm{h}}\right)}=\frac{1}{x_{\mathrm{h}}} \sqrt{\left\langle p_{\text {out }}^{2}\right\rangle-\left\langle j_{\mathrm{Ty}}^{2}\right\rangle\left(1+x_{\mathrm{h}}^{2}\right)} . \tag{22}
\end{equation*}
$$

All quantities on the right-hand side of Eq. (22) can be directly extracted from the correlation function. The correlation functions are measured in the variable $\Delta \phi$ in bins of $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$ (e.g. see Fig. 8), and the rms of the near and away peaks $\sigma_{\mathrm{N}}$ and $\sigma_{\mathrm{A}}$ are extracted. We tabulated $\sigma_{\mathrm{N}}$ and $\sigma_{\mathrm{A}}$ for many combinations of $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$ (see Figs. 9 and 10 and Tables II and III).

Initially, we used the approximation $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle} \sim$ $p_{\mathrm{Ta}} \sin \sigma_{\mathrm{A}}$ in Eq. (22). However, we have noticed that this approximation and other approximations for $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ proposed e.g. in Ref. [48] (see Appendix B) are inadequate for $\sigma_{\mathrm{A}}>0.4$ radians, so we do not use $\sigma_{\mathrm{A}}$ to calculate $k_{\mathrm{T}}$.

We extract $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ directly for all values of $p_{\mathrm{Tt}} p_{\mathrm{Ta}}$ (even for wide bins in $p_{\mathrm{Ta}}$ using the $\left\langle p_{\mathrm{Ta}}\right\rangle$ of the bin) by fitting the correlation function in the $\pi / 2<\Delta \phi<3 \pi / 2$ region by

$$
\begin{align*}
\left.\frac{d N_{\text {away }}}{d \Delta \phi}\right|_{\pi / 2} ^{3 \pi / 2} & =\frac{d N}{d p_{\text {out }}} \frac{d p_{\text {out }}}{d \Delta \phi} \\
& =\frac{-p_{\mathrm{Ta}} \cos \Delta \phi}{\sqrt{2 \pi\left\langle p_{\text {out }}^{2}\right\rangle} \operatorname{Erf}\left(\frac{\sqrt{2} p_{\text {Ta }}}{\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}}\right)} \exp \left(-\frac{p_{\mathrm{Ta}}^{2} \sin ^{2} \Delta \phi}{2\left\langle p_{\text {out }}^{2}\right\rangle}\right), \tag{23}
\end{align*}
$$

where we assumed a Gaussian distribution in $p_{\text {out }}$. We still use a Gaussian function in $\Delta \phi$ in the near-angle peak to

TABLE II. Measured widths of the near- and away-angle $\pi^{0}-h^{ \pm}$correlation peaks for various trigger particle momenta, as shown in Fig. 9. Only the statistical errors are shown.

| $p_{\mathrm{Tt}}=3.39 \mathrm{GeV} / c$ |  |  | $p_{\mathrm{Tt}}=4.40 \mathrm{GeV} / c$ |  |  | $p_{\mathrm{Tt}}=5.41 \mathrm{GeV} / c$ |  |  | $p_{\mathrm{Tt}}=6.40 \mathrm{GeV} / c$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p_{\text {Ta }}$ | $\sigma_{\mathrm{N}} \mathrm{rad}$ | $\sigma_{A} \mathrm{rad}$ | $p_{\text {Ta }}$ | $\sigma_{\mathrm{N}} \mathrm{rad}$ | $\sigma_{A} \mathrm{rad}$ | $p_{\text {Ta }}$ | $\sigma_{\mathrm{N}} \mathrm{rad}$ | $\sigma_{A} \mathrm{rad}$ | $p_{\text {Ta }}$ | $\sigma_{\mathrm{N}} \mathrm{rad}$ | $\sigma_{A} \mathrm{rad}$ |
| 1.59 | $0.27 \pm 0.01$ | $0.58 \pm 0.05$ | 1.72 | $0.28 \pm 0.02$ | $0.50 \pm 0.03$ | 1.51 | $0.26 \pm 0.01$ | $0.49 \pm 0.03$ | 1.34 | $0.40 \pm 0.03$ | $0.68 \pm 0.05$ |
| 1.84 | $0.24 \pm 0.01$ | $0.52 \pm 0.03$ | 2.14 | $0.18 \pm 0.01$ | $0.47 \pm 0.06$ | 2.22 | $0.21 \pm 0.02$ | $0.39 \pm 0.05$ | 1.64 | $0.30 \pm 0.02$ | $0.58 \pm 0.05$ |
| 2.22 | $0.23 \pm 0.01$ | $0.52 \pm 0.03$ | 2.53 | $0.20 \pm 0.01$ | $0.47 \pm 0.04$ | 2.88 | $0.17 \pm 0.01$ | $0.37 \pm 0.05$ | 1.94 | $0.23 \pm 0.02$ | $0.52 \pm 0.06$ |
| 2.73 | $0.19 \pm 0.01$ | $0.46 \pm 0.04$ | 3.17 | $0.16 \pm 0.01$ | $0.38 \pm 0.04$ | 4.01 | $0.14 \pm 0.02$ | $0.34 \pm 0.07$ | 2.29 | $0.23 \pm 0.02$ | $0.40 \pm 0.03$ |
| 3.24 | $0.19 \pm 0.01$ | $0.47 \pm 0.04$ | 4.36 | $0.14 \pm 0.01$ | $0.39 \pm 0.07$ |  |  |  | 2.74 | $0.17 \pm 0.01$ | $0.41 \pm 0.05$ |
| 3.93 | $0.17 \pm 0.01$ | $0.41 \pm 0.03$ |  |  |  |  |  |  | 3.36 | $0.17 \pm 0.02$ | $0.36 \pm 0.04$ |
| 5.04 | $0.12 \pm 0.01$ | $0.38 \pm 0.05$ |  |  |  |  |  |  |  |  |  |



FIG. 11. Extracted values of $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ for $3.0<p_{\mathrm{Tt}}<4.0$ and $5.0<p_{\mathrm{Tt}}<10.0 \mathrm{GeV} / c$ for various values of $p_{\mathrm{Ta}}$ using the direct $p_{\text {out }}$ extraction method based on fitting the away-side peak by Eq. (23).
extract $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}$. The $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ values extracted from the fit of the functional form (23) are shown in Figs. 11 and 12 and Table IV.

The per-trigger yields as a function of $p_{\mathrm{Tt}}$ for fixed associated $1.4<p_{\mathrm{Ta}}<5.0 \mathrm{GeV} / c$ bin are shown in


FIG. 12. (solid circles) Extracted values of $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ for $1.4<$ $p_{\mathrm{Ta}}<5.0 \mathrm{GeV} / c$ for various values of $p_{\mathrm{Tt}}$ from Eq. (23). (open diamonds) Indirect $p_{\text {out }}=p_{\mathrm{Ta}} \sin \left(\sigma_{\mathrm{A}}\right)$ values.

TABLE IV. The $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ values shown in Fig. 11 and 12. All units in $\mathrm{GeV} / c$. Only the statistical errors are shown.

| $1.4<p_{\mathrm{Ta}}<5.0$ | $3<p_{\mathrm{Tt}}<4$ |  | $5<p_{\mathrm{Tt}}<10$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $p_{\mathrm{Tt}}$ | $\left.\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}\right\rangle$ | $p_{\mathrm{Ta}}$ | $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ | $p_{\mathrm{Ta}}$ | $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ |
| 2.23 | $1.315 \pm 0.043$ | 1.72 | $0.996 \pm 0.056$ | 1.85 | $0.960 \pm 0.102$ |
| 2.72 | $1.250 \pm 0.046$ | 2.22 | $1.244 \pm 0.079$ | 2.24 | $1.100 \pm 0.103$ |
| 3.22 | $1.182 \pm 0.049$ | 2.73 | $1.222 \pm 0.095$ | 2.73 | $1.088 \pm 0.110$ |
| 3.89 | $1.011 \pm 0.038$ | 3.23 | $1.496 \pm 0.105$ | 3.44 | $1.285 \pm 0.136$ |
| 4.90 | $0.953 \pm 0.052$ | 3.93 | $1.793 \pm 0.115$ | 4.65 | $1.268 \pm 0.210$ |
| 5.91 | $0.868 \pm 0.064$ | 5.04 | $1.675 \pm 0.141$ |  |  |
| 7.24 | $0.798 \pm 0.068$ |  |  |  |  |

Fig. 13 and Table V. There is a distinct behavior of the near-side yield which varies with trigger $p_{\mathrm{Tt}}$ much less than the away-side yield. For the away-side, this reflects the fact that the particle detected in the fixed associated bin are produced from the lower $z$ region of the fragmentation function for events with higher trigger $p_{\mathrm{Tt}}$. For the nearside jet, this multiplicity increase is reduced due to the fact that with increasing $p_{\mathrm{Tt}}$ the near-side jet energy increases; however, at the same time the larger fraction of this energy is taken away by the more energetic trigger particle. Thus the relative change in $z$ is smaller on the near side.

In order to extract $\left\langle z_{\mathrm{t}}\right\rangle$ and $\hat{x}_{\mathrm{h}}$ knowledge of the fragmentation function is needed; a detailed discussion is given in following sections.


FIG. 13. Measured yield of charged hadrons associated with one trigger $\pi^{0}$ with transverse momenta indicated in Table I and associated charged hadron with $1.4<p_{\mathrm{T}<} 5.0 \mathrm{GeV} / \mathrm{c}$. Dashed lines represent the linear fit. The data values are given in Table V .

JET PROPERTIES FROM DIHADRON CORRELATIONS ...
TABLE V. The near and away-side conditional-yield per number of triggers for $1.4<p_{\mathrm{Ta}}<5.0 \mathrm{GeV} / c$ shown in Fig. 13. All units in rad and $\mathrm{GeV} / c$. Only the statistical errors are shown.

| $p_{\mathrm{Tt}}$ | $Y_{N}$ | $Y_{A}$ |
| :--- | :---: | :---: |
| 2.23 | $1.911 \pm 0.018$ | $1.717 \pm 0.044$ |
| 2.72 | $1.863 \pm 0.022$ | $1.908 \pm 0.055$ |
| 3.22 | $2.032 \pm 0.032$ | $2.130 \pm 0.071$ |
| 3.89 | $1.966 \pm 0.033$ | $2.360 \pm 0.074$ |
| 4.90 | $2.120 \pm 0.061$ | $2.611 \pm 0.123$ |
| 5.91 | $2.153 \pm 0.098$ | $2.992 \pm 0.196$ |
| 7.24 | $2.174 \pm 0.125$ | $3.690 \pm 0.242$ |

## A. $\sqrt{\left\langle\boldsymbol{j}_{\mathbf{T}}^{2}\right\rangle}$ and $\hat{\boldsymbol{x}}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle\boldsymbol{k}_{\boldsymbol{T}}^{2}\right\rangle}$ results

The measurement is performed in two different kinematical regimes; first the transverse momentum of the trigger particle, $p_{\mathrm{Tt}}$, is fixed and the peak width is measured for different values of associated particle transverse momenta $p_{\text {Ta }}$ (Fig. 9). (Note that in the region of overlap, the data are in excellent agreement with a previous measurement [9].) In the second case, particle pairs with a fixed associated bin $1.4<p_{\mathrm{Ta}}<5.0 \mathrm{GeV} / c$ and various $p_{\mathrm{Tt}}$ are selected (Fig. 10). It is evident that both near and away-side correlation peaks in all cases reveal a decreasing trend with $p_{\mathrm{Ta}}$ and $p_{\mathrm{Tt}}$.

However, the asymptotic behavior of $\sigma_{\mathrm{N}}$ and $\sigma_{\mathrm{A}}$ is different. Whereas the magnitude of $\sigma_{\mathrm{N}}$, according to Eq. (16), should vanish for large values of $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$, the $\sigma_{\mathrm{A}}$ according to Eq. (22) should be approximately constant around $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle} / p_{\mathrm{Tt}}$ for large values of $p_{\mathrm{Ta}}$.


FIG. 14 (color online). $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}$ values calculated according Eq. (16). The dashed line represents a fit to a constant in the $1.5<p_{\mathrm{Ta}}<5 \mathrm{GeV} / c$ region. The data values are shown in Table VI.

TABLE VI. The $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}$ values shown in Figs. 14 and 15. All units in rad and $\mathrm{GeV} / c$. Only the statistical errors are shown.

| $1.4<p_{\mathrm{Ta}}<5.0$ | $3<p_{\mathrm{Tt}}<4$ |  |  |
| :--- | :---: | :---: | :---: |
| $p_{\mathrm{Tt}}$ | $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}$ | $p_{\mathrm{Ta}}$ | $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}$ |
| 3.22 | $0.587 \pm 0.009$ | 1.72 | $0.562 \pm 0.011$ |
| 3.89 | $0.577 \pm 0.009$ | 2.22 | $0.597 \pm 0.014$ |
| 4.90 | $0.600 \pm 0.017$ | 2.73 | $0.572 \pm 0.017$ |
| 5.91 | $0.596 \pm 0.026$ | 3.23 | $0.590 \pm 0.020$ |
| 7.24 | $0.597 \pm 0.038$ | 3.93 | $0.603 \pm 0.017$ |
| 8.34 | $0.632 \pm 0.085$ | 5.04 | $0.506 \pm 0.029$ |
| $4<p_{\mathrm{Tt}}<5$ | $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}$ | $5<p_{\mathrm{Tt}}<6$ |  |
| $p_{\mathrm{Ta}}$ | $p_{\mathrm{Ta}}$ | $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}$ |  |
| 1.72 | $0.643 \pm 0.036$ | 1.52 | $0.529 \pm 0.030$ |
| 2.14 | $0.492 \pm 0.032$ | 2.22 | $0.581 \pm 0.049$ |
| 2.53 | $0.608 \pm 0.035$ | 2.88 | $0.590 \pm 0.047$ |
| 3.17 | $0.590 \pm 0.032$ | 4.01 | $0.603 \pm 0.063$ |
| 4.36 | $0.631 \pm 0.052$ |  |  |

The $\left\langle z_{\mathrm{t}}\right\rangle$ and $\hat{x}_{\mathrm{h}}$ quantities are implicitly $p_{\mathrm{Ta}}$ dependent, however, their ratio is roughly $\sim 1$ so that the asymptotic value of $\left.\sigma_{\mathrm{A}}\right|_{p_{\mathrm{Ta}} \rightarrow \infty} \sim\left\langle k_{\mathrm{T}}^{2}\right\rangle / p_{\mathrm{Tt}}$.

Extracted values of $\left\langle j_{\mathrm{T}}^{2}\right\rangle$ as a function of $p_{\mathrm{Ta}}$ and $p_{\mathrm{Tt}}$ according to Eq. (16) are shown in Fig. 14 and 15 and Table VI. All $\left\langle j_{\mathrm{T}}^{2}\right\rangle$ values are constant in the explored region $\left(p_{\mathrm{Ta}}>1.5 \mathrm{GeV} / c\right)$. It is expected that $\left\langle j_{\mathrm{T}}^{2}\right\rangle$ can not remain constant for arbitrarily small values of $p_{\mathrm{Ta}}$ because of the phase space limitation. In the region where


FIG. 15. Averaged values of $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}$ in $\left(1.5<p_{\mathrm{Ta}}<5 \mathrm{GeV} / c\right)$ as a function of the trigger transverse momentum $p_{\mathrm{Tt}}$ (solid triangles). The CCOR values measured at $\sqrt{s}=62.4 \mathrm{GeV}$ shown by open triangles. The data values are shown in Table VI.
$p_{\mathrm{Ta}} \leq \sqrt{\left\langle j_{T}^{2}\right\rangle}$, the magnitude of the $j_{\mathrm{T}}$-vector is truncated, similar to the "seagull effect" [49]. Since the $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}$ values are on the order of $600 \mathrm{MeV} / c$, we assume that the phase space limitation can be safely neglected for $p_{\mathrm{Ta}}>$ $1.5 \mathrm{GeV} / c$ and extract the value of $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}$ averaged over $p_{\mathrm{Ta}}$ and $p_{\mathrm{Tt}}$ :

$$
\begin{equation*}
\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}=585 \pm 6(\text { stat }) \pm 15(\text { sys }) \mathrm{MeV} / \mathrm{c} \tag{24}
\end{equation*}
$$

The systematic error originates from the finite momentum resolution and Eq. (16) where we assume that the arcsine function can be approximated by its argument. For the angular width of the near-angle peak (see Fig. 9 and 10) it corresponds to an uncertainty of order of $3 \%$.

The independence of $\left\langle j_{T}^{2}\right\rangle$ on either $p_{\mathrm{Tt}}$ or $\sqrt{s}$ has been observed by the CCOR experiment in the range $\sqrt{s}=$ $31-62.4 \mathrm{GeV}$ [1]. The $\left\langle j_{\mathrm{T}}^{2}\right\rangle$ values at $\sqrt{s}=62.4 \mathrm{GeV}$ (open triangles on Fig. 15) are systematically larger then values found in this analysis. The discrepancy should not be taken as significant, as CCOR used a slightly different technique than in this paper. CCOR extracted the $\left\langle j_{\mathrm{T}}^{2}\right\rangle$ values from measurements of $\langle | p_{\text {out }}| \rangle$ for different values of the $x_{E}$ variable Eq. (19). According to Eq. (18) the $\langle | p_{\text {out }}| \rangle^{2}$ magnitude should depend linearly on $x_{\mathrm{E}}^{2}$; and the $\langle | j_{\mathrm{Ty}}| \rangle$ value was extracted from the intercept of the $\left\langle p_{\text {out }}^{2}\right\rangle\left(x_{\mathrm{E}}\right)$ fit at $x_{\mathrm{E}}=0$, rather than from a measurement of $\sigma_{\mathrm{N}}$.


FIG. 16 (color online). $\left.\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}\right\rangle$ values calculated according to Eq. (22) for trigger $\pi^{0}$ in $3<p_{\mathrm{Tt}}<4 \mathrm{GeV} / c$ and $5<$ $p_{\mathrm{Tt}}<6 \mathrm{GeV} / c$ as a function of $p_{\mathrm{Ta}}$. The systematic uncertainties are indicated by the shaded boxes.


FIG. 17 (color online). The same calculation according Eq. (22) for fixed associated bin $1.4<p_{\mathrm{Ta}}<5.0 \mathrm{GeV} / c$ as a function of $p_{\mathrm{Tt}}$. The systematic errors indicated by colored rectangles. The $\sqrt{\left\langle k_{T}^{2}\right\rangle}$ results obtained by CCOR Collaboration at $\sqrt{s}=62.4 \mathrm{GeV}$ [1] displayed by solid triangles.

Knowing the $\left\langle j_{\mathrm{T}}^{2}\right\rangle$ and $\left\langle p_{\text {out }}^{2}\right\rangle$ values, we used Eq. (22) to determine $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{t}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ (see Fig. 16 and Fig. 17). The systematic error was estimated with Monte Carlo simulations to be on the order of $5 \%$. The main source of systematic error originates from the assumption [Eqs. (16) and (21)] of the relative smallness of $\left\langle j_{T}^{2}\right\rangle$, collinearity between $p_{\text {out }}$ and $j_{\text {Tay }}$ and from the limited momentum resolution discussed in Sec. III.

The $p_{\mathrm{Ta}}$ dependence of the extracted $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{t}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ (Fig. 16) reveals a strikingly decreasing trend. It was originally expected that by fixing the value of $p_{\mathrm{Tt}}$, the kinematics of the hard scattering (i.e. $\hat{p}_{T t} \simeq \hat{p}_{\mathrm{Ta}}$ ) would be fixed, independently of the value of $p_{\mathrm{Ta}}$. Various values of $p_{\mathrm{Ta}}$ would then sample the $\hat{p}_{\mathrm{Ta}}$ fragmentation function, and the value of $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle\left\langle k_{\mathrm{T}}^{2}\right\rangle$ was expected to be constant. It is evident that this assumption is not quite correct.

A similar line of argument applies also for the rising trend when $p_{\mathrm{Ta}}$ is fixed and $p_{\mathrm{Tt}}$ varies (Fig. 17). It is interesting to note that the CCOR $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ values measured at $\sqrt{s}=62.4 \mathrm{GeV}$ (open triangles on Fig. 17) reveal a similar rising trend. However, the rising trend of $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \times$ $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ with $p_{\mathrm{Tt}}$ and falling with $p_{\mathrm{Ta}}$ suggests that the variation of $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ with $p_{\mathrm{Tt}}$ seen by the CCOR Collaboration [1] may be indicative of the $\left\langle z_{\mathrm{t}}\right\rangle \hat{x}_{\mathrm{h}}^{-1}$ variation which was there neglected [50]. In order to understand
variation of $\left\langle z_{\mathrm{t}}\right\rangle$ and $\hat{x}_{\mathrm{h}}^{-1}$ we have to explore the process of dijet fragmentation.

## V. FRAGMENTATION FUNCTIONS

We have shown in Eq. (22) that the width of the awayside correlation peak is related to the product of $\hat{x}_{h}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \times$ $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$. In order to evaluate $\left\langle z_{\mathrm{t}}\right\rangle$, knowledge of the scattered parton $\hat{p}_{\mathrm{T}}$ spectrum and fragmentation function is required.

Fragmentation functions from $e^{+} e^{-}$collisions, weighted by the appropriate hard-scattering constituent cross sections and $Q^{2}$ evolution could in principle be used. However, it was originally thought that the shape of the fragmentation function could be deduced from present measurements using the combined analysis of the inclusive trigger $p_{\mathrm{Tt}}$ and associated particle $p_{\mathrm{Ta}}$ distributions. Although this idea turned out to be incorrect, we will follow this line of reasoning for a while as it is instructive.

Generally, the invariant cross section for inclusive hadron production from jets can be parametrized in the following way. First, we assume that the number of parton fragments (consider only pions for simplicity) at a given $p_{\mathrm{T}}$ corresponds to the sum over all contributions from parton momenta, $\hat{p}_{\mathrm{T}}$ from $p_{\mathrm{T}}<\hat{p}_{\mathrm{T}}<\sqrt{s} / 2$. The joint probability of detecting a pion with $p_{\mathrm{T}}=z \hat{p}_{\mathrm{T}}$ originating from a parton with $\hat{p}_{T}$ can be written as

$$
\begin{equation*}
\frac{d^{2} \sigma_{\pi}}{\hat{p}_{\mathrm{T}} d \hat{p}_{\mathrm{T}} d z}=\frac{d \sigma_{q}}{\hat{p}_{\mathrm{T}} d \hat{p}_{\mathrm{T}}} \times D_{\pi}^{q}(z)=f_{q}\left(\hat{p}_{\mathrm{T}}\right) \times D_{\pi}^{q}(z) \tag{25}
\end{equation*}
$$

Here we use $f_{q}\left(\hat{p}_{\mathrm{T}}\right)$ to represent the final state scatteredparton invariant spectrum $d \sigma_{q} / \hat{p}_{\mathrm{T}} d \hat{p}_{\mathrm{T}}$ and $D_{\pi}^{q}(z)$ to represent the fragmentation function. The first term in Eq. (25) can be viewed as a probability of finding a parton with transverse momentum $\hat{p}_{\mathrm{T}}$ and the second term corresponds to the probability that the parton fragments into a particle of momentum $p_{\mathrm{T}}=z \hat{p}_{\mathrm{T}}$. With a simple change of variables from $\hat{p}_{\mathrm{T}}$ to $p_{\mathrm{T}}=z \hat{p}_{\mathrm{T}}$, we obtain the joint probability of a pion with $p_{\mathrm{T}}$ which is a fragment with momentum fraction $z$ from a parton with $\hat{p}_{\mathrm{T}}=p_{\mathrm{T}} / z$ :

$$
\begin{equation*}
\frac{d^{2} \sigma_{\pi}}{p_{\mathrm{T}} d p_{\mathrm{T}} d z}=f_{q}\left(\frac{p_{\mathrm{T}}}{z}\right) \cdot D_{\pi}^{q}(z) \frac{1}{z^{2}} \tag{26}
\end{equation*}
$$

The $p_{\mathrm{T}}$ and $z$ dependences do not factorize. However, the $p_{\mathrm{T}}$ spectrum may be found by integrating over all values of $\hat{p}_{\mathrm{T}} \geq p_{\mathrm{T}}$ to $\hat{p}_{\mathrm{T}} \max =\sqrt{s} / 2$, which corresponds to values of $z$ from $x_{\mathrm{T}}=2 p_{\mathrm{T}} / \sqrt{s}$ to 1 .

$$
\begin{equation*}
\frac{1}{p_{\mathrm{T}}} \frac{d \sigma_{\pi}}{d p_{\mathrm{T}}}=\int_{x_{T}}^{1} f_{q}\left(\frac{p_{\mathrm{T}}}{z}\right) \cdot D_{\pi}^{q}(z) \frac{d z}{z^{2}} \tag{27}
\end{equation*}
$$

Alternatively, for any fixed value of $p_{T}$ one can evaluate the $\left\langle z\left(p_{\mathrm{T}}\right)\right\rangle$, integrated over the parton spectrum:

$$
\begin{equation*}
\left\langle z\left(p_{\mathrm{T}}\right)\right\rangle=\frac{\int_{x_{T}}^{1} z D_{\pi}^{q}(z) f_{q}\left(p_{\mathrm{T}} / z\right) \frac{d z}{z^{2}}}{\int_{x_{T}}^{1} D_{\pi}^{q}(z) f_{q}\left(p_{\mathrm{T}} / z\right) \frac{d z}{z^{2}}} \tag{28}
\end{equation*}
$$

From the scaling properties of QCD and from the shape of the $\pi^{0}$ invariant cross section itself, which is a pure power law for $p_{T} \geq 3 \mathrm{GeV} / c$ [43], one can deduce that $f_{q}\left(\hat{p}_{\mathrm{T}}\right)$ should have a power law shape, $f_{q}\left(\hat{p}_{\mathrm{T}}\right)=A \hat{p}_{\mathrm{T}}^{-n}$. In this case the hadron spectrum also has a power law shape because

$$
\begin{align*}
\frac{1}{p_{\mathrm{T}}} \frac{d \sigma_{\pi}}{d p_{\mathrm{T}}} & \approx \int_{x_{T}}^{1} A D_{\pi}^{q}(z) \cdot\left(\frac{p_{\mathrm{T}}}{z}\right)^{-n} \frac{d z}{z^{2}} \\
& \approx \frac{A}{p_{\mathrm{T}}^{n}} \int_{x_{T}}^{1} D_{\pi}^{q}(z) \cdot z^{n-2} d z \tag{29}
\end{align*}
$$

and the last integral depends only weakly on $p_{\mathrm{T}}$ due to the small value of $x_{T}$. For small parton $\hat{p}_{\mathrm{T}}$ (below $3-4 \mathrm{GeV} / c$ ) the power law shape is no longer valid, but the region $p_{\mathrm{T}}<$ $3 \mathrm{GeV} / c$ is outside the scope of this paper. The $f_{q}\left(\hat{p}_{\mathrm{T}}\right)$ should also diminish for very high $\hat{p}_{T} \rightarrow \sqrt{s} / 2$ where the phase space available for hard parton production diminishes, again not relevant for the present purposes.

We used the power law parameterization for the final state scattered-parton invariant spectrum $f_{q}\left(\hat{p}_{T}\right) \propto \hat{p}_{\mathrm{T}}^{-n}$ where $n$ is a free parameter which can be determined from the fit of Eq. (27) to the measured $\pi^{0}$ cross section. There is, however, one more missing piece of informa-tion-the shape of the fragmentation function $D_{q}^{\pi}$. In an attempt to extract this information from the data, we have analyzed associated $x_{\mathrm{E}}$-distributions, as shown in Table VII and Figs. 18-22.

## A. "Scaling" variable $\boldsymbol{x}_{\mathrm{E}}$

It was expected [2] that the $x_{\mathrm{E}}$ variable, defined by Eq. (19), to first order, approximates the fragmentation function in the limit of high values of $p_{\mathrm{Tt}}$, where there is sufficient collinearity between the trigger particle and the fragmenting parton. In this case where $j_{\mathrm{T}} \ll p_{\mathrm{Tt}}$ and $k_{\mathrm{T}} \ll$ $p_{\mathrm{Tt}}$ one can assume that $p_{\mathrm{Tt}}=\hat{p}_{\mathrm{Tt}} / z_{\mathrm{t}}$ and $x_{\mathrm{E}} z_{\mathrm{t}}=$ $\hat{x}_{\mathrm{h}} p_{\mathrm{Ta}} \cos \Delta \phi / \hat{p}_{\mathrm{Ta}} \simeq \hat{x}_{\mathrm{h}} z_{\mathrm{a}}$, and thus the slopes of $D\left(z_{\mathrm{a}}\right)$ and $x_{\mathrm{E}}$ are related as

$$
\begin{equation*}
\left\langle z_{\mathrm{a}}\right\rangle \approx\left\langle x_{\mathrm{E}}\right\rangle\left\langle z_{\mathrm{t}}\right\rangle \hat{x}_{\mathrm{h}}^{-1} \tag{30}
\end{equation*}
$$

The $x_{\mathrm{E}}$ distributions of particles associated with trigger particles in the $3-8 \mathrm{GeV} / c$ range of transverse momentum are plotted in Fig. 18. The dashed lines represent exponential fits. The slopes of these exponentials range from $-5.8\left(3<p_{\mathrm{Tt}}<4 \mathrm{GeV} / c\right)$ to -7.8 (open symbols on Fig. 19). This is qualitatively and quantitatively different from the similar measurement done by CCOR Collaboration at $\sqrt{s}=62.4 \mathrm{GeV}$ where the slopes of exponential fits to the $x_{\mathrm{E}}$ distributions were found to be $\approx$ -5.3 and independent of the trigger transverse momenta. That observation also supported the hypothesis of the $x_{\mathrm{E}}$ distribution being a good approximation of the fragmentation function. We also note that the $x_{\mathrm{E}}$ distributions are not quite exponential and at large values of $x_{\mathrm{E}}$ there is a tail similar to the power law tail of the single inclusive $p_{\mathrm{T}}$ distribution.

TABLE VII. Measured $x_{\mathrm{E}}$ distributions associated with various transverse momenta of the trigger $\pi^{0}$. Only the statistical errors are shown. See also Figs. 18-22.

| $\begin{aligned} & p_{\mathrm{Tt}}= \\ & x_{\mathrm{E}} \end{aligned}$ | $\begin{aligned} & 3.39 \mathrm{GeV} / c \\ & d n / d x_{\mathrm{E}} \end{aligned}$ | $x_{\mathrm{E}}$ | $\begin{gathered} p_{\mathrm{Tt}}=4.40 \mathrm{GeV} / c \\ d n / d x_{\mathrm{E}} \end{gathered}$ | $x_{\mathrm{E}}$ | $\begin{gathered} p_{\mathrm{Tt}}=5.41 \mathrm{GeV} / c \\ d n / d x_{\mathrm{E}} \end{gathered}$ | $x_{\mathrm{E}}$ | $\begin{gathered} p_{\mathrm{Tt}}=6.40 \mathrm{GeV} / c \\ d n / d x_{\mathrm{E}} \end{gathered}$ | $x_{\mathrm{E}}$ | $\begin{gathered} p_{\mathrm{Tt}}=7.39 \mathrm{GeV} / c \\ d n / d x_{\mathrm{E}} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.32 | $2.7 \pm 4.7 \times 10^{-2}$ | 0.23 | $2.2 \pm 7.5 \times 10^{-2}$ | 0.22 | $2.3 \pm 1.2 \times 10^{-1}$ | 0.18 | $2.7 \pm 2.1 \times 10^{-1}$ | 0.17 | $1.8 \pm 3.1 \times 10^{-1}$ |
| 0.37 | $1.9 \pm 4.0 \times 10^{-2}$ | 0.27 | $2.3 \pm 7.5 \times 10^{-2}$ | 0.27 | $1.4 \pm 9.6 \times 10^{-2}$ | 0.22 | $1.5 \pm 1.6 \times 10^{-1}$ | 0.24 | $9.0 \times 10^{-1} \pm 1.5 \times 10^{-1}$ |
| 0.42 | $1.4 \pm 3.3 \times 10^{-2}$ | 0.32 | $1.6 \pm 6.2 \times 10^{-2}$ | 0.32 | $9.4 \times 10^{-1} \pm 7.7 \times 10^{-2}$ | 0.27 | $1.0 \pm 1.3 \times 10^{-1}$ | 0.33 | $4.4 \times 10^{-1} \pm 1.0 \times 10^{-1}$ |
| 0.47 | $9.6 \times 10^{-1} \pm 2.8 \times 10^{-2}$ | 0.37 | $9.5 \times 10^{-1} \pm 4.8 \times 10^{-2}$ | 0.37 | $5.7 \times 10^{-1} \pm 6.0 \times 10^{-2}$ | 0.35 | $5.5 \times 10^{-1} \pm 6.6 \times 10^{-2}$ | 0.45 | $2.8 \times 10^{-1} \pm 8.1 \times 10^{-2}$ |
| 0.52 | $7.3 \times 10^{-1} \pm 2.4 \times 10^{-2}$ | 0.42 | $7.2 \times 10^{-1} \pm 4.1 \times 10^{-2}$ | 0.43 | $4.1 \times 10^{-1} \pm 5.0 \times 10^{-2}$ | 0.44 | $2.7 \times 10^{-1} \pm 4.6 \times 10^{-2}$ | 0.55 | $6.9 \times 10^{-2} \pm 4.0 \times 10^{-2}$ |
| 0.57 | $5.2 \times 10^{-1} \pm 2.0 \times 10^{-2}$ | 0.47 | $4.9 \times 10^{-1} \pm 3.4 \times 10^{-2}$ | 0.47 | $2.8 \times 10^{-1} \pm 4.2 \times 10^{-2}$ | 0.54 | $1.3 \times 10^{-1} \pm 3.1 \times 10^{-2}$ | 0.64 | $4.5 \times 10^{-2} \pm 3.2 \times 10^{-2}$ |
| 0.62 | $3.8 \times 10^{-1} \pm 1.7 \times 10^{-2}$ | 0.52 | $2.7 \times 10^{-1} \pm 2.5 \times 10^{-2}$ | 0.52 | $2.3 \times 10^{-1} \pm 3.8 \times 10^{-2}$ | 0.64 | $8.1 \times 10^{-2} \pm 2.4 \times 10^{-2}$ |  |  |
| 0.67 | $3.0 \times 10^{-1} \pm 1.5 \times 10^{-2}$ | 0.57 | $2.9 \times 10^{-1} \pm 2.6 \times 10^{-2}$ | 0.57 | $1.9 \times 10^{-1} \pm 3.4 \times 10^{-2}$ | 0.81 | $3.1 \times 10^{-2} \pm 8.6 \times 10^{-3}$ |  |  |
| 0.75 | $2.1 \times 10^{-1} \pm 9.0 \times 10^{-3}$ | 0.62 | $1.9 \times 10^{-1} \pm 2.1 \times 10^{-2}$ | 0.63 | $1.1 \times 10^{-1} \pm 2.5 \times 10^{-2}$ |  |  |  |  |
| 0.85 | $1.1 \times 10^{-1} \pm 6.5 \times 10^{-3}$ | 0.68 | $1.6 \times 10^{-1} \pm 1.9 \times 10^{-2}$ | 0.67 | $1.1 \times 10^{-1} \pm 2.5 \times 10^{-2}$ |  |  |  |  |
| 0.95 | $8.2 \times 10^{-2} \pm 5.5 \times 10^{-3}$ | 0.75 | $1.1 \times 10^{-1} \pm 1.1 \times 10^{-2}$ | 0.76 | $5.6 \times 10^{-2} \pm 1.3 \times 10^{-2}$ |  |  |  |  |
| 1.04 | $5.4 \times 10^{-2} \pm 4.5 \times 10^{-3}$ | 0.85 | $6.5 \times 10^{-2} \pm 8.4 \times 10^{-3}$ | 0.85 | $2.9 \times 10^{-2} \pm 9.2 \times 10^{-3}$ |  |  |  |  |
| 1.15 | $3.6 \times 10^{-2} \pm 3.6 \times 10^{-3}$ | 0.94 | $5.2 \times 10^{-2} \pm 7.5 \times 10^{-3}$ | 0.97 | $2.3 \times 10^{-2} \pm 8.1 \times 10^{-3}$ |  |  |  |  |
| 1.25 | $2.8 \times 10^{-2} \pm 3.2 \times 10^{-3}$ | 1.04 | $2.3 \times 10^{-2} \pm 5.0 \times 10^{-3}$ | 1.07 | $8.3 \times 10^{-3} \pm 4.8 \times 10^{-3}$ |  |  |  |  |



FIG. 18. The distribution of associated particles with $x_{\mathrm{E}}$ variable for various trigger particle $p_{\mathrm{Tt}}$ indicated in the legend. Exponential fits indicated by dashed lines.

The reason why the $x_{\mathrm{E}}$ distributions do not have the same slope for different $p_{\mathrm{Tt}}$ and why there is a "power law" tail at large $x_{\mathrm{E}}$ is the same as that which causes $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ to decrease with the associated particle transverse momentum. It turns out that by sampling different regions of $p_{\mathrm{Ta}}$ for fixed $p_{\mathrm{Tt}}$, the average momentum of the parton fragmenting into a trigger particle, $\left\langle z_{\mathrm{t}}\right\rangle$, also changes. This kind of trigger bias causes the hard-


FIG. 19. The negative slope parameters extracted from the fit of a plain exponential function into a $x_{\mathrm{E}}$ (see Fig. 18) and $p_{\mathrm{Ta}} / p_{\mathrm{Tt}}$ (see Fig. 20) distributions.


FIG. 20. The distribution of associated particles with $p_{\mathrm{Ta}} / p_{\mathrm{Tt}}$ variable for various trigger particle $p_{\mathrm{Tt}}$ indicated in the legend. The distribution were fitted in the limited range $0.2<p_{\mathrm{Ta}} / p_{\mathrm{Tt}}<$ 0.4 by an exponential function (dashed lines).
scattering kinematics, the value of $\hat{p}_{T}$, to not be fixed for the case where $p_{\mathrm{Tt}}$ is fixed but $p_{\mathrm{Ta}}$ varies.

Taking this into account, one can not treat the associated $x_{\mathrm{E}}$ distribution as a rescaled fragmentation function, but rather as a folding of the two fragmentation processes of trigger and associated jets. The same line of arguments applies also for other two-particle variables, e.g. $p_{\mathrm{Ta}} / p_{\mathrm{Tt}}$, [51] used for an approximation of the fragmentation variable $z$ (see Fig. 20). The negative slopes of an exponential fit in the $0.2<p_{\mathrm{Ta}} / p_{\mathrm{Tt}}<0.4$ range (solid symbols on Fig. 19) are, within the error bars, the same as for $x_{\mathrm{E}}$.

In conclusion: the slope parameters extracted from associated $x_{\mathrm{E}}$ distributions reveal the rising trend with $p_{\mathrm{Tt}}$ which reflects the fact, that the different $p_{\mathrm{Ta}}$ samples not only different $z_{\mathrm{a}}$ but also different $z_{\mathrm{t}}$.

The description of an associated distribution detected under the condition of the existence of a trigger particle requires an extension of the formulae discussed in Sec. V and is a subject of the next section.

## VI. DIJET FRAGMENTATION

For the description of the detection of a single particle which is the result of jet fragmentation, recall Eq. (25)

$$
\begin{align*}
\frac{d^{2} \sigma_{\pi}}{d \hat{p}_{\mathrm{T}} d z_{\mathrm{t}}} & =\frac{d \sigma_{q}}{d \hat{p}_{\mathrm{T}}} \times D_{\pi}^{q}\left(z_{\mathrm{t}}\right)=\hat{p}_{\mathrm{T}} f_{q}\left(\hat{p}_{\mathrm{T}}\right) \times D_{\pi}^{q}\left(z_{\mathrm{t}}\right) \\
& \equiv \Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right) \times D_{\pi}^{q}\left(z_{\mathrm{t}}\right) \tag{31}
\end{align*}
$$

where we have now explicitly labeled the $z$ of the trigger particle as $z_{\mathrm{t}}$, and defined

$$
\begin{equation*}
\Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right) \equiv \hat{p}_{\mathrm{T}} f_{q}\left(\hat{p}_{\mathrm{T}}\right)=\frac{d \sigma_{q}}{d \hat{p}_{\mathrm{T}}} \tag{32}
\end{equation*}
$$

When $k_{\mathrm{T}}$ smearing is introduced, configurations for which the high $p_{\mathrm{T}}$ parton pair is on the average moving towards the trigger particle are favored due to the steeply falling $\hat{p}_{T}$ spectrum, such that

$$
\left\langle\hat{p}_{\mathrm{Tt}}-\hat{p}_{\mathrm{T}}\right\rangle \simeq \frac{1}{2}\left\langle\hat{p}_{\mathrm{Tt}}-\hat{p}_{\mathrm{Ta}}\right\rangle \equiv s\left(k_{\mathrm{T}}\right)
$$

with small variance $\sigma_{s}^{2}$, and we explicitly introduced $\hat{p}_{\mathrm{Tt}}$ and $\hat{p}_{T a}$ to represent the transverse momenta of the trigger and away partons. The single inclusive $p_{\mathrm{Tt}}$ spectrum is now given by

$$
\begin{equation*}
\frac{d^{2} \sigma_{\pi}}{d \hat{p}_{T \mathrm{t}} d z_{\mathrm{t}}}=\Sigma_{q}^{\prime}\left(\hat{p}_{\mathrm{Tt}}\right) \times D_{\pi}^{q}\left(z_{\mathrm{t}}\right)=\frac{z_{t} d^{2} \sigma_{\pi}}{d p_{\mathrm{T}_{\mathrm{t}}} d z_{t}}, \tag{33}
\end{equation*}
$$

where the trigger parton $\hat{p}_{\mathrm{Tt}}$ spectrum after $k_{\mathrm{T}}$ smearing is

$$
\begin{equation*}
\Sigma_{q}^{\prime}\left(\hat{p}_{\mathrm{Tt}}\right) \equiv \hat{p}_{\mathrm{Tt}} f_{q}^{\prime}\left(\hat{p}_{\mathrm{Tt}}\right)=\frac{d \sigma_{q}}{d \hat{p}_{\mathrm{Tt}}} \tag{34}
\end{equation*}
$$

Then, the conditional probability for finding the away-side parton with $\hat{p}_{\mathrm{Ta}}$ and $z_{\mathrm{a}}$, given $\hat{p}_{\mathrm{Tt}}\left(\right.$ and $\left.z_{\mathrm{t}}\right)$, is

$$
\left.\frac{d P\left(\hat{p}_{\mathrm{Ta}}, z_{\mathrm{a}}\right)}{d \hat{p}_{\mathrm{Ta}} d z_{\mathrm{a}}}\right|_{\hat{p}_{\mathrm{Tt}}}=C\left(\hat{p}_{\mathrm{Ta}}, \hat{p}_{\mathrm{Tt}}, k_{T}\right) D_{\pi}^{q}\left(z_{\mathrm{a}}\right)
$$

where $C\left(\hat{p}_{\mathrm{Ta}}, \hat{p}_{\mathrm{Tt}}, k_{\mathrm{T}}\right)$ represents the distribution of the transverse momentum of the away parton $\hat{p}_{\mathrm{Ta}}$, given $\hat{p}_{\mathrm{Tt}}$ and $k_{\mathrm{T}}$, which can be written as

$$
\begin{align*}
C\left(\hat{p}_{\mathrm{Ta}}, \hat{p}_{\mathrm{Tt}}, k_{\mathrm{T}}\right)= & \frac{1}{\sqrt{2 \pi \sigma_{s}^{2}}} \\
& \times \exp \left(\frac{-\left[\hat{p}_{\mathrm{Ta}}-\left(\hat{p}_{\mathrm{Tt}}-2 s\left(k_{\mathrm{T}}\right)\right)\right]^{2}}{2 \sigma_{s}^{2}}\right) \tag{35}
\end{align*}
$$

Then

$$
\frac{d^{4} \sigma_{\pi}}{d \hat{p}_{\mathrm{Tt}} d z_{\mathrm{t}} d \hat{p}_{\mathrm{Ta}} d z_{\mathrm{a}}}=\frac{d^{2} \sigma_{\pi}}{d \hat{p}_{\mathrm{Tt}} d z_{\mathrm{t}}} \times\left.\frac{d P\left(\hat{p}_{\mathrm{Ta}}, z_{\mathrm{a}}\right)}{d \hat{p}_{\mathrm{Ta}} d z_{\mathrm{a}}}\right|_{\hat{p}_{\mathrm{Tt}}} .
$$

In general, $\sigma_{s} / s\left(k_{T}\right)$ is small (see Sec. VIB) so that $C\left(\hat{p}_{\mathrm{Ta}}, \hat{p}_{\mathrm{Tt}}, k_{T}\right)$ is well approximated by a $\delta$ function and we may take

$$
\hat{p}_{\mathrm{Ta}}=\hat{p}_{\mathrm{Tt}}-2 s\left(k_{\mathrm{T}}\right)=\hat{x}_{\mathrm{h}} \hat{p}_{\mathrm{Tt}},
$$

so that

$$
\frac{d^{3} \sigma_{\pi}}{d \hat{p}_{\mathrm{Tt}} d z_{\mathrm{t}} d z_{\mathrm{a}}}=\Sigma_{q}^{\prime}\left(\hat{p}_{\mathrm{Tt}}\right) D_{\pi}^{q}\left(z_{\mathrm{t}}\right) D_{\pi}^{q}\left(z_{\mathrm{a}}\right)
$$

where

$$
z_{\mathrm{a}}=\frac{p_{\mathrm{Ta}}}{\hat{p}_{\mathrm{Ta}}}=\frac{p_{\mathrm{Ta}}}{\hat{x}_{\mathrm{h}} \hat{p}_{\mathrm{Tt}}}=\frac{z_{\mathrm{t}} p_{\mathrm{Ta}}}{\hat{x}_{\mathrm{h}} p_{\mathrm{Tt}}} .
$$

Changing variables from $\hat{p}_{\mathrm{Tt}}, z_{\mathrm{t}}$ to $p_{\mathrm{Tt}}, z_{\mathrm{t}}$ as above, and similarly from $z_{\mathrm{a}}$ to $p_{\mathrm{Ta}}$, we obtain

$$
\begin{equation*}
\frac{d^{3} \sigma_{\pi}}{d p_{\mathrm{Tt}} d z_{\mathrm{t}} d p_{\mathrm{Ta}}}=\frac{1}{\hat{x}_{\mathrm{h}} p_{\mathrm{Tt}}} \Sigma_{q}^{\prime}\left(\frac{p_{\mathrm{Tt}}}{z_{\mathrm{t}}}\right) D_{\pi}^{q}\left(z_{\mathrm{t}}\right) D_{\pi}^{q}\left(\frac{z_{\mathrm{t}} p_{\mathrm{Ta}}}{\hat{x}_{\mathrm{h}} p_{\mathrm{Tt}}}\right), \tag{36}
\end{equation*}
$$

where for integrating over $z_{\mathrm{t}}$ or finding $\left\langle z_{\mathrm{t}}\right\rangle$ for fixed $p_{\mathrm{Tt}}$, $p_{\mathrm{Ta}}$, the minimum value of $z_{\mathrm{t}}$ is $z_{\mathrm{t}}^{\mathrm{min}}=2 p_{\mathrm{Tt}} / \sqrt{s}=x_{\mathrm{Tt}}$ and the maximum value is

$$
z_{\mathrm{t}}^{\max }=\hat{x}_{\mathrm{h}} \frac{p_{\mathrm{Tt}}}{p_{\mathrm{Ta}}}=\frac{\hat{x}_{\mathrm{h}}}{x_{\mathrm{h}}}
$$

where $\hat{x}_{\mathrm{h}}\left(p_{\mathrm{Tt}}, p_{\mathrm{Ta}}\right)$ is also a function of $k_{\mathrm{T}}$ [Eq. (20)].
Thus, in order to evaluate $\hat{x}_{\mathrm{h}}\left(p_{\mathrm{Tt}}, p_{\mathrm{Ta}}\right)$ for use in Eq. (36), $k_{\mathrm{T}}$ must be known. We attack this problem by successive approximations. First we solve for $k_{\mathrm{T}}$ and $D_{\pi}^{q}(z)$ assuming $\hat{x}_{h}=1$ as done at the ISR where the smearing correction was small. Then we solve for $\hat{x}_{\mathrm{h}}\left(p_{\mathrm{Tt}}, p_{\mathrm{Ta}}\right)$ with this value of $k_{T}$ and iterate. On the first solution we solve only for $\Sigma_{q}^{\prime}\left(\hat{p}_{\mathrm{Tt}}\right)$ while on the iteration we include the $k_{\mathrm{T}}$ smearing to solve for the unsmeared parton spectrum $\Sigma_{q\left(\hat{p}_{T}\right)}=$ $\hat{p}_{\mathrm{T}} f_{q}\left(\hat{p}_{\mathrm{T}}\right)$ [Eq. (32)].

## A. Sensitivity of the associated spectra to the fragmentation function

As discussed in Sec. VA, the associated $x_{\mathrm{E}}$ distribution was thought to approximate the fragmentation function of the away jet. Equation (36) can be transformed to the $x_{\mathrm{E}}$ distribution at fixed $p_{\mathrm{Tt}}$ with a change of variables from $p_{\mathrm{Ta}}$ to $x_{\mathrm{E}}$ followed by integration over $z_{\mathrm{t}}$ :

$$
\begin{align*}
\frac{d^{2} \sigma}{d p_{\mathrm{Tt}} d x_{\mathrm{E}}} & =\frac{d p_{\mathrm{Ta}}}{d x_{\mathrm{E}}} \times \frac{d^{2} \sigma}{d p_{\mathrm{Tt}} d p_{\mathrm{Ta}}} \\
& \simeq \frac{1}{\hat{x}_{\mathrm{h}}} \int_{x_{\mathrm{Tt}}}^{\hat{x}_{\mathrm{h}}\left(p_{\mathrm{Tt}} / p_{\mathrm{Ta}}\right)} D_{\pi}^{q}\left(z_{\mathrm{t}}\right) D_{\pi}^{q}\left(\frac{z_{\mathrm{t}} p_{\mathrm{Ta}}}{\hat{x}_{\mathrm{h}} p_{\mathrm{Tt}}}\right) \Sigma_{q}^{\prime}\left(\frac{p_{\mathrm{Tt}}}{z_{\mathrm{t}}}\right) d z_{\mathrm{t}} . \tag{37}
\end{align*}
$$

We at first attempted to solve for the fragmentation function by simultaneous fits of the measured $x_{\mathrm{E}}$ distributions to Eq. (37) constrained by a fit of the inclusive invariant $\pi^{0}$ cross section to Eq. (27). There were difficulties with convergence.

The reason for the lack of convergence became apparent when we calculated $x_{\mathrm{E}}$ distributions according to Eq. (37) (Fig. 21) for two different fragmentation functions corresponding to quark and gluon jet fragmentation. A simple exponential parameterization was used and the slopes were obtained from the fit to the LEP data [52,53] (Fig. 22). For quark and gluon jets, we found $D_{q}(z) \approx \exp (-8.2 \cdot z)$ and $D_{g}(z) \approx \exp (-11.4 \cdot z)$ respectively. For the parton final state spectrum, we used $\Sigma_{q}^{\prime} \propto \hat{p}_{\mathrm{T}}^{-8}$. It is evident that the $x_{\mathrm{E}}$ distributions calculated for the quite different quark and gluon fragmentation functions do not differ significantly (the difference between solid and dashed lines on Fig. 21). Clearly, the $x_{\mathrm{E}}$ distributions are rather insensitive to the fragmentation functions of the away jet in contradiction to the previous conventional wisdom. The evidence of this explicit counter example led to attempts to perform the


FIG. 21 (color online). The same $x_{\mathrm{E}}$ distributions as on Fig. 18 shown with calculations according to Eq. (37) for quark (solid lines) and for gluon (dashed lines) $D(z)$. An exponential approximation was used and the slopes for quark and gluon fragmentation function were obtained by fitting to LEP data [52,53] (see Fig. 22).
integrals of Eq. (33) and (36) analytically which straightforwardly confirmed the observation that the $x_{\mathrm{E}}$ distribution is not sensitive to the fragmentation function.

If the smeared trigger parton spectrum is taken as a power law,

$$
\Sigma_{q}^{\prime}\left(\frac{p_{\mathrm{Tt}}}{z_{\mathrm{t}}}\right)=A\left(\frac{p_{\mathrm{Tt}}}{z_{\mathrm{t}}}\right)^{-(n-1)}
$$



FIG. 22. Fragmentation function measured in $e^{+} e^{-}$collisions at $\sqrt{s}=180 \mathrm{GeV}$ for gluon and quark jets. The solid and dashed lines represent the exponential fit in the $0.2<z<1$ region.
and the fragmentation function as an exponential, $D(z)=$ $B \exp (-b z)$, then the integral of Eq. (36) over $z_{\mathrm{t}}$ becomes

$$
\begin{align*}
\frac{d \sigma_{\pi}}{d p_{\mathrm{Tt}} d p_{\mathrm{Ta}}}= & \frac{B^{2}}{\hat{x}_{\mathrm{h}}} \frac{A}{p_{\mathrm{T}_{\mathrm{t}}}^{n}} \int_{x_{\mathrm{T}_{\mathrm{t}}}}^{\hat{x}_{\mathrm{h}}\left(p_{\mathrm{Tt}} / p_{\mathrm{Ta}}\right)} d z_{\mathrm{t}} z_{\mathrm{t}}^{n-1} \\
& \times \exp \left[-b z_{\mathrm{t}}\left(1+\frac{p_{\mathrm{Ta}}}{\hat{x}_{\mathrm{h}} p_{\mathrm{Tt}}}\right)\right], \tag{38}
\end{align*}
$$

which is an incomplete gamma function. Since $\hat{x}_{h} \sim 1$, we make the assumption that it is constant. Similarly, the integrals of Eqs. (29) and (33) are also incomplete gamma functions:

$$
\begin{equation*}
\frac{d \sigma_{\pi}}{d p_{\mathrm{Tt}}}=\frac{A B}{p_{\mathrm{T}_{\mathrm{t}}}^{n-1}} \int_{x_{\mathrm{T}_{\mathrm{t}}}}^{1} d z_{\mathrm{t}} z_{\mathrm{t}}^{n-2} \exp \left[-b z_{\mathrm{t}}\right] . \tag{39}
\end{equation*}
$$

A reasonable approximation for the inclusive single, and two-particle cross sections is obtained by taking the lower limit to zero and the upper limit to infinity, leading to the replacement of the incomplete gamma functions by gamma functions, with the result that

$$
\begin{align*}
\frac{d^{2} \sigma_{\pi}}{d p_{\mathrm{Tt}} d p_{\mathrm{Ta}}} & \approx \frac{\Gamma(n)}{b^{n}} \frac{B^{2}}{\hat{x}_{\mathrm{h}}} \frac{A}{p_{\mathrm{T}_{\mathrm{t}}}^{n}} \frac{1}{\left(1+\frac{p_{\mathrm{Ta}}}{\hat{x}_{\mathrm{h}} p_{\mathrm{Tt}}}\right)^{n}}  \tag{40}\\
\frac{d \sigma_{\pi}}{d p_{\mathrm{Tt}}} & \approx \frac{\Gamma(n-1)}{b^{n-1}} \frac{A B}{p_{\mathrm{T}_{\mathrm{t}}}^{n-1}} \tag{41}
\end{align*}
$$

where $\Gamma(n)=(n-1) \Gamma(n-1)$.
The conditional probability is just the ratio of the joint probability Eq. (40) to the inclusive probability Eq. (41), or

$$
\begin{equation*}
\left.\frac{d P_{\pi}}{d p_{\mathrm{Ta}}}\right|_{p_{\mathrm{Tt}}} \approx \frac{B(n-1)}{b p_{\mathrm{Tt}}} \frac{1}{\hat{x}_{\mathrm{h}}} \frac{1}{\left(1+\frac{p_{\mathrm{T}_{\mathrm{a}}}}{\hat{x}_{\mathrm{h}} p_{\mathrm{Tt}}}\right)^{n}} \tag{42}
\end{equation*}
$$

In the collinear limit, where $p_{\mathrm{Ta}}=x_{\mathrm{E}} p_{\mathrm{Tt}}$ :

$$
\begin{equation*}
\left.\frac{d P_{\pi}}{d x_{\mathrm{E}}}\right|_{p_{\mathrm{Tt}}} \approx \frac{B(n-1)}{b} \frac{1}{\hat{x}_{\mathrm{h}}} \frac{1}{\left(1+\frac{x_{\mathrm{E}}}{\hat{x}_{\mathrm{h}}}\right)^{n}} \tag{43}
\end{equation*}
$$

The only dependence on the fragmentation function, in this approximation, is in the normalization constant $B / b$ which equals $\langle m\rangle$, the multiplicity in the away-jet from the integral of the fragmentation function. The dominant term in Eq. (43) is the Hagedorn function $1 /\left(1+x_{\mathrm{E}} / \hat{x}_{\mathrm{h}}\right)^{n}$, so that at fixed $p_{\mathrm{Tt}}$ the $x_{\mathrm{E}}$ distribution is predominantly a function only of $x_{\mathrm{E}}$ and thus does exhibit " $x_{\mathrm{E}}$ " scaling. Also, the Hagedorn function explains the "power law" tail at large $x_{\mathrm{E}}$ noted in Sec. VA. The reason that the $x_{\mathrm{E}}$ distribution is not very sensitive to the fragmentation function is that the integral over $z_{\mathrm{t}}$ for fixed $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$ [Eq. (38)] is actually an integral over the jet transverse momentum $\hat{p}_{T_{t}}$. However since both the trigger and away jets are always roughly equal and opposite in transverse momentum, integrating over $\hat{p}_{T_{t}}$ simultaneously integrates over $\hat{p}_{T_{a}}$, and thus also integrates over the away jetfragmentation function. This can be seen directly by the
presence of $z_{\mathrm{t}}$ in both the same and away fragmentation functions in Eqs. (36) and (37), so that the integral over $z_{\mathrm{t}}$ integrates over both fragmentation functions simultaneously.

## B. $\boldsymbol{k}_{\mathrm{T}}$ smearing

In order to evaluate $\hat{x}_{\mathrm{h}}\left(p_{\mathrm{Tt}}, p_{\mathrm{Ta}}\right)$ and $\left\langle z_{\mathrm{t}}\right\rangle, k_{\mathrm{T}}$ must be known. We attack this problem by successive approximations: first we solve for $k_{\mathrm{T}}$ assuming $\hat{x}_{\mathrm{h}}=1$ as done at the ISR, where the smearing correction was small. Then we iterate for finite $k_{\mathrm{T}}$. The Gaussian approximation for the smearing function Eq. (35) does not work so well in the low $\hat{p}_{\mathrm{T}}$ region. The product of the steeply falling parton distribution function and the fragmentation function is peaked at $z \approx 1$ preferring "small" parton momenta. We have developed more accurate description of the conditional yields taking into account the $k_{\mathrm{T}}$ smearing.

Let us consider the configuration depicted on Fig. 23. The two back-to-back partons in $\hat{s}$ frame undergo the Lorentz boost determined by net pair momentum

$$
\begin{equation*}
\vec{p}_{\mathrm{n}} \equiv \vec{p}_{\mathrm{Tpair}} \equiv \overrightarrow{\hat{p}}_{\mathrm{Tt}}+\overrightarrow{\hat{p}}_{\mathrm{Ta}}=\vec{k}_{\mathrm{Tt}}+\vec{k}_{\mathrm{Ta}} \tag{44}
\end{equation*}
$$

If we denote an angle between the unsmeared parton momentum and $k_{\mathrm{T}}$-vector (or $\vec{p}_{\mathrm{n}}$ ) as $\alpha$ (see Fig. 23) then we can write the conditional probability distribution of trigger parton momenta, $\hat{p}_{\mathrm{Tt}}$, as

$$
\begin{align*}
\left.\frac{d^{3} \sigma}{d \hat{p}_{\mathrm{Tt}} d \alpha d \hat{p}_{\mathrm{T}}}\right|_{p_{\mathrm{Tv}}, p_{\mathrm{Ta}}}= & \hat{p}_{\mathrm{Tt}} \cdot \Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right) \cdot \hat{p}_{\mathrm{n}} \cdot G\left(\hat{p}_{\mathrm{n}}\left(\vec{r}_{t}\right)\right) \\
& \cdot D_{\pi}^{q}\left(\frac{p_{\mathrm{Tt}}}{\hat{p}_{\mathrm{Tt}}}\right) \frac{p_{\mathrm{Tt}}}{\hat{p}_{\mathrm{Tt}}^{2}} \cdot D_{\pi}^{q}\left(\frac{p_{\mathrm{Ta}}}{\hat{p}_{\mathrm{Ta}}\left(\vec{r}_{t}\right)}\right) \\
& \times \frac{p_{\mathrm{Ta}}}{\hat{p}_{\mathrm{Ta}}^{2}\left(\vec{r}_{t}\right)}, \tag{45}
\end{align*}
$$

where $G\left(\hat{p}_{\mathrm{n}}\right)=\exp \left(-\hat{p}_{\mathrm{n}}^{2} / 2\left\langle k_{\mathrm{T}}^{2}\right\rangle\right)$ describes the Gaussian probability distribution of the net pair momentum magnitude distribution, $\Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right)$ is the unsmeared parton momentum distribution, $D_{\pi}^{q}$ is the fragmentation function and $\vec{r}_{t}=\left(\hat{p}_{\mathrm{Tt}}, \phi, \hat{p}_{\mathrm{T}}, k_{\mathrm{T}}\right)$ is the phase space vector. The $\hat{p}_{\mathrm{Tt}}$ is


FIG. 23 (color online). Back-to-back partons in hardscattering rest frame (back-to-back dashed arrows) with fourmomenta ( $\hat{p}_{\mathrm{T}}, 0,0, \hat{p}_{\mathrm{T}}$ ) and ( $-\hat{p}_{\mathrm{T}}, 0,0, \hat{p}_{\mathrm{T}}$ ) in (,,,---+ ) metrics moving along $\hat{p}_{\mathrm{n}}\left(\hat{p}_{\mathrm{n}}=\hat{p}_{\text {Tpair }}\right)$ for an event where detection of $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$ is required (the $j_{\mathrm{T}}$ contribution is neglected). The $p_{\mathrm{Tt}}>p_{\mathrm{Ta}}$ condition implies that the events with $\hat{p}_{\mathrm{n}}$ pointing more in the direction of $p_{\mathrm{Tt}}$ are selected.
chosen to be an integration variable and $\hat{p}_{\mathrm{Ta}}$ is fully determined by given values of $\hat{p}_{\mathrm{Tt}}, \hat{p}_{\mathrm{T}}$, angle $\phi$ and by the requirement of Lorentz invariance.

In order to evaluate $\left.\left\langle z_{\mathrm{t}}\left(k_{\mathrm{T}}\right)\right\rangle\right|_{p_{\mathrm{T}}, p_{\mathrm{Ta}}}$ and $\left.\hat{x}_{\mathrm{h}}\left(k_{\mathrm{T}}\right)\right|_{p_{\mathrm{T}}, p_{\mathrm{Ta}}}$ we have to evaluate first the parton distribution for events where given $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$ are detected. This conditional cross section can be expressed as a definite integral over the unobserved variables $\phi$ and $\hat{p}_{\mathrm{T}}$ (see Fig. 23)

$$
\begin{align*}
\left.\frac{d \sigma}{d \hat{p}_{\mathrm{Tt}}}\right|_{p_{\mathrm{T},}, p_{\mathrm{Ta}}}= & \left.2 \int_{0}^{\sqrt{s} / 2} \int_{0}^{\pi} \frac{d^{3} \sigma}{d \hat{p}_{\mathrm{Tt}} d \hat{p}_{\mathrm{T}} d \phi}\right|_{p_{\mathrm{T}}, p_{\mathrm{Ta}}} d \phi d \hat{p}_{\mathrm{T}} \\
= & D_{\pi}^{q}\left(\frac{p_{\mathrm{Tt}}}{\hat{p}_{\mathrm{Tt}}}\right) \frac{2}{\hat{p}_{\mathrm{Tt}}} \int_{0}^{\sqrt{s} / 2} \Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right) \\
& \times \int_{0}^{\pi} \hat{p}_{\mathrm{n}}\left(\vec{r}_{t}\right) G\left(\hat{p}_{\mathrm{n}}\left(\vec{r}_{t}\right)\right) \\
& \cdot D_{\pi}^{q}\left(\frac{p_{\mathrm{Ta}}}{\hat{p}_{\mathrm{Ta}}\left(\vec{r}_{t}\right)}\right) \frac{1}{\hat{p}_{\mathrm{Ta}}^{2}\left(\vec{r}_{t}\right)} d \phi d \hat{p}_{\mathrm{T}} \tag{46}
\end{align*}
$$

The $d \sigma /\left.d \hat{p}_{\text {Ta }}\right|_{p_{\text {Tı }}, p_{\text {Ta }}}$ distribution can be derived from Eq. (46) just by rotation $\hat{p}_{\mathrm{Tt}} \rightarrow \hat{p}_{\mathrm{Ta}}$ and $\hat{p}_{\mathrm{Ta}} \rightarrow \hat{p}_{\mathrm{Tt}}$. The $\left.\left\langle z_{\mathrm{t}}\left(k_{T}\right)\right\rangle\right|_{p_{\mathrm{T}}, p_{\mathrm{Ta}}}$ and $\left.\hat{x}_{\mathrm{h}}\left(k_{\mathrm{T}}\right)\right|_{p_{\mathrm{T}}, p_{\mathrm{Ta}}}$ quantities can then be evaluated as

$$
\begin{equation*}
\left.\left\langle z_{\mathrm{t}}\left(k_{\mathrm{T}}\right)\right\rangle\right|_{p_{\mathrm{T},}, p_{\mathrm{Ta}}}=\frac{Z(1)}{Z(0)}, \tag{47}
\end{equation*}
$$

where

$$
\begin{aligned}
Z(n)= & \int_{x_{\mathrm{Tt}}}^{1} z_{\mathrm{t}}^{n-1} D_{\pi}^{q}\left(z_{\mathrm{t}}\right) \int_{0}^{\sqrt{s} / 2} \Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right) \\
& \times \int_{0}^{\pi} \hat{p}_{\mathrm{n}} G\left(\hat{p}_{\mathrm{n}}\left(\vec{r}_{z t}\right)\right) \cdot D_{\pi}^{q}\left(\frac{p_{\mathrm{Ta}}}{\hat{p}_{\mathrm{Ta}}\left(\vec{r}_{z t} t\right)}\right) \\
& \times \frac{1}{\hat{p}_{\mathrm{Ta}}^{2}\left(\vec{r}_{z t}\right)} d \phi d \hat{p}_{\mathrm{T}} d z_{\mathrm{t}}
\end{aligned}
$$

and $\vec{r}_{z t}=\left(p_{\mathrm{Tt}} / z_{\mathrm{t}}, \phi, \hat{p}_{\mathrm{T}}, k_{\mathrm{T}}\right)$. The $\left.\hat{x}_{\mathrm{h}}\left(k_{\mathrm{T}}\right)\right|_{p_{\mathrm{T}}, p_{\mathrm{Ta}}}$ is evaluated as

$$
\begin{equation*}
\left.\hat{x}_{\mathrm{h}}\left(k_{\mathrm{T}}\right)\right|_{p_{\mathrm{T},}, p_{\mathrm{Ta}}}=\left.\frac{\left\langle\hat{p}_{\mathrm{Ta}}\right\rangle}{\left\langle\hat{p}_{\mathrm{Tt}}\right\rangle}\right|_{p_{\mathrm{T},}, p_{\mathrm{Ta}}}=\frac{\mathcal{X}_{a}(1)}{\mathcal{X}_{a}(0)} \frac{\mathcal{X}_{t}(0)}{\mathcal{X}_{t}(1)} \tag{48}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathcal{X}_{t}(n)= & \int_{p_{\mathrm{Tt}}}^{\sqrt{s} / 2} \hat{p}_{\mathrm{Tt}}^{n-1} D_{\pi}^{q}\left(\frac{p_{\mathrm{Tt}}}{\hat{p}_{\mathrm{Tt}}}\right) \int_{0}^{\sqrt{s} / 2} \Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right) \\
& \times \int_{0}^{\pi} \hat{p}_{\mathrm{n}}\left(\vec{r}_{t}\right) G\left(\hat{p}_{\mathrm{n}}\left(\vec{r}_{t}\right)\right) \cdot D_{\pi}^{q}\left(\frac{p_{\mathrm{Ta}}}{\hat{p}_{\mathrm{Ta}}\left(\vec{r}_{t}\right)}\right) \\
& \times \frac{1}{\hat{p}_{\mathrm{Ta}}^{2}\left(\vec{r}_{t}\right)} d \phi d \hat{p}_{\mathrm{T}} \hat{p}_{\mathrm{Tt}} \\
\mathcal{X}_{a}(n)= & \int_{p_{\mathrm{Ta}}}^{\sqrt{s} / 2} \hat{p}_{\mathrm{Ta}}^{n-1} D_{\pi}^{q}\left(\frac{p_{\mathrm{Ta}}}{\hat{p}_{\mathrm{Ta}}}\right) \int_{0}^{\sqrt{s} / 2} \Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right) \\
& \times \int_{0}^{\pi} \hat{p}_{\mathrm{n}}\left(\vec{r}_{a}\right) G\left(\hat{p}_{\mathrm{n}}\left(\vec{r}_{a}\right)\right) \cdot D_{\pi}^{q}\left(\frac{p_{\mathrm{Tt}}}{\hat{p}_{\mathrm{Tt}}\left(\vec{r}_{a}\right)}\right) \\
& \times \frac{1}{\hat{p}_{\mathrm{Tt}}^{2}\left(\vec{r}_{a}\right)} d \phi d \hat{p}_{\mathrm{T}} \hat{p}_{\mathrm{Ta}} .
\end{aligned}
$$

We have tested the above formulae on PYTHIA simulation. We have generated events with $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}=3 \mathrm{GeV} / c$ and evaluated the partons' momenta unbalance variation with $p_{\mathrm{Tt}}$ for fixed $3<p_{\mathrm{Ta}}<4 \mathrm{GeV} / c$ bin. The results from the PYTHIA simulation (solid points on Fig. 24) are compared to calculation based on Eq. (48) (solid line on Fig. 24). The magnitude of momentum unbalance saturates at $p_{\mathrm{Tt}} \approx 10 \mathrm{GeV} / c$ around $\sqrt{\left\langle k_{\mathrm{Tx}}^{2}\right\rangle}$ and then starts to decrease. The maximum value depends on the $k_{\mathrm{T}}$ magnitude and on the asymmetry between $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$. Eventually, the unbalance should vanish at high $p_{\mathrm{Tt}}$ as a consequence of $\Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right)$ flattening.

The comparison of $\left\langle z_{\mathrm{t}}\right\rangle$ and $\left\langle z_{\mathrm{a}}\right\rangle$ found in PYTHIA and derived according to Eq. (47) is shown in Fig. 25. The overall agreement between the PYTHIA simulations and the calculation is excellent. The small deviations may be attributed to the fact that in the PYTHIA simulation, $1 \mathrm{GeV} / c$-wide bins were used for trigger and associated particle identification, whereas the calculation was performed for fixed values of $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$.

The last missing piece of information needed before solving Eq. (22) is the fragmentation function $D_{\pi}^{q}$ and unsmeared $\Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right)$. The description of how this knowledge was extracted from the data is a subject of the next section.


FIG. 24. PYTHIA simulated average momentum unbalance for the associated particles in $3.0<p_{\mathrm{Ta}}<4.0 \mathrm{GeV} / c$ bin and calculated according Eq. (48). The two vertical dashed line indicates the range where $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$ bins are equal and the parton momenta unbalance vanishes (fixed correlations).


FIG. 25. Average $z$ of a trigger and associated particle as a function of $p_{\mathrm{Ta}}$ from PYTHIA and according Eq. (47).

## VII. CORRECTED $\left\langle k_{\boldsymbol{T}}\right\rangle$ RESULTS

The $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ extracted according to Eq. (22) for various $p_{\mathrm{Tt}}$ and $p_{\mathrm{Ta}}$ are shown in Fig. 16 and 17. In order to extract $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ values we have solved

$$
\begin{equation*}
x_{\mathrm{h}}^{-1} \sqrt{\left\langle p_{\mathrm{out}}^{2}\right\rangle-\left\langle j_{\mathrm{Ty}}^{2}\right\rangle\left(x_{\mathrm{h}}^{2}+1\right)}-\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}=0 \tag{49}
\end{equation*}
$$

for $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ where the $\left\langle z_{\mathrm{t}}\right\rangle$ and $\hat{x}_{\mathrm{h}}=\left\langle\hat{p}_{\mathrm{Ta}}\right\rangle /\left\langle\hat{p}_{\mathrm{Tt}}\right\rangle$ are evaluated according Eq. (47) and (48) respectively. These two quantities depend on $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ so we solved Eq. (49) iteratively by varying a $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ value and in every step the $\left\langle z_{\mathrm{t}}\right\rangle$ and $\hat{x}_{h}$ were recalculated. To do so we need to know the unsmeared final state parton spectrum $\Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right)$ and the fragmentation function. For the latter we used the LEP data (see Fig. 22) where the fragmentation functions of gluon and quark jets were measured in $e^{+} e^{-}$collision at $\sqrt{s}=180 \mathrm{GeV}$. We have chosen

$$
\begin{equation*}
D_{\pi}^{q} \propto z^{-\alpha}(1-z)^{\beta}(1+z)^{-\gamma} \tag{50}
\end{equation*}
$$

form used e.g. in [52] and extracted $\alpha, \beta$, and $\gamma$ parameters from the fit to distributions shown in Fig. 22 and Table VIII.

For a given set of parameters $\alpha, \beta$, and $\gamma$ the power of the unsmeared final state parton spectra $\Sigma_{q}\left(\hat{p}_{T}\right)$ was evaluated from the fit formula Eq. (27) to the single inclusive $\pi^{0}$ invariant cross section [43]. Here we used the simplified $k_{\mathrm{T}}$ smearing

$$
f_{q}^{\prime}\left(\hat{p}_{\mathrm{Tt}}\right)=\frac{1}{\hat{p}_{\mathrm{Tt}}} \Sigma_{q}^{\prime}\left(\hat{p}_{\mathrm{Tt}}\right)=\frac{1}{\hat{p}_{\mathrm{T}}} \Sigma_{q}\left(\hat{p}_{\mathrm{T}}\right) \otimes \exp \frac{-\left(\hat{p}_{\mathrm{T}}-\hat{p}_{\mathrm{Tt}}\right)^{2}}{\left\langle k_{\mathrm{Tx}}^{2}\right\rangle}
$$

and for the fixed value of $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}=\sqrt{2} \sqrt{\left\langle k_{\mathrm{Tx}}^{2}\right\rangle}=2.5 \mathrm{GeV} / c$ the power $n$ of $\Sigma q\left(\hat{p}_{T}\right)$ distribution was determined.

The measurement of the fragmentation functions at LEP was done separately for quark and gluon jets and the slopes of these two $D(z)$ distributions are different. Quark jets produce a significantly harder spectrum than gluon jets (see Fig. 22). Since the relative abundance of quark and gluon jets at $\sqrt{s}=200 \mathrm{GeV}$ is not known, for the final results we assumed that the numbers of quark and gluon jets are equal; the final $D(z)$ uses the averaged parameter values between quark and gluon and the difference was used as a measure of the systematic uncertainty.

Resulting $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ values for $3<p_{\mathrm{Tt}}<4 \mathrm{GeV} / c$ and $5<$ $p_{\mathrm{Tt}}<10 \mathrm{GeV} / c$ as a function of $p_{\mathrm{Ta}}$ are shown in Fig. 26 and Table IX. The solid and dashed lines bracket the systematic error due to the unknown ratio of quark and gluon jets. These data points correspond to the uncorrected $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ values shown in Fig. 16. The $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ values for varying $p_{\mathrm{Tt}}$ corresponding to the data of Fig. 17 are shown in Fig. 27 and Table X. The solid lines bracket the systematic error due to the unknown ratio of quark and gluon jets. It is evident that unfolded $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ values reveal, within the error bars, no dependence neither on $p_{\mathrm{Ta}}$ nor on $p_{\mathrm{Tt}}$.

We compared the $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ data obtained in this analysis to $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ values found by the CCOR Collaboration at $\sqrt{s}=$ 62.4 GeV [1] (empty triangles on Fig. 27). Although the trend with $p_{\mathrm{Tt}}$ seems to be similar the overall magnitude at $\sqrt{s}=200 \mathrm{GeV}$ is significantly higher.

The $\left\langle z_{\mathrm{t}}\right\rangle$ and $\hat{x}_{\mathrm{h}}$ values from the iterative solution of Eq. (49) as a function of the $\pi^{0}$ trigger momenta $p_{\mathrm{Tt}}$ and associated momenta $p_{\mathrm{Ta}}$ are shown in Fig. 28 and 29 and Tables XI and XII. There is an opposite trend; whereas the $\left\langle z_{\mathrm{t}}\right\rangle$ rises with $p_{\mathrm{Tt}}$ it is falling with $p_{\mathrm{Ta}}$. It is an interesting consequence of two effects: competition between steeply falling final state parton spectra and rising fragmentation

TABLE VIII. Extracted values of $D(z)$ parameters according Eq. (50) from the fit to the LEP data (see Fig. 22) and power $n$ of the unsmeared final state parton spectra $\Sigma_{q}\left(\hat{p}_{T}\right)$ extracted from the fit to the single inclusive $\pi^{0}$ invariant cross section [43] for corresponding fragmentation function and fixed value of $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}=2.5 \mathrm{GeV} / c$.

|  | Gluon | Quark | (gluon + quark) $/ 2$ |
| :---: | :---: | :---: | :---: |
| $\alpha$ | 0.16 | 0.49 | 0.32 |
| $\beta$ | 0.88 | 0.57 | 0.72 |
| $\gamma$ | 13.29 | 8.00 | 10.65 |
| $n$ | 7.53 | 7.28 | 7.40 |



FIG. 26. $\sqrt{\left\langle k_{T}^{2}\right\rangle}$ values corresponding to Fig. 16 as a solution to Eq. (49) for trigger $\pi^{0}$ in $3<p_{\mathrm{Tt}}<4 \mathrm{GeV} / c$ (solid symbols) and $5<p_{\mathrm{Tt}}<10 \mathrm{GeV} / c$ (open symbols) range. The solid and dashed lines bracket the systematic uncertainty due to the unknown ratio of quark and gluon jets, for the solid and open symbols, respectively.
function with parton momentum. Secondly, the detection of trigger particle biases the $\vec{k}_{\mathrm{T}}$ vector in the direction of the trigger jet as discussed in Sec. VIB.

The $p_{\mathrm{Tt}}$ averaged value of $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ (Fig. 27) is compared to the average parton-pair momentum, $\left\langle\hat{p}_{\mathrm{n}}\right\rangle=\left\langle p_{\mathrm{T}}\right\rangle$ pair, presented in [33] (see Fig. 30). The value of $\left\langle p_{\mathrm{T}}\right\rangle$ pair is determined as a sum of the two partons' $\left\langle k_{\mathrm{T}}\right\rangle$. In the present

TABLE IX. The $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ and $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ values as a function of $p_{\mathrm{Ta}}$ for two different trigger $\pi^{0}$ transverse momentum bins shown in Fig. 16 and Fig. 26. All units in rad and $\mathrm{GeV} / c$.

| $3<p_{\mathrm{Tt}}<4$ | $\frac{\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}}{\hat{x}_{\mathrm{h}}}$ |  |
| :--- | :---: | :---: |
| $p_{\mathrm{Ta}}$ | $1.76 \pm 0.12$ | $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ |
| 1.7 | $1.74 \pm 0.13$ | $2.66 \pm 0.19$ |
| 2.2 | $1.37 \pm 0.13$ | $2.94 \pm 0.22$ |
| 2.7 | $1.45 \pm 0.12$ | $2.57 \pm 0.23$ |
| 3.2 | $1.44 \pm 0.11$ | $2.93 \pm 0.23$ |
| 3.9 | $1.04 \pm 0.10$ | $3.19 \pm 0.23$ |
| 5.0 |  | $2.68 \pm 0.25$ |
| $5<p_{\mathrm{Tt}}<10$ | $\frac{\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}}{\hat{x}_{\mathrm{h}}}$ |  |
| $p_{\mathrm{Ta}}$ | $2.69 \pm 0.37$ | $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ |
| 1.9 | $2.54 \pm 0.31$ | $3.09 \pm 0.30$ |
| 2.2 | $2.13 \pm 0.26$ | $3.19 \pm 0.30$ |
| 2.7 | $1.89 \pm 0.27$ | $3.04 \pm 0.30$ |
| 3.4 | $1.41 \pm 0.30$ | $3.04 \pm 0.38$ |
| 4.7 |  | $2.64 \pm 0.56$ |

JET PROPERTIES FROM DIHADRON CORRELATIONS ...
TABLE X. Values of $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ and $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ for various trigger particle $p_{\mathrm{Tt}}$ and associated momenta in the $1.4<p_{\mathrm{Ta}}<$ $5.0 \mathrm{GeV} / c$ region shown in Fig. 17 and 27.

| $p_{\mathrm{Tt}}$ <br> $\mathrm{GeV} / c$ | $\hat{x}_{\mathrm{h}}^{-1}\left\langle z_{\mathrm{t}}\right\rangle \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ <br> $\mathrm{GeV} / c$ | $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ <br> $\mathrm{GeV} / c$ |
| :--- | :---: | :---: |
| 3.22 | $1.63 \pm 0.08$ | $2.79 \pm 0.13 \pm 0.35$ |
| 3.89 | $1.66 \pm 0.08$ | $2.57 \pm 0.11 \pm 0.33$ |
| 4.90 | $1.89 \pm 0.13$ | $2.66 \pm 0.17 \pm 0.35$ |
| 5.91 | $2.06 \pm 0.19$ | $2.74 \pm 0.20 \pm 0.34$ |
| 7.24 | $2.17 \pm 0.25$ | $2.83 \pm 0.25 \pm 0.32$ |
| 8.34 | $2.53 \pm 0.62$ | $3.11 \pm 0.60 \pm 0.33$ |

analysis the $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ is determined and thus the value of $\left\langle p_{\mathrm{T}}\right\rangle$ pair is evaluated as $\left\langle p_{\mathrm{T}}\right\rangle_{\text {pair }}=\sqrt{2} \times\left\langle k_{\mathrm{T}}\right\rangle=\sqrt{\pi / 2} \times \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$.

The present value of $\left\langle p_{\mathrm{T}}\right\rangle$ pair

$$
\left\langle p_{\mathrm{T}}\right\rangle_{\text {pair }}=3.36 \pm 0.09(\text { stat }) \pm 0.43(\text { sys }) \mathrm{GeV} / \mathrm{c}
$$

appears to be in a good agreement with the lower energy dijet and dilepton measurements or the higher energy measurement in diphoton production [54]. A UA2 measurement of $\left\langle p_{\mathrm{T}}\right\rangle$ of $Z^{0}$ production at $\sqrt{s} \sim 600 \mathrm{GeV}$ gives $8.6 \pm 1.5 \mathrm{GeV} / c[55,56]$.

## VIII. SUMMARY

We have made the first measurement of jet $j_{\mathrm{T}}$ and $k_{\mathrm{T}}$ for $p+p$ collisions at $\sqrt{s}=200 \mathrm{GeV}$ using the method of two-particle correlations. Analysis of the angular widths of


FIG. 27. $\left.\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}\right\rangle$ values corresponding to Fig. 17 as a solution to Eq. (49) for associated particles in $1.4<p_{\mathrm{Ta}}<5 \mathrm{GeV} / c$ region (solid symbols). The solid lines bracket the systematic error due to the unknown ratio of quark and gluon jets. The CCOR measurement at $\sqrt{s}=62.4 \mathrm{GeV}$ [1] (empty triangles).


FIG. 28. $\left\langle z_{\mathrm{t}}\right\rangle$ and $\hat{x}_{\mathrm{h}}$ as a function of $p_{\mathrm{Tt}}$ for the $1.4<p_{\mathrm{Ta}}<$ $5.0 \mathrm{GeV} / c$ associated region. The data values are shown in Table XI.
the near-side peak in the correlation function has determined that the jet-fragmentation transverse momentum $j_{\mathrm{T}}$ is constant with trigger particle $p_{\mathrm{Tt}}$ and the extracted value $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}=585 \pm 6$ (stat) $\pm 15$ (sys) $\mathrm{MeV} / c$ is comparable with previous lower $\sqrt{s}$ measurements. The width of the


FIG. 29. The values of $\hat{x}_{\mathrm{h}}$ (upper two curves) and $\left\langle z_{\mathrm{t}}\right\rangle$ (lower two curves) as a function of $p_{\mathrm{Ta}}$ are shown as solutions of Eq. (48) for $3<p_{\mathrm{Tt}}<4 \mathrm{GeV} / c$ and $5<p_{\mathrm{Tt}}<10 \mathrm{GeV} / c$. Equation (20) defines $\left\langle z_{\mathrm{t}}\right\rangle$ and $\hat{x}_{\mathrm{h}}$. The data values are shown in Table XII.

TABLE XI. The $\left\langle z_{\mathrm{t}}\right\rangle$ and $\hat{x}_{\mathrm{h}}$ values with $p_{\mathrm{Tt}}$ shown in Fig. 28.

| $p_{\mathrm{Tt}}(\mathrm{GeV} / c)$ | $\left\langle z_{\mathrm{t}}\right\rangle$ | $\hat{x}_{\mathrm{h}}$ |
| :--- | :---: | :---: |
| 3.22 | $0.51 \pm 4 \times 10^{-3} \pm 0.06$ | $0.88 \pm 0.01$ |
| 3.89 | $0.56 \pm 2 \times 10^{-3} \pm 0.07$ | $0.87 \pm 0.01$ |
| 4.90 | $0.61 \pm 1 \times 10^{-3} \pm 0.07$ | $0.85 \pm 0.01$ |
| 5.91 | $0.64 \pm 1 \times 10^{-4} \pm 0.07$ | $0.85 \pm 0.02$ |
| 7.24 | $0.66 \pm 1 \times 10^{-3} \pm 0.07$ | $0.86 \pm 0.02$ |
| 8.34 | $0.68 \pm 5 \times 10^{-3} \pm 0.06$ | $0.84 \pm 0.05$ |

TABLE XII. The $\left\langle z_{\mathrm{t}}\right\rangle$ and $\hat{x}_{\mathrm{h}}$ values with $p_{\mathrm{Ta}}$ for two trigger $\pi^{0}$ momenta bins as shown on Fig. 29.

| $\begin{aligned} & 3< \\ & p_{\mathrm{Ta}} \end{aligned}$ | $\mathrm{eV} / c \quad\left\langle z_{\mathrm{t}}\right\rangle$ | $\hat{x}_{\text {h }}$ |
| :---: | :---: | :---: |
| 1.72 | $0.54 \pm 8 \times 10^{-3} \pm 0.06$ | $0.81 \pm 0.01$ |
| 2.22 | $0.52 \pm 6 \times 10^{-3} \pm 0.06$ | $0.88 \pm 0.01$ |
| 2.73 | $0.51 \pm 1 \times 10^{-3} \pm 0.07$ | $0.95 \pm 0.01$ |
| 3.23 | $0.49 \pm 1 \times 10^{-3} \pm 0.06$ | $0.99 \pm 0.01$ |
| 3.93 | $0.47 \pm 5 \times 10^{-3} \pm 0.06$ | $1.04 \pm 0.01$ |
| 5.04 | $0.41 \pm 6 \times 10^{-3} \pm 0.06$ | $1.06 \pm 0.01$ |
| $5<p_{\mathrm{Tt}}<10 \mathrm{GeV} / c$ |  |  |
|  | $\left\langle z_{\mathrm{t}}\right\rangle$ | $\hat{x}_{\text {h }}$ |
| 1.85 | $0.66 \pm 4 \times 10^{-3} \pm 0.06$ | $0.75 \pm 0.04$ |
| 2.24 | $0.64 \pm 1 \times 10^{-3} \pm 0.06$ | $0.80 \pm 0.03$ |
| 2.73 | $0.61 \pm 2 \times 10^{-3} \pm 0.07$ | $0.87 \pm 0.02$ |
| 3.44 | $0.57 \pm 2 \times 10^{-3} \pm 0.07$ | $0.92 \pm 0.02$ |
| 4.65 | $0.52 \pm 5 \times 10^{-3} \pm 0.08$ | $0.98 \pm 0.01$ |



FIG. 30 (color online). Compilation of mean pair $p_{\text {T }}$ measurements [33] and comparisons to the $\left\langle p_{\mathrm{T}}\right\rangle$ pair measured in this analysis.
away-side peak is shown to be a measure of the convolution of $j_{\mathrm{T}}$ with the jet momentum fraction $z$ and the partonic transverse momentum $k_{\mathrm{T}}$. $\left\langle z_{\mathrm{t}}\right\rangle$ is determined through a combined analysis of the measured $\pi^{0}$ inclusive and associated spectra using the jet-fragmentation functions from $e^{+} e^{-}$measurements. The average of $\left\langle z_{\mathrm{t}}\right\rangle$ from the gluon and quark fragmentation functions is used and the difference is taken as the measure of the systematic error. The final extracted values of $k_{\mathrm{T}}$ are then determined to be also independent of the transverse momentum of the trigger $\pi^{0}$, in the range measured, with values of $\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}=$ $2.68 \pm 0.07$ (stat) $\pm 0.34$ (sys) $\mathrm{GeV} / c$.

## ACKNOWLEDGMENTS

We thank the staff of the Collider-Accelerator and Physics Departments at Brookhaven National Laboratory and the staff of the other PHENIX participating institutions for their vital contributions. We acknowledge support from the Department of Energy, Office of Science, Office of Nuclear Physics, the National Science Foundation, Abilene Christian University Research Council, Research Foundation of SUNY, and Dean of the College of Arts and Sciences, Vanderbilt University (USA), Ministry of Education, Culture, Sports, Science, and Technology and the Japan Society for the Promotion of Science (Japan), Conselho Nacional de Desenvolvimento Científico e Tecnológico and Fundação de Amparo à Pesquisa do Estado de São Paulo (Brazil), Natural Science Foundation of China (People's Republic of China), Centre National de la Recherche Scientifique, Commissariat à l'Énergie Atomique, and Institut National de Physique Nucléaire et de Physique des Particules (France), Ministry of Industry, Science and Tekhnologies, Bundesministerium für Bildung und Forschung, Deutscher Akademischer Austausch Dienst, and Alexander von Humboldt Stiftung (Germany), Hungarian National Science Fund, OTKA (Hungary), Department of Atomic Energy (India), Israel Science Foundation (Israel), Korea Research Foundation, Center for High Energy Physics, and Korea Science and Engineering Foundation (Korea), Ministry of Education and Science, Russia Academy of Sciences, Federal Agency of Atomic Energy (Russia), V. R. and the Wallenberg Foundation (Sweden), the U.S. Civilian Research and Development Foundation for the Independent States of the Former Soviet Union, the USHungarian NSF-OTKA-MTA, and the US-Israel Binational Science Foundation.

## APPENDIX A: FIRST AND SECOND MOMENTS OF NORMALLY DISTRIBUTED QUANTITIES

Let $x$ be a 1D variable with normal (Gaussian) distribution and $r=\sqrt{x^{2}+y^{2}}$ is a 2D variable with $x$ and $y$ of
normal distribution then the following relations can be easily derived.

| $\langle x\rangle$ | $=$ | 0 | $\langle r\rangle$ | $=$ | $\sqrt{\frac{\pi}{2}} \sigma_{1}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\langle \| x\rangle$ | $=$ | $\sqrt{\frac{2}{\pi}} \sigma_{1}$ | $\langle \| r\rangle$ | $=$ | $\langle r\rangle$ |
| $\langle x\rangle^{2}$ | $=$ | $\sigma_{1}^{2}$ | $\langle r\rangle^{2}$ | $=$ | $2 \sigma_{1}^{2} \equiv \sigma_{2}^{2}$ | Both $\vec{j}_{\mathrm{T}}$ and $\vec{k}_{\mathrm{T}}$ are two-dimensional vectors. We assume Gaussian distributed $x$ and $y$ components and thus the mean value $\left\langle k_{\mathrm{Tx}}\right\rangle$ and $\left\langle k_{\mathrm{Ty}}\right\rangle$ is equal to zero. The nonzero moments of 2D Gaussian distribution are e.g. the root mean squares $\sqrt{\left\langle j_{\mathrm{T}}^{2}\right\rangle}, \sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}$ or the mean absolute values of the $\vec{j}_{\mathrm{T}}, \vec{k}_{\mathrm{T}}$ projections into the perpendicular plane to the jet axes $\langle | j_{\text {Ty }}| \rangle$ and $\langle | k_{\text {Ty }}| \rangle$. Note that there are a trivial correspondences

$$
\begin{equation*}
\sqrt{\left\langle k_{\mathrm{T}}^{2}\right\rangle}=\frac{2}{\sqrt{\pi}}\left\langle k_{\mathrm{T}}\right\rangle=\sqrt{\pi}\langle | k_{\mathrm{Ty}}| \rangle . \tag{51}
\end{equation*}
$$

## APPENDIX B: THE CORRECT WAY TO ANALYZE THE AZIMUTHAL CORRELATION FUNCTION

Construction and fitting of the two-particle azimuthal correlation function is discussed in Sec. IV. Traditionally the correlation function is fitted by two Gaussian functions - one for intrajet correlation (near peak) and one for the interjet correlations (away-side peak). From the extracted variances of the Gaussian functions the $j_{\mathrm{T}}$ and $p_{\text {out }}$ magnitudes are extracted.

There is, however, a fundamental problem with this approach. The $p_{\text {out }}$-vector defined in Eq. (17) is equal to $p_{T \mathrm{a}} \sin \Delta \phi$ event by event. However, we measure the width of the correlation peak and this corresponds to $\sqrt{\left\langle\overline{\left.\Delta \phi^{2}\right\rangle}\right.}=$ $\sigma_{\mathrm{A}}$. The relation $\left.\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}\right\rangle p_{\mathrm{Ta}} \sin \sigma_{\mathrm{A}}$ is not a good approximation for $\sigma_{\mathrm{A}}>0.4 \mathrm{rad}$ (see Fig. 31). The assumption that the away-side correlation has a Gaussian shape is also good only for small values of $\sigma_{\mathrm{A}}$ (see Fig. 31).

One way of relating $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ and $\sigma_{\mathrm{A}}$ was proposed e.g. by Peter Levai [48] and used in several other analyzes. Since $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}=p_{\mathrm{Ta}} \sqrt{\left\langle\sin ^{2} \Delta \phi\right\rangle}$ one possibility how to relate $p_{\text {out }}$ and $\sigma_{\mathrm{A}}$ is to expand

$$
\begin{aligned}
\left\langle\sin ^{2} \Delta \phi\right\rangle & =\left\langle\Delta \phi^{2}-\frac{1}{3} \Delta \phi^{4}+\frac{2}{45} \Delta \phi^{6} \ldots\right\rangle \\
& =\sigma_{\mathrm{A}}^{2}-\sigma_{\mathrm{A}}^{4}+\frac{2}{3} \sigma_{\mathrm{A}}^{6} \ldots,
\end{aligned}
$$

where we assumed a Gaussian distribution of $\Delta \phi$. The comparison of $p_{\mathrm{Ta}} \cdot\left(\sigma_{\mathrm{A}}^{2}-\sigma_{\mathrm{A}}^{4}+\frac{2}{3} \sigma_{\mathrm{A}}^{6} \ldots\right)$ with the true $p_{\text {out }}$ magnitude (simple monte carlo) for various $\sigma_{\mathrm{A}}$ values is shown in Fig. 31. It is obvious that there is only a little


FIG. 31 (color online). The relative error on $p_{\text {out }}$ determination from the azimuthal correlation function based on the Taylor expansion of $\left\langle\sin \Delta \phi^{2}\right\rangle$ (dashed line), with an assumption of $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}=p_{\mathrm{Ta}} \quad \sin \sigma_{\mathrm{A}} \quad($ dotted $\quad$ line $)$ and $\quad \sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}=p_{\mathrm{Ta}} \sigma_{\mathrm{A}}$ (dotted-dashed line). The solid red line corresponds to $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}$ from Eq. (23).
difference between $\sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}=p_{\text {Ta }} \sin \sigma_{\mathrm{A}}, \quad \sqrt{\left\langle p_{\text {out }}^{2}\right\rangle}=$ $p_{\mathrm{Ta}} \sigma_{\mathrm{A}}$ and the Taylor series. In the region where $\sigma_{\mathrm{A}}>$ 0.4 rad , all approximations seems to be equally bad.

However, $p_{\text {out }}$, the only quantity with a truly Gaussian distribution (if we neglect the radiative corrections responsible for non-Gaussian tails in the $p_{\text {out }}$ distribution which are anyway not relevant for the $k_{\mathrm{T}}$ analysis) can be directly extracted from the correlation function. With the assumption of Gaussian distribution in $p_{\text {out }}$, we can write the away-side $\Delta \phi$-distribution (normalized to unity) as

$$
\begin{aligned}
\left.\frac{d N_{\text {away }}}{d \Delta \phi}\right|_{\pi / 2} ^{3 \pi / 2}= & \frac{d N}{d p_{\text {out }}} \frac{d p_{\text {out }}}{d \Delta \phi}=\frac{-p_{\mathrm{Ta}} \cos \Delta \phi}{\left.\sqrt{2 \pi\left\langle p_{\text {out }}^{2}\right\rangle}\right\rangle \operatorname{Erf}\left(\frac{\sqrt{2} p_{\text {Ta }}}{\left.\sqrt{\left\langle p_{\text {out }}^{2}\right.}\right)}\right.} \\
& \times \exp \left(-\frac{p_{\text {Ta }}^{2} \sin ^{2} \Delta \phi}{2\left\langle p_{\text {out }}^{2}\right\rangle}\right) .
\end{aligned}
$$

This is the correct way of extracting a dimensional quantity from the azimuthal correlation function in the case of narrow associated bin. Similar line of arguments can be drawn also in the case of near peak. However, given the narrowness of the near-angle peak, the simple Gaussian approximation is good enough.
[1] A.L.S. Angelis et al. (CERN-Columbia-OxfordRockefeller Collaboration), Phys. Lett. B 97, 163 (1980).
[2] P. Darriulat et al., Nucl. Phys. B107, 429 (1976).
[3] M. Della Negra et al. (CERN-College de France-Heidelberg-Karlsruhe Collaboration), Nucl. Phys. B127, 1 (1977).
[4] K. Adcox et al. (PHENIX Collaboration), Phys. Rev. Lett. 88, 022301 (2001).
[5] C. Adler et al. (STAR Collaboration), Phys. Rev. Lett. 89, 202301 (2002).
[6] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 91, 072301 (2003).
[7] B. B. Back et al. (PHOBOS Collaboration), Phys. Rev. Lett. 91, 072302 (2003).
[8] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 91, 072303 (2003).
[9] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 91, 072304 (2003).
[10] I. Arsene et al. (BRAHMS Collaboration), Phys. Rev. Lett. 91, 072305 (2003).
[11] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 91, 182301 (2003).
[12] C. Adler et al. (STAR Collaboration), Phys. Rev. Lett. 90, 032301 (2003).
[13] C. Adler et al. (STAR Collaboration), Phys. Rev. Lett. 90, 082302 (2003).
[14] A. B. Migdal, Phys. Rev. 103, 1811 (1956).
[15] X.-N. Wang and M. Gyulassy, Phys. Rev. Lett. 68, 1480 (1992).
[16] X.-N. Wang, Phys. Rev. C 58, 2321 (1998).
[17] R. Baier, Y. L. Dokshitzer, S. Peigne, and D. Schiff, Phys. Lett. B 345, 277 (1995).
[18] B. G. Zakharov, JETP Lett. 63, 952 (1996).
[19] C. A. Salgado and U. A. Wiedemann, Phys. Rev. Lett. 93, 042301 (2004).
[20] J.-w. Qiu and I. Vitev, Phys. Lett. B 570, 161 (2003).
[21] X.-N. Wang, Nucl. Phys. A702, 238 (2002).
[22] S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. D 4, 3388 (1971).
[23] J. F. Owens and J. D. Kimel, Phys. Rev. D 18, 3313 (1978).
[24] J. F. Owens, E. Reya, and M. Gluck, Phys. Rev. D 18, 1501 (1978).
[25] R. P. Feynman, R. D. Field, and G. C. Fox, Phys. Rev. D 18, 3320 (1978).
[26] J. F. Owens, Rev. Mod. Phys. 59, 465 (1987).
[27] G. Bunce, N. Saito, J. Soffer, and W. Vogelsang, Annu. Rev. Nucl. Part. Sci. 50, 525 (2000).
[28] R. Cutler and D. W. Sivers, Phys. Rev. D 17, 196 (1978).
[29] R. Cutler and D. W. Sivers, Phys. Rev. D 16, 679 (1977).
[30] B. L. Combridge, J. Kripfganz, and J. Ranft, Phys. Lett. B

70, 234 (1977).
[31] R. P. Feynman, R. D. Field, and G. C. Fox, Nucl. Phys. B128, 1 (1977).
[32] Y.L. Dokshitzer, V. A. Khoze, A.H. Mueller, and S.I. Troian, Basics of Perturbative QCD (Editions Frontieres, Gif-sur-Yvette, France, 1991), p. 274.
[33] L. Apanasevich et al., Phys. Rev. D 59, 074007 (1999).
[34] A. Kulesza, G. Sterman, and W. Vogelsang, Nucl. Phys. A721, C591 (2003).
[35] F. W. Busser et al., Phys. Lett. B 46, 471 (1973).
[36] R. Blankenbecler, S.J. Brodsky, and J.F. Gunion, Phys. Lett. B 42, 461 (1972).
[37] D. Antreasyan et al., Phys. Rev. D 19, 764 (1979).
[38] P. Darriulat, Annu. Rev. Nucl. Part. Sci. 30, 159 (1980).
[39] K. Adachi et al. (TOPAZ Collaboration), Phys. Lett. B 451, 256 (1999).
[40] K. Adcox et al. (PHENIX Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 499, 469 (2003).
[41] L. Aphecetche et al. (PHENIX Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 499, 521 (2003).
[42] K. Adcox et al. (PHENIX Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 499, 489 (2003).
[43] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 91, 241803 (2003).
[44] M. Aizawa et al. (PHENIX Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A 499, 508 (2003).
[45] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. C 69, 034910 (2004).
[46] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 94, 082301 (2005).
[47] For relations between $\sqrt{\left\langle X^{2}\right\rangle}$ and $\langle | X_{\mathrm{y}}| \rangle$, where $X$ is any 2D quantity, see Appendix A
[48] P. Levai, G. Fai, and G. Papp, Phys. Lett. B 634, 383 (2006).
[49] G. W. van Apeldoorn et al., Nucl. Phys. B91, 1 (1975).
[50] Note, however that the method was different. CCOR determined $j_{T}$ and $k_{T}$ from the slope and intercept of Eq. (21) with respect to $p_{T \mathrm{a}}$ at each value of $p_{T \mathrm{t}}$, with the implicit assumption that $\left\langle z_{t}\right\rangle \hat{x}_{h}^{-1}=1$.
[51] X.-N. Wang, Phys. Lett. B 595, 165 (2004).
[52] P. Abreu et al. (DELPHI Collaboration), Eur. Phys. J. C 13, 573 (2000).
[53] G. Alexander et al. (OPAL Collaboration), Z. Phys. C 69, 543 (1996).
[54] F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. 70, 2232 (1993).
[55] R. Ansari et al. (UA2 Collaboration), Phys. Lett. B 194, 158 (1987).
[56] R. Ansari et al. (UA2 Collaboration), Z. Phys. C 41, 395 (1988).


[^0]:    ${ }^{1}$ Abilene Christian University, Abilene, Texas 79699, USA
    ${ }^{2}$ Institute of Physics, Academia Sinica, Taipei 11529, Taiwan
    ${ }^{3}$ Department of Physics, Banaras Hindu University, Varanasi 221005, India
    ${ }^{4}$ Bhabha Atomic Research Centre, Bombay 400 085, India

[^1]:    *Deceased.
    ${ }^{\dagger}$ PHENIX Spokesperson: zajc@nevis.columbia.edu

