# Joint assignment, scheduling and routing models to Home Care optimization: a pattern based approach

Paola Cappanera [*]   Maria Grazia Scutellà [†]

March 19, 2014

**Abstract**

The design of efficient Home Care Services is a quite recent and challenging field of study. We propose an integrated approach that jointly addresses: ($i$) the assignment of operators to patients so as to guarantee the compatibility between skills associated with operators and patient visits; ($ii$) the scheduling of the visits in a given planning horizon; and ($iii$) the determination of the operator tours in every day of the planning horizon. The main Home Care Problem we investigate refers to providers dedicated to palliative care and terminal patients. In this context, balancing objective functions are particularly relevant. Therefore, two balancing functions are studied, i.e. *maxmin*, which maximizes the minimum operator utilization factor, and *minmax*, which minimizes the maximum operator utilization factor. In both cases, the concept of *pattern* is introduced as a key tool to jointly address assignment, scheduling and routing decisions, where a pattern specifies a possible schedule for skilled visits. The approach we propose is however able to cope with peculiarities from other home care contexts. Model extensions to handle scenarios other than the palliative one are discussed in the paper.

Extensive computational results are reported both on palliative home care instances based on real data, and on two real-world data sets from the literature, related to contexts very different from the palliative one. For both data sets the proposed approach is able to find solutions of good quality. In the palliative context, the results show that the selection of the pattern generation policy is crucial to solve large instances efficiently. Furthermore, the maxmin criterion is able to return more balanced solutions, i.e. the difference between the maximum and the minimum operator utilization factors is very small. On the other hand, the minmax criterion is more suitable for minimizing the operating costs, since it computes solutions with smaller total travelled time.

**keywords:** Home Care, joint optimization, pattern, computational analysis

[*]Dipartimento di Ingegneria dell'Informazione, Università degli Studi di Firenze, Firenze, Italy - email: `paola.cappanera@unifi.it`

[†]Dipartimento di Informatica, Università di Pisa, Pisa, Italy - email: `scut@di.unipi.it`

# Introduction

Nowadays, the ever increasing average age of population, at least in industrialized countries, and the increased costs for the consequently required care, compel the medical care units (hospitals and so on) to offer home care services in an attempt to limit costs. Elderly people have in fact varying degrees of need for assistance and medical treatment, and it may be advantageous to allow them to live in their own homes as long as possible, since a long-term stay in a nursing home can be much more costly for the social insurance system than a treatment at home providing assistance to the required level. Even more important, medical treatments carried out at patients home impact favorably on their quality of life. Therefore, home care services are a cost-effective and flexible instrument in the social system.

In this paper we address relevant optimization problems arising in Home Care; specifically, given a planning horizon $W$, usually a week, a set of patients with an associated *care plan*, i.e. weekly requests each of them demanding a specific skill to be operated, and a set of operators also characterized by a skill, the problems ask to schedule the patient requests during the planning horizon, assign the operators to the patients by taking into account the compatibility between request and operator skills, and determine the tour each operator has to perform in every day of the planning horizon.

Indeed, many significant differences do exist in Home Care, which induce the study of different problems and the development of different models. The main Home Care problem we investigate in this paper refers to providers which are mainly dedicated to palliative care and terminal patients. In this setting, an objective typically used to guide the decisions is the balancing of the utilization factor among the operators, where the *operator utilization factor* is the total workload of the operator in the considered planning horizon over his/her maximum possible workload. In order to achieve this objective, we study two alternative functions: (*i*) *maxmin*, which maximizes the minimum operator utilization factor, and (*ii*) *minmax*, which consists in minimizing the maximum operator utilization factor.

Relevant Quality of Service (QoS) requisites are taken into account during the optimization process. One of them is the so-called *continuity of care*, expressed by imposing that at most $T$ operators can be assigned to each patient in the considered planning horizon, where $T$ is given. Another relevant requisite, the *maximum daily workload*, states that the workload of each operator in each day, i.e. the sum of the daily service times and traveling times of the operator, can not exceed the duration of the operator workday.

In the state-of-the-art literature Home Care problems are usually solved in cascade: first the operators are assigned to the patients on a geographical basis so as to match the skills required by a service at patient's home with the skill owned by the operator; second, the schedule of each operator is determined, usually operator-wise. Some optimization models that extend Vehicle Routing Problem (VRP) formulations have been proposed in the literature, but generally they deal with a daily planning horizon. To the best of our knowledge there are only three exceptions which deal with a longer time horizon. In [2] the weekly planning is viewed as the union of independent daily plannings. The other two studies ([17], [22]) address a problem which is similar to the one analysed in this paper, seeking for an optimal weekly plan. However, no integrated approach is proposed to solve the overall problem, but two-stage solution approaches are presented. In fact, in the literature tailored metaheuristic approaches are usually proposed to solve Home Care problems, which address the complex decisions via multiple steps. On the other hand, here the studied Home Care problem is formulated by jointly addressing assignment, scheduling and routing decisions, over a given planning horizon, by generalizing the VRP, and specifically the Skill VRP ([6], [7]), from which it inherits the skill based structure. In the Skill VRP papers, however, only daily routing decisions have been addressed.

We propose a new Integer Linear Programming (ILP) formulation to state the Home Care problem referring to the palliative context, under the two kinds of balancing objective functions mentioned above. An innovative modelling device to combine the three levels of decisions is that services are offered according to a set of a priori given *patterns*. Three policies to generate patterns are designed and will be discussed in the paper. The relevant QoS constraints mentioned before, i.e. continuity of care and maximum daily workload, have been incorporated into the models. In addition, the models have been enhanced by means of valid inequalities aimed at breaking the symmetric structure that usually characterizes the underlying logistics network, and by means of cuts whose aim is to take under control the length of the operator tours.

The issues that feature the palliative context, however, do not narrow the applicability of the models we propose to different home care scenarios. Model extensions are in fact presented that may help in coping with different home care organisations. They include the satisfaction of time windows indicated by the patients and those relative to operator shifts (like a morning and an afternoon shift in each day), and overtime handling, with possibilities of overtime compensation within the time horizon.

The approach has been validated, over a weekly planning horizon, on a set of palliative home care instances based on real data and publicly available. Specifically, the set includes both two real instances which are related to the largest division of a northern Italian medical care unit, and which are characterized by 11 operators and by 128 and 162 patients, respectively, and a pool of 24 smaller instances, which have been generated starting from the two real ones. Although smaller than the previous ones, the size of these 24 instances (characterized by a number of patients varying in $\{40, 50, 60, 80\}$ and by a number of operators varying in $\{4, 5, 6\}$) reflects the size of real home care districts in Italy, into which divisions are usually organized. Furthermore, in order to give computational evidence of the generalizability of the models proposed, the approach has been validated on two real-world data sets from the literature, pertaining to contexts very different from the palliative one, where some of the aforementioned model extensions are required.

One of the objectives of the experimental analysis was to study the impact of the pattern selection policy in solving the proposed models. At this regard, all the experiments on the palliative data set show that determining a good but limited set of patterns is crucial for efficiently solving the Linear Programming (LP) relaxation, thus helping in computing very good quality solutions. In particular, one of the proposed pattern generation policies, which is based on the solution of an auxiliary network flow problem, proved to be very effective in selecting a small number of patterns of good quality. Also, the proposed symmetry management valid inequalities proved to be crucial for the efficiency of the approach. Therefore the first achievement is that, by properly selecting the pattern generation mechanism and suitably using the symmetry management tool, in almost all the cases the proposed models were able to compute very good quality solutions, near to the optimal ones. This is true for both the 24 smaller instances and for the two biggest ones. In the latter case, such good quality solutions have been computed within the stated time and memory limits, which are 12 hours and 1 GB for the branch and bound tree, respectively. Concerning the required time and memory resources, notice that a large but affordable consumption of such resources does not seem to be an issue when the focus is a difficult planning problem that has to be solved once a week.

Another objective of the computational analysis was to compare the two balancing objective functions, i.e. *maxmin* and *minmax*, by showing their impact on diverse relevant QoS indicators. This set of indicators includes the mean operator utilization factor over the considered planning horizon and the corresponding range, i.e. the difference between the maximum and the minimum operator utilization factors. In addition, in order to provide a hint about the influence of such equity measures on the Home Care Services efficiency, QoS indicators related to the operator travelled time have been investigated. At this regard, by selecting the most promising pattern generation policy, i.e. the one involving an auxiliary network flow problem, the interesting result is that *maxmin* is able to return more balanced solutions, in the sense that the operator utilization range is smaller than the one returned by *minmax*. Such a stronger equity achievement is obtained for not too high a price in the increased mean operator utilization factor and mean operator traveling time. On the other hand, the minmax criterion is more suitable for the minimization of the operating costs, since it always returns solutions with the smaller total travelled time for the operators. Therefore, *minmax* is a criterion better pursuing home care efficiency, while *maxmin* appears to be more suitable for equity in Home Care.

We performed an additional set of experiments on the 24 palliative home care instances, by relaxing the care continuity constraint. The results in some way contrast the main conclusions of previous studies in the home care sector. In fact, it appears that relaxing the care continuity constraint does not deteriorate too much the computed solutions in terms of number of operators assigned to the same patient. Furthermore, such a relaxation often allows to solve critical instances in a more efficient way.

Finally, to show the applicability of the approach to contexts different from the palliative one, we tested the proposed formulation, with some of the presented model extensions, on two real-world data sets from the literature. These two data sets have been experimented in the two most recent works dealing with a weekly planning horizon in Home Care, i.e. [17] and [22]. Even if the two instances seem quite comparable to the palliative biggest instances according to their size, they have a very different structure and require specific extensions, due to the different home care context they address. The computational results show that our approach seems to be able to find solutions of good quality, near to the optimal ones, also on such real instances.

The determination of very good quality solutions in almost all the performed experiments thus reveals, in our opinion, the potentiality of the proposed ILP models in successfully addressing real home care instances. Further, the joint approach appears to be suitable for the design of decomposition methods, relying on the proposed ILP models, and tailored to solve even larger home care instances.

The plan of the paper is the following. In Section 1 we introduce the studied Home Care problems. In Section 2 we describe the main results from the literature on modelling and solution approaches to Home Care optimization problems. In Section 3 we present the proposed ILP models to Home Care in the palliative scenario, with the related balancing criteria *maxmin* and *minmax*. Valid inequalities to enhance the models are also presented, which exploit the peculiar symmetric cost structure of the real instances. In addition, constraints aiming at avoiding long tours are introduced. The main similarities and dissimilarities between our models and the models in [17] and [22] are explained in Section 4. In Section 5 we present some relevant extensions to the ILP Home Care models previously introduced, that allow to cope with different home care scenarios. Section 6 contains the description of some pattern generation approaches. In Section 7 we discuss the results of the computational experiments aimed at validating the proposed models on real home care instances. In particular, the main achievements of our computational study are summarized in Section 7.7. Finally, Section 8 provides conclusions and discussion about possible avenues of research.

# 1 The Home Care problems addressed

Home Care Services aim to satisfy the health and social needs of people by providing appropriate and high quality home based healthcare and social services within a balanced and affordable continuum of care [26], [27]. Anyway, many significant differences do exist, which induce the development of different models in the planning of home care organisation operations. An analysis of the main types of organisations is depicted in [20], where a distinction among public service providers, and private non for profit/for profit providers is outlined depending on their corporate status. Another classification of home care providers is related to the pathologies suffered by their patients and the patient characteristics.

The main Home Care Problem we investigate in this paper refers to providers which are mainly dedicated to palliative care and terminal patients. For the sake of clarity, it will be referred to as the *Palliative Home Care Problem* (PHCP). This is the most common case in Italy as witnessed by the analysis shown in [20]. However the characteristics of PHCP are very general; in fact, most of them constitute a common kernel to other types of health care service organisations (such as the ones classified as polyvalent in [20]). In contrast, the issues that characterize PHCP are mainly the objective function used, the low importance of time windows constraints, the care continuity management and a hierachical structure of the skills associated with patients and caregivers. In the following a brief discussion on these key features is given. Nevertheless the models we propose are able to cope with additional peculiarities from different contexts as well.

Home Care problems are multicriteria in nature due to the large number of stakeholders involved and due to a large number of interwined activities. Typical criteria used to drive decisions are the minimization of the operating costs (essentially the total travel time), the minimization of unscheduled visits, the control of the overtime, the regularity of the service offered, the continuity between patients and operators, the minimization of masterplan disruptions, and a fair distribution of the workload among the operators. Each of the aforementioned criterion can give rise to a component of the objective function or to a constraint. These two situations are commonly referred to as soft and hard constraints respectively. Objective functions that combine together some of these criteria by means of weights are widespread in the Home Care literature. In the oncology and terminal diseases context, balancing objective functions are particularly relevant. In fact, balancing the operator workload is crucial to try to mitigate the so-called burnout syndrome, typical of home care nurses who have developed special relationships with terminal patients [20]. Objective functions that aim at balancing the operator workload are very common in more general human resource planning problems and specifically in those dealing with home care organisations; as an example they are used when providers have to assign operators to patients (see [20]). See also [4] where, in a different context, the problem of partitioning a territory into districts is addressed by balancing the operator workload.

Time windows imposed on patient's visits can be crucial in several home care scenarios, especially for patients requiring a timely administration of drugs. However their management is very complex and in the practice time windows are often the results of a negotiation with the patients [22] or they are quite large. In contrast, in the palliative context the satisfaction of time windows indicated by the patients is typically not pursued.

The continuity of care aims to ensure that the same caregiver, or at least a limited set of caregivers, delivers service to a patient. The advantages of this practice are that loss of information among operators is reduced, and patients perceive a better quality of care [20]. In the literature, several ways to manage the care continuity are provided. As an example, the *patient-nurse loyalty* addressed in [22] is modeled as a term of the objective function which controls the number of different operators which can serve a certain job of a certain patient during the planning horizon. As a further example, *the employee regularity* studied in [17] is modeled as an objective function that minimizes the sum, over the patients, of the number of different operators assigned to each patient. In the palliative care context the continuity of care is particularly relevant and we model it via a constraint imposing that at most $T$ operators can be assigned to each patient in the planning horizon ($T = 2$ in our experiments). Although stronger forms of care continuity can be devised, what matters is the number of operators that a patient has to become acquainted with.

Another issue that characterizes the palliative context is that patient's visits and operators are associated with levels of skill that are managed hierarchically. In our case study, for example, there are two levels of skill where skill 1 refers to ordinary requests, whereas skill 2 corresponds to palliative requests. A specialized operator (i.e. an operator with skill 2) can visit a patient requiring a less skilled visit (i.e. a visit with skill 1) and in such a case an *overskilled* visit occurs. This opportunity is perceived positively by the patients since it increases their confidence in the service they receive [21].

The issues that feature the palliative context, however, do not narrow the applicability of the models we propose to different home care contexts. In Section 5 we will present model extensions that may help in coping with different scenarios. Specifically, we will present extensions addressing the request to satisfy time windows indicated by the patients and those relative to operator shifts (i.e. a morning and an afternoon shift in each day); we will discuss the handling of overtime, with possibilities of overtime compensation within the time horizon, and finally we will deal with scenarios where skills are not organised hierarchically. In order to give computational evidence of the generalizability of the models proposed, in Section 7.6 we will present the results obtained on two real-world data sets from different contexts, where some of the aforementioned model extensions are required.

# 2 The literature

Planning Home Care Services is a rather young but quickly evolving research area. As indicated before, in the state-of-the-art Home Care literature problems are usually solved in cascade: first the operators are assigned to the patients on a geographical basis so as to match the skills required by a service at patient's home with the skill owned by the operator; second, the schedule of each operator is determined, usually operator-wise.

In 1997, Begur et al. [2] clearly stated the importance of jointly addressing scheduling and routing decisions in Home Care. Furthermore, they firstly suggested the use of patterns in order to schedule the patient requests during the considered planning horizon. However, they used the concept of pattern as a means to decompose the overall problem into a set of daily independent routing problems, and used a heuristic approach to generate the operator tours. Specifically, the heuristic proposed in [2] uses a sequential savings algorithm as well as a nearest neighbor heuristic to reoptimize each single route. The authors estimated large savings potentials for 40 patients and 7 operators per day.

By distinguishing between salaried operators (full-time nurses) and non-salaried operators (part-time nurses), in about the same period Cheng and Rich [10] studied the problem of determining an optimal schedule for each operator such that each nurse leaves from his/her home, visits a set of compatible patients within associated time windows, takes a lunch break and returns home, all within the nurse time window and the maximum duration of a shift. The objective function to be minimized is the amount of the overtime and part-time worked. Compared to the Home Care problems studied in this paper, there is no concept of skill. Pairs of compatible nurse-patient are considered instead. In addition, the planning horizon is the day. Two and three-index ILP formulations are presented. However, the problem is solved by means of a two phase algorithm that first builds parallel tours that are improved afterwards. The authors compared their heuristic results with the optimal solutions found by the solver software Cplex for some random test instances with up to 10 patients and 4 operators, and showed some heuristic results for larger instances.

A mathematical programming model for the combined vehicle routing and scheduling problem with time windows and additional temporal constraints is presented in [5]. The temporal constraints allow for imposing pairwise synchronization and pairwise temporal precedence between customer visits, independently of the vehicles. The authors describe some real world applications, among which the daily planning of home care staff. They propose an optimization based heuristic to solve real size instances for up to 80 visits and 16 nurses, and compare a direct usage of a commercial solver against the proposed heuristic. Again, the considered planning horizon is the day. The description of an analogous combined scheduling and routing problem, with a related decision support system to solve it, called Laps Care, can be found in [13]. The problem is formulated as a set partitioning problem and solved by repeated matching on a daily horizon. Requirements that two operators occur simultaneously or in given order are also taken into account by the decision support system. The presented case studies deal with 86 - 123 patients and 12 - 21 nurses. See [14] for a description of the advantages of using Laps Care in Swedish municipalities.

Syncronized visits in the home care context are addressed also in [18]. The authors study a routing problem where some cares have to be performed by several operators and some cares cannot be performed with others. If a patient needs several cares, he/she may want to be treated by the same person. Moreover, some skill constraints and time windows have to be satisfied. Again, the planning horizon is the day. The authors showed that the studied Home Care problem is equivalent to a multiple traveling salesman problem with time windows (mTSPWT) with some specific constraints, and proposed an ILP model with some technical improvements. The proposed ILP model was not able to deal with instances of real size, but it gave rapidly solutions of good quality.

A multiple vehicle routing model with time windows and additional constraints has been proposed also in [12]. Here the concept of skill is not present, but the patients are assigned to a geographical sector depending on their home address. Whenever possible, an operator visits only patients from his/her sector, but he/she may however have to visit patients from other sectors to balance the operator workloads. Continuity of care and blood sample related constraints (the specific application context concerns in fact blood sample requests) are also taken into account. Although a mathematical model is proposed, a metaheuristic approach based on Tabu Search is indeed used to solve the specific Home Care problem under consideration. Other specific Home Care problems have been addressed in the literature, such as the planning of operations related to chemiotherapy at home in [9], and the daily planning of Home Health Care in times of flood disasters in [25]. In [25], a model formulation has been presented which includes assignment constraints, working time restrictions, time windows, and mandatory break times. A feasible solution has also to consider qualification levels, language skills, and rejections due to personal reasons. The model has been implemented with the solver software Xpress 7.0 and solved for small problem instances. Real life-sized instances have been tackled with a variable neighborhood search (VNS)-based heuristic that is capable of solving even large instances covering 512 requests and 75 operators.

[11] and [23] present a Branch and Price framework for a daily Home Health Care scheduling problem in Denmark. Instances with up to 150 requests and 15 operators are presented and compared to current practice. See also [1] for a collaborative population-based metaheuristic technique to the scheduling of home care workers in the UK.

A daily assignment and routing Home Care problem is also addressed in [3], in [24] and in [16]. The problem in [3] considers the skill of patients and operators, and includes a variety of hard and soft constraints, plus preferences. The proposed two-phase approach interweaves two parts: finding a partition of requests to operators, and finding an optimal sequencing for each such partition. A combination of linear programming, constraint programming, and heuristics is used to develop a software tool to home care applications. The authors solved instances with 200-600 requests and 20-50 operators. In [24] a new insertion method is developed on the basis of an already known insertion heuristic where the assignment of visits to operators and the generation of routes is done in the same process. The solution found by the insertion heuristic is then used as the initial solution of a tabu search approach. In [24], sometimes the operators have to visit the same patient more than once on the same day. Furthermore, shared visits are allowed. However, all operators are assumed to have the same qualifications, and the visits are not characterized by skills. The instances used for parameter tuning and performance testing have 17 operators and 166.5 visits on average, and they are based on data from a municipality in Denmark. In [16], a general framework for solving a real-world multimodal home-healthcare scheduling problem from a major Austrian provider is presented. The goal is to assign homecare staff to customers by respecting various side constraints, like (preferred) time windows, and taking into account staff and patient satisfaction as well as the modality of the routes. Multi-modality is a key issue in the considered context, since nurses use different modes of transportation (cars or public transportation), which inherently leads to travel times dependent on the chosen modality. A two stage heuristic approach is proposed to solve the studied problem: in the first stage, initial solutions are generated either via constraint programming or by a random procedure; during the second stage, the initial solutions are then iteratively improved by applying one of four metaheuristics (variable neighborhood search, a memetic algorithm, scatter search and a simulated annealing hyper-heuristic). An extensive computational comparison shows that the approach is capable of solving real-world instances in reasonable time and produces valid solutions within only a few seconds.

To the best of our knowledge, the only works dealing with a weekly planning horizon (in addition to the already mentioned [2]) are [22] and [17]. In [22] Nickel et al. present a two phase heuristic approach. In the first stage, a constraint programming heuristic guarantees the quick calculation of a feasible, applicable solution. Afterwards, an adaptive large neighborhood search (ALNS) seeks to improve the initial solution if further computational time is available. The approach has been evaluated with two real-world data sets from Germany and the Netherlands. The results show that it is possible to solve practical instances of home care operations planning in reasonable time, with up to 12 operators and 95 patient jobs.

A two-level approach is used also by Jensen [17]. The first phase constructs a masterplan, which is a long-term plan, via a five-phase heuristic, essentially based on local search routines. The second phase is a daily planning which uses the masterplan as a starting point, but incorporates last minute changes such as employees falling in sick, ad hoc visits, and other unforeseen events. A real-life instance characterized by 5 operators and 37 patients is tested, together with some large randomly generated instances. An ILP formulation of the key parts of the masterplan problem is also presented.

With respect to the reviewed literature, a key contribution of our study is to address a long term (say weekly) planning horizon in Home Care, rather than the daily planning horizon commonly investigated. In addition, with respect to the aforementioned [2], [17] and [22], that also consider such a long time horizon, here assignment, scheduling and routing decisions are addressed in a joint way, i.e. without heuristically decomposing the problem by means of a two-level approach. For the sake of completeness the main differences between the ILP models proposed in this paper and the models in [17] and [22] will be emphasized in Section 4.

In order to coordinate the diverse decision levels, crucial and original is, in our opinion, the use of a pattern modelling device. Partial and preliminary computational results related to such a modelling device can be found in [8], showing that pattern generation policies may have a strong impact on the efficiency of the optimization approach and on the quality of the returned solutions. Notice that the ILP models presented in this paper strictly generalize some of the models in [6] and [7]. In fact, they incorporate the skill hierarchy structure introduced in [6] and [7] where, however, only daily routing decisions have been addressed. Furthermore, assignment and scheduling aspects are not present in [6] and [7].

# 3 ILP Home Care models

This section is devoted to the Palliative Home Care Problem (PHCP) introduced in Section 1. Suitable extensions will be then presented in Section 5.

PHCP is defined on a complete directed network $G = (N, A)$, having $n$ nodes, where each node $j$ corresponds to a patient. There is an extra node (node 0), which is used to denote the basis where the operators start their tour from and arrive in.

A set $K$ of $\overline{k}$ levels of skill is assumed for both patients and operators, where skill $\overline{k}$ corresponds to the highest ability and skill 1 to the lowest one. Skill levels are organized hierarchically so that an operator with skill $k$ can work all the requests characterized by a skill up to $k$. Under this assumption, requests of the highest skill $\overline{k}$ can be operated only by the most skilled operators.

A *care plan* is given for each patient. Specifically, for each patient $j \in N$ and for each level of skill $k \in K$, $r_{jk}$ denotes the number of visits of skill $k$ required by $j$ in the planning horizon $W$.

A new modelling device, called *pattern*, plays a crucial role to address jointly scheduling, assignment and routing decisions. A pattern specifies a possible schedule for skilled visits; patient requests are then operated according to a set $P$ of a priori given patterns. Specifically, for each pattern $p \in P$ and for each day $d \in W$, we define $p(d) = 0$ if no service is offered at day $d$, while it is $p(d) = k$ if a visit of skill $k$ is operated on day $d$ according to $p$. Only one visit per day can be operated. For example, if a patient requires three visits a week, they may be scheduled on Monday-Wednesday-Friday or Monday-Tuesday-Thursday: in this case two patterns will be considered with the appropriate skill level for the visits.

Given the input data above, PHCP consists in assigning a pattern from $P$ to each patient $j$, so scheduling the requests of $j$, expressed by the $r_{jk}$, over the planning horizon (*care plan scheduling*), in assigning operators to each patient $j$, for each day where a request of $j$ has been scheduled (*operator assignment*), and in determining the tour of each operator for each scheduled day (*routing decisions*), so as to balance the operator workload. In addressing these three groups of decisions, the skill constraints (i.e. the compatibility between the skills associated with the patient requests and the skills of the operators), the continuity of care (i.e. at most $T$ operators can be assigned to each patient in $W$, for given $T$) and the maximum daily workload constraints have to be taken into account.

In order to state PHCP in a more formal way, let us introduce some additional notation:

| | |
|---|---|
| $O$ | the set of (skilled) available operators |
| $O_d \subseteq O$ | the set of the operators available on day $d$, for each $d \in W$ |
| $s_\omega$ | the skill of operator $\omega$, $\omega \in O$ |
| $D_\omega$ | the workday length of operator $\omega$, $\omega \in O$ |
| $t'_j$ | the service time at node $j$, $j \in N$ ($t'_0 = 0$) |
| $t_{ij}$ | the traveling time from node $i$ to node $j$, $(i, j) \in A$. |

Then let us define the following three families of variables in order to model the care plan scheduling, the operator assignment and the routing decisions:

$$z_{jp} = \begin{cases} 1 \text{ if pattern } p \text{ is assigned to patient } j \\ 0 \text{ otherwise} \end{cases} \qquad j \in N \ (j \neq 0),\ p \in P$$

$$u_{\omega j} = \begin{cases} 1 \text{ if operator } \omega \text{ is assigned to patient } j \\ 0 \text{ otherwise} \end{cases} \qquad \omega \in O,\ j \in N\ (j \neq 0)$$

$$x_{ij}^{\omega d} = \begin{cases} 1 \text{ if operator } \omega \text{ travels along } (i,j) \text{ on day } d \\ 0 \text{ otherwise.} \end{cases} \qquad (i,j) \in A,\ d \in W,\ \omega \in O_d.$$

Furthermore, let:

$y_{ij}^d$ auxiliary flow variable which represents the number of patients visited after patient $i$ by the operator moving along $(i,j)$ on day $d$, $(i,j) \in A$, $d \in W$.

Using the variables above, the feasibility set of PHCP can then be modeled as follows:

$$\sum_{i \in N} \sum_{\omega \in O} x_{ij}^{\omega d} \leq \sum_{p:p(d) \geq 1} z_{jp} \qquad \forall j \in N \setminus \{0\}, \forall d \in W \tag{1}$$

$$\sum_{i \in N} \sum_{\omega : s_\omega \geq k} x_{ij}^{\omega d} \geq \sum_{p:p(d)=k} z_{jp} \qquad \forall j \in N \setminus \{0\}, \forall d \in W, \forall k \in K \tag{2}$$

$$\sum_{p \in P} z_{jp} = 1 \qquad \forall j \in N \setminus \{0\} \tag{3}$$

$$\sum_{\omega \in O} u_{\omega j} \leq T \qquad \forall j \in N \setminus \{0\} \tag{4}$$

$$x_{ij}^{\omega d} \leq u_{\omega j} \qquad \forall (i,j) \in A, \forall j \in N \setminus \{0\}, \forall d \in W, \forall \omega \in O_d \tag{5}$$

$$u_{\omega j} \leq \sum_{i \in N : i \neq j} \sum_{d} x_{ij}^{\omega d} \qquad \forall j \in N \setminus \{0\}, \forall \omega \in O \tag{6}$$

$$D_{\omega d} = \sum_{(i,j) \in A} (t_{ij} + t'_j) \cdot x_{ij}^{\omega d} \leq D_\omega \qquad \forall d \in W, \forall \omega \in O_d \tag{7}$$

$$\sum_{i \in N} x_{ij}^{\omega d} = \sum_{i \in N} x_{ji}^{\omega d} \qquad \forall j \in N \setminus \{0\}, \forall d \in W, \forall \omega \in O_d \tag{8}$$

$$\sum_{j \in N} y_{0j}^d = \sum_{j \in N} \sum_{p:p(d) \geq 1} z_{jp} \qquad \forall d \in W \tag{9}$$

$$\sum_{i \in N} y_{ij}^d - \sum_{i \in N} y_{ji}^d = \sum_{p:p(d) \geq 1} z_{jp} \qquad \forall j \in N, \forall d \in W \tag{10}$$

$$y_{ij}^d \leq n \sum_{\omega \in O_d} x_{ij}^{\omega d} \qquad \forall (i,j) \in A, \forall d \in W \tag{11}$$

$$z_{jp} \in \{0,1\} \qquad \forall j \in N (j \neq 0), \forall p \in P \tag{12}$$

$$u_{\omega j} \in \{0,1\} \qquad \forall \omega \in O, \forall j \in N (j \neq 0) \tag{13}$$

$$x_{ij}^{\omega d} \in \{0,1\} \qquad \forall (i,j) \in A, \forall d \in W, \forall \omega \in O_d \tag{14}$$

$$y_{ij}^d \geq 0 \qquad \forall (i,j) \in A, \forall d \in W \tag{15}$$

Constraints (1) state that at most one operator per day can visit patient $j$, if a visit has been scheduled on that day for node $j$. Constraints (2) guarantee that, on day $d$ and for each level of skill, exactly one operator, of adequate skill, must visit patient $j$ if a service of that skill has been scheduled for $j$ on day $d$. Constraints (3) assure that exactly one pattern is assigned to each patient, thus scheduling his requests in the planning horizon. Constraints (4) assure that at most $T$ operators are assigned to each patient during a week, where $T$ is assumed to be given. This models the continuity of care requisite described in Section 1. Constraints (5) guarantee that an operator can visit a patient only if he has been assigned to that patient (linking between routing and assignment variables). Furthermore, constraints (6) force variables $u_{\omega j}$ to zero if operator $\omega$ never visits patient $j$ during the planning horizon. Constraints (7) assure that the workload of each operator in each day, expressed as the sum of the service times and the traveling times, does not exceed the duration of a workday. Constraints (8) are the classical flow conservation constraints on the routing variables. Constraints (9) and (10) are the flow conservation constraints on the auxiliary $y$ variables, which are introduced to avoid subtours in the model solutions. They also guarantee the correct linking between scheduling decisions and auxiliary flow variables. Finally, constraints (11) link together routing variables and auxiliary flow variables.

Observe that a pattern variable $z_{jp}$ can assume a value other than zero only if pattern $p$ is compatible with patient $j$, i.e.:

$$|\{d : p(d) = k\}| = r_{jk} \quad \forall k \in K. \tag{16}$$

Therefore, in the preprocessing phase $z_{jp} = 0$ if anyone of the $\overline{k}$ constraints (16) is not satisfied. Furthermore, $x_{ij}^{\omega d} = 0$ if patients $i$ and $j$ have only requests of skill at least $k$ during the planning horizon, and $\omega$ is an operator of skill less than $k$.

In order to balance the operator workload, the first objective function we address, *maxmin*, maximizes the minimum *operator utilization factor*, expressed as the total workload of the operator during the planning horizon over his maximum possible workload. The corresponding ILP model is thus:

$$\begin{array}{ll} \max & m \\ & (1) - (15) \\ & \dfrac{\sum_{d \in W} D_{\omega d}}{|W| \cdot D_\omega} \geq m, \quad \forall \omega \in O, \end{array} \tag{17}$$

where $|W|$ denotes the width of the planning horizon. In contrast, the alternative balancing objective function investigated in this paper, *minmax*, minimizes the maximum operator utilization factor. The resulting ILP model thus differs from the one stated above in the kind of optimization to be pursued (minimization rather than maximization), and in the form of constraints (17), that is to say:

$$\begin{array}{ll} \min & m \\ & (1) - (15) \\ & \dfrac{\sum_{d \in W} D_{\omega d}}{|W| \cdot D_\omega} \leq m, \quad \forall \omega \in O. \end{array}$$

In real home care instances usually the set of the patients is clusterised. Precisely, patients located in the same municipality or in a small rural area form a cluster of nodes, such that the distance between any pair of nodes within the cluster is very small compared to the distance between nodes belonging to different clusters. Such intra-cluster distances can be usually assumed equal to a certain (small) value $\delta$. In addition, the inter-cluster distances depend only on the clusters, and not on the specific patients within the clusters. For example, if patients $i$ and $j$ belong to a certain cluster (or municipality) $C_1$, while patients $v$ and $w$ are resident in $C_2$, then the distance between $i$ and $v$ is equal to the distance between $j$ and $w$, since it only depends on the considered municipalities. This is the case of the Palliative Home Care dataset we used. Hereafter these instances will be referred to as *Clusterized Home Care instances*. These clusterized instances may have a huge number of symmetric feasible solutions, due to the structure of their logistics network. This, in turn, may slow the solution of the ILP models. To overcome these difficulties, we studied the following families of *symmetry management valid inequalities*, which are tailored to Clusterized Home Care instances (in these inequalities $C$ denotes a cluster):

$$x_{ij}^{\omega d} = 0 \qquad \forall d \in W, \forall \omega \in O_d, \forall C, \forall i, j \in C : i > j. \tag{18}$$

Due to the distance structure, any permutation of a subset of patients belonging to the same cluster induces the same identical cost contribution to the objective function. Therefore, the aim of inequalities (18) is to break these symmetries by forcing an ordered visit of the patients within each cluster $C$. As shown in Section 7, (18) proved to be crucial for the efficiency of the optimization process on PHCP instances. Furthermore, the following inequalities have been proposed and computationally analysed to take under control the length of the operator tours:

$$\sum_{i \in (N \setminus C)} \sum_{j \in C} x_{ij}^{\omega d} \leq 1 \qquad \forall C, \forall d \in W, \forall \omega \in O_d. \tag{19}$$

Inequalities (19) impose that, for each cluster of patients $C$, and in each day of the planning horizon, each available operator enters $C$ at most once. Hereafter these cuts will be referred to as *Exploit-Cluster*, or simply *EC*. (19) model a reasonable requisite in PHCP, since for the peculiar structure of the logistics network, and in the absence of time windows associated with the patient visits, stakeholders usually require that operators visit each municipality at most once per day.

# 4 Comparison with models from the literature

Before presenting some model extensions allowing to cope with different home care scenarios, here we shall better explain the main similarities and dissimilarities between our models and the two most recent works in the literature dealing with a weekly planning horizon in Home Care, i.e. [17] and [22]. The comparison is shown synthetically in Figure 1, where the columns represent the three afore mentioned studies, while each row describes a particular issue of the problem addressed. It is worth noticing that both in [17] and [22] more than one problem is addressed depending on the typology of the decisions involved (tactical vs operational decisions). Here the comparison is done between our models and the most similar problem studied in [17] and [22]. In fact, in their paper Nickel et al. [22] distinguish between two cases: the case in which the provider needs to construct the next week schedule from scratch, and the case in which the provider prefers the application of the same basic or master schedule from week to week, in order to ensure continuity of the service, also called time continuity. This last problem is referred to as Master Schedule Problem (MSP). MSP is less complicated than the former problem, since several decisions and constraints are relaxed. In the table, the column corresponding to [22] refers to the construction of the schedule from scratch which is also the case addressed in our study. Similarly, in Jensen [17] two problems are addressed: the MSP problem and a simplified version of MSP. In the table, the column corresponding to [17] refers to the former problem, i.e. to the most general one.

| | **Our study** | **Nickel [22]** | **Jensen [17]** |
|---|---|---|---|
| **Approach (joint vs decomposition)** | Joint | Decomposition | Decomposition |
| **Scheduling decisions (tool used)** | Pattern | Shift combination | No specific tools: satisfaction of time lags constraints |
| **Time windows** | * | Hard TW associated with operators and patients | Hard: days organized in not-overlapping TW within which visits must occur |
| **Overtime** | * | Allowed and penalized (soft) | Not specified |
| **Care Continuity** | Hard | Patient-nurse loyalty (soft) | Employee regularity (soft) |
| **Covering of demand** | Guaranteed (hard) | Not guaranteed (soft) | Guaranteed (hard) |
| **Patient-operator compatibility** | Hierarchical skills | Set of qualifications for operators<br>Single skill for visits | Prioritized list (soft) |
| **Objective function** | Balancing | Combination of :<br>number of unscheduled visits<br>patient-nurse loyalty penalty<br>overtime penalty<br>total distance traveled | Combination of:<br>total travel time<br>employee regularity<br>time continuity<br>employee priority<br>busyness |

Figure 1: Comparison with the literature (a "*" denotes an issue that can be managed)

In regards to the comparison between PHCP and [22], the main differences concern: ($i$) the time windows, ($ii$) the overtime, and ($iii$) the fact that in [22] it is not mandatory to assign each patient job to an operator. In contrast the main similarities regard: ($i$) the tool used to address scheduling decisions (pattern vs shift combination); ($ii$) the care continuity management; ($iii$) the compatibility constraints between qualifications required by the visits and those owned by the operators. Specifically, in [22] each operator provides a set of qualifications, while each job requires a specific qualification; then the assignment of an operator to a job $j$ is feasible if the operator provides the qualification required by $j$; notice that such a qualification mechanism differs from ours since it is not hierarchical (although, as indicated in [22], typically the qualification levels are hierarchical), and since more qualifications can be associated with an operator. Model extensions able to cope with this alternative form of qualification mechanism will be addressed in Section 5.

In regards to the comparison between PHCP and [17], the main differences concern the time windows associated with each visit and with each operator. Specifically, for each operator a number of intervals in which the operator is on duty per day is supported; usually there are two such intervals, one before and one after the lunch break. Model extensions able to cope with this form of time windows will be addressed in Section 5. In contrast, the main similarities regard: (*i*) the compatibility constraints between visits and operators, expressed by prioritized lists associated with visits; (*ii*) the management of visit frequency; and (*iii*) the care continuity management. Concerning the frequency of the visits, a time lag is associated with each visit, which specifies the distance in time between the starting time of two visit repetitions. Indeed, time lags are not addressed in our models in an explicit way; anyway, observe that patterns allow to incorporate the requirement of time distances between visits; for example, if the provider does not want to visit a patient on two consecutive days, then it is sufficient not to include such kind of visiting templates into the set of patterns $P$. Also observe that the care continuity addressed in [17] is a relaxation of the one considered in PHCP.

Besides the MSP discussed above, Jensen proposed a Branch and Price approach for a simplified version of MSP where the time continuity objective is disregarded and QoS requisites related to the feasibility of the planned tours and the compatibility between the qualification of operators and visits are relaxed and introduced, in a soft way, into the objective function. The adoption of this approach allowed to solve instances with up to 55 visits, whereas CPLEX was unable to solve an instance of the simplified MSP with more than 11 visits. In any cases, the instances are characterized by one or two operators. On larger instances the Branch and Price algorithm, however, exceeded 20 CPU hours by reporting percentage optimality gaps up to about 8%. This is mainly due to the large number of visit combinations to be taken into account. Such a behavior outlines that selecting suitably combinations of the patient visits, or patterns in our terminology, is crucial to the efficiency of the Home Care optimization process. This will be the subject of Section 6.

# 5 Model extensions

Here we shall present some relevant extensions to the ILP Home Care models introduced in Section 3.

## 5.1 Overtime handling

Overtime can be handled by replacing constraints (7) by:

$$D_{\omega d} = \sum_{(i,j)\in A} (t_{ij} + t'_j) \cdot x_{ij}^{\omega d} \leq D_\omega + Over_{\omega d} \qquad \forall d \in W, \forall \omega \in O_d \tag{20}$$

$$c(\sum_{d\in W} \sum_{\omega \in O_d} Over_{\omega d}) \leq B, \tag{21}$$

where $Over_{\omega d}$ are newly introduced variables, which specify the overtime of operator $\omega$ on day $d$, $c$ denotes the unit overtime penalty, and $B$ is the budget which is available for overtime. Notice that, by replacing (7) by (20) − (21) within the stated ILP models, then overtime compensation within a week is allowed, since the weekly workload $\sum_{d\in W} D_{\omega d}$ contributes to the definition of the operator utilization factor.

## 5.2 Skills not organised hierarchically

As in Section 3, let $K$ be the set of $\overline{k}$ skills characterizing both patients and operators. However, consider now the situation where the skills are not organised hierarchically. Let $S_\omega$ denote the set of the skills characterizing the abilities of operator $\omega$: as in [22], assume that a visit of skill $k$ can be operated by $\omega$ only if $k \in S_\omega$. By defining a pattern $p$ as before, i.e. $p(d) = 0$ if no service is offered at day $d$, while $p(d) = k$ if a visit of skill $k$ is operated according to pattern $p$ on day $d$, then the alternative skill organisation can be modelled by replacing constraints (2) by:

$$\sum_{i \in N} \sum_{\omega : k \in S_\omega} x_{ij}^{\omega d} \geq \sum_{p:p(d)=k} z_{jp} \qquad \forall j \in N \setminus \{0\}, \forall d \in W, \forall k \in K \tag{22}$$

Skill organisations of this type have been introduced in [7] for skill-based routing problems.

## 5.3 Time windows

First observe that the management of two time windows per operator, i.e. a morning and an afternoon shift, can be easily handled by the proposed ILP models by replacing each single original operator with two copies of it, one relative to the morning shift and the other relative to the afternoon shift (however, the utilization factor of the operators must be still computed operator-wise, i.e. considering the two copies of each original operator together). Splitting each operator therefore allows us to manage loose time windows, simply guaranteeing the compatibility between a shift (morning or afternoon) and a visit.

Assume now that a hard time window, say $[a_h^d, b_h^d]$, is associated with patient $h$ on day $d$. Then, by extending the modelling concepts introduced in [7], we can perform a disaggregation of the flow variables $y_{ij}^d$ by patient. Specifically, replace the flow variables $y_{ij}^d$ (definition (15)) by the disaggregated flow variables $y_{ij}^{hd}$, $h \in N \setminus \{0\}$, that specify the amount of flow traversing the arc $(i, j)$ on day $d$ that is destined to node $h$. Using these disaggregated flow variables, then it is possible to replace constraints (9)-(11) and (15) by:

$$\begin{aligned}
&\sum_{i \in N} y_{ij}^{hd} - \sum_{i \in N} y_{ji}^{hd} = \begin{cases} \sum_{p:p(d)\geq 1} z_{hp} & \text{if } j = h \\ 0 & \text{if } j \neq 0, h \\ -\sum_{p:p(d)\geq 1} z_{hp} & \text{if } j = 0 \end{cases} && h \in N \setminus \{0\},\ d \in W \\
&y_{ij}^{hd} \leq \sum_{\omega \in O_d} x_{ij}^{\omega d} && (i,j) \in A, h \in N \setminus \{0\},\ d \in W \qquad (23) \\
&y_{ij}^{hd} \geq 0 && (i,j) \in A,\ h \in N \setminus \{0\},\ d \in W
\end{aligned}$$

The disaggregated flow constraints (23) state that one unit of flow must be sent from node 0 to patient $h$ on day $d$, if a visit has been scheduled for $h$ on that day. Thus, the flow variables $y_{ij}^{hd}$ can be reinterpreted as indicating whether the arc $(i, j)$ is in the path from node 0 to patient $h$ on day $d$. By exploiting such disaggregated variables, the time window constraints can be expressed as:

$$a_h^d \sum_{p:p(d)\geq 1} z_{hp} \leq \sum_{(i,j)\in A} (t'_i + t_{ij}) y_{ij}^{hd} \leq b_h^d \sum_{p:p(d)\geq 1} z_{hp} \qquad \forall h \in N \setminus \{0\},\, d \in W. \tag{24}$$

### 5.4 More visits per day

By considering the model comparison in Section 4, it may be worthy to observe that, since jobs respectively visits are scheduled in [17] and [22] rather than patients, then more visits per day can be planned for some patients in the corresponding solutions. More visits per day can indeed be handled also by the ILP formulations in Section 3. To this end, just modify the home care graph interpretation, so that each node in $N \setminus \{0\}$ represents a job or a visit rather than a patient. This modelling interpretation will be experimented in Section 7.

## 6 Pattern generation policies

Three policies to generate the set of patterns $P$ (see Section 3) have been analyzed:

1. a greedy heuristic procedure, $Heur$, which is based on the frequency of the request types: it firstly orders the patient requests according to their numbers of requirements for increasing levels of skill and then, by scanning the ordered list, generates patterns that can accomplish with the frequency of such requests;

2. a procedure based on the extraction of pattern information from the solution actually implemented at the Palliative Home Care provider; this will be referred to as $ImplSol$;

3. a flow based model, or $FB$, described in the following in details.

**Flow based patterns.** As a third approach to generate a set of a priori patterns, let us consider an auxiliary layered network $G_W = (N_W, A_W)$ with $|N_W| = n_w$, having one layer $L_d$ for each considered day $d$ in the planning horizon $W$, plus a source node and a destination node. For matter of convenience the source node will be denoted by index 1 and the destination by $n_w$. Each layer is composed of $\overline{k}+1$ nodes: node 0, which indicates that no visit is scheduled in the day corresponding to the layer, and a node $k$, for each $k \in K$, which represents the scheduling of a visit of skill $k$. In $G_W$ there exists a directed arc from the source node to the nodes in first layer, from each node in the last layer to the destination node, and from each node in layer $L_d$ to each node in the layer corresponding to the day after $d$ (if any), for each $d \in W$. The figure below provides a picture of this auxiliary layered network in the case of a weekly planning horizon.

Any directed source-destination path in $G_W$ corresponds to a potential pattern. Therefore, let us introduce a binary commodity for each patient $j$, having node 1 as the origin and node $n_w$ as its destination, and state the following auxiliary multicommodity flow problem on $G_W$ as a tool to generate a set of feasible patterns:

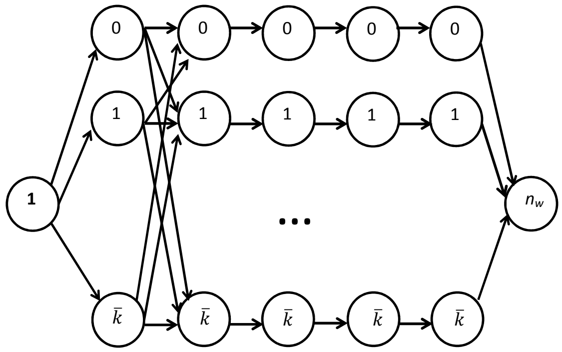


Figure 2: The auxiliary layered network

$$\min \sum_{(h,i)\in A_W} q_{hi}$$

$$\sum_{(h,i)\in A_W} f^j_{hi} - \sum_{(i,h)\in A_W} f^j_{ih} = \begin{cases} -1 & \text{if } i = 1 \\ 1 & \text{if } i = n_w \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in N_W, \forall j \in N \setminus \{0\} \tag{25}$$

$$\sum_{d\in W} \sum_{(h,k):k\in L_d} f^j_{hk} = r_{jk} \qquad \forall j \in N \setminus \{0\}, \forall k \in K \tag{26}$$

$$\sum_{j\in N\setminus\{0\}} t'_j \sum_{k'\geq k} \sum_{(h,k'):k'\in L_d} f^j_{hk'} \leq \sum_{\omega\in O_d:s_\omega\geq k} D_\omega \qquad \forall d \in W, \forall k \in K \tag{27}$$

$$\sum_{j\in N\setminus\{0\}} f^j_{hi} \leq n q_{hi} \qquad \forall (h,i) \in A_W \tag{28}$$

$$f^j_{hi} \in \{0,1\} \qquad \forall (h,i) \in A_W, \forall j \in N \setminus \{0\} \tag{29}$$

$$q_{hi} \in \{0,1\} \qquad \forall (h,i) \in A_W \tag{30}$$

For each patient $j$, the multicommodity flow variables $f^j_{hi}$ model a directed path in the layered graph, which starts from node 1 and ends to $n_w$ (see (25)). These variables satisfy conditions (16), thanks to constraints (26), and therefore they model a feasible pattern for $j$, that is a pattern which is compatible with the care plan of $j$. Constraints (27) take into account, skill by skill, the operators availability in each day of the planning horizon. In fact, they impose that the total service time of scheduled visits of skill $k$ per day does not exceed the daily availability in that day of the set of the operators of skill at least $k$. Finally, constraints (28) link together the flow variables $f^j_{hi}$ with the design variables $q_{hi}$, and they guarantee that if arc $(h,i)$ is used, i.e. $q_{hi} = 1$, it can be crossed by any number of patients, whereas if that arc is not used, i.e. $q_{hi} = 0$, then it cannot be traversed.

Such auxiliary variables $q_{hi}$ are introduced to discover which arcs are used to design the patterns: by minimizing the total number of used arcs, the model thus tends to minimize, in an implicit way, the number of generated patterns.

It is worth saying that, in the experimental campaign related to PHCP, this flow based policy has been used in combination with a parameter that reduces the right-hand-side (r.h.s.) in constraints (27). In fact, since the flow based model neglects the traveling times, it may occur that the patterns thus provided generate an infeasible solution when the routing issue is also considered. A reduction of the r.h.s. in constraints (27) is then used as a means to prevent some undesirable infeasibilities. Specifically, in the experimental campaign three values of the aforesaid parameter are considered: 0.5 which halves the r.h.s., 0.75 which reduces the r.h.s. by 25%, and 1 which maintains the r.h.s. as stated in (27). These policies will be referred to as *FB-0.50*, *FB-0.75* and *FB-1.00*, respectively.

# 7 Computational results

We generated a set of PHCP instances starting from real data, and analyzed the behavior of the proposed ILP models depending on: (*i*) pattern characteristics; (*ii*) kind of objective function selected; (*iii*) symmetry management; and (*iv*) management of care continuity constraints.

We also analyzed both the computational efficiency and the quality of the solutions obtained in terms of operator utilization factor and travel time. Moreover, we tested two real-world data sets from the literature. A summary on the main achievements of our computational campaign will be provided in Section 7.7.

## 7.1 The PHCP data set

The real data have been provided by one of the largest Italian public medical care unit, mainly devoted to palliative care and terminal patients, and they have been already used in [19]. An instance of PHCP is characterized by: (*i*) the geographical area where the service is provided; (*ii*) a time period; (*iii*) the set of patients; (*iv*) the set of operators; (*v*) the set of patterns. The considered provider operates in the north of Italy and its services cover a region of about 800 km$^2$ that is organized in three divisions. In turn, each division is organized in districts. The instances we used consider the largest district of the Merate area which corresponds to the largest division in the considered area and comprises 10 municipalities. As already indicated, two skills are considered for operators and patient requests: *ordinary*, corresponding to skill 1 in the proposed models, and *palliative*, corresponding to skill 2. In regards to the patients, we selected two weeks in the historical database that are considered by the provider representative of typical situations, i.e. a week in January 2006 (hereafter denoted by *January 2006*) and a week in April 2007 (hereafter denoted by *April 2007*). Patient demands had been computed by looking at the scheduling implemented by the provider in the period considered: specifically, for each skill, the requested number of visits in our instances is set equal to the real number of visits performed by operators of that particular skill. This choice is supported by the fact that the provider never used operators with skill different from the skill required by a visit.

For each selected week we generated both a complete instance involving all the patients requiring a service in that week and restricted smaller instances comprising subsets of municipalities and patients. Specifically, the restricted instances are characterized approximately by either one third or one half of the patients present in the complete instance. The dimension of the smaller instances is however considered representative of real application contexts. Summarizing, for the January 2006 week, the number of patients is 40, 60 or 128, whereas for the April 2007 week, the number of patients is 50, 80 or 162. For each week the biggest number of patients refers to the complete instance. In regards to the municipalities, in the complete instance related to January 2006 all the 10 municipalities are considered, whereas in the complete instance related to April 2007 there were no requests regarding two of the 10 municipalities. On the contrary, the restricted instances comprise either 5 or 8 municipalities. The district under consideration is characterized by 11 operators, 8 of which of skill 1 and 3 of skill 2. The workday durations ($D_t$) are 4, 6 or 8 hours. In restricted instances the number of operators depends on the number of patients considered. Specifically, when the number of patients is 40, 4 operators are chosen; when the number of patients is 50 or 60, the corresponding number of operators is fixed to 5, while for 80 patients 6 operators are selected. In all the restricted instances only one operator with skill 2 and a workday duration of 6 hours is considered, while the remaining operators are all characterized by a workday duration of 8 hours and skill 1. For a given combination of number of municipalities, number of patients and number of operators, three restricted instances are generated by randomly selecting the desired number of patients among the available patients. The instances are thus identified by a string reporting the following fields separated by a "-" character: the week, the number of municipalities, the number of patients, the number of operators, and the instance identifier in the group (i.e. 0, 1 or 2). As an example "0106-5-40-4-0" refers to the week in January 2006, 5 municipalities, 40 patients, 4 operators, and instance number 0.

Summarizing, for each of the two weeks, 12 restricted instances are generated in addition to the complete instance. In fact, 2 values for the number of municipalities are combined with 2 values for the number of patients and for each of these combinations 3 instances are generated, thus giving rise to 12 instances for each week. The total number of instances is thus equal to 24 restricted instances + 2 complete instances (i.e. the biggest ones). These instances are available at `http://www.di.unipi.it/optimize/`.

In all the generated instances, the traveling times $t_{ij}$ have been computed via Google Maps for the inter-municipalities distances, while they have been set equal to 3 minutes ($\delta = 3$) for the intra-municipalities distances, consistently with the provider indications. Furthermore, according to the medical care unit indications the service time has been fixed to 30 minutes (i.e. $t'_j = 30$ min), while the maximum number of operators per patient has been initially set to the value 2 (i.e. $T = 2$). Summary information of the complete instances are given in Table 1, while for restricted instances the information about number of patients, municipalities and operators are contained in the string used to denote the name of the instance as described above.

In regards to the patterns, which are a peculiarity of our approach, the generation policies described in Section 6 have been used. These policies may produce a number of patterns very different the one from each other; these values are reported in Table 2 and commented hereafter.

In addition to the pattern selection, the other two features that characterize our approach to solve PHCP are the selection of the objective function (*maxmin* vs *minmax*) and the management of symmetries. Moreover, the models can be equipped with EC constraints or not; the impact of these cuts has been evaluated experimentally only in combination with *maxmin*, since in this case the cost control may be more critical than in the minmax case.

Summarizing, the experimental campaign analyzes the behavior of the models on the considered 26 PHCP instances both in terms of efficiency and quality of the solutions provided

Table 1: Instances features

| Week | Patients | Operators | Municipalities |
|---|---|---|---|
| January 2006 | 128 | 11 | 10 |
| April 2007 | 162 | 11 | 8 |

Table 2: Number of patterns used

| Week | Heur | ImplSol | FB-0.50 | FB-0.75 | FB-1.00 |
|---|---|---|---|---|---|
| January 2006 | 20 | 29 | 13 | 11 | 11 |
| April 2007 | 27 | 33 | 17 | 14 | 14 |

when all of the pattern generation policies (5 choices) are combined with the two alternative objective functions, the symmetries are managed or not by means of constraints (18) and the cluster structure is exploited or nor by means of constraints (19).

The experiments have been performed on a AMD Opteron(tm) Dual Core Processor 246 (CPU MHz 1991.060). The solver is CPLEX 12.4 with a time limit of 12 hours and a memory limit for the branch and bound tree of 1 GB. In the following the computational times are expressed in seconds of CPU time.

## 7.2 Assessing the impact of pattern selection

In this section a comparison among the different pattern selection strategies is done on the biggest instances in the test bed, i.e. the complete ones, when the maxmin objective function is used. The impact of both symmetry management and EC constraints is evaluated. Computational results for the minmax case are not reported since for the biggest instances the solver failed to provide a feasible solution within the time or memory limit imposed in the experimental campaign.

Tables 3 and 4 report, respectively for the maxmin and the maxmin with EC variants, the performance of the models in terms of time required to solve the Linear Programming relaxation (LPTime) and in terms of LP objective function (LPValue).

For both weeks the impact of the pattern generation policies (see the last 5 columns of Tables 3 and 4) is evaluated against the symmetry management (see the rows).

It is worth observing the following two facts: (*i*) the introduction of the EC constraints makes the model more difficult to solve; (*ii*) the flow based policy with parameter set to 0.50 exhibits an LPTime that is significantly smaller than the one required by policies Heur and ImplSol and it is a viable option also to address the EC variant. While the LPTime can almost reach the time limit for Heur policy, it is at most about one hour for FB-0.50 which is characterized by a much smaller number of generated patterns with respect to the policies Heur and ImplSol (see Table 2). Determining a good and limited set of patterns seems thus to be crucial in our study. Particular attention should be given to the LPValue: observe that, when EC are not inserted, the LPValue is the same (when available) for policies Heur, ImplSol and FB-0.50, thus suggesting that the selected patterns are sufficient to obtain almost the same solution quality (but, as observed, with very different computational times).

Table 3: Maxmin - LP results

| | | | Heur | ImplSol | FlowBased 0.50 | FlowBased 0.75 | FlowBased 1.00 |
|---|---|---|---|---|---|---|---|
| January 2006 | NoSymm | LPTime | 9329.13 | 18031.67 | 1235.19 | 210.89 | 258.93 |
| | | LPValue | 0.3552 | 0.3552 | 0.3552 | 0.2922 | 0.2922 |
| | Symm | LPTime | 10487.83 | 14097.05 | 1049.34 | 107.32 | 193.21 |
| | | LPValue | 0.3552 | 0.3552 | 0.3552 | 0.2922 | 0.2922 |
| April 2007 | NoSymm | LPTime | 21292.40 | 17638.89 | 1973.86 | 1491.31 | 585.62 |
| | | LPValue | 0.5393 | 0.5393 | 0.5393 | 0.5393 | inf |
| | Symm | LPTime | 11154.98 | 13355.74 | 1557.08 | 1359.73 | 446.32 |
| | | LPValue | 0.5393 | 0.5393 | 0.5393 | 0.5393 | inf |

Table 4: Maxmin with EC - LP results

| | | | Heur | ImplSol | FlowBased 0.50 | FlowBased 0.75 | FlowBased 1.00 |
|---|---|---|---|---|---|---|---|
| January 2006 | NoSymm | LPTime | 14660.29 | 18781.83 | 1964.06 | 201.69 | 951.19 |
| | | LPValue | 0.3552 | 0.3552 | 0.3452 | 0.2922 | 0.2922 |
| | Symm | LPTime | 10984.84 | 20297.67 | 2623.68 | 288.76 | n.a. |
| | | LPValue | 0.3552 | 0.3552 | 0.3452 | 0.2922 | 0.2922 |
| April 2007 | NoSymm | LPTime | 27802.37 | 30989.28 | 4132.02 | 2933.63 | 813.12 |
| | | LPValue | 0.5382 | 0.5382 | 0.5334 | 0.5251 | inf |
| | Symm | LPTime | 41295.70 | 27141.74 | 3001.12 | 2511.57 | 668.26 |
| | | LPValue | 0.5382 | 0.5382 | 0.5334 | 0.5251 | inf |

On the contrary, it may happen that equal sized sets of patterns provide different continuous solution values. This is the case for example of the selected week in April 2007, where FB-0.75 and FB-1.00 both return a set of 11 patterns. However, while the patterns obtained with policy FB-1.00 provides an infeasible solution regardless of the symmetry management (string *inf* in the presented tables), all of the other policies give the same solution value.

The highlights above mentioned become even more evident when integer solutions are analyzed. In Tables 5 and 6, respectively for the maxmin and the maxmin with EC cases, the results obtained when the integer problem is solved are reported for all the combinations of pattern selection policies and symmetry management in terms of percentage relative gap computed with respect to the best upper bound obtained in the branch and bound tree (string "n.a." is used to point out that no integer solution is found). The stopping criterion that determines the termination of the algorithm is also given in columns Stop, where the character "T" is used to indicate that the time limit has been exceeded while a "M" is used to indicate an out of memory condition. Specifically, the FB-0.50 policy is able to find solutions very close to the best upper bound even when the EC are considered, while the other policies often exceed the time limit without reporting a feasible solution.

Tables 5 and 6 also reveal that symmetry management is, almost everywhere, a very effective tool to reduce the optimality gap and even to find feasible solutions for the most critical approach combinations. Indeed, the symmetry management used in combination with Heur policy allows to more than halve the gap when the base maxmin case is considered; moreover, in the maxmin with cuts case, for the January 2006 instance, the symmetry management allows to obtain a feasible solution with a gap equal to 3.22% whereas no feasible solutions are obtained without the symmetry management. On the contrary the impact of

Table 5: Maxmin - ILP results

| | | Heur | | ImplSol | | FlowBased 0.50 | | FlowBased 0.75 | | FlowBased 1.00 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | %Gap | Stop | %Gap | Stop | %Gap | Stop | %Gap | Stop | %Gap | Stop |
| January 2006 | NoSymm | 1.74 | T | 0.66 | T | 0.07 | T | n.a. | T | n.a. | T |
| | Symm | 0.66 | T | 0.30 | T | 0.18 | M | n.a. | T | n.a. | T |
| April 2007 | NoSymm | 1.59 | T | 1.99 | M | 0.17 | M | 0.17 | M | n.a. | inf |
| | Symm | 0.69 | T | n.a. | T | 0.22 | T | 0.22 | M | n.a. | inf |

symmetries on ImplSol policy is less clear since there are cases where their introduction worsens the performance obtained.

However, it is worth observing that the symmetry management is not an issue when the LP problem can be solved efficiently, as it happens with the flow based policies. In addition, whereas the FB-0.50 policy provides very good results (without the help of the symmetry management tool), and thus seems to be a viable option to address also the most difficult instances, the flow based policy fails almost everywhere to find feasible solutions when the number of patterns is too small (parameter set to 0.75 and 1). For this reason the last two policies will not be commented further in the following.

We also observed that the symmetry management generally allows one to explore more branch and bound nodes, and that the very good solutions found by policy FB-0.50 are computed quite early within the branch and bound approach. Details on this aspect can be found in the Online Supplement. There, comments and explanations in terms of dimension of the solved problems are also reported. In particular, the symmetry management allows to reduce the LP dimension. However, despite the reduction in terms of constraints and variables, the time required to solve the LP may increase when the symmetries are treated. Anyway Tables 5 and 6 show that the symmetry management is crucial almost everywhere either to halve the ILP gap or to find feasible solutions in the most critical cases.

In regards to the quality of the solutions we report that on instance January 2006 the introduction of EC constraints allows to halve the gap between the most and the least loaded operators, thus providing more balanced solutions. In addition, the total travel time spent weekly by the operators decreases from 3120 minutes in the base maxmin case to 2849 minutes when EC are introduced, whereas the weekly number of crossings between different municipalities decreases from 296 to 267. On April 2007 instance the figures are the following: the total travel time is 3734 in the base maxmin case and 3598 in the maxmin with EC case, whereas the numbers of crossings are respectively 321 and 297. Indeed, the number of crossings represents a quality indicator: the bigger it is the greater the occurrence of operators traveling among municipalities solely to increase their utilization factor. The EC constraints seem thus to represent an effective tool to control the travel time; motivated by these promising results, from now on we assume that, under *maxmin*, the models are always equipped with the EC constraints.

Table 6: Maxmin with EC - ILP results

| | | Heur | | ImplSol | | FlowBased | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | 0.50 | | 0.75 | | 1.00 | |
| | | %Gap | Stop | %Gap | Stop | %Gap | Stop | %Gap | Stop | %Gap | Stop |
| January 2006 | NoSymm | n.a. | T | 3.09 | T | 0.17 | T | n.a. | T | n.a. | T |
| | Symm | 3.22 | T | n.a. | T | 0.17 | T | n.a. | T | n.a. | T |
| April 2007 | NoSymm | n.a. | T | n.a. | T | 1.29 | T | 0.82 | T | n.a. | inf |
| | Symm | n.a. | T | n.a. | T | 0.25 | M | 0.25 | T | n.a. | inf |

## 7.3 Assessing the impact of the alternative objective functions

In this section we analyze the impact of the two objective functions on computational efficiency. The computational results are related to the 24 restricted instances. Specifically, Table 7 refers to the minmax objective function and it reports a comparison among the pattern generation policies Heur, ImplSol and FB-0.50 in terms of time required to solve the linear programming relaxation (LPTime), time required to solve the integer problem (IPTime) and gap of the best feasible solution with respect to the best bound computed in the branch and bound tree (%Gap). Finally, Table 8 shows the performance of the maxmin model when the FB-0.50 pattern generation policy is used: the computational results presented in previous sections, in fact, indicate that such a policy is the best performing among the investigated ones.

The computational results presented in Tables 7 and 8 allow to draw the following conclusions: (*i*) the maxmin objective function makes the problem significantly easier to solve than the minmax one; (*ii*) the FB-0.50 seems to be crucial also to address the minmax objective function.

Indeed, for the maxmin case (Table 8) the time limit is exceeded only for three of the restricted instances and the optimality gap is very close to or zero everywhere. Moreover, for 13 instances *maxmin* returns a gap $\leq 0.57\%$ in less than 1.5 hours. In regards to the goodness of the flow based policy, it is worth observing that, for the minmax case (Table 7), when FB-0.50 is used the percentage gap is below 0.53 for 17 over 24 instances. In addition, always looking at Table 7, the computational time increases remarkably with the number of patients, quite independently of the number of municipalities. The instances with 80 patients seem thus to be the most difficult in the test bed, and Heur and ImplSol fail to provide a feasible solution for all of them. Conversely, FB-0.50 allows to find good quality feasible solutions for 5 over 6 of these difficult instances. As for the complete instances, the computational results relative to *minmax* thus corroborate the efficacy of the flow based pattern selection strategy to address hard settings. We also observe that the value of the best integer solution (IPValue) returned by FB-0.50 is not greater than the one given by the other two policies except that in two cases and it can be remarkably smaller. Table 7 also reveals that the smaller instances in the test bed are by no way trivial to solve (see the results relative to the 0106-5-40-4-0 instance). Additional information about the efficiency of the approach for the *minmax* objective function can be found in the Online Supplement.

Table 7: minmax - LP and ILP results

| | Heur | | | ImplSol | | | FB-0.50 | | |
|---|---|---|---|---|---|---|---|---|---|
| | LPTime | IPTime | %Gap | LPTime | IPTime | %Gap | LPTime | IPTime | %Gap |
| 0106-5-40-4-0 | 265.46 | 43200.00 | 14.22 | 332.95 | 43200.00 | 15.53 | 19.92 | 43200.00 | 6.41 |
| 0106-5-40-4-1 | 16.90 | 3045.74 | 0.00 | 109.84 | 25485.74 | 0.25 | 8.82 | 279.49 | 0.53 |
| 0106-5-40-4-2 | 35.65 | 6371.47 | 0.00 | 38.01 | 43200.00 | 6.48 | 6.07 | 380.96 | 0.00 |
| 0106-8-40-4-0 | 1098.74 | 15072.77 | 0.00 | 984.01 | 43200.00 | 0.88 | 4.15 | 1946.90 | 0.00 |
| 0106-8-40-4-1 | 47.41 | 2924.98 | 0.00 | 31.24 | 43200.00 | 3.81 | 0.87 | 113.01 | 0.00 |
| 0106-8-40-4-2 | 367.71 | 2006.08 | 0.15 | 56.25 | 7242.51 | 0.00 | 1.06 | 295.91 | 0.00 |
| 0106-5-60-5-0 | 336.28 | 43200.00 | 9.78 | 485.77 | 43200.00 | 11.88 | 23.07 | 7454.13 | 0.38 |
| 0106-5-60-5-1 | 285.47 | 43200.00 | 9.18 | 692.85 | 43200.00 | 28.47 | 6.83 | 3169.17 | 0.32 |
| 0106-5-60-5-2 | 558.72 | 43200.00 | 17.21 | 456.01 | 43200.00 | 13.01 | 89.63 | 16959.97 | 0.50 |
| 0106-8-60-5-0 | 370.14 | 43200.00 | 23.74 | 276.88 | 43200.00 | 19.31 | 4.75 | 1086.50 | 0.00 |
| 0106-8-60-5-1 | 338.93 | 43200.00 | 13.31 | 462.46 | 43200.00 | 7.51 | 33.61 | 5470.26 | 0.23 |
| 0106-8-60-5-2 | 196.77 | 43200.00 | 10.78 | 274.80 | 43200.00 | 16.65 | 4.14 | 3570.36 | 0.00 |
| 0407-5-50-5-0 | 141.63 | 9644.53 | 0.25 | 302.23 | 11331.98 | 0.45 | 9.97 | 685.17 | 0.07 |
| 0407-5-50-5-1 | 1820.38 | 43200.00 | 20.92 | 1955.30 | 43200.00 | 21.64 | 184.77 | 43200.00 | 11.01 |
| 0407-5-50-5-2 | 106.74 | 22949.32 | 0.00 | 150.03 | 15451.70 | 0.00 | 235.35 | 787.64 | 0.31 |
| 0407-8-50-5-0 | 76.28 | 10171.30 | 0.27 | 81.94 | 9626.55 | 0.00 | 108.61 | 4306.24 | 0.00 |
| 0407-8-50-5-1 | 69.53 | 35082.61 | 0.00 | 37.93 | 38268.08 | 0.00 | 379.89 | 2013.19 | 0.00 |
| 0407-8-50-5-2 | 2564.60 | 43200.00 | 51.41 | 2110.19 | 43200.00 | 57.76 | 112.94 | 43200.00 | 9.70 |
| 0407-5-80-6-0 | 1720.48 | 43200.00 | n.a. | 1633.70 | 43200.00 | n.a. | 235.88 | 39240.70 | 0.00 |
| 0407-5-80-6-1 | 1686.18 | 43200.00 | n.a. | 11162.44 | 43200.00 | n.a. | 164.30 | 43200.00 | 18.78 |
| 0407-5-80-6-2 | 3183.52 | 43200.00 | n.a. | 2254.02 | 43200.00 | n.a. | 91.10 | 32900.24 | 0.00 |
| 0407-8-80-6-0 | 519.95 | 43200.00 | n.a. | 2243.43 | 43200.00 | n.a. | 38.02 | 43200.00 | n.a. |
| 0407-8-80-6-1 | 3266.31 | 43200.00 | n.a. | 3048.89 | 43200.00 | n.a. | 100.56 | 43200.00 | 6.62 |
| 0407-8-80-6-2 | 1244.38 | 43200.00 | n.a | 999.79 | 43200.00 | n.a. | 65.11 | 43200.00 | 1.02 |

## 7.4 Assessing the solution quality

Figures 3 and 4, respectively for the restricted instances related to January 2006 and April 2007, provide some information on the quality of the solutions obtained in terms of operator utilization factor and travel time. Specifically, we considered the solutions obtained with the FB-0.50 policy and in each figure, for each instance, the results obtained for *maxmin* and *minmax* are reported consecutively so as to facilitate the comparison. The figures report the minimum, the maximum, the average operator utilization factor (min UF, max UF, avg UF) and the minimum, the maximum, the average fraction of travel time with respect to the maximum weekly workload (min TTF, max TTF, avg TTF), computed over the available operators. The utilization factor is computed as the weekly time an operator globally spends in servicing the patients and traveling among them over the weekly availability time (as defined in the l.h.s. of constraints (17)).

As expected, we observe that the minmax case consumes less travel time than the maxmin counterpart to visit the patients. Anyway the cost savings are not particularly high but on some instances. Conversely, it is worth observing that these savings seem to happen at the expenses of a worse balancing of the solutions in terms of operator utilization factor, i.e. in terms of the difference between the most and the least loaded operators. Specifically, in regards to restricted instances related to January 2006, the average difference between max UF and min UF computed over the 24 instances is 0.0755 in the maxmin case and 0.1251 in the minmax case. The minimum differences are 0.0007 and 0.0021 in the two cases, while the maximum differences are respectively 0.1912 and 0.3196. In the restricted instances related to April 2007 this behavior is even more evident; in this case the statistics are computed

Table 8: maxmin - LP and ILP results

| | FB-0.50 | | | | |
|---|---|---|---|---|---|
| | LPTime | IPTime | %Gap | IPValue | NodeInf |
| 0106-5-40-4-0 | 0.89 | 1813.52 | 0.25 | 0.2646 | 16811/100089 |
| 0106-5-40-4-1 | 0.64 | 1896.11 | 0.11 | 0.2588 | 1596/188762 |
| 0106-5-40-4-2 | 0.79 | 1689.32 | 0.46 | 0.2438 | 11077/113218 |
| 0106-8-40-4-0 | 0.77 | 893.02 | 0.11 | 0.3072 | 788/90607 |
| 0106-8-40-4-1 | 0.65 | 5645.37 | 0.05 | 0.2558 | 794/592484 |
| 0106-8-40-4-2 | 0.29 | 43200.00 | 0.11 | 0.2550 | 1545/3646097 |
| 0106-5-60-5-0 | 4.20 | 1873.59 | 0.19 | 0.2758 | 303/66839 |
| 0106-5-60-5-1 | 5.69 | 75.40 | 0.00 | 0.2471 | 784/784 |
| 0106-5-60-5-2 | 8.34 | 4258.55 | 0.25 | 0.2863 | 996/97146 |
| 0106-8-60-5-0 | 6.02 | 2121.53 | 0.10 | 0.3275 | 1126/73981 |
| 0106-8-60-5-1 | 3.46 | 43200.00 | 0.04 | 0.2800 | 43415/1878404 |
| 0106-8-60-5-2 | 2.95 | 58.71 | 0.00 | 0.3017 | 818/818 |
| 0407-5-50-5-0 | 2.19 | 43200.00 | 0.06 | 0.3412 | 1525/2878969 |
| 0407-5-50-5-1 | 2.26 | 14283.88 | 0.14 | 0.3342 | 3200/431512 |
| 0407-5-50-5-2 | 3.93 | 10467.40 | 0.17 | 0.3658 | 758/744031 |
| 0407-8-50-5-0 | 1.73 | 508.48 | 0.00 | 0.3767 | 17850/17850 |
| 0407-8-50-5-1 | 4.06 | 11546.51 | 0.18 | 0.3383 | 1730/900536 |
| 0407-8-50-5-2 | 2.85 | 21220.87 | 0.09 | 0.2950 | 401/2385155 |
| 0407-5-80-6-0 | 35.27 | 2534.66 | 0.32 | 0.4142 | 630/39149 |
| 0407-5-80-6-1 | 45.74 | 3281.84 | 0.57 | 0.4667 | 5817/35044 |
| 0407-5-80-6-2 | 34.39 | 13511.08 | 0.10 | 0.4317 | 44417/134691 |
| 0407-8-80-6-0 | 45.10 | 12246.63 | 0.11 | 0.4550 | 16322/110239 |
| 0407-8-80-6-1 | 39.10 | 11224.26 | 0.04 | 0.4075 | 26386/190312 |
| 0407-8-80-6-2 | 22.17 | 20819.45 | 0.06 | 0.3850 | 1576/1824514 |

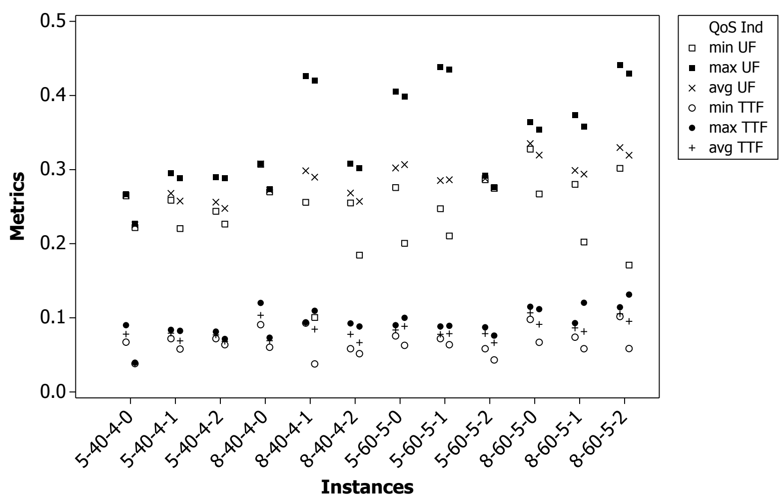


Figure 3: Solution quality analysis on 0106 instances

over 23 instances instead over 24 since in the minmax case instance 0407-8-80-6-0 failed to provide a feasible solution. In regards to the average difference between the extreme utilization factors we observed a value of 0.1071 in the maxmin case and a value of 0.2048 in the minmax case. The minimum differences are both equal to 0.0088 whereas the figures

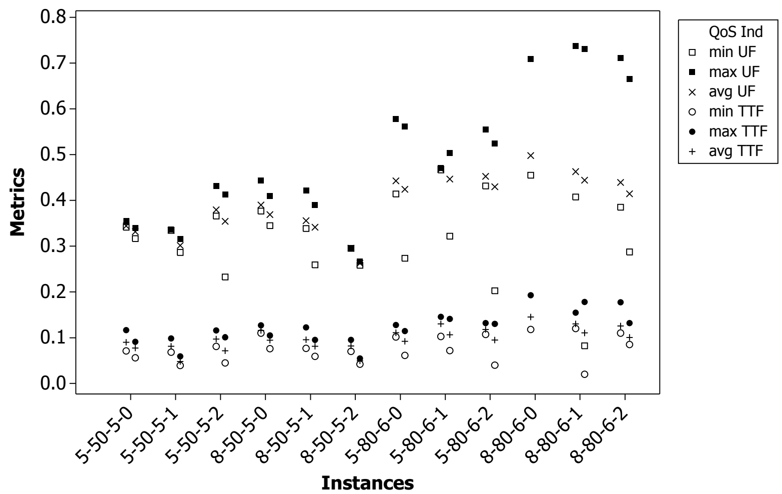


Figure 4: Solution quality analysis on 0407 instances

for the maximum differences are respectively 0.3303 and 0.6481 in the maxmin and minmax cases.

Finally we give some information on the number of times operators are used to service a request with a skill less specialized than their skill. The number of overskilled visits is quite low for both groups of instances and for both the alternative objective functions, even if it is slightly bigger in April 2007 than in January 2006. In January 2006, this indicator is zero almost everywhere except that in three cases - two times for *maxmin* and one time for *minmax*. Conversely, for April 2007 the indicator is bigger for *minmax* than for *maxmin* (14 vs 9).

## 7.5 Impact of care continuity constraints

The aim of this section is to investigate the effect of the care continuity constraints on the efficiency of the resolution method and on the solution quality for both the alternative objective functions. Motivated by the results analyzed in previous sections, the FB-0.50 strategy is used to select the patterns. For the 24 restricted instances, the care continuity constraints have been removed thus allowing that a patient is visited by up to 5 operators each week; then the solutions obtained when the continuity of care constraints are imposed, are compared with the solutions obtained without care continuity. First of all we report that the elimination of care continuity constraints had no effect on the LPValue, at least on this data set. Furthermore, by looking at the lower bound value (in the minmax case), there was no potential decrease of the operators utilization factor in increasing the number of operators that can service a client beyond two. In general, we observe that relaxing the care continuity constraints has no impact on the value of the solutions except in few cases; on the other hand, disregarding the care continuity, the model seems to be faster in finding feasible solutions with respect to the case where the care continuity is ensured. In regards to the IPTime, we observe a non monotonic behavior since the relaxation of the care continuity constraints can involve either a reduction or an increase of computational time with respect to the more constrained model, at least on instances with up to 60 patients. On the most critical instances in the test bed, i.e. on instances characterized by 80 patients, we report either a valuable reduction of the IPTime or of the optimality gap; indeed, the elimination of care continuity constraints allows to find a feasible solution also for instance 0407-8-80-6-0 whereas no feasible solution is found in the other case. Similar results are obtained in the maxmin case. See the Online Supplement for detailed results about the impact of care continuity on efficiency and efficacy in the minmax case.

Interestingly, the solutions returned when care continuity is disregarded are not very different from the ones which take into account care continuity in terms of operator utilization factor and travel time. These observations are supported by the results shown in Figures 5 and 6, which report information on the quality of the solutions respectively for the maxmin and the minmax restricted instances related to January 2006. Analogous information for the instances related to April 2007 can be found in the Online Supplement. In these figures, for each instance, information relative to the case where the care continuity is imposed and the case where it is relaxed are reported consecutively. The elimination of care continuity constraints seems thus to represent an useful tool to address the most critical instances since it allows to find quickly feasible solutions preserving the quality of the solutions obtained when the care continuity is guaranteed. Furthermore, we observe that the number of patients assigned to more than two operators is quite low when the care continuity constraints are ignored, for both the alternative objective functions. For restricted instances referring to January 2006 this behavior is more evident than for instances related to April 2007. See the Online Supplement for an analysis of the number of operators per patient in both scenarios (*maxmin* and *minmax*), under the care continuity constraints and their removal.

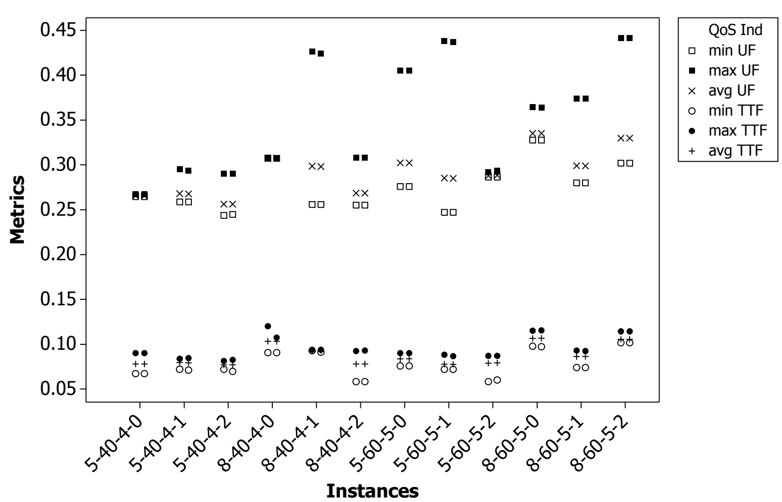


Figure 5: maxmin - quality analysis on 0106 instances with and without CC

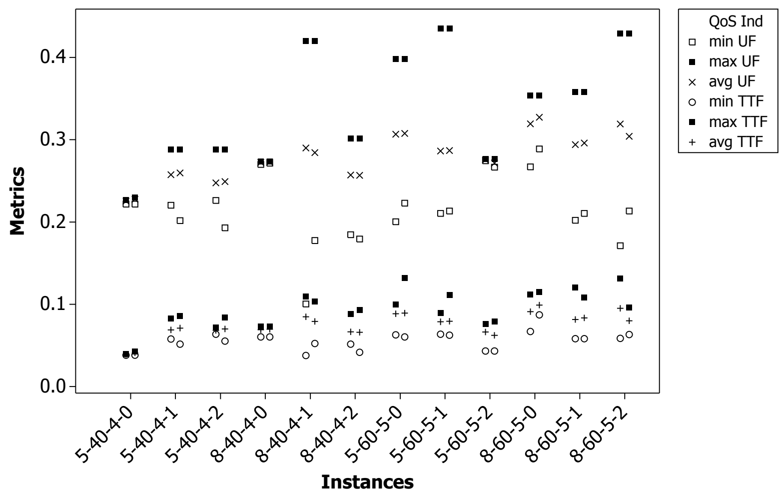


Figure 6: minmax - quality analysis on 0106 instances with and without CC

## 7.6 Results on real instances from the literature

In this section we report the computational results obtained on two real-world data sets from the literature, referred to as Nickel_et_al [22] and Jensen [17] instances. In Table 9 we give the main characteristics of the two instances, i.e. the length of the planning horizon ($|W|$), the number of patients (Patients), the total number of typologies of visits that the patients require (Jobs), the total number of visits required (Visits), and the number of operators (Operators). In fact, differently from the Italian home care provider context, in these two instances a patient may require more than one typology of visits each day and each visit typology may have to be repeated several times in the planning horizon. As an example a patient may require an application of medication three times a week and washing five times a week; in such a case we have to consider two jobs (medication and washing) for a total of 8 (3+5) visits. As anticipated in Section 5, in our study this issue is modeled by associating a node of the network with each job instead than with each patient.

Finally, we give also the number of patterns we generated to reflect information relative to the frequency of the visits in the two real instances. Specifically, in regards to the Nickel_et_al instance patterns reflect the shift combinations used by the authors, whereas for Jensen instance patterns reflect the requirements expressed in term of time lags, i.e. in terms of number of days that must elapse between two repetitions of the same visit typology. Observe that in the computational campaign performed in [22] only a shift combination for each job is given. This implies that, for each job, the days in the week where the visits occur are given.

Even if the two instances seem quite comparable according to their size, they have a very different structure. In fact, Nickel_et_al instance arises from a German provider operating in a rural area; hence traveling distances range from 0 to 45 minutes and most of them range in [15,45] minutes. On the contrary, the Jensen instance comes from a Danish provider operating in a urban area where the operators travel by bike. In this case traveling times

Table 9: Instances features

| Instance | $\|W\|$ Length | Patients | Jobs | Visits | Operators | Patterns |
|---|---|---|---|---|---|---|
| Nickel_et_al | 7 | 36 | 53 | 287 | 11 | 18 |
| Jensen | 5 | 37 | 65 | 256 | 5 | 9 |

are quite short and range from 0 to 18 minutes. A second relevant difference between the two instances concerns the duration of the visits: in the Nickel_et_al instance the service time ranges from a minimum of 10 minutes to a maximum of 510 minutes. Indeed while most visits require a service time not greater than one hour, there are 28 visits with a service time in [180,510] minutes. In contrast, in Jensen instance the service time always varies in [10,120] minutes. A third difference between the two instances lies in the number of skills which is equal to two in the Nickel_et_al instance and equal to one in the Jensen instance. Summarizing, the Nickel_et_al instance presents a heterogenous structure and as clearly reported by the authors, this irregularity further complicates the problem solving.

In regards to operators, in Nickel_et_al instance the weekly maximum number of minutes an operator should work is known, whereas in Jensen instance the workday length is given. In both cases we assume two daily shifts (morning and afternoon) for each operator. As discussed in Section 5, this is implemented in our model by replacing a single original operator with two copies of it, one relative to the morning shift and the other relative to the afternoon shift. However the utilization factor of the operators is still computed operator-wise, i.e. considering the two copies of each original operator together. Splitting each operator allows us to manage loose time windows, simply guaranteeing the compatibility between a shift (morning or afternoon) and a visit. Other kinds of time windows constraints are not considered in this computational campaign. Observe, however, that the operator splitting allows us (at least theoretically) to manage exactly the same time windows constraints considered in [17]; there, each day of the planning horizon is organized in not-overlapping time windows within which visits must occur. In the same manner we could plan several copies of each operator, where each copy refers to a time window; then we can handle the compatibility between time windows and visits properly. In contrast, in Nickel_et_al instance quite narrow time windows are considered that we do not address in the present experimentation. It is however worth recalling that in Nickel_et_al instance the time windows do not comply with patients' needs; rather, they are extracted ex-post from the solution implemented by the provider, weakly enlarged for feasibility reasons, and then imposed in the mathematical model to make the comparison between the automated solution and the provider's solution possible.

Tables 10 and 11 show the computational results obtained on the two real-world data sets by means of proper extensions of the models in Section 3. Specifically, we performed three tests on Nickel_et_al instance and two tests on Jensen instance. In all of the five tests, the care continuity is not addressed since the corresponding data are unavailable. The objective function *maxmin* is used for all the tests coming from the Nickel_et_al instance; *minmax*, in fact, failed to provide a feasible solution within time and memory limits. In the first test (referred to as Nickel_et_al), as in the campaign performed in [22], only a shift combination for each job is given; in our model this corresponds to fix a pattern for each node of the

Table 10: LP/IP information

| | LPTime | LPRows | LPCols | %Gap | Stop | BestNode/TotalNodes |
|---|---|---|---|---|---|---|
| Nickel_et_al | 390.39 | 14757 | 82935 | 0.40 | T | 1810/1971 |
| Nickel_et_al_free_p53 | 374.95 | 14918 | 84165 | 0.30 | T | 0/0 |
| Nickel_et_al_free_all | 648.00 | 18432 | 111215 | 1.01 | T | 651/651 |
| Jensen_maxmin | 120.88 | 15344 | 66939 | 0.30 | M | 29258/67898 |
| Jensen_minmax | 139.89 | 15344 | 66939 | 29.88 | T | 0/8 |

Table 11: Solution quality analysis

| | TT | ST | 0T | min UF | max UF | avg UF | min TTF | max TTF | avg TTF |
|---|---|---|---|---|---|---|---|---|---|
| Nickel_et_al | 10070 | 13090 | 5017 | 1.2734 | 1.2814 | 1.2761 | 0.2987 | 0.8864 | 0.6157 |
| Nickel_et_al_free_p53 | 7298 | 13090 | 2245 | 1.0260 | 1.3081 | 1.1297 | 0.1234 | 0.8262 | 0.4358 |
| Nickel_et_al_free_all | 9990 | 13090 | 4937 | 1.2651 | 1.2839 | 1.2721 | 0.2359 | 0.8606 | 0.5989 |
| Jensen_maxmin | 2422 | 6489 | 0 | 0.7920 | 0.7924 | 0.7921 | 0.2027 | 0.2373 | 0.2153 |
| Jensen_minmax | 1977 | 6489 | 0 | 0.6502 | 0.9338 | 0.7525 | 0.1453 | 0.2027 | 0.1757 |

network. In the other two tests a degree of freedom is added by letting that the model takes job pattern decisions. In particular, in test referred to as Nickel_et_al_free_p53, schedule decisions are fixed for all nodes except that for a job (number 53) that is characterized by a long service time. This test is motivated by the fact that the computational campaign performed in [22] and some of our preliminary tests as well, confirm that the irregularity of service time makes problem solving complicate. In this sense, the test we perform follows the approach delineated in [22] where some of the long jobs are treated aside. In contrast, in the third test relevant to Nickel_et_al instance, the model is free to select a pattern for each node. Preliminary computational tests also revealed that a feasible solution to our model does not exist if overtime is not allowed; thus, the model is extended as described in Section 5. On the other side the two tests performed on Jensen instance use respectively the maxmin (Jensen_maxmin) and the minmax (Jensen_minmax) objective function.

Table 10 reports for the 5 tests performed the following information: the time required to solve the Linear Programming relaxation (LPTime), the dimension of the instances in terms of number of constraints (LProws) and variables (LPCols), the optimality gap (%Gap), the stopping criterion (Stop: "T" for time limit and "M" for memory limit) and finally the branch and bound node where the best integer solution is found with respect to the total number of nodes explored (BestNode/TotalNodes).

Table 11 reports the following information: the total travel time (TT), the total service time (ST) and the total overtime (OT), all of them expressed in minutes. Then information relative to the utilization factor and travel time factor of the operators is given as for the results relative to the Italian provider. Specifically, the minimum, maximum and average values are given for each metric.

In regards to Nickel_et_al tests, we observe that all of the solutions are pretty well balanced in terms of utilization factor while they are quite different the one from the other in terms of total travel time and overtime. All of the patient requests are covered (this is a hard constraint in our model). Test Nickel_et_al_free_p53 represents a good compromise between efficiency and balancing being characterized by the smallest values of travel time and overtime among the three tests. We can also observe that the reduction of travel time and overtime obtained when the pattern is not a priori fixed is quite limited (compare the results obtained on tests Nickel_et_al and Nickel_et_al_free_all). It is worth noticing that in Nickel_et_al instance and in Jensen instance as well, the clustering of patients according to their geographical location is not given; hence the EC cuts have not been exploited and the cost containment is not addressed in these tests.

In regards to Jensen instance, the computational tests confirm the same trend emerged for the instances from the Italian home care provider. Specifically, *maxmin* is better suited to balance the operators utilization factor than *minmax* and it allows for a very good optimality gap; in contrast, *minmax* allows to contain the total travel time (see column TT). In these tests the overtime never occurs and we can also observe that, as expected, the travel times play a role less crucial than in Nickel_et_al instance (see column avg TTF). Finally we observe that consistently with the time lags requirements, in this instance the patterns are generated exhaustively, i.e. they comprise all the feasible scheduling combinations. Interestingly, we performed a test with a smaller number of patterns (6 instead of 9) and we observed a very low detriment of the optimality gap with respect to the run characterized by the complete set of patterns (0.47% instead of 0.30%). The corresponding solution however was obtained more efficiently and the LPValue remains the same.

In conclusion observe that the results we obtained are not directly comparable to those reported in [22] and in [17]. In fact, although we used the same real-world data sets, the problem we solved presents some dissimilarities with respect to those addressed in such two papers, as already emphasized in Section 4. Anyway, by referring to the computational results in [22], we can report that the solution found in [22] for the first real-world data set has four ten minute requests unscheduled. In contrast, all of the patient requests are covered in our solution. Furthermore, the solution found in [22] is characterized by 8745 overtime units, which reduces to 8180 by fixing two long jobs in advance. In addition, according to the indications in [22], it seems that the overtime is not equally split among the operators. On the other hand, our solutions are characterized by a sensibly lower total overtime, and are pretty well balanced in terms of utilization factor (see Table 11). Concerning the computational results reported in [17], again a direct comparison can not be done. However observe that, as reported in [17], after the daily planning step the masterplan computed by Jensen's approach on the second real-world data set is characterized by a total travel time of 1305 minutes; therefore, each operator spends 52 minutes on average on the road per working day. Furthermore, according to the indications in [17] the total number of different operators assigned to the patients, i.e. the the employee regularity, is 74; therefore, each patient is visited on average by 2 different operators over the considered five day period. Both under *maxmin* and *minmax*, the total travel time returned by our computational campaign is bigger (see Table 11) and the average number of operators per job is 2.82 in the maxmin case and 2.83 in the minmax case.

Summarizing, we conclude that our approach seems to be able to find solutions of quite good quality also on real instances with an irregular structure and pertaining to context very different from the one characterizing our case study. In addition, the two real-world data sets allow us to validate experimentally two of the extensions addressed theoretically in Section 5, i.e. those pertaining to (weak) time windows and to overtime management.

## 7.7 Main achievements

The main achievements supported by the compuutational results can be summarized as follows:

**Pattern selection plays a crucial role in the optimization process.** Determining a good but limited set of patterns is crucial for efficiently solving the LP relaxation, and for finding good quality solutions to the ILP models; in particular, the flow based procedure, and specifically the one halving the operators availability, proved to be very effective in selecting a small number of ad-hoc patterns.

**Symmetry management may help in finding quickly feasible solutions.** The symmetry management allowed either to halve the optimum percentage gap, or to find feasible solutions in the most critical cases; however, it was not an issue for the flow based pattern generation policy, which proved to be very efficient independently of the symmetry management tool.

**Minmax and maxmin objectives reflect different but equally relevant stakeholders' perspectives.** Both objective functions provide interesting results. Specifically, the maxmin objective allows to meet equity and computational efficiency criteria. Indeed, balanced solutions are obtained rapidly where balancing refers to the gap between the most and the least loaded operators. On the contrary, the minmax objective meets the cost efficiency criterion which is the typical perspective assumed by the providers in an attempt to limit transportation costs, whereas *maxmin* tends to provide, more than *minmax*, the undesirable effect of operators spending time in traveling between municipalities to increase their utilization factor. However, *minmax* requires on average longer computational time than the *maxmin* counterpart and is not able to provide feasible solutions for the biggest instances in the test bed. In summary, both objective functions are capable of capture essential features that characterize real settings and can support the decision makers providing them with alternative solutions among which selecting the one that best fits their preferences: when the focus is on equity criterion the *maxmin* is preferable; on the contrary the *minmax* should be used when the focus is on cost control.

**The introduction of the EC cuts allows to tradeoff equity vs efficiency.** The maxmin objective function equipped with the EC constraints represents a very good compromise between efficiency and equity criteria. Specifically, the EC constraints allow to indirectly control travel costs, thus letting the model be distance aware. It still preserves the good balancing level of the solutions provided by *maxmin* and allows to address also the biggest instances.

**The care continuity constraints may be used as a tool to control computational efficiency.** When care continuity constraints are relaxed, patients may be assigned to more than two operators. In general, the less constrained variants of the models exhibit a greater capability than the more constrained counterparts in finding quickly feasible solutions. In addition, the quality of the solutions obtained does not experience an important detriment in the less constrained models, in the sense that the number of patients assigned to more than two operators is overall limited.

**Some biggest instances in the test bed can be solved only for some combinations of objective function and cost control mechanism.** The biggest instances from the palliative home care context and the real instance from [22] can be solved only under *maxmin* equipped with the EC constraints. The study of decomposition approaches thus represents a challenging line of research we are currently pursuing.

**The models proposed can be extended so as to cope successfully with problems from different home care contexts.** We run some variants of our models on two real big instances from the literature which are very different in nature the one from the other. On these instances, within the time and memory limits imposed, we are able to provide good quality solutions both in terms of optimality gap and of workload balancing.

# 8 Conclusions

In this paper we addressed weekly planning horizon Home Care Problems. With respect to [2], [17] and [22] that, to best of our knowledge, are the only papers which extend the daily planning horizon usually investigated in Home Care, here assignment, scheduling and routing decisions are addressed in a joint way, i.e. without heuristically decomposing the problem by means of a two-level approach. To this end, crucial and original is, in our opinion, the use of the pattern modelling device as a means to coordinate the diverse decision levels. The main Home Care Problem we have investigated refers to providers dedicated to palliative care and terminal patients. In this context, balancing objective functions are particularly relevant. Therefore, two balancing functions have been studied, *maxmin* and *minmax*. Relevant QoS constraints such as continuity of care and maximum daily workload have been incorporated into the proposed Integer Linear Programming formulations. In addition, the models have been enhanced by means of valid inequalities aimed at breaking the symmetric structure that usually characterizes the underlying logistics network. The approach we proposed is however able to cope with peculiarities from different home care contexts. Model extensions to handle scenarios other than the palliative one have been discussed in the paper. These extensions include the satisfaction of time window constraints, the handling of overtime and scenarios where skills are not organised hierarchically, which is instead typical of the palliative context.

The computational experiments performed reveal, in our opinion, the potentiality of the proposed ILP models in successfully addressing home care instances. Crucial, to this end, is the selection of a limited set of patterns of good quality, such as the ones produced by the flow based pattern generation approach, which is another contribution of this paper. The experimental results have been obtained on palliative home care instances based on real data, and on two real-world data sets from the literature, related to contexts very different from the palliative one, where some of the aforementioned model extensions are required. In the palliative context, the comparison of the two balancing objective functions, i.e. *maxmin* and *minmax*, shows that *maxmin* is able to return more balanced solutions, in the sense that the operator utilization range is smaller than the one returned by *minmax*. Such a stronger equity achievement is obtained for not too high a price in the increased mean operator utilization factor and mean operator traveling time. On the other hand, the minmax criterion is more suitable for the minimization of the operating costs, since it always returns solutions with the smaller total travelled time for the operators. The results on the two data sets from the literature show that the proposed approach is able to find solutions of good quality also on instances pertaining to contexts very different from the palliative one.

The design of decomposition methods based on the proposed ILP models, and tailored to solve even larger home care instances, is in progress.


## Acknowledgments

We are very grateful to Prof. S. Nickel and to Dr. T.S. Jensen for having provided the real instances experimented in Section 7.6 and for the helpful discussion about their use. We also thank the three anonymous referees and the Associate Editor for the interesting comments and suggestions which allowed us to greatly improve the first version of the paper.


## References


[1] Akjiratikarl, C., Yenradee, P. and P.R. Drake, "PSO-based algorithm for home care worker scheduling in the UK", *Computers & Industrial Engineering*, 53, 559583, 2007.

[2] Begur, S.V., Miller, D.M. and J.R. Weaver, "An Integrated Spatial DSS for Scheduling and Routing Home-Health-Care Nurses", *Interfaces*, 27(4), 35-48, 1997.

[3] Bertels, S. and Fahle T., "A hybrid setup for a hybrid scenario: combining heuristics for the home health care problem", *Computers & Operations Research*, 33, 2866-2890, 2006

[4] M. Blais, S.D. Lapierre and G. Laporte, "Solving a home care districting problem in an urban setting", *J. Oper. Res. Soc.*, 54, 1141-1147, 2003

[5] Bredstrom, D. and M. Ronnqvistr, "Combined vehicle routing and scheduling with temporal precedence and synchronization constraints", *European Journal of Operational Research*, 191, 1931, 2008.

[6] P. Cappanera, L. Gouveia and M.G. Scutellà, "The Skill Vehicle Routing Problem", in *Network Optimization. 5th International Conference, INOC 2011. Hamburg, Germany, June 2011*, J. Pahl, T. Reiners and S. Voss (eds), Lecture Notes in Computer Science, vol. LNCS 6701, 354-364, Springer-Verlag Berlin Heidelberg, 2011.

[7] P. Cappanera, L. Gouveia and M.G. Scutellà, "Models and valid inequalities to Asymmetric Skill-Based Routing Problems", *EURO Journal on Transportation and Logistics*, 2(1-2), 29-55, 2013.

[8] P. Cappanera and M.G. Scutellà, "Home Care optimization: impact of pattern generation policies on scheduling and routing decisions", *Electronic Notes in Discrete Mathematics*, 41, 53-60, 2013.

[9] Chahed, S., Marcon, E., Sahin, E. Feillet, D. and Y. Dallery, "Exploring new operational research opportunities within the Home Care context: the chemotherapy at home", *Health Care Manag Sci*, 12, 179-191, 2009.

[10] Cheng E. and J.L. Rich, "A Home Health Care Routing and Scheduling Problem", Technical Report, 1998.

[11] Dohn, A., Rasmussen, M.S., Justesen, T. and J. Larsen, "The home care crew scheduling problem", In: SheibaniK (ed) Proceedings of the 1st international conference on applied operational research (ICAOR 08), Yerevan, Armenia. *Lecture Notes in Management Science*, 1, 1-8, 2008.

[12] Elbenani, B., Ferland, J.A. and V. Gascon, "Mathematical Programming Approach for Routing Home Care Nurses", *IEEE*, 107-111, 2008.

[13] Eveborn, P., Flisberg, P. and M. Ronnqvist, "Laps care - and operational system for staff planning of home care", *European Journal of Operational Research*, 171(3), 962-976, 2006.

[14] Eveborn, P., Ronnqvist, M., Almroth, M., Eklund, M., Einarsdottir, H. and K. Liden, "Operations Research (O.R.) Improves Quality and Efficiency in Home Care", *Interfaces*, 39 (1), 18-34, 2009.

[15] A. Hertz and N. Lahrichi, "A patient assignment algorithm for home care services", *Journal of the Operational Research Society* 60(4), 481-495, 2009.

[16] G. Hiermann, M. Prandtstetter, A. Rendl, J. Puchinger and G.R. Raidl, "Metaheuristics for Solving a Multimodal Home-Healthcare Scheduling Problem", *Central European Journal of Operations Research*, Online First, May 2013.

[17] T.S. Jensen, "Application of Metaheuristics to Real-life Scheduling Problems", Ph.D. Thesis, Department of Mathematics and Computer Science, University of Southern Denmark, 2012.

[18] Kergosien, Y., Lentè, C. and J.C. Billaut, "Home health care problem. An extended multiple Traveling Salesman Problem", *MISTA2009*, 2009.

[19] Lanzarone, E. and A. Matta, "A cost assignment policy for home care patients", *Flexible Services and Manufacturing Journal* 24(4), 465-495, 2012.

[20] A. Matta, S. Chahed, E. Sahin and Y. Dallery, "Modelling home care organisations from an operations management perspective", *Flexible Services and Manufacturing Journal*, Published online: 24 August 2012.

[21] M.A. Neergaard, "Palliative home care for cancer patients in Denmark - with a particular focus on the primary care sector, GPs and community nurses", Ph.D. Thesis, Faculty of Health Sciences, Aarhus University, 2009.

[22] Nickel, S., Schroder, M. and J. Steeg, "Mid-term and short-term planning support for home health care services", *European Journal of Operational Research* 219, 574-587, 2012.

[23] Rasmussen, M.S., Justesen, T., Dohn, A. and J. Larsen, "The Home Care Crew Scheduling Problem: Preference-based visit clustering and temporal dependencies", *European Journal of Operational Research* 219, 598-610, 2012.

[24] K. Thomsen, "Optimization on Home Care", M.Sc. Report, Department of Informatics and Mathematical Modelling, Technical University of Denmark, 2006.

[25] Trautsamwieser, A., Gronalt, M. and P. Hirsch, "Securing home health care in times of natural disasters", *OR Spectrum*, 33, 787813, 2011.

[26] World Health Organization (WHO), "Home-based and long-term care: home care issues and evidence", WHO Technical Report Series, 2000.

[27] World Health Organization (WHO), "Community home-based care in resource-limited settings: a framework for action", WHO Technical Report Series, 2002.