JOINT DECISIONS ON INVENTORY REPLENISHMENT AND EMISSION REDUCTION INVESTMENT UNDER DIFFERENT EMISSION REGULATIONS

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By

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August, 2013

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Carbon emission regulation policies have emerged as mechanisms to control firms' carbon emissions. To meet regulatory requirements, firms can change their operations or invest in green technologies. In this thesis, we analyze a retailer's joint decisions on inventory replenishment and carbon emission reduction investment under three carbon emission regulation policies. Particularly, we first study the economic order quantity model to consider carbon emissions reduction investment availability under carbon cap, tax, and cap-and-trade policies. We analytically show that carbon emission reduction investment opportunities, additional to reducing emissions as per regulations, further reduce carbon emissions while reducing costs. We also provide an analytical comparison between various investment opportunities and compare different carbon emission regulation policies in terms of costs and emissions. We document the results of a numerical study to further illustrate the effects of investment availability and regulation parameters. We later extend our analysis to a retailer operating in a newsvendor setting, taking into account the existence of environmentally sensitive customers.

Keywords: Green technology, carbon emissions, investment, economic order quantity.

ÖZET

FARKLI EMİSYON DÜZENLEMELERİ ALTINDA ENVANTER YENİLEME VE EMİSYON AZALTMA YATIRIMININ ORTAK KARARI

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Karbon emisyonu düzenleme politikaları firmaların karbon emisyonlarını kontrol etmek için ortaya çıkan araçlardır. Bu araçların firmalara getirdiği gereksinimleri karşılamak için, operasyonel işlemler değiştirilebilir ya da temiz teknolojilere yatırım yapılabilir. Bu tezde, üç farklı emisyon düzenleme politikası altında bir perakendecinin envanter yenileme ve emisyon azaltma yatırımlarının ortak kararı analiz edilmiştir. Spesifik olarak, iktisadi sipariş verme modelinin bir uzantısı, emisyon üst sınırı, emisyon vergisi, ve emisyon üst sınırı ve ticareti politikaları altında, temiz teknolojilere yatırım olanağı düşünülerek çalışılmıştır. Emisyon azaltma yatırımlarının, düzenleme politikalarının sağlamış olduğu emisyon azaltımına ilaveten, hem maliyetleri hem de karbon emisyonunu azalttığı analitik olarak gösterilmiştir. Ayrıca, çeşitli yatırım fırsatları arasında analitik karşılaştırmalar yapılmış ve farklı karbon emisyon düzenleme politikaları maliyet ve emisyon bakımından birbiriyle karşılaştırılmıştır. Temiz teknolojilere yatırım fırsatının ve düzenleme politikalarına ait parametrelerin etkilerini daha iyi göstermek için yapılan bir sayısal çalışmanın sonuçları da sunulmuştur. Son olarak, benzer bir analiz, literatürde gazete satıcısı problemi olarak bilinen bir ortama sahip perakendeci için, çevresel duyarlı müşteriler de göz önünde bulundurularak, yapılmıştır.

Anahtar sözcükler: Temiz teknoloji, karbon emisyonu, yatırım, en kazançlı ısmarlama miktarı.

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Chapter 1

Introduction

Global warming, environmental disasters, and increased public awareness about environmental issues are encouraging countries to reduce greenhouse gas (GHG) emissions. The Kyoto Protocol, signed in 1997 by 37 industrialized countries and European Union (EU) members, enabled nations to aggregately focus on GHG emission abatement. Several government programs (e.g., the EU Emissions Trading System, the New Zealand Emissions Trading Scheme, the U.S.' Regional Greenhouse Gas Initiative), private voluntary-membership organizations (e.g., the Chicago Climate Exchange, the Montreal Climate Exchange), and many emissions-offset companies have emerged as control mechanisms over firms' GHG emissions, primarily carbon emissions (other GHG emissions can be measured in terms of equivalent carbon emissions, see, e.g., EPA [1]). To reduce carbon emissions, policy makers either provide incentives to achieve emission reduction or impose costs on carbon emissions.

Under carbon emission regulation policies, firms seek cost-efficient methods to decrease emissions, mainly through replanning (changing) their operations and investing in carbon emission abatement (Bouchery et al. [2]). A firm can reduce its carbon emissions level via changing its production, inventory, warehousing, logistics, and transportation operations (Benjaafar et al. [3], Hua et al. [4]). For instance, after 60,000 suppliers of Wal-Mart decreased their packaging by

5% upon Wal-Mart's request, they achieved 667,000 m^3 of CO_2 emission reduction (Hoffman [5]). Hewlett-Packard (HP) reported that they decreased toxic inventory release to the air from 26.1 tonnes to 18.3 tonnes in 2010 by adjusting operations (HP [6]).

A firm can also reduce its carbon emissions level by directly investing in carbon emission reduction projects such as greener transportation fleets (see, e.g., Bae et al. [7]), energy-efficient warehousing (see, e.g., Ilic et al. [8]), and environmentally friendly manufacturing processes (see, e.g., Liu et al. [9]). McKinsey & Company reports that U.S. carbon emissions can be reduced by three to 4.5 gigatons in 2030 using tested approaches and high-potential technologies (Creyts et al. [10]). Additional to directly investing in carbon emission reduction projects that decrease emissions from internal operations, companies can indirectly invest in carbon emission reduction by purchasing carbon offsets (see, e.g., Benjaafar et al. [3], Song and Leng [11]), which can compensate for a company's carbon emissions and be used to increase its carbon emissions cap. Carbon-offset projects are referred to as clean development mechanisms (CDM) under the Kyoto Protocol. The United Nations Framework for Convention on Climate Change provides a list of CDM (See [12]). The World Bank reports that the global carbon market, including traded allowances and offset transactions, reached $176 billion in 2011 (Kossoy and Guigon [13]).

Examples of how emission abatement increases companies' competitiveness and profitability can be extended. Some retailers follow environmental friendly supply chain operations via new technologies to boost their demands and to decrease their operational costs. Carrefour uses a new refrigeration system to reduce both emission and energy consumption (Schotter et al. [14]). They also invest in solar panels for some of their hypermarkets in Italy and France (Jacobs and Smits [15]). Similarly, Wall-Mart has assigned $500 million to sustainability projects to improve the effectiveness of its vehicle fleet, decrease the energy usage in its store and mitigate solid waste in U.S. stores (Robb et al. [16]). Lindeman reports that a 10% energy reduction in a grocery store may lead to 6% increase in the retailer's profit ([17]).

In this thesis, we consider three different carbon emission regulations; cap, cap-and-trade, and tax. There is an ongoing debate about how these regulations compare to one another in terms of their effectiveness. While a significant number of economists favor cap-and-trade or tax policies, environmental advocacy groups consider these policies as "licences to pollute" and they favor cap policy (Stavins [18]). Under the cap policy, a firm's carbon emissions should not exceed a pre-determined amount, which is referred to as a carbon cap. The cap can be determined by a government agency and/or the firm's green goals (Chen et al. [19]). The US Environmental Protection Agency (EPA) regulated SO˙2 emission between the years 1970 and 1990 using a cap policy (see Popp [20]). Furthermore, in a recent New York Times article (Broder [21]), it is reported that "President Obama is preparing regulations limiting carbon dioxide emissions from existing power plants..."

Cap-and-trade policy is the most common regulation instrument due to its market-based structure. Under the cap-and-trade policy, carbon emissions are tradable through a system such as the EU Emissions Trading System or the New Zealand Emissions Trading Scheme; a firm can buy or sell carbon allowances at a specified market price. Under the tax policy, a firm is charged for its carbon emissions through taxes. While some countries are enacted a state based emission tax (e.g. USA and China), others choose to introduce a product-based emission tax (e.g. coal tax in India and fossil fuel tax in Japan (SBS [22])). It is reported that South Africa government is planning to implement a tax policy in 2015 (Galbraith [23]). Since South Africa has an oligopoly in energy market, they thought that tax policy is more appropriate than cap-and-trade policy for their short and medium carbon emission goals (National Treasury: Republic of South Africa [24]). In this thesis, we study a retailer's joint decisions on inventory replenishment and carbon emission reduction investment under these three policies.

As the world economy becomes increasingly conscious of the environmental concerns, evidence suggests that companies who make better business decisions to consider the interests of other stakeholders, including the human and natural environments, will succeed (Jaber [25]). While the environmental regulation policies aim to protect consumers, employees and the environment, cost of compliance should not deter companies to do business. Inventories play an important role in the operations and the profitability of a company. Therefore, one of our goals in this thesis is to provide guidance to companies to make better inventory decisions while utilizing the available environmental technologies under different regulation policies. Our other purpose is to help policy makers understand the implications of each regulation policy on the profitability of a company, and the role that green technologies play in the resulting carbon emissions and costs of the company.

In light of the above objectives, two main problems are studied. In the first part of the thesis (mainly in Chapters 2, 3, 4), which is the core of the thesis, we consider a retailer operating under the conditions of the classical EOQ model. We provide a solution method for the retailer's joint inventory control and carbon emission reduction investment decisions for each carbon regulation policy considered. The resulting optimal values of the order quantity and the yearly investment amount under a certain policy simultaneously minimize the retailers average annual costs if that policy is in place. This analysis is later extended to the Newsboy setting in the second part of the thesis (i.e., Chapter 5). Different than the first problem, in this part of the thesis, we also model the existence of customers who are environmentally sensitive. That is, an investment in green technology not only decreases the carbon emission, but it also increases the customers' willingness to buy the product.

In our analysis of the first problem, we compare the retailer's annual costs and carbon emissions with and without investment availability under each carbon regulation policy. We analytically show that availability of carbon emission reduction investment, additional to the reductions achieved by carbon emission regulation policies, further reduces carbon emissions while reducing costs under the tax and cap-and-trade policies. Under the cap policy, emissions level does not decrease due to investment, however, the same emissions level is achieved with lower costs. Therefore, we conclude that it is more important for governments to stimulate green technology under the tax and cap-and-trade policies. Several investment options with varying cost and carbon emission reduction characteristics may be available to the retailer. The retailer may thus need to select one investment opportunity. We provide analytical and numerical comparisons of the resulting costs and carbon emissions between different investment opportunities available to the retailer under each carbon emission regulation policy.

Our analysis enables comparing carbon emission regulation policies with the carbon emission reduction investment option. Our results indicate that when the retailer can invest in carbon emission reduction, compared to a given tax policy, a cap policy that will lower costs and not increase carbon emissions is possible. Furthermore, we show that for any given cap policy, there exists a cap-and-trade policy that will lower costs and carbon emissions. Further analytical and numerical results are discussed about the effects of policy parameters on the retailer's costs and emissions. These results can be utilized by policy makers in legislating carbon emissions or in constructing specific carbon emission regulation policies.

The rest of the thesis is organized as follows: In Chapter 2, we present a review of the studies in the literature. Then, we describe the first problem in more detail in Chapter 3, and provide solutions for the retailer's order quantity and carbon emission reduction investment decisions under cap, tax, and cap-and-trade policies. In this chapter, we also present the analytical results on the benefits of the carbon emission reduction investment option, the comparison of different carbon emission reduction investment opportunities and comparison of the carbon regulation policies. We summarize our numerical studies concerning the first problem in Chapter 4. We describe the second problem in Chapter 5 and provide some preliminary analysis. We conclude the thesis with some final remarks in Chapter 6.

Chapter 2

Literature Review

Environmental considerations in supply chains have drawn the attention of many researchers in recent years. Most of the papers in the operations research and the management science literature concerning this area are published in the last five years since it is a progressing research area. In this chapter, we present a survey of the related literature with an emphasis on the following four attributes: (i) what the research question of the study is about, (ii) in what ways the study differs from others, (iii) what the basic models and solution methods in the study are, and (iv) how the study contributes to the literature.

Our review of the literature is based on a classification of the studies into two groups (see Table 2.1 and Table 2.2). First group of papers propose emission reduction through better production/inventory related decisions. Second group of studies consider investing in green technologies for emission reduction.

Table 2.1: Studies in the Literature (Part I)

Studies on Emission Reduction via Replanning Inventory Replenishment Decisions							
Paper	**Problem/s**	**Demand Property**	**♯ of Items**	**Planning Horizon**	**Backlogging**	**Components of Emission**	**Investment Function**
Hoen et al. (2010)	Transport Mode Selection Problem	Stochastic (Normal)	Single-item	Infinite Horizon	Allowed	Distance, Volume and Product Density	–
Chen et al. (2013)	EOQ Model	Deterministic	Single-item	Infinite Horizon	Not Allowed	Transportation, Inventory Holding and Production	–
	The Facility Location Model	Stochastic (Uniform)	–	–	–	Facility and Distance	–
	The Newsvendor Model	Stochastic	Single-item	Finite Horizon	Allowed	Shortage and Overage	Cap Offset
Arslan and Türkay (2013)	EOQ Model	Deterministic	Single-item	Infinite Horizon	Not Allowed	Setup, Transportation and Production	–
Bouchery et al. (2012)	Multi-objective EOQ and Two Echelon Sustainable EOQ model	Deterministic	Single-item	Infinite Horizon	Not Allowed	Ordering and Inventory Holding	–
Letmathe and Balakrishnan (2005)	Lot Sizing Problem	Deterministic	Multi-item	Finite Horizon	Not Allowed	Production	–
Absi et al. (2013)	Lot Sizing Problem	Deterministic	Multi-item	Finite Horizon	Not Allowed	Production	–
Song and Leng (2012)	The Newsvendor Problem	Stochastic	Single-item	Finite Horizon	Allowed	Production	Cap Offset
Jaber et al. (2013)	The Buyer-Vendor Coordination Problem	Deterministic	Single-item	Infinite Horizon	Not Allowed	Quadratic Function of Production Rate	–

Table 2.1 – continued from previous page

Paper	Problem/s	Demand Property	♯ of Items	Planning Horizon	Backlogging	Components of Emission	Investment Function
Kim et al. (2009)	Transportation Cost and Emission Relationship for Inter-Modal and Truck-Only Networks	Deterministic	-	Finite Horizon	Not Allowed	Transportation and Transshipment	–
Benjaafar et al. (2013)	Lot Sizing Problem	Deterministic	Single/Multi-item	Finite Horizon	Not Allowed	Ordering, Production and Inventory Holding	Carbon Offset

Table 2.2: Studies in the Literature (Part II)

Studies on Emission Reduction via Investment Opportunities							
Paper	**Problem/s**	**Demand Property**	**♯ of Items**	**Planning Horizon**	**Backlogging**	**Components of Emission**	**Investment Function**
Zavanella et al. (2013)	The buyer-Vendor Coordination Problem	Deterministic (Price and Environmentally Performance Dependent)	Single-item	Infinite Horizon	Not Allowed	–	Nonlinear
Swami and Shah (2013)	The Channel Coordination Problem	Deterministic (Price and Environmentally Performance Dependent)	Single-item	Finite Horizon	Not Allowed	–	Quadratic
Raz et al. (2013)	Life Cycle Approach Using The Newsvendor Problem	Stochastic (Price and Environmentally Effort Dependent)	Single-item	Finite Horizon	Allowed	–	Quadratic
Krass et al. (2013)	The Firms Green Technology Choice Under Tax Policy	Deterministic (Price Dependent)	Single-item	Finite Horizon	Not Allowed	Production	Discrete
Jiang and Klabjan (2012)	Single/Multi Period Carbon Emission Reduction Investment	Stochastic	Single-item	Finite Horizon	Allowed	Production	Linear

2.1 Studies on Emission Reduction via Better Production/Inventory Related Decisions

Most papers focusing on replanning production/inventory related decisions for environmental considerations, study the classic economic order quantity (EOQ) setting. In Arslan and Türkay [26], EOQ model is examined under environmental and social criteria. Firstly, optimal order quantities are found for five different carbon emission control policies which are direct accounting, carbon tax, direct cap, cap-and-trade, and carbon offset. Secondly, labor working hours are used as social criterion for evaluating EOQ model. Then, an analysis is made for an integrated model that takes into account both the environmental and the social criteria. Based on their analytical and numerical results, the authors give recommendations about which actions should be taken by organizations and governments to reduce carbon emission. This article contributes to the literature by considering EOQ with different emission policies and incorporation of social criteria.

Hua et al. [4] construct an environmental inventory model based on the single-product EOQ model. This paper examines inventory operations under the cap-and-trade system in which a firm sells or buys carbon capacity according to its carbon emission cap. Optimal order quantity under the cap-and-trade system is compared to EOQ and minimum emission solutions. A detailed analysis is made to investigate the behavior of the optimal order quantity with varying levels of carbon price and carbon cap. This article contributes to the literature by proposing a solution algorithm for an environmental EOQ model under cap-and-trade policy and by providing a detailed analysis about ordering policies under different parameters of the problem.

Chen et al. [19] examine an environmentally sensitive EOQ model under an emission cap in order to derive analytical results about carbon emission and inventory related cost. The quantity intervals where emission is reduced are derived, and it is concluded that it is possible to maximize the difference between emission reduction and cost by adjusting operational decisions. In addition, the classical facility location model and the newsvendor model are extended in this paper under environmental considerations. It is found that a significant emission reduction can be achieved at a reasonable cost increase. This article contributes to the literature by pointing out that reduction in emission is possible for different operational models at an acceptable cost increase.

It should be noted that while Hua et al. [4], Chen et al. [19] and Arslan and Türkay [26] consider the existence of a carbon regulation policy, there are also studies that propose extensions of the EOQ model with environmental considerations in the absence of carbon emission regulation policies. For instance, Bonney and Jaber [2] question the necessity of classical inventory modeling system because of the emerging environmental problems and emphasize the importance of environmentally responsible inventory models to cope with environmental problems. This paper examines results and causes of environmental problems in the scope of inventory systems and proposes what actions should be taken by stakeholders. Bonney and Jaber [2] also suggest some possible performance metrics for environmental inventory systems and exemplify an environmental-EOQ model indicating the effects of transportation on environment. This article contributes to the literature by evaluating the environmentally responsible inventory system in a broader sense and by pointing out the importance of taking precautions.

Similarly, Bouchery et al. [27] study how the firms can improve sustainability of their inventory systems by making operational adjustments. They integrate sustainability criteria into EOQ model and call it sustainable order quantity (SOQ) model. Then, they extend SOQ model for a two-echelon system consisting of a retailer and a warehouse. For both the SOQ model and its two-echelon extension, Pareto optimal solutions are provided. The authors find out that the firms can decrease their carbon emission in an important amount by small cost increase. They also compare the different emission regulation policies and make some suggestions for policy makers about how they can decrease carbon emission. This study contributes to the literature by considering multiple objectives in the EOQ model.

It is worthwhile noting that along with the ordering decisions in the EOQ setting, some classical supply chain problems have been revisited in regard to environmental considerations. For instance, Letmathe and Balakrishnan [28] analyze the product mix problem under cap and cap-and-trade policies. They consider the product mix problem with a single operating procedure, and a multiple operating procedure which has multiple available resources, production yields and emission outputs. Unlike most of the studies in the literature, they model the customer demand as dependent on emission output of products (i.e., demand decreases with emission amount of the the firm). This study contributes to the literature by explicitly modeling multiple products and finite capacities on production resources within the context of production planning under environmentally regulations.

Benjaafar et al. [3] consider the integration of environmental regulations into operational models. They evaluate single and multi-stage lot sizing problems under some regulation options such as mandatory cap, emission tax, cap-and-trade policy and carbon offset. Benjaafar et al. [3] present some insightful recommendations for both the firms and the policy makers to decrease environmental effects of the firms at minimum cost. This paper contributes to the literature by suggesting managerial results to understand the emission reduction by operational adjustments.

Similar to Benjaafar et al. [3], Absi et al. [29] focus on the environmental constraints on the production and distribution planning of the firms. They analyze a multi-sourcing lot-sizing problem under different carbon emission constraints such as periodic carbon emission constraint, cumulative carbon emission constraint, global carbon emission constraint and rolling carbon emission constraint. In their setting, the firm's unitary environmental effect is subject to a maximum emission amount per period. They find a polynomial dynamic programming algorithm for the uncapacitated lot sizing problem with periodic carbon emission constraint and show that the problem with any of the other emisssion constraints is *NP*-hard. This study contributes to the literature by integrating different carbon emission constraints into lot-sizing problem.

Song and Leng [11] discuss the single-period stochastic replenishment problem

(The Newsvendor Problem) for perishable products with short lifespan under cap, tax, and, cap-and-trade policies. They investigate the impact of emission regulations on carbon emission reduction and expected profit of the firm. Song and Leng [11] examine the single-period problem for the low-margin, the moderate-margin and high-margin firms and give different managerial advices to the firms under different emission policies. They also propose basic results for policy makers to abate carbon emission. The authors make a scenario analysis to observe the influence of policy parameters on the firm's emission and expected total cost. This article contributes to the literature by drawing some managerial advices for both policy makers and the firms with different profit margins.

In Hoen et al. [30], transport mode selection problem (TMSP) is analyzed under carbon emission constraint (ETMSP) and carbon emission cost minimization (ECTMSP) policies. Carbon emissions for different transportation types are calculated based on Network for Transport and Environment (NTM) method. Then, the choice of transport mode for the ranges of emission cost is found for TMSP, ETMSP, and ECTMSP, and the effect of parameters (distance, volume and product density) on ECTMSP and indifference emission cost is examined. It is concluded that road is the preferable transport mode for TMSP, ETMSP and ECTMSP by a numerical example. This article contributes to the literature by presenting a detailed analysis about transport mode selection problem under some possible environmental regulations. Hoen et al. [31] extend the study of Hoen et al. [30] by further analyzing ECTMSP. They present more detailed analytical results for ECTMSP.

Jaber et al. [32] examine the buyer-vendor coordination problem under different environmental cost schemes. In addition to buyer's emission related parameters, they also model the fact that carbon is emitted due to manufacturing operations of the vendor and excessive emission is penalized with a carbon cost. Jaber et al. [32] incorporate carbon tax and emission penalty cost simultaneously into total supply chain cost function, and present an algorithm for finding the vendor's optimal production rate and optimal vendor-buyer coordination multiplier. Then, they numerically analyze the effects of carbon tax, emission penalty and manufacturer-retailer coordination on the total supply chain cost and total carbon emission. They find that combination of emission tax and emission penalty may be the most effective in reducing carbon emission. This article contributes to the literature by studying a two-level supply chain under European Union Emission Trading System.

In Kim et al. [33], the relationship between transportation cost and carbon emission is analyzed for intermodal and truck-only freight networks. A multi-objective optimization model with the objectives of minimizing freight cost and carbon emission is constructed and a procedure is proposed for estimating pareto-optimal solutions. In addition, a case study is presented to compare different inter-modal transportation networks under different market situations. This article contributes to the literature by examining the trade-offs between freight cost and carbon emission for intermodal networks.

2.2 Studies on Emission Reduction via Investment Opportunities

As noted in Chapter 1, leading companies in their sectors invest to decrease the environmental effects of their products and production and logistical processes, or to curb emissions through offset projects. Although investment decisions for environmental considerations is still a developing area in the operations research and the management science literature, it is possible to classify the related studies in three groups. The first group of papers (e.g., Zavanenella et al. [34], Swami and Shah [35], Raz et al. [36]) study the ordering and investment decisions in settings where consumer demand is sensitive to the environmental quality of the product, which in turn, can be increased through investment. Zavanenella et al. [34] study the coordination problem in a single-buyer, single-vendor system under environmental considerations. They decide the order quantity of the buyer, number of batches sent by the vendor, selling price of product and investment amount made by vendor to increase environmental quality of product. Their model assumes that demand is decreasing in the product's retail price and increasing in its environmental performance. They use a nonlinear investment function which has decreasing return in environmental quality. They also model production cost as increasing in the ratio of investment amount to the customer demand. Zavanenella et al. [34] compare the solution of independent policy and coordinated policy numerically and conclude that coordination leads to increase in supply chain profitability and improvement in the product's environmental quality. This study contributes to the literature by modeling demand that is dependent on both the price and the environmental quality of the product within the context of buyer-vendor coordination problem.

Swami and Shah [35] also study the channel coordination problem from a perspective of green supply chain management. They consider a setting in which the manufacturer decides the wholesale price and sells the product to the single retailer who determines the retail price. In their setting, customer demand is linearly decreasing in retail price and increasing in environmental efforts of both the retailer and manufacturer. They assume that cost of environmental effort is quadratically increasing in the efforts of the retailer and the manufacturer. The authors investigate the effects of problem parameters on cost of environmental efforts and pricing decisions. This study contributes to the literature by examining nonobligatory environmental efforts in supply chain coordination problem.

Raz et al. [36] study the economical and environmental impacts of innovation investments made by firms to change environmental performance of the product. They assume that manufacturing stage innovations reduce the cost of the product while use stage innovations increase the customer demand by lowering price sensitivity of customer. The authors evaluate the newsvendor problem by considering two aspects of product type (i.e. functional or innovative products) and environmental effect in life-cycle stage (i.e. manufacturing or use stage). They also present some analytical results on the firm's ordering and investment decisions, and ex-ante environmental effect of decisions. This article contributes to the literature by integrating environmental friendly design innovations into the firm's production decisions.

We would like to note that the above group of studies do not consider any regulation policies; the only motivation for investing in greening efforts is to increase demand by improving customers' perception of the product. The second group of papers model carbon offset investments when a cap-and-offset policy is in place (e.g., Benjaafar et al. [3], Song and Leng [11], and Chen et al. [19]). A cap-and-offset policy can be considered as a mix of cap and cap-and-trade policies. It differs from a cap policy in that the carbon allowance can be increased with offset investments. It differs from a cap-and-trade policy in that it does not allow carbon allowances to be tradable. The second group of studies exhibit two important characteristics. First, all three papers (i.e., Benjaafar et al. [3], Song and Leng [11], and Chen et al. [19]) assume unit reduction in carbon emissions per unit investment (which is included as an additional component in the cost function). Second, this type of investment modeling (i.e., offset investments) is not relevant within the context of other regulation policies.

The final group of studies consider investing in technology to reduce emissions under a regulation policy. We have identified two papers that fall into this group, i.e., Jiang and Klabjan [37] and Krass et al. [38], taking a firm's perspective to analyze the effects of investment decisions on the profitability and carbon emissions. This thesis also contributes to the third group of literature by modeling and solving a retailer's joint inventory replenishment and carbon emission reduction investment decisions under each of the three stated carbon emission regulation policies.

Jiang and Klabjan [37] analyze production and carbon emission reduction investment decisions under different regulation policies (i.e, cap-and-trade, command-and-control). They consider a setting in which carbon trading price and demand are stochastic, and assume a linear investment function. The decision maker first decides on production capacity and carbon emission reduction investment, and then, after the carbon trading price and demand are realized, the operations are adjusted. The authors extend this model to analyze investment timing decisions in two periods. They also investigate the effects of production cost change due to carbon emission reduction under cap-and-trade policy.

Krass et al. [38] discuss the firm's green technology choice under emission tax.

They model a Stackelberg game between a firm (i.e., the follower) and a policy maker (i.e., the leader) where the firm decides the product price and makes an investment over finite technology opportunities with different costs and emission reduction amounts to maximize its profit while the policy maker determines the tax price. Krass et al. [38] claim that higher tax does not always lead to lower emission, it may force the firm to choose the dirtier technology. They also model a social welfare problem which depends on firm's profit, consumer surplus and environmental damage. Then they investigate the effects of governmental subsidies and consumer rebates on the firm's emission and profit. This article contributes to the literature by analyzing taxation of emission over available technology choices.

Our study differs from Jiang and Klabjan [37] and Krass et al. [38] in two major ways. First, we analyze the classic EOQ model with an investment option under cap, tax, and cap-and-trade policies and provide an extension to understand the retailer's behavior under stochastic demand. Second, we consider a nonlinear investment function. We treat the investment amount as capital expenditure, similar to Billington [39], that is, some amount of money is invested per unit time and the reduction in carbon emissions per unit time is a function of the invested money. We benefit from Huang and Rust [40] in creating a correlation between investment and carbon emission reduction. Huang and Rust [40] note that spending on green technologies has decreasing marginal returns in pollution/environmental damage reduction. Therefore, the firm's carbon emission reduction per unit time is assumed to be an increasing concave function of the investment money per unit time. Through this functional form, we generalize the linear relation (i.e., constant marginal returns of the investment amount in carbon emission reduction) assumed by Benjaafar [3], Song and Leng [11], Chen et al. [19], and Jiang and Klabjan [37], and discrete relation (i.e. specific emission reduction for fixed investment over available green technologies) assumed by Krass et al. [38].

Chapter 3

Problem Definition and Analysis Under Different Carbon Emission Policies

3.1 Problem Definition

In this part of the thesis, a retailer's emission reduction investment and inventory replenishment decisions are analyzed under different government regulations on carbon emissions. It is assumed that the retailer operates under the conditions of the classical EOQ model. That is, the retailer orders Q units at each replenishment to meet deterministic and steady demand on time in the infinite horizon. In the setting of interest, there is significant carbon emission due to ordering, inventory holding, and procurement. The carbon emitted per replenishment, per-unit purchase and per-unit per-year inventory holding amount to $\hat{A}$, $\hat{c}$, and $\hat{h}$, respectively.

We consider three different carbon emission policies: cap, tax, and cap-and-trade. Under the cap policy, the retailer's carbon emissions per year cannot exceed an emission cap, denoted by C. Under the tax policy, the retailer is taxed p monetary units for unit carbon emission. Under the cap-and-trade policy, the retailer can trade a unit carbon emission for a value of c_p monetary units. These policies are intended to reduce carbon emissions by affecting the retailer's operations, however, the retailer can also reduce his/her carbon emissions by investing in new technology, equipment, or machinery. Mainly, annual carbon emission can be decreased in an amount of $\alpha G - \beta G^2$ in return for G monetary units invested per year ($0 \leq G \leq \frac{\alpha}{\beta}$). Here, α reflects the efficiency of green technology in reducing emissions, and β is a decreasing return parameter (Huang and Rust [40]). In each case, the problem is to find the order quantity and the investment amount that jointly minimize the retailer's total average annual costs. Table 3.1 summarizes the notation used in this part of the thesis. Additional notation will be defined as needed.

Without any carbon emission policy in place, the total average annual costs due to ordering, inventory holding, procurement, and investment is given by

$$TC(Q, G) = \frac{AD}{Q} + \frac{hQ}{2} + cD + G, \tag{3.1}$$

and the total average annual emission amount is given by

$$E(Q, G) = \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2. \tag{3.2}$$

When the retailer makes no investment, i.e., $G = 0$, Expression (3.1) provides the total average annual costs in the EOQ model, and its value is minimized at $Q^0 = \sqrt{\frac{2AD}{h}}$, which we refer to as the "cost-optimal quantity". If there is no carbon emission policy in place, $(Q^0, 0)$ will in fact be the optimizing pair of order quantity and investment amount for the retailer. Furthermore, it follows from Expression (3.2) that $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ is the minimum average annual carbon emission possible without investment, and is achieved when the retailer orders $Q^e = \sqrt{\frac{2\hat{A}D}{\hat{h}}}$ units, which we refer to as the "emission-optimal quantity".

The problem parameters are assumed to satisfy the following conditions:

(A1) The minimum annual carbon emission possible due to ordering decisions is more than the maximum yearly emission reduction possible due to investment decisions. That is,

$$\sqrt{2\hat{A}\hat{h}D} + \hat{c}D > \frac{\alpha^2}{4\beta}. \tag{3.3}$$

Table 3.1: Problem Parameters and Decision Variables

Retailer's Parameters	
A	fixed cost of inventory replenishment
h	cost of holding one unit inventory for a year
c	unit procurement cost
D	demand per year
$\hat{A}$	carbon emission amount due to inventory replenishment
$\hat{h}$	carbon emission amount due to holding one unit inventory for a year
$\hat{c}$	carbon emission amount due to unit procurement
Policy Parameters	
i	carbon policy index; $i = 1$ for cap, $i = 2$ for tax, and $i = 3$ for cap-and-trade policies
C	annual carbon emission cap
p	tax paid for one unit of emission
c_p	unit carbon emission trading price
Retailer's Decision Variables	
Q	order quantity
G	annual investment amount for carbon emission reduction
X	traded quantity of emission capacity in cap-and-trade policy
Functions and Optimal Values of Decision Variables	
$TC(Q, G)$	total average annual costs as a function of Q and G without a carbon policy
$E(Q, G)$	carbon emissions per year as a function of Q and G
$TC_i(Q, G)$	total average annual costs as a function of Q and G under carbon policy i
Q_i^*	optimal order quantity under carbon policy i
G_i^*	optimal investment amount under carbon policy i

(A2) For the tax policy under consideration, there exists a value of $G > 0$ at which savings in taxes when G monetary units are invested in new technology to reduce carbon emissions exceeds the cost of investment. Hence, we have

$$\alpha p > 1. \tag{3.4}$$

(A3) For the cap-and-trade policy under consideration, there exists a value of investment amount $G > 0$ at which more reduction in carbon emissions can be achieved by investing in new technology rather than purchasing carbon capacity at a total value of G monetary units. Hence, we have

$$\alpha c_p > 1. \tag{3.5}$$

(A4) For the cap policy under consideration, there exist values of the investment amount that can reduce the annual carbon emission to below carbon capacity. Hence, we have

$$\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C. \tag{3.6}$$

The right hand side of Inequality (3.3), that is, $\frac{\alpha^2}{4\beta}$, is the maximum possible value of annual carbon emission reduction and is achieved when $G = \frac{\alpha}{2\beta}$. Recall that $\sqrt{2\hat{A}\hat{h}D}+\hat{c}D$ is the minimum possible value of yearly carbon emissions due to ordering decisions. An implication of Assumption (A1), therefore, is that carbon emissions cannot be completely eliminated with new technology. Assumption (A2), in mathematical terms, is equivalent to saying that there exists some $G > 0$ at which $(\alpha G - \beta G^2)p > G$. Dividing both sides of this inequality by G and considering the fact that $\beta G p > 0$ leads to $\alpha p > 1$. If Assumption (A2) does not hold, then any investment to reduce carbon emissions does not pay off, and hence, an investment decision should not be of concern. Similarly, Assumption (A3) can be written as $\alpha G - \beta G^2 > \frac{G}{c_p}$ for some positive value of G, which in turn implies $\alpha c_p > 1$. Finally, Assumption (A4) is necessary for the retailer to be in business under the current cap policy. If the minimum carbon emission possible (i.e., $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta}$) due to ordering and investment decisions were more than the cap C, then there would be no feasible solution to the retailer's inventory problem.

3.2 Analysis Under Different Carbon Emission Policies

In this section, we solve the retailer's integrated problem of finding the optimal order quantity and carbon emission reduction investment under the three carbon emission regulation policies: cap, tax, and cap-and-trade. We represent the optimal solution under each policy i as a pair of values (Q_i^*, G_i^*).

Recall that, by definition of the investment function, there exists an upper bound on G, that is, $G \leq \frac{\alpha}{\beta}$. We do not include this restriction as a constraint because the nature of our formulations for all emission regulations makes it redundant. That is, the investment value in all optimal solutions without incorporating $G \leq \frac{\alpha}{\beta}$ already satisfies this constraint. In fact, due to the strict concavity of $\alpha G - \beta G^2$ with respect to G and the fact that $\frac{\alpha}{2\beta}$ is its unique maximizer, for every investment value that is greater than $\frac{\alpha}{2\beta}$, the corresponding reduction in annual carbon emission can be achieved by a smaller investment amount within the range $0 \leq G \leq \frac{\alpha}{2\beta}$. Therefore, the optimal investment value will always be less than or equal to $\frac{\alpha}{2\beta}$. The optimal solutions for the cap, tax, and cap-and-trade policies, as they are stated in Theorems 1, 2, and 3, justify these observations.

3.2.1 Cap Policy

Under a cap policy, the retailer is subject to an upper bound, that is an "emission cap", on the total average annual carbon emission. The retailer's problem is to find the optimal order quantity and the investment amount to minimize average annual total cost without exceeding the emission cap C. This problem can be formulated as follows:

$$\begin{aligned} \min \quad & TC_1(Q,G) = \tfrac{AD}{Q} + \tfrac{hQ}{2} + cD + G \\ \text{s.t.} \quad & \tfrac{\hat{A}D}{Q} + \tfrac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2 \leq C \ , \\ & Q \geq 0, G \geq 0. \end{aligned}$$

Note that, when $G = 0$, there exists a feasible solution to the above problem as long as $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$. Given that $G = 0$, the feasible region consists of all pairs $(Q, 0)$ such that $Q_1 \geq Q \geq Q_2$, where

$$Q_1 = \frac{C - \hat{c}D + \sqrt{(C - \hat{c}D)^2 - 2\hat{A}\hat{h}D}}{\hat{h}} \tag{3.7}$$

and

$$Q_2 = \frac{C - \hat{c}D - \sqrt{(C - \hat{c}D)^2 - 2\hat{A}\hat{h}D}}{\hat{h}}. \tag{3.8}$$

Q_1 and Q_2 are the two roots of $\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D = C$. It is important to note that the existence of Q_1 and Q_2 depend on how $(C - \hat{c}D)$ compares to $\sqrt{2\hat{A}\hat{h}D}$, and is not guaranteed. In fact, in Theorem 1, we characterize the optimal solution to the retailer's problem in two parts, considering the following two cases: (i) $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ and (ii) $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$. In the latter case, the restriction on the maximum carbon emission cannot be overcome only by ordering decisions, the retailer must also take advantage of investment opportunities. Assumption (A4) guarantees that there exists a feasible solution in this case. Prior to stating the retailer's optimal order quantity and investment decisions under a cap policy, let us also introduce the following solution pairs:

$$(Q_3, G_3) = \left(\frac{(C - \hat{c}D + \alpha G_3 - \beta G_3^2) + \sqrt{(C - \hat{c}D + \alpha G_3 - \beta G_3^2)^2 - 2\hat{A}\hat{h}D}}{\hat{h}}, \frac{2D(A\alpha + \hat{A}) - Q_3^2(\alpha h + \hat{h})}{2\beta(2AD - Q_3^2 h)} \right),$$

$$(Q_4, G_4) = \left(\frac{(C - \hat{c}D + \alpha G_4 - \beta G_4^2) - \sqrt{(C - \hat{c}D + \alpha G_4 - \beta G_4^2)^2 - 2\hat{A}\hat{h}D}}{\hat{h}}, \frac{2D(A\alpha + \hat{A}) - Q_4^2(\alpha h + \hat{h})}{2\beta(2AD - Q_4^2 h)} \right),$$

$$(Q_5, G_5) = \left(Q^e, \frac{\alpha - \sqrt{\alpha^2 - 4\beta\left(-C + \hat{c}D + \sqrt{2\hat{A}D\hat{h}}\right)}}{2\beta} \right).$$

Note that $\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2 = C$ when (Q, G) is any one of the pairs (Q_3, G_3), (Q_4, G_4), and (Q_5, G_5). For $0 \leq G \leq \frac{\alpha}{2\beta}$, it can be shown that

$$Q_3 \geq Q_1 \geq Q_2 \geq Q_4. \tag{3.9}$$

As characterized in the next theorem and its proof, the optimal solution to the retailers problem under the cap policy is given by one of the following pairs:

$(Q^0, 0)$, $(Q_1, 0)$, $(Q_2, 0)$, (Q_3, G_3), (Q_4, G_4), (Q_5, G_5). If $(Q_1^*, G_1^*) = (Q^0, 0)$, then the cost-optimal solution satisfies the emission constraint already. If $(Q_1^*, G_1^*) = (Q_1, 0)$ or $(Q_1^*, G_1^*) = (Q_2, 0)$, then the retailer is able to satisfy the emission constraint by ordering a quantity other than the cost-optimal one while not making any investment. In other cases where $G_1^* > 0$, the retailer minimizes his/her costs under the emission constraint by investing in new technology besides carefully-made ordering decisions.

Theorem 1 *Under a cap policy, the optimal pair of the retailer's replenishment quantity and his/her investment amount is as follows:*

If $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ *then,*

$$(Q_1^*, G_1^*) = \begin{cases} (Q^0, 0) & \text{if} \quad Q_2 \leq Q^0 \leq Q_1, \\ (Q_1, 0) & \text{if} \quad Q^\alpha < Q_1 < Q^0, \\ (Q_3, G_3) & \text{if} \quad Q^e < Q_3 \leq Q^\alpha, \\ (Q_2, 0) & \text{if} \quad Q^0 < Q_2 < Q^\alpha, \\ (Q_4, G_4) & \text{if} \quad Q^\alpha \leq Q_4 < Q^e, \\ \end{cases}$$

and if $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$, *then*

$$(Q_1^*, G_1^*) = \begin{cases} (Q_3, G_3) & \text{if} \quad Q^e < Q_3 \leq Q^\alpha, \\ (Q_4, G_4) & \text{if} \quad Q^\alpha \leq Q_4 < Q^e, \\ (Q_5, G_5) & o.w., \end{cases}$$

where $Q^\alpha = \sqrt{\frac{2(\hat{A}+A\alpha)D}{\hat{h}+h\alpha}}$.

Proof: The proof will follow by making use of the Karush-Kuhn-Tucker (KKT) conditions. The objective function is differentiable, and it is convex because its Hessian matrix $\begin{pmatrix} \frac{2AD}{Q^3} & 0 \\ 0 & 0 \end{pmatrix}$ is positive semi-definite. Emission cap constraint is also differentiable, and it is strictly convex in Q and G because its Hessian matrix

$\begin{pmatrix} \frac{2\hat{A}D}{Q^3} & 0 \\ 0 & 2\beta \end{pmatrix}$ is positive definite. In addition, Assumption (A4) implies that there exists a feasible point in the set $\{\ \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2 < C,\ Q \geq 0, G \geq 0\ \}$. As a result, we conclude that the KKT conditions listed below guarantee global optimality along with feasibility conditions.

$$\frac{-AD}{Q^2} + \frac{h}{2} + \lambda_1 \left(\frac{-\hat{A}D}{Q^2} + \frac{\hat{h}}{2} \right) - \mu_1 = 0, \tag{3.10}$$

$$1 + \lambda_1(-\alpha + 2\beta G) - \mu_2 = 0, \tag{3.11}$$

$$\lambda_1 \left(C - \frac{\hat{A}D}{Q} - \frac{\hat{h}Q}{2} - \hat{c}D + \alpha G - \beta G^2 \right) = 0, \tag{3.12}$$

$$\mu_1 Q = 0, \tag{3.13}$$

$$\mu_2 G = 0, \tag{3.14}$$

$$\lambda_1 \geq 0, \quad \mu_1 \geq 0, \quad \mu_2 \geq 0. \tag{3.15}$$

The multipliers λ_1, μ_1, and μ_2 may be equal to zero or be greater than zero. Considering these alternatives, there are eight possible cases, however, only the following three may lead to feasible solutions.

Case 1: $\lambda_1 = 0$, $\mu_1 = 0$, $\mu_2 > 0$

Expression (3.12) and Expression (3.13) are satisfied because $\lambda_1 = 0$ and $\mu_1 = 0$. Expression (3.11) implies $\mu_2 = 1$. Because $\mu_2 > 0$, Expression (3.14) leads to $G = 0$. Finally, evaluating Expression (3.10) at $\lambda_1 = 0$ and $\mu_1 = 0$, we obtain $Q = Q^0 = \sqrt{\frac{2AD}{h}}$.

Now, let us check the feasibility of $Q = \sqrt{\frac{2AD}{h}}$ and $G = 0$. When $G = 0$, to find a feasible order quantity, we should have $C \geq \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, because the contrary implies that even the minimum carbon emission possible by ordering decisions would exceed the emission cap. In addition, any feasible order quantity Q should satisfy $\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D \leq C$. This inequality further yields $Q_2 \leq Q \leq Q_1$, where Q_1 and Q_2 are defined in (3.7) and (3.8). Observe that since $C \geq \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, both Q_1 and Q_2 exist. Therefore, if $C \geq \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$ and $Q_2 \leq Q^0 \leq Q_1$, then $Q_1^* = Q^0$ and $G_1^* = 0$.

Case 2: $\lambda_1 > 0$, $\mu_1 = 0$, $\mu_2 > 0$

Using the fact that $\mu_1 = 0$, Expression (3.10) can be rewritten as

$$-\frac{AD}{Q^2} + \frac{h}{2} + \lambda_1 \left(-\frac{\hat{A}D}{Q^2} + \frac{\hat{h}}{2} \right) = 0. \tag{3.16}$$

Since $\mu_2 > 0$, Expression (3.14) implies $G = 0$. Therefore, Expression (3.11) reduces to

$$1 - \alpha\lambda_1 - \mu_2 = 0. \tag{3.17}$$

Because $\lambda_1 > 0$ and $G = 0$, Expression (3.12) implies

$$C - \frac{\hat{A}D}{Q} - \frac{\hat{h}Q}{2} - \hat{c}D = 0.$$

Note that, Q_1 and Q_2 are the two values of Q that satisfy the above equality. Since $G = 0$, we should have $C \geq \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$ for the same reason as discussed in Case 1, which in turn, implies that Q_1 and Q_2 exist. In the rest of our analysis for Case 2, we will consider the following two possibilities:

Case 2.1: $C = \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$

It can be shown that if $C = \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, then $Q_1 = Q_2 = \sqrt{\frac{2\hat{A}D}{\hat{h}}}$. In this case, Expression (3.16) holds for any positive value of λ_1 as long as $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$. However, due to the relationship between λ_1 and μ_2 as stated in Expression (3.17) and the fact that $\mu_2 > 0$, λ_1 should be chosen such that $\lambda_1 < \frac{1}{\alpha}$. Therefore, if $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$, then $Q_1^* = Q^0$ and $G_1^* = 0$.

Case 2.2: $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$

If $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, then $Q_1 \neq Q_2$. For $Q = Q_1$ or $Q = Q_2$ to be optimal, there must exist positive values of λ_1 and μ_2 that satisfy Expression (3.16) and Expression (3.17). Using Expression (3.16), we obtain

$$\lambda_1 = \frac{\frac{AD}{Q^2} - \frac{h}{2}}{-\frac{\hat{A}D}{Q^2} + \frac{\hat{h}}{2}} = \frac{2AD - hQ^2}{-2\hat{A}D + \hat{h}Q^2}.$$

Note that, since $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, it turns out that the denominator of the above expression is different than zero for $Q = Q_1$ and $Q = Q_2$, therefore, λ_1 is finite. Utilizing this expression in (3.17) further leads to

$$\mu_2 = 1 - \alpha \frac{2AD - hQ^2}{-2\hat{A}D + \hat{h}Q^2}.$$

Since $\lambda_1 > 0$ and $\mu_2 > 0$, any optimal Q should then satisfy

$$0 < \frac{2AD - hQ^2}{-2\hat{A}D + \hat{h}Q^2} < \frac{1}{\alpha}. \tag{3.18}$$

Now, let us check the conditions for Q_1 to satisfy the above expression, and hence, to be optimal. Since $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, we have

$$2(C - \hat{c}D)^2 - 4\hat{A}D\hat{h} > 0.$$

Combining $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$ with the fact that $\sqrt{2\hat{A}D\hat{h}} > 0$, we conclude

$$2(C - \hat{c}D)^2 + 2(C - \hat{c}D)\sqrt{(C - \hat{c}D)^2 - 2\hat{A}D\hat{h}} - 4\hat{A}D\hat{h} > 0,$$

which can be rewritten as

$$\left[C - \hat{c}D + \sqrt{(C - \hat{c}D)^2 - 2\hat{A}D\hat{h}}\right]^2 - 2\hat{A}D\hat{h} > 0.$$

The above inequality implies

$$-2\hat{A}D + \hat{h}\frac{\left[C - \hat{c}D + \sqrt{(C - \hat{c}D)^2 - 2\hat{A}D\hat{h}}\right]^2}{\hat{h}^2} > 0.$$

Observe from Expression (3.7) that, the fractional term in the above expression is equal to Q_1^2, therefore, we have

$$-2\hat{A}D + \hat{h}Q_1^2 > 0.$$

Based on the above result, for Expression (3.18) to hold for $Q = Q_1$, we should have $2AD - hQ_1^2 > 0$ and $\frac{2AD - hQ_1^2}{-2\hat{A}D + \hat{h}Q_1^2} < \frac{1}{\alpha}$. Evaluating these two expressions, we conclude that if $Q_1 < Q^0 = \sqrt{\frac{2AD}{h}}$ and $Q_1 > Q^\alpha = \sqrt{\frac{2(\hat{A} + A\alpha)D}{\hat{h} + h\alpha}}$, then $Q_1^* = Q_1$ and $G_1^* = 0$.

To check the conditions for optimality of Q_2, we use a similar methodology. Since $C > \sqrt{2\hat{A}D\hat{h}} + \hat{c}D$, we have

$$\left((C - \hat{c}D)^2 - 2\hat{A}D\hat{h}\right)^2 < (C - \hat{c}D)^2 \left((C - \hat{c}D)^2 - 2\hat{A}D\hat{h}\right),$$

which, in turn, implies that

$$(C - \hat{c}D)^2 - 2\hat{A}D\hat{h} - (C - \hat{c}D)\sqrt{(C - \hat{c}D)^2 - 2\hat{A}D\hat{h}} < 0.$$

Multiplying both sides of the above expression with $\frac{2}{\hat{h}}$ leads to

$$-2\hat{A}D + \hat{h}\frac{\left[(C - \hat{c}D) - \sqrt{(C - \hat{c}D)^2 - 2\hat{A}D\hat{h}}\right]^2}{\hat{h}^2} < 0.$$

Observe from Expression (3.8) that, the fractional term in the above expression is equal to Q_2^2, therefore, we have

$$-2\hat{A}D + \hat{h}Q_2^2 < 0.$$

Based on the above result, for Expression (3.18) to hold for $Q = Q_2$, we should have $2AD - hQ_2^2 < 0$ and $\frac{2AD - hQ_2^2}{-2\hat{A}D + \hat{h}Q_2^2} < \frac{1}{\alpha}$. Evaluating these two expressions, we conclude that if $Q_2 > Q^0 = \sqrt{\frac{2AD}{h}}$ and $Q_2 < Q^\alpha = \sqrt{\frac{2(\hat{A}+A\alpha)D}{\hat{h}+h\alpha}}$, then $Q_1^* = Q_2$ and $G_1^* = 0$.

Case 3: $\lambda_1 > 0$, $\mu_1 = 0$, $\mu_2 = 0$

Expression (3.13) and Expression (3.14) are satisfied because $\mu_1 = 0$ and $\mu_2 = 0$. Using the fact that $\mu_1 = 0$, Expression (3.10) can be rewritten as

$$\frac{-AD}{Q^2} + \frac{h}{2} + \lambda_1 \left(\frac{-\hat{A}D}{Q^2} + \frac{\hat{h}}{2}\right) = 0. \qquad (3.19)$$

Since $\mu_2 = 0$, Expression (3.13) reduces to

$$1 + \lambda_1(-\alpha + 2\beta G) = 0. \qquad (3.20)$$

As $\lambda_1 > 0$, Expression(3.12) implies

$$\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - C - \alpha G + \beta G^2 = 0. \qquad (3.21)$$

Now, we should find nonnegative values of Q and G, and a positive value of λ_1 that solve the system of equations as given by (3.19), (3.20), and (3.21). It follows from Expression (3.20) that $G < \frac{\alpha}{2\beta}$. For any value of G, Expression (3.21) is satisfied at the following two values of Q, which we refer to as $Q_3(G)$ and $Q_4(G)$:

$$Q_3(G) = \frac{(C - \hat{c}D + \alpha G - \beta G^2) + \sqrt{(C - \hat{c}D + \alpha G - \beta G^2)^2 - 2\hat{A}\hat{h}D}}{\hat{h}}, \quad (3.22)$$

$$Q_4(G) = \frac{(C - \hat{c}D + \alpha G - \beta G^2) - \sqrt{(C - \hat{c}D + \alpha G - \beta G^2)^2 - 2\hat{A}\hat{h}D}}{\hat{h}}, \quad (3.23)$$

For the existence of such $Q_3(G)$ and $Q_4(G)$, we should have $C - \hat{c}D + \alpha G - \beta G^2 \geq \sqrt{2\hat{A}\hat{h}D}$. In the rest of our analysis for Case 3, we will consider the following two possibilities:

Case 3.1: $C - \hat{c}D + \alpha G - \beta G^2 = \sqrt{2\hat{A}D\hat{h}}$

In this case, $Q_3(G) = Q_4(G) = Q^e = \sqrt{\frac{2\hat{A}D}{\hat{h}}}$. When $Q = Q^e$, Expression (3.19) holds for any $\lambda_1 > 0$ as long as $\frac{\hat{A}}{\hat{h}} = \frac{A}{h}$. Now, for any value of G that satisfies $C - \hat{c}D + \alpha G - \beta G^2 = \sqrt{2\hat{A}D\hat{h}}$ to be optimal, we should have $0 \leq G < \frac{\alpha}{2\beta}$. Although there are two real roots of this equation, these conditions only hold at $G = G_5 = \frac{\alpha - \sqrt{\alpha^2 - 4\beta\left(-C + \hat{c}D + \sqrt{2\hat{A}D\hat{h}}\right)}}{2\beta}$. Therefore, if $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ and $\frac{\hat{A}}{\hat{h}} = \frac{A}{h}$, then $Q_1^* = Q^e$ and $G_1^* = G_5$.

Case 3.2: $C - \hat{c}D + \alpha G - \beta G^2 > \sqrt{2\hat{A}D\hat{h}}$

If $C - \hat{c}D + \alpha G - \beta G^2 > \sqrt{2\hat{A}D\hat{h}}$, then $Q_3(G) \neq Q_4(G)$. For any $(Q_3(G), G)$ or $(Q_4(G), G)$ pair to be optimal, there must exist corresponding positive values of λ_1 that satisfy Expression (3.16). That is, we should have $\lambda_1 = \frac{2AD - hQ^2}{-2\hat{A}D + \hat{h}Q^2} > 0$. Now, let us check the conditions for $Q_3(G)$ to satisfy this inequality. It can be shown that $-2\hat{A}D + \hat{h}Q_3^2(G) > 0$, or equivalently $Q_3(G) > Q^e$, for any given value of G that satisfies $C - \hat{c}D + \alpha G - \beta G^2 > \sqrt{2\hat{A}D\hat{h}}$. Combining the condition of having $\lambda_1 > 0$ with the fact that $-2\hat{A}D + \hat{h}Q_3^2(G) > 0$, we conclude $2AD - hQ_3^2(G) > 0$. This implies $Q_3(G) < Q^0$.

Next, utilizing $\lambda_1 = \frac{2AD - hQ^2}{-2\hat{A}D + \hat{h}Q^2}$ in Expression (3.20), we obtain

$$G = \frac{2(\alpha A + \hat{A})D - (\alpha h + \hat{h})Q_3^2(G)}{2\beta(2AD - hQ_3^2(G))}. \quad (3.24)$$

At this point, the above expression with Expression (3.22) lead to a unique pair of (Q, G), which we refer to as (Q_3, G_3). The condition that $G \geq 0$, jointly with $2AD - hQ_3^2 > 0$, implies that $2(\alpha A + \hat{A})D - (\alpha h + \hat{h})Q_3^2 \geq 0$. This, in turn, leads to $Q_3 \leq Q^\alpha = \sqrt{\frac{2D(\alpha A + \hat{A})}{\alpha h + \hat{h}}}$.

We have shown that the optimality of Q_3 is due to the following conditions: $Q_3 > Q^e$, $Q_3 < Q^0$ and $Q_3 \leq Q^\alpha$. Note that $Q_3 > Q^e$ and $Q_3 < Q^0$ simultaneously hold only if $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$. Having $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ further implies that $Q^\alpha < Q^0$. Therefore, we conclude that if $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ and $Q^e < Q_3 \leq Q^\alpha$, then $Q_1^* = Q_3$ and $G_1^* = G_3$.

With a similar approach, it can be shown that (Q_4, G_4) obtained by solving Expression (3.23) and $G = \frac{2(\alpha A + \hat{A})D - (\alpha h + \hat{h})Q_4^2(G)}{2\beta(2AD - hQ_4^2(G))}$ simultaneously, is optimal if $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} < C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ and $Q^\alpha \leq Q_4 < Q^e$. ■

The result that will be highlighted next, applies to the special case of the problem where $\frac{\hat{A}}{\hat{h}} = \frac{A}{h}$, and is a consequence of Theorem 1 and its proof.

Remark 1 *If $\frac{\hat{A}}{\hat{h}} = \frac{A}{h}$, the optimal replenishment quantity is always given by the cost-optimal solution Q^0, which is equal to the emission-optimal solution Q^e. However, if $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$, then $G_1^* = 0$, and if $C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$, then $G_1^* > 0$.*

It is worthwhile to note that, when there is no investment opportunity for carbon emissions reduction, Theorem 1 coincides with the results of Chen et al. [19]. The next corollary presents the annual carbon emission level resulting from the retailer's optimal decisions as given in Theorem 1.

Corollary 1 *The average annual carbon emission resulting from the retailer's* *optimal solution under a cap policy is*

$$E(Q_1^*, G_1^*) = \begin{cases} \frac{\sqrt{D}(\hat{A}h+\hat{h}A)}{\sqrt{2Ah}} + \hat{c}D & \text{if} \quad Q_2 \leq Q^0 \leq Q_1, \\ \\ C & o.w. \end{cases}$$

As seen in Corollary 1, the maximum carbon emissions per year are bounded by C. However, as long as C is not binding such that $Q_2 \leq Q^0 \leq Q_1$, annual carbon emissions are linearly increasing with $\hat{A}$ and $\hat{h}$. For those nonbinding C values, annual carbon emissions are also dependent on an A/h ratio, and in fact, increases with A/h if $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$. Furthermore, the carbon emissions level is not dependent on investment parameters α and β.

In the next lemma, we investigate the impact of having an investment option for carbon emission reduction on the retailer's annual emission level under a cap policy. In doing this, we consider the following two measures: $E\left(Q_1^*(0), 0\right) - E\left(Q_1^*, G_1^*\right)$ and $TC_1\left(Q_1^*(0), 0\right) - TC_1\left(Q_1^*, G_1^*\right)$. We use the notation $Q_1^*(0)$ to refer to the retailer's optimal replenishment quantity under a cap policy, given that the investment amount is zero. Note that, a feasible value for $Q_1^*(0)$ may not always exist, specifically when $C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$. The lemma, which will be presented without a proof, follows from Corollary 1 and the expression for $E\left(Q_1^*(0), 0\right)$ provided in Chen et al. [19]. The result applies to cases in which a feasible value of $Q_1^*(0)$ can be found.

Lemma 1 *Having an investment opportunity for carbon emission reduction does not change the annual carbon emission level under a cap policy, however, it may lead to lower average annual costs for the retailer. That is,* $E\left(Q_1^*(0), 0\right) - E\left(Q_1^*, G_1^*\right) = 0$ *and* $TC_1\left(Q_1^*(0), 0\right) - TC_1\left(Q_1^*, G_1^*\right) \geq 0$.

If $C < \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$ and an investment option is not available for the retailer to reduce his/her carbon emissions, there is no feasible replenishment quantity, and therefore it does not make sense for him/her to be in business. Therefore, in such cases, the savings in costs due to having an investment option may as well be considered as infinity. Note that when $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$, $Q_1^*(0)$ is given by Q^0

if $Q_2 \leq Q^0 \leq Q_1$, by Q_2 if $Q^0 < Q_2$, and by Q_1 if $Q_1 < Q^0$. The optimal (Q, G) pairs in the problems with and without the investment option coincide in those cases. Therefore, the savings in costs due to investment can be strictly positive only under the circumstances in which $C \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$, and the solution to the problem with investment option is given by either (Q_3, G_3) or (Q_4, G_4).

Next, we study the effects of a cap policy on the retailer's annual carbon emissions and costs in comparison to a case where there is no governmental regulation. In the latter case, the retailer orders Q^0 units and makes no investment for emission reduction.

Lemma 2 *Under a cap policy, the retailer's optimal decisions for replenishment quantity and investment amount may reduce the yearly carbon emissions with an annual cost that is no less than what it would be when no emission policy is in place. That is,* $TC_1\left(Q_1^*, G_1^*\right) \geq TC\left(Q^0, 0\right)$ *and* $E\left(Q_1^*, G_1^*\right) \leq E\left(Q^0, 0\right)$.

Proof: It follows from the expressions for $TC(Q, G)$ and $TC_1(Q, G)$, and the definition of Q^0, that $TC(Q^0, 0) \leq TC_1(Q_1^*, 0)$. Furthermore, we have $TC_1(Q_1^*, 0) \leq TC_1(Q_1^*, G_1^*)$; thus, $TC_1(Q_1^*, G_1^*) \geq TC(Q^0, 0)$. The result about the annual emission levels follows from Corollary 1 and the fact that $E\left(Q^0, 0\right) = \frac{\sqrt{D}(\hat{A}h + \hat{h}A)}{\sqrt{2Ah}} + \hat{c}D$.
■

Under any of the emission regulation policies, there may exist investment options with different parameters α and β. If this is the case, then the retailer must choose among different investment options. The result presented in the next lemma may help the retailer to make such a decision when a cap policy is in place.

Lemma 3 *Let us consider two feasible investment options (i.e., they satisfy Assumption (A4)): one with parameters* α_1 *and* β_1*, and the other with parameters* α_2 *and* β_2*. Let* $(\bar{Q}_2, \bar{G}_2)$ *be the retailer's optimal solution if the second investment option (i.e., the one with parameters* α_2 *and* β_2*) is adopted. If* $\beta_2 \geq \beta_1$ *and* $\alpha_2 \leq \alpha_1$*, then under the first investment option, there exists a solution which leads to the same annual emission level with no more costs.*

Proof: First, we will show that there exists a feasible solution under the first investment option, say $\left(\bar{Q}_1, \bar{G}_1\right)$, that leads to the same annual emissions level as that of $\left(\bar{Q}_2, \bar{G}_2\right)$ under the second investment option. Second, we will show that the annual costs at $\left(\bar{Q}_1, \bar{G}_1\right)$, when the first investment option is adopted, are lower than or equal to the annual costs at $\left(\bar{Q}_2, \bar{G}_2\right)$ under the second investment option.

Let us set $\bar{Q}_1 = \bar{Q}_2$. The two conditions for $\left(\bar{Q}_1, \bar{G}_1\right)$ along with the first investment option to lead to the same annual emissions level as that of $\left(\bar{Q}_2, \bar{G}_2\right)$ under the second investment option are:

$$\alpha_1 \bar{G}_1 - \beta_1 \left(\bar{G}_1\right)^2 = \alpha_2 \bar{G}_2 - \beta_2 \left(\bar{G}_2\right)^2 \tag{3.25}$$

and

$$\bar{G}_1 \leq \frac{\alpha_1}{2\beta_1}. \tag{3.26}$$

We will show that there exists a unique solution to Expression (3.25) that also satisfies Expression (3.26).

The two values of $\bar{G}_1$ that satisfy Expression (3.25) are:

$$\frac{\alpha_1 + \sqrt{(\alpha_1)^2 - 4\beta_1 \left(\alpha_2 \bar{G}_2 - \beta_2 \left(\bar{G}_2\right)^2\right)}}{2\beta_1}, \tag{3.27}$$

and

$$\frac{\alpha_1 - \sqrt{(\alpha_1)^2 - 4\beta_1 \left(\alpha_2 \bar{G}_2 - \beta_2 \left(\bar{G}_2\right)^2\right)}}{2\beta_1}. \tag{3.28}$$

Note that $\frac{(\alpha_2)^2}{4\beta_2}$ is the maximum of the annual emission reduction under the second investment option. Therefore, $\alpha_2 \bar{G}_2 - \beta_2 \left(\bar{G}_2\right)^2 \leq \frac{(\alpha_2)^2}{4\beta_2}$. Since $\alpha_1 \geq \alpha_2$ and $\beta_1 \leq \beta_2$, we have $\frac{(\alpha_2)^2}{4\beta_2} \leq \frac{(\alpha_1)^2}{4\beta_1}$. This in turn implies that $\frac{(\alpha_1)^2}{4\beta_1} \geq \alpha_2 \bar{G}_2 - \beta_2 \left(\bar{G}_2\right)^2$, and hence, $(\alpha_1)^2 \geq 4\beta_1 \left(\alpha_2 \bar{G}_2 - \beta_2 \left(\bar{G}_2\right)^2\right)$. Therefore, Expression (3.27) and Expression (3.28) lead to positive values. However, value of $\bar{G}_1$ provided by Expression (3.28) leads to lower annual costs, therefore, we set $\bar{G}_1 = \frac{\alpha_1 - \sqrt{(\alpha_1)^2 - 4\beta_1 \left(\alpha_2 \bar{G}_2 - \beta_2 \left(\bar{G}_2\right)^2\right)}}{2\beta_1}$, which also satisfies Expression (3.26).

We show above the feasibility of $\left(\bar{Q}_1, \bar{G}_1\right)$ for the retailer's problem if the first investment option is adopted. Note that in this solution, $\bar{Q}_1 = \bar{Q}_2$ and

$\bar{G}_1 = \frac{\alpha_1 - \sqrt{(\alpha_1)^2 - 4\beta_1\left(\alpha_2\bar{G}_2 - \beta_2\left(\bar{G}_2\right)^2\right)}}{2\beta_1}$. Now, assume that $\left(\bar{Q}_1, \bar{G}_1\right)$ leads to greater annual costs. Then, due to the objective function under the cap policy, it must be that $G_2 < G_1$. Since $\alpha_1 G - \beta_1 G^2$ is strictly increasing over those values of G such that $G \leq \frac{\alpha_1}{2\beta_1}$, it follows that

$$\alpha_1\bar{G}_1 - \beta_1\left(\bar{G}_1\right)^2 > \alpha_1\bar{G}_2 - \beta_1\left(\bar{G}_2\right)^2.$$

As $\alpha_2 \leq \alpha_1$ and $\beta_2 \geq \beta_1$, we have

$$\alpha_1\bar{G}_2 - \beta_1\left(\bar{G}_2\right)^2 \geq \alpha_2\bar{G}_2 - \beta_2\left(\bar{G}_2\right)^2.$$

The above two inequalities jointly imply that $\alpha_1\bar{G}_1 - \beta_1\left(\bar{G}_1\right)^2 > \alpha_2\bar{G}_2 - \beta_2\left(\bar{G}_2\right)^2$, which contradicts with Expression (3.25). Therefore, in contrary to our assumption, we must have $G_2 \geq G_1$. This implies the annual costs of $\left(\bar{Q}_1, \bar{G}_1\right)$ along with the first investment option are lower than or equal to the optimum costs under the second investment option. ■

The above lemma implies that between two different investment options, the retailer should choose the one with higher α and smaller β. If the investment option with higher α does not also have smaller β, we will show, in the numerical analysis in Chapter 4, that the problem parameters determine which investment option is better in terms of costs. Recall from Corollary 1 that the annual carbon emissions level under the cap policy is independent of the investment parameters α and β. Therefore, annual costs due to each investment option is the only criterion that determines which investment option is better.

3.2.2 Tax Policy

Under a tax policy, the retailer pays p monetary units in taxes for unit carbon emission. There is no restriction on the maximum carbon emissions. The retailer's problem can be formulated as follows:

$$\begin{aligned}
\min \quad & TC_2(Q,G) = \tfrac{AD}{Q} + \tfrac{hQ}{2} + cD + G + pE(Q,G) \\
\text{s.t.} \quad & E(Q,G) = \tfrac{\hat{A}D}{Q} + \tfrac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2 \ , \\
& Q \geq 0, G \geq 0.
\end{aligned}$$

The following theorem characterizes the solution to the above problem:

Theorem 2 *Under a tax policy, the optimal pair of retailer's replenishment quantity and his/her investment amount is given by*

$$(Q_2^*, G_2^*) = \left(\sqrt{\frac{2(A+\hat{A}p)D}{h+\hat{h}p}}, \frac{\alpha p - 1}{2p\beta} \right).$$

Proof: Plugging $\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2$ in place of $E(Q,G)$ in the objective function, it turns out be

$$\frac{(A+p\hat{A})D}{Q} + \frac{(h+\hat{h}p)Q}{2} + (c+\hat{c}p)D + G - \alpha pG + p\beta G^2.$$

The Hessian matrix corresponding to the above function is

$$\begin{pmatrix} \frac{2D(A+\hat{A}p)}{Q^3} & 0 \\ 0 & 2p\beta \end{pmatrix},$$

with a determinant $\frac{4(A+p\hat{A})Dp\beta}{Q^3}$, which is greater than zero. Combined with the fact that $\frac{2D(A+\hat{A}p)}{Q^3} > 0$, this result implies the objective function is jointly and strictly convex in Q and G, and hence, Q_2^* and G_2^* should satisfy the following system of equations:

$$\frac{\partial TC_2}{\partial Q}(Q_2^*, G_2^*) = -\frac{(A+p\hat{A})D}{(Q_2^*)^2} + \frac{(h+p\hat{h})}{2} = 0,$$

$$\frac{\partial TC_2}{\partial G}(Q_2^*, G_2^*) = 1 - \alpha p + 2p\beta G_2^* = 0.$$

Solving for Q_2^* and G_2^* in the above two expressions leads to the result in the theorem. ■

It can be observed that G_2^* is increasing with p. Furthermore, Q_2^* is increasing with p when $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, Q_2^* is decreasing with p when $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$, and Q_2^* is not affected by p when $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$. In fact, when $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$, we have $Q_2^* = Q^0 = Q^e$. The next corollary follows from plugging the expressions for Q_2^* and G_2^* in the emission function and the cost function.

Corollary 2 *The average annual carbon emission and the average annual cost resulting from the retailer's optimal solution under a tax policy are*

$$E\left(Q_2^*, G_2^*\right) = \frac{\sqrt{D}\left[\hat{A}(h+p\hat{h}) + \hat{h}(A+p\hat{A})\right]}{\sqrt{2(A+p\hat{A})(h+p\hat{h})}} + \frac{1-\alpha^2p^2}{4p^2\beta} + \hat{c}D, \tag{3.29}$$

$$TC_2\left(Q_2^*, G_2^*\right) = \sqrt{2(A+p\hat{A})(h+p\hat{h})D} + D(c+\hat{c}p) - \frac{(\alpha p-1)^2}{4p\beta}. \tag{3.30}$$

Proof: When $(Q_2^*, G_2^*) = \left(\sqrt{\frac{2(A+\hat{A}p)D}{h+\hat{h}p}}, \frac{\alpha p-1}{2p\beta}\right)$ is plugged in $E(Q,G) = \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2$, we have

$$E(Q_2^*, G_2^*) = \frac{\hat{A}D}{\sqrt{\frac{2(A+\hat{A}p)D}{h+\hat{h}p}}} + \frac{\hat{h}\sqrt{\frac{2(A+\hat{A}p)D}{h+\hat{h}p}}}{2} + \hat{c}D - \alpha\frac{\alpha p-1}{2p\beta} + \beta\left(\frac{\alpha p-1}{2p\beta}\right)^2,$$

which can be rewritten as

$$E(Q_2^*, G_2^*) = \hat{A}\sqrt{\frac{(h+\hat{h}p)D}{2(A+\hat{A}p)}} + \hat{h}\sqrt{\frac{(A+\hat{A}p)D}{2(h+\hat{h}p)}} + \hat{c}D + \frac{-\alpha^2p+\alpha}{2p\beta} + \frac{\alpha^2p^2-2\alpha p+1}{(2p)^2\beta}.$$

Writing the first and the second terms of the above expression under a common denominator, similarly writing the fourth and the fifth terms under a common denominator, and doing some cancellation of terms leads to

$$E(Q_2^*, G_2^*) = \frac{\sqrt{D}\left[\hat{A}(h+p\hat{h}) + \hat{h}(A+p\hat{A})\right]}{\sqrt{2(A+p\hat{A})(h+p\hat{h})}} + \hat{c}D + \frac{1-\alpha^2p^2}{4p^2\beta}.$$

Now, let us continue with deriving a closed form expression for $TC_2(Q_2^*, G_2^*)$. Plugging $\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2$ in place of $E(Q,G)$ in the objective function of the model for tax policy, leads to

$$\frac{(A+p\hat{A})D}{Q} + \frac{(h+\hat{h}p)Q}{2} + (c+\hat{c}p)D + (1-\alpha p)G + p\beta G^2.$$

When we put $Q_2^* = \sqrt{\frac{2(A+\hat{A}p)D}{h+\hat{h}p}}$ and $G_2^* = \frac{\alpha p - 1}{2p\beta}$ in place of Q and G, respectively, the above expression turns out to be

$$\frac{(A + p\hat{A})D}{\sqrt{\frac{2(A+\hat{A}p)D}{h+\hat{h}p}}} + \frac{(h + \hat{h}p)\sqrt{\frac{2(A+\hat{A}p)D}{h+\hat{h}p}}}{2} + (c + \hat{c}p)D - \frac{(\alpha p - 1)^2}{2p\beta} + \frac{(\alpha p - 1)^2}{4p\beta},$$

which can be rewritten as

$$\sqrt{\frac{(A + \hat{A}p)(h + \hat{h}p)D}{2}} + \sqrt{\frac{(A + \hat{A}p)(h + \hat{h}p)D}{2}} + (c + \hat{c}p)D - \frac{(\alpha p - 1)^2}{4p\beta}.$$

The above expression is equal to

$$\sqrt{2(A + \hat{A}p)(h + \hat{h}p)D} + (c + \hat{c}p)D - \frac{(\alpha p - 1)^2}{4p\beta}.$$

■

It can be verified by Assumptions (A1) and A(3) that $E(Q_2^*, G_2^*)$ and $TC_2(Q_2^*, G_2^*)$ are positive. $E(Q_2^*, G_2^*)$ is decreasing in p and $TC_2(Q_2^*, G_2^*)$ is increasing in p. In the next lemma, we quantify the reduction in emissions and the savings in costs due to the investment option. For this purpose, we consider the following two measures: $E(Q_2^*(0), 0) - E(Q_2^*, G_2^*)$ and $TC_2(Q_2^*(0), 0) - TC_2(Q_2^*, G_2^*)$. Here, $Q_2^*(0)$ refers to the retailer's optimal replenishment quantity under the tax policy, given that the investment amount is zero.

Lemma 4 *Under a tax policy, having an investment opportunity for carbon emission reduction leads to positive savings in annual carbon emissions and in annual costs, as quantified by the following:*

$$E(Q_2^*(0), 0) - E(Q_2^*, G_2^*) = \frac{\alpha^2 p^2 - 1}{4p^2\beta},$$

$$TC_2(Q_2^*(0), 0) - TC_2(Q_2^*, G_2^*) = \frac{(\alpha p - 1)^2}{4p\beta}.$$

Proof: Under a tax policy, if there is no investment opportunity to reduce carbon emissions, the retailer minimizes the following function to find Q:

$$TC_2(Q,0) = \frac{(A+p\hat{A})D}{Q} + \frac{(h+p\hat{h})Q}{2} + (c+p\hat{c})D.$$

$TC_2(Q,0)$ is minimized at $Q_2^*(0) = \sqrt{\frac{2(A+p\hat{A})D}{(h+p\hat{h})}}$. In turn, the retailer's annual costs at $Q_2^*(0)$ are

$$TC_2(Q_2^*(0),0) = \sqrt{2(A+p\hat{A})(h+p\hat{h})D} + (c+p\hat{c})D,$$

and his/her annual carbon emissions are

$$E(Q_2^*(0),0) = \frac{\sqrt{D}[\hat{A}(h+p\hat{h}) + \hat{h}(A+p\hat{A})]}{\sqrt{2(A+p\hat{A})(h+p\hat{h})}} + \hat{c}D.$$

Expressions (3.29) and (3.30) are then utilized to compute the differences $E(Q_2^*(0),0) - E(Q_2^*,G_2^*)$ and $TC_2(Q_2^*(0),0) - TC_2(Q_2^*,G_2^*)$. ■

Lemma 4 along with Assumption (A2) imply that the reduction in annual costs and the reduction in annual carbon emissions due to utilizing the investment opportunity are both increasing in p. The reduction in annual carbon emissions is bounded by $\frac{\alpha^2}{4\beta}$ and its rate of change with increasing p decreases. This, in turn, implies that if the government further increases the tax for one unit of emission at its already large values, a retailer investing in new technology does very little to reduce emissions. However, the retailer still invests in new technology because he/she can reduce his/her costs significantly by means of tax savings. Note that the total taxes the retailer must pay may be very large at high values of p, therefore, even a marginal reduction in emissions may save the retailer a lot of money.

In the next lemma, we study the effects of the carbon tax policy on the retailer's annual carbon emissions and costs. Without a carbon emission policy in place, the retailer minimizes Expression (3.1), and he/she orders Q^0 units and makes no investment in emissions reduction.

Lemma 5 *Under a tax policy, the retailer's cost-optimal decisions for replenishment quantity and investment amount lead to lower annual emissions and* *higher annual costs, in comparison to a case with no emission policy. That is,* $TC_2(Q_2^*, G_2^*) > TC(Q^0, 0)$ *and* $E(Q_2^*, G_2^*) < E(Q^0, 0)$.

Proof: By definitions of $TC(Q, G)$ and $TC_2(Q, G)$, we know that $TC(Q, G) \leq TC_2(Q, G)$ as $E(Q, G) \geq 0$. It then follows that $TC(Q_2^*, G_2^*) < TC_2(Q_2^*, G_2^*)$ because $E(Q_2^*, G_2^*) > 0$, as noted in Corollary 2. Furthermore, we have $TC(Q^0, 0) < TC(Q_2^*, G_2^*)$ because $(Q^0, 0)$ minimizes $TC(Q, G)$. Combining this with the fact that $TC(Q_2^*, G_2^*) < TC_2(Q_2^*, G_2^*)$ leads to $TC_2(Q_2^*, G_2^*) > TC(Q^0, 0)$.

Now, let us prove the second part of the lemma. We have from Theorem 2 and Assumption (A2) that $E(Q_2^*, G_2^*) < E(Q_2^*, 0)$. The remaining part of the proof will follow by showing that $E(Q_2^*, 0) < E(Q^0, 0)$ in case $\frac{A}{h} \neq \frac{\hat{A}}{\hat{h}}$, and that $E(Q_2^*, 0) = E(Q^0, 0)$, in case $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$. Therefore, we will conclude that $E(Q_2^*, G_2^*) < E(Q^0, 0)$ in all cases.

If $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$, we have $Q_2^* = Q^0$, which implies $E(Q_2^*, 0) = E(Q^0, 0)$. We will analyze the case of $\frac{A}{h} \neq \frac{\hat{A}}{\hat{h}}$ in two parts. First, suppose that $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$. In this case, we have $Q^e < Q_2^* < Q^0$. This further leads to $E(Q_2^*, 0) < E(Q^0, 0)$ due to the strict convexity of $E(Q, 0)$ and the fact that Q^e is the unique minimizer of $E(Q, 0)$. Now, suppose that $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$. In this case, we have $Q^e > Q_2^* > Q^0$. It again follows from the strict convexity of $E(Q, 0)$ and the definition of Q^e that we have $E(Q_2^*, 0) < E(Q^0, 0)$. ■

The above lemma implies that a tax policy is effective in reducing a retailer's annual carbon emissions, but it increases the retailer's annual costs even if he/she has access to an investment opportunity for carbon emission reduction. In what follows, we compare two investment opportunities under the tax policy.

Lemma 6 *Let us consider two investment options: one with parameters* α_1 *and* β_1*, and the other with parameters* α_2 *and* β_2*. When a tax policy is in place, the retailer's annual costs and emissions under one option compare to those under another in the following way:*

- *If* $\beta_2 \geq \beta_1$ *and* $\alpha_2 \leq \alpha_1$*, then the first investment option (i.e., the one with* *parameters α_1 and β_1) leads to no greater annual emissions and no greater annual costs for the retailer than the second investment option does.*

- *If $\beta_2 \geq \beta_1$ and $\alpha_2 > \alpha_1$, then*
  - *If the second investment option leads to greater annual costs than the first one does, then it also results in greater annual emissions.*
  - *If the second investment option leads to annual costs lower than or equal to the first one, then it results in lower annual emissions if $\frac{1-\alpha_2{}^2p^2}{\beta_2} < \frac{1-\alpha_1{}^2p^2}{\beta_1}$ holds, otherwise, it results in no lower annual emissions than the first investment option does.*

Proof: We will prove the different parts of the lemma in the following two cases.

Case 1: $\beta_2 \geq \beta_1$, $\alpha_2 \leq \alpha_1$

It follows from $\beta_2 \geq \beta_1$ that we have $\frac{\alpha_2 p-1}{\sqrt{\beta_2}} \leq \frac{\alpha_2 p-1}{\sqrt{\beta_1}}$. Also, the fact that $\alpha_2 \leq \alpha_1$ leads to $\frac{\alpha_2 p-1}{\sqrt{\beta_1}} \leq \frac{\alpha_1 p-1}{\sqrt{\beta_1}}$. Combining these two results, we have $\frac{\alpha_2 p-1}{\sqrt{\beta_2}} \leq \frac{\alpha_1 p-1}{\sqrt{\beta_1}}$, and hence $\frac{(\alpha_2 p-1)^2}{4p\beta_2} \leq \frac{(\alpha_1 p-1)^2}{4p\beta_1}$. Expression (3.30) and the fact that $\frac{(\alpha_2 p-1)^2}{4p\beta_2} \leq \frac{(\alpha_1 p-1)^2}{4p\beta_1}$ jointly imply that the annual costs under the first investment option is lower than or equal to the annual costs under the second investment option.

Now, let us compare the annual emissions under the two investment options. It follows from $\frac{\alpha_2 p-1}{\sqrt{\beta_2}} \leq \frac{\alpha_1 p-1}{\sqrt{\beta_1}}$ that $\alpha_1 p\sqrt{\beta_2} - \sqrt{\beta_2} \geq \alpha_2 p\sqrt{\beta_1} - \sqrt{\beta_1}$. Because $\beta_2 \geq \beta_1$, we have $2\sqrt{\beta_2} \geq 2\sqrt{\beta_1}$. Combining this with $\alpha_1 p\sqrt{\beta_2} - \sqrt{\beta_2} \geq \alpha_2 p\sqrt{\beta_1} - \sqrt{\beta_1}$ leads to $\alpha_1 p\sqrt{\beta_2} + \sqrt{\beta_2} \geq \alpha_2 p\sqrt{\beta_1} + \sqrt{\beta_1}$, which in turn implies $\frac{\alpha_1 p+1}{\sqrt{\beta_1}} \geq \frac{\alpha_2 p+1}{\sqrt{\beta_2}}$. Since $\frac{\alpha_1 p-1}{\sqrt{\beta_1}} \geq \frac{\alpha_2 p-1}{\sqrt{\beta_2}}$ and $\frac{\alpha_1 p+1}{\sqrt{\beta_1}} \geq \frac{\alpha_2 p+1}{\sqrt{\beta_2}}$, it follows that $\frac{\alpha_1^2 p^2-1}{\beta_1} \geq \frac{\alpha_2^2 p^2-1}{\beta_2}$, or equivalently, $\frac{1-\alpha_1^2 p^2}{\beta_1} \leq \frac{1-\alpha_2^2 p^2}{\beta_2}$. This implies, due to Expression (3.29), that annual emissions under the first investment option are lower than or equal to annual emissions under the second investment option.

Case 2: $\beta_2 \geq \beta_1$, $\alpha_2 > \alpha_1$

If the second investment option leads to greater annual costs than the first one does, then Expression (3.30) implies that $\frac{(\alpha_2 p-1)^2}{4p\beta_2} < \frac{(\alpha_1 p-1)^2}{4p\beta_1}$, or equivalently, that $\alpha_2 p\sqrt{\beta_1} - \sqrt{\beta_1} < \alpha_1 p\sqrt{\beta_2} - \sqrt{\beta_2}$. Now, in contrary to the lemma, assume that the annual emissions level resulting from the second investment option is lower than or equal to that of the first investment option. In mathematical terms, assume that $\frac{1-\alpha_2^2p^2}{4p^2\beta_2} \leq \frac{1-\alpha_1^2p^2}{4p^2\beta_1}$, which is equivalent to

$$\frac{\alpha_2 p - 1}{\sqrt{\beta_2}} \times \frac{\alpha_2 p + 1}{\sqrt{\beta_2}} \geq \frac{\alpha_1 p - 1}{\sqrt{\beta_1}} \times \frac{\alpha_1 p + 1}{\sqrt{\beta_1}}.$$

Due to $\frac{(\alpha_2p-1)^2}{4p\beta_2} < \frac{(\alpha_1p-1)^2}{4p\beta_1}$, we have $\frac{\alpha_2p-1}{\sqrt{\beta_2}} < \frac{\alpha_1p-1}{\sqrt{\beta_1}}$. Therefore, in order for the above inequality to hold, we should have $\frac{\alpha_2p+1}{\sqrt{\beta_2}} > \frac{\alpha_1p+1}{\sqrt{\beta_1}}$, or equivalently, $\alpha_2p\sqrt{\beta_1}+\sqrt{\beta_1} > \alpha_1p\sqrt{\beta_2}+\sqrt{\beta_2}$. Since $\beta_2 \geq \beta_1$, this implies $\alpha_2p\sqrt{\beta_1}-\sqrt{\beta_1} > \alpha_1p\sqrt{\beta_2}-\sqrt{\beta_2}$, which contradicts $\alpha_2p\sqrt{\beta_1} - \sqrt{\beta_1} < \alpha_1p\sqrt{\beta_2} - \sqrt{\beta_2}$. Therefore, if the second investment option leads to greater annual costs than the first one, it must be that the annual emissions level resulting from the second investment is greater than that of the first investment.

If the second investment option leads to lower than or equal annual costs than the first one, the annual emission levels of the two investment options depend on the second term of Expression (3.29). If $\frac{1-\alpha_2^2p^2}{4p^2\beta_2} < \frac{1-\alpha_1^2p^2}{4p^2\beta_1}$, or equivalently, $\frac{1-\alpha_2^2p^2}{\beta_2} < \frac{1-\alpha_1^2p^2}{\beta_1}$, holds, then the second investment option is better in terms of the retailer's annual emissions; otherwise, the annual emissions level is greater than or equal to that of the first investment option. ■

3.2.3 Cap-and-Trade Policy

Under a cap-and-trade policy, similar to the cap policy, the retailer is subject to an emissions cap, C, on the total carbon emissions per year. However, if the annual carbon emission is more than the cap C, the firm can buy carbon permits equivalent to its excess demand for carbon capacity, at a market price of c_p monetary units per unit emission. On the other hand, if the retailer's annual carbon emission is lower than the carbon cap, she/he can sell the extra carbon capacity at the same market price, i.e., c_p. It is assumed that carbon permits are always available for buying and selling. In particular, let X denote the carbon amount the retailer trades annually. $X > 0$ indicates a case in which the retailer sells his/her carbon permits, whereas $X < 0$ implies a case in which the retailer purchases carbon permits. The retailer's problem of deciding the replenishment quantity and the investment amount is formulated below.

$$\begin{aligned}
\min \quad & TC_3(Q,G) = \tfrac{AD}{Q} + \tfrac{hQ}{2} + cD + G - Xc_p \\
\text{s.t.} \quad & \tfrac{\hat{A}D}{Q} + \tfrac{\hat{h}Q}{2} + \hat{c}D - \alpha G + \beta G^2 + X = C, \\
& Q \geq 0, G \geq 0.
\end{aligned}$$

In the following theorem, we present the solution to the above problem:

Theorem 3 *Under a cap-and-trade policy, the optimal pair of retailer's replenishment quantity and his/her investment amount is given by*

$$(Q_3^*, G_3^*) = \left(\sqrt{\frac{2(A+\hat{A}c_p)D}{h+\hat{h}c_p}}, \frac{\alpha c_p - 1}{2c_p\beta} \right).$$

It then follows that $X^* = C - E(Q_3^*, G_3^*)$*, where* X^* *is the retailer's optimal traded carbon amount per year.*

Proof: Plugging $C - \frac{\hat{A}D}{Q} - \frac{\hat{h}Q}{2} - \hat{c}D + \alpha G - \beta G^2$ in place of X, the objective function turns out be

$$\frac{(A+c_p\hat{A})D}{Q} + \frac{(h+\hat{h}c_p)Q}{2} + c_p\beta G^2 + (1-\alpha c_p)G + (c+\hat{c}c_p)D - c_pC.$$

Following similar steps to those in the proof of Theorem 2 for checking the structural properties of $TC_2(Q,G)$, it can be shown that $TC_3(Q,G)$ is also jointly and strictly convex in Q and G, and hence, Q_3^* and G_3^* should satisfy the following system of equations:

$$\frac{\partial TC_3}{\partial Q}(Q_3^*, G_3^*) = -\frac{(A+c_p\hat{A})D}{(Q_3^*)^2} + \frac{(h+\hat{h}c_p)}{2} = 0,$$

$$\frac{\partial TC_3}{\partial G}(Q_3^*, G_3^*) = 1 - \alpha c_p + 2c_p\beta G_3^* = 0.$$

Solving for Q_3^* and G_3^* in the above two expressions leads to the result in the theorem. ■

Using the expression for G_3^*, one can show that G_3^* is increasing with c_p. Furthermore, Q_3^* is increasing with c_p when $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, Q_3^* is decreasing with c_p when $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$, and it is not affected by c_p when $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$. In case $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$, we have $Q_3^* = Q^0 = Q^e$. The next three corollaries follow from Theorem 3.

Corollary 3 *If $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta} > C$, then the retailer does not sell any carbon permits (i.e., $X \leq 0$), regardless of what the carbon trading price c_p is.*

At high values of c_p, the retailer may want to sell his/her permits in the market for extra revenue. However, Corollary 3 implies that if the cap is smaller than the minimum carbon emissions possible due to ordering and investment decisions, the retailer must purchase carbon permits to be within the allowed limits of annual carbon emissions at any value of c_p.

Corollary 4 *The average annual carbon emissions and the average annual costs resulting from the retailer's optimal decisions under a cap-and-trade policy are*

$$E(Q_3^*, G_3^*) = \frac{\sqrt{D}(\hat{A}(h + c_p\hat{h}) + \hat{h}(A + c_p\hat{A}))}{\sqrt{2(A + c_p\hat{A})(h + c_p\hat{h})}} + \frac{1 - \alpha^2 c_p^2}{4c_p^2\beta} + \hat{c}D, \tag{3.31}$$

$$TC_3(Q_3^*, G_3^*) = \sqrt{2(A + c_p\hat{A})(h + c_p\hat{h})D} + D(c + \hat{c}c_p) - \frac{(\alpha c_p - 1)^2}{4c_p\beta} - c_pC. \tag{3.32}$$

Equation (3.31) implies that the carbon emissions level does not change with carbon cap C. Hua et al. [4] obtain a similar result for the case when there is no investment option. It can be shown using Assumption (A3) that $E(Q_3^*, G_3^*) > 0$; however, $TC_3(Q_3^*, G_3^*)$ may assume any value depending on the magnitude of C. If $TC_3(Q_3^*, G_3^*) < 0$, then the retailer has excess carbon capacity in such a large amount that by selling this amount he/she covers the inventory-related costs and even makes a profit. (In practice, this should be avoided for the cap and trade policy to be effective.) Based on this result, the next corollary proposes an upper bound on the value of C that the policy maker should impose on the retailer in this setting.

Corollary 5 *Under a cap-and-trade policy with a carbon trading price c_p, an upper bound on the carbon capacity C is given by*

$$C < \frac{\sqrt{2(A + c_p\hat{A})(h + c_p\hat{h})D} + D(c + \hat{c}c_p) - \frac{(\alpha c_p - 1)^2}{4c_p\beta}}{c_p}.$$

To quantify the reduction in emissions and the savings in costs due to the investment option under a cap-and-trade policy, in the next lemma we consider the following two measures: $E\left(Q_3^*(0), 0\right) - E\left(Q_3^*, G_3^*\right)$ and $TC_3\left(Q_3^*(0), 0\right) - TC_3\left(Q_3^*, G_3^*\right)$. Here, $Q_3^*(0)$ refers to the retailer's optimal replenishment quantity under the cap-and-trade policy, given that the investment amount is zero.

Lemma 7 *Under a cap-and-trade policy, having an investment opportunity for carbon emission reduction leads to positive savings in annual carbon emissions and in annual costs, as quantified by the following:*

$$E\left(Q_3^*(0), 0\right) - E\left(Q_3^*, G_3^*\right) = \frac{\alpha^2 {c_p}^2 - 1}{4{c_p}^2\beta},$$

$$TC_3\left(Q_3^*(0), 0\right) - TC_3\left(Q_3^*, G_3^*\right) = \frac{(\alpha c_p - 1)^2}{4c_p\beta}.$$

Proof: Under a cap-and-trade policy, if there is no investment opportunity to reduce carbon emissions, the retailer minimizes the following function to find Q:

$$TC_3(Q, 0) = \frac{(A + c_p\hat{A})D}{Q} + \frac{(h + c_p\hat{h})Q}{2} + (c + c_p\hat{c})D.$$

$TC_3(Q, 0)$ is minimized at $Q_3^*(0) = \sqrt{\frac{2(A+c_p\hat{A})D}{(h+c_p\hat{h})}}$. In turn, the retailer's annual costs at $Q_3^*(0)$ are

$$TC_3(Q_3^*(0), 0) = \sqrt{2(A + c_p\hat{A})(h + c_p\hat{h})D} + (c + c_p\hat{c})D,$$

and his/her annual carbon emissions are

$$E(Q_3^*(0),0) = \frac{\sqrt{D}[\hat{A}(h+c_p\hat{h})+\hat{h}(A+c_p\hat{A})]}{\sqrt{2(A+c_p\hat{A})(h+c_p\hat{h})}} + \hat{c}D.$$

Expressions (3.31) and (3.32) are then utilized to compute the differences $E(Q_3^*(0),0) - E(Q_3^*,G_3^*)$ and $TC_3(Q_3^*(0),0) - TC_3(Q_3^*,G_3^*)$. ■

Lemma 7 and Assumption (A3) jointly imply that the reduction in annual costs and the reduction in annual carbon emissions due to utilizing the investment opportunity are both increasing in c_p. The reduction in annual carbon emissions is again bounded by $\frac{\alpha^2}{4\beta}$, as in the case of the tax policy, and, its rate of change with increasing c_p decreases. With an interpretation similar to the one we developed for Lemma 4, it can be concluded that the incremental benefit of retailer's one-unit investment on emission reduction diminishes at large values of unit carbon emission trading prices. However, the retailer still invests in new technology, because he/she can reduce his/her costs significantly either by creating excess carbon capacity and selling it at high prices, or by avoiding the need to purchase excess capacity at high prices with the capacity generated from new technology.

In the next lemma, we study the effects of the cap-and-trade policy on the retailer's annual carbon emissions and costs. For this purpose, we compare the annual carbon emissions and the annual costs in case of no government regulation to the results in Corollary 4. Note that, in the former case, the retailer orders Q^0 units and makes no investment in emission reduction.

Lemma 8 *Under a cap-and-trade policy, the retailer's cost-optimal decisions for replenishment quantity and investment amount lead to lower annual emissions in comparison to a case with no emission policy. That is,* $E(Q_3^*,G_3^*) < E(Q^0,0)$. *However, annual costs may increase or decrease depending on* C. *Specifically, we have* $TC_3(Q_3^*,G_3^*) \leq TC(Q^0,0)$ *if* $C \geq \frac{\sqrt{2(A+\hat{A}c_p)(h+\hat{h}c_p)D}-\sqrt{2AhD}}{c_p} - \frac{(\alpha c_p-1)^2}{4c_p^2\beta} + \hat{c}D$, *and we have* $TC_3(Q_3^*,G_3^*) > TC(Q^0,0)$ *otherwise.*

Proof: The first part of the lemma follows from a similar discussion to the proof of Lemma 5 and Assumption A(3). The second part follows from comparing Equation (3.32) to $TC(Q^0,0)$. ■

The next lemma presents a result for the cap-and-trade policy, similar to the one in Lemma 6 for the tax policy.

Lemma 9 *Let us consider two investment options: one with parameters* α_1 *and* β_1*, and the other with parameters* α_2 *and* β_2*. The retailer's annual costs and emissions under one option compare to those under the other in the following way:*

- *If* $\beta_2 \geq \beta_1$ *and* $\alpha_2 \leq \alpha_1$*, then the first investment option (i.e., the one with parameters* α_1 *and* β_1*) leads to no greater annual emissions and no greater annual costs for the retailer than the second investment option does.*
- *If* $\beta_2 \geq \beta_1$ *and* $\alpha_2 > \alpha_1$*, then*
  - *If the second investment option leads to greater annual costs than the first one does, then it also results in greater annual emissions.*
  - *If the second investment option leads to annual costs lower than or equal to the first one, then it results in lower annual emissions if* $\frac{1-{\alpha_2}^2 c_p^2}{\beta_2} < \frac{1-{\alpha_1}^2 c_p^2}{\beta_1}$ *holds, otherwise, it results in no lower annual emissions than the first investment option does.*

Proof: We will prove the different parts of the lemma in the following two cases.

Case 1: $\beta_2 \geq \beta_1$, $\alpha_2 \leq \alpha_1$

It follows from $\beta_2 \geq \beta_1$ that we have $\frac{\alpha_2 c_p - 1}{\sqrt{\beta_2}} \leq \frac{\alpha_2 c_p - 1}{\sqrt{\beta_1}}$. Also, the fact that $\alpha_2 \leq \alpha_1$ leads to $\frac{\alpha_2 c_p - 1}{\sqrt{\beta_1}} \leq \frac{\alpha_1 c_p - 1}{\sqrt{\beta_1}}$. Combining these two results, we have $\frac{\alpha_2 c_p - 1}{\sqrt{\beta_2}} \leq \frac{\alpha_1 c_p - 1}{\sqrt{\beta_1}}$, and hence, $\frac{(\alpha_2 c_p - 1)^2}{4 c_p \beta_2} \leq \frac{(\alpha_1 c_p - 1)^2}{4 c_p \beta_1}$. Expression (3.32) and the fact that $\frac{(\alpha_2 c_p - 1)^2}{4 c_p \beta_2} \leq \frac{(\alpha_1 c_p - 1)^2}{4 c_p \beta_1}$ jointly imply that the annual costs under the first investment option is less than or equal to the annual costs under the second investment option.

Now, let us compare the annual emissions under the two investment options. It follows from $\frac{\alpha_2 c_p - 1}{\sqrt{\beta_2}} \leq \frac{\alpha_1 c_p - 1}{\sqrt{\beta_1}}$ that $\alpha_1 c_p \sqrt{\beta_2} - \sqrt{\beta_2} \geq \alpha_2 c_p \sqrt{\beta_1} - \sqrt{\beta_1}$. Since $\beta_2 \geq \beta_1$, we have $2\sqrt{\beta_2} \geq 2\sqrt{\beta_1}$. Combining this with $\alpha_1 c_p \sqrt{\beta_2} - \sqrt{\beta_2} \geq$

$\alpha_2 c_p\sqrt{\beta_1} - \sqrt{\beta_1}$ leads to $\alpha_1 c_p\sqrt{\beta_2} + \sqrt{\beta_2} \geq \alpha_2 c_p\sqrt{\beta_1} + \sqrt{\beta_1}$, which in turn, implies $\frac{\alpha_1 c_p+1}{\sqrt{\beta_1}} \geq \frac{\alpha_2 c_p+1}{\sqrt{\beta_2}}$. Since $\frac{\alpha_1 c_p-1}{\sqrt{\beta_1}} \geq \frac{\alpha_2 c_p-1}{\sqrt{\beta_2}}$ and $\frac{\alpha_1 c_p+1}{\sqrt{\beta_1}} \geq \frac{\alpha_2 c_p+1}{\sqrt{\beta_2}}$, it follows that $\frac{\alpha_1^2 c_p^2-1}{\beta_1} \geq \frac{\alpha_2^2 c_p^2-1}{\beta_2}$, or equivalently $\frac{1-\alpha_1^2 c_p^2}{\beta_1} \leq \frac{1-\alpha_2^2 c_p^2}{\beta_2}$. This implies, due to Expression (3.31), that the annual emissions under the first investment option is less than or equal to the annual emissions under the second investment option.

Case 2: $\beta_2 \geq \beta_1$, $\alpha_2 > \alpha_1$

If the second investment option leads to more annual costs than the first one does, then Expression (3.32) implies that $\frac{(\alpha_2 c_p-1)^2}{4c_p\beta_2} < \frac{(\alpha_1 c_p-1)^2}{4c_p\beta_1}$, or equivalently that $\alpha_2 c_p\sqrt{\beta_1} - \sqrt{\beta_1} < \alpha_1 c_p\sqrt{\beta_2} - \sqrt{\beta_2}$. Now, assume in contrary to the lemma, that the annual emissions level resulting from the second investment option is less than or equal to that of the first investment option. In mathematical terms, assume that $\frac{1-\alpha_2^2 c_p^2}{4c_p^2\beta_2} \leq \frac{1-\alpha_1^2 c_p^2}{4c_p^2\beta_1}$, which can be rewritten as

$$\frac{\alpha_2 c_p - 1}{\sqrt{\beta_2}} \times \frac{\alpha_2 c_p + 1}{\sqrt{\beta_2}} \geq \frac{\alpha_1 c_p - 1}{\sqrt{\beta_1}} \times \frac{\alpha_1 c_p + 1}{\sqrt{\beta_1}}.$$

Due to $\frac{(\alpha_2 c_p-1)^2}{4c_p\beta_2} < \frac{(\alpha_1 c_p-1)^2}{4c_p\beta_1}$, we have $\frac{\alpha_2 c_p-1}{\sqrt{\beta_2}} < \frac{\alpha_1 c_p-1}{\sqrt{\beta_1}}$. Therefore, in order for the above inequality to hold, we should have $\frac{\alpha_2 c_p+1}{\sqrt{\beta_2}} > \frac{\alpha_1 c_p+1}{\sqrt{\beta_1}}$, or equivalently $\alpha_2 c_p\sqrt{\beta_1} + \sqrt{\beta_1} > \alpha_1 c_p\sqrt{\beta_2} + \sqrt{\beta_2}$. Since $\beta_2 \geq \beta_1$, this implies $\alpha_2 c_p\sqrt{\beta_1} - \sqrt{\beta_1} > \alpha_1 c_p\sqrt{\beta_2} - \sqrt{\beta_2}$, which contradicts with $\alpha_2 c_p\sqrt{\beta_1} - \sqrt{\beta_1} < \alpha_1 c_p\sqrt{\beta_2} - \sqrt{\beta_2}$. Therefore, if the second investment option leads to more annual costs than the first one does, it must be that the annual emissions level resulting from the second investment is more than that of the first investment.

If the second investment option leads to less than or equal to annual costs than the first one does, the annual emissions levels of the two investment options depend on the second term of Expression (3.31). If $\frac{1-\alpha_2^2 c_p^2}{4c_p^2\beta_2} < \frac{1-\alpha_1^2 c_p^2}{4c_p^2\beta_1}$, or equivalently $\frac{1-\alpha_2^2 c_p^2}{\beta_2} < \frac{1-\alpha_1^2 c_p^2}{\beta_1}$, holds, then the second investment option is better in terms of retailer's annual emissions, otherwise, its annual emissions level is more than or equal to that of the first investment option. ■

3.2.4 Analytical Results on the Comparison of the Three Emission Policies

In Sections 3.2.1, 3.2.2 and 3.2.3, we derived analytical solutions to the retailer's problem of finding the replenishment quantity and the investment amount under the three carbon regulation policies. We obtained two sets of results: one about the impact of an investment opportunity on the annual costs and emissions (see Lemmas 1, 4, and 7), and the other about how the different emission policies change the retailer's annual costs and emissions in comparison to a no-policy case (see Lemmas 2, 5, and 8). Looking into the first set of results, we arrive at the following conclusions:

- Under any of the three carbon regulation policies, total annual costs without the investment option are greater than or equal to the total annual costs with the investment option.
- While annual carbon emissions levels with and without the investment option are equal under the cap policy, carbon emissions level without the investment option is greater than the carbon emissions level with the investment option under the tax policy and cap-and-trade policy.

The above results imply that having an investment option under a cap policy does not reduce the retailer's emission level in comparison to a case with no such option; however, it may help him/her achieve the same carbon amount with lower costs. On the other hand, having an investment option under a tax policy or a cap-and-trade policy has a more pronounced effect on the retailer's annual carbon emissions and costs: the retailer can take advantage of the investment option and reduce both his/her emissions and costs. From an environmental point of view, the above implies that an investment option along with a tax policy or a cap-and-trade policy as an emission regulation further enhances emission reduction. Therefore, governments should enable opportunities for companies to invest in emission reduction, particularly if a tax policy or a cap-and-trade policy is in place.

The second set of results leads to the following conclusion:

- In comparison to the case where there is no emission regulation in place, the cap policy and the tax policy reduce annual carbon emissions at the expense of increased annual total costs. (If the cap is not binding, annual costs and emissions do not change under the cap policy.) On the other hand, it is possible to reduce carbon emissions with decreased annual total costs under a cap-and-trade policy.

In the next two lemmas, we present some results following a direct comparison of the different regulation policies.

Lemma 10 *For any tax policy with parameter $p > 0$, a better cap policy can be designed by an appropriate choice of parameter $C > 0$ so that $TC_1(Q_1^*, G_1^*) < TC_2(Q_2^*, G_2^*)$ and $E(Q_1^*, G_1^*) \leq E(Q_2^*, G_2^*)$. On the other hand, for a cap policy with parameter $C > 0$, a better tax policy with parameter $p > 0$ cannot be found to result in $TC_2(Q_2^*, G_2^*) < TC_1(Q_1^*, G_1^*)$ and $E(Q_2^*, G_2^*) \leq E(Q_1^*, G_1^*)$.*

Proof: Consider a tax policy with parameter $p > 0$. Let $C = E(Q_2^*, G_2^*)$. Note that $C > 0$ because $E(Q_2^*, G_2^*) > 0$. It follows from the expressions for $TC_1(Q, G)$ and $TC_2(Q, G)$, and the fact that $E(Q_2^*, G_2^*) > 0$ and $p > 0$, that we have $TC_1(Q_2^*, G_2^*) < TC_2(Q_2^*, G_2^*)$. Furthermore, as $C = E(Q_2^*, G_2^*)$, the optimal solution of the tax policy (i.e., (Q_2^*, G_2^*)), is also a feasible solution for the newly designed cap policy. Let (Q_1^*, G_1^*) be the retailer's optimal solution under the cap policy. It follows from this definition that $TC_1(Q_1^*, G_1^*) \leq TC_1(Q_2^*, G_2^*)$. Combining this with $TC_1(Q_2^*, G_2^*) < TC_2(Q_2^*, G_2^*)$ leads to $TC_1(Q_1^*, G_1^*) < TC_2(Q_2^*, G_2^*)$. Also, note that $E(Q_1^*, G_1^*) \leq C$, therefore, $E(Q_1^*, G_1^*) \leq E(Q_2^*, G_2^*)$.

For the second part of the proof, consider a cap policy with parameter $C > 0$. Suppose that a tax policy with parameter $p > 0$ can be found so that $TC_2(Q_2^*, G_2^*) < TC_1(Q_1^*, G_1^*)$ and $E(Q_2^*, G_2^*) \leq E(Q_1^*, G_1^*)$. By definition of (Q_1^*, G_1^*), $E(Q_1^*, G_1^*) \leq C$, thus $E(Q_2^*, G_2^*) \leq C$ as well. This implies that (Q_2^*, G_2^*) is a feasible solution to the retailer's problem under the cap policy. Because (Q_1^*, G_1^*) is the optimal solution under the cap policy, it must be that

$TC_1(Q_1^*, G_1^*) \leq TC_2(Q_2^*, G_2^*)$. This contradicts $TC_2(Q_2^*, G_2^*) < TC_1(Q_1^*, G_1^*)$, therefore a tax policy with the assumed characteristics cannot be found. ■

Lemma 10 indicates that for any tax policy, it is possible to design a lower-cost cap policy for the retailer without increasing his/her emissions levels. It is worthwhile noting that Lemma 10 takes the perspective of the retailer by consideration of annual costs and emissions as comparison criteria, and disregards the government's financial gains. A tax policy may benefit to the society in the long run if the government uses the revenues from environmental taxes in subsidizing green technologies. In the next lemma, we present the result of a similar comparison between the cap policy and the cap-and-trade policy.

Lemma 11 *Consider a cap policy with parameter $C > 0$, and a cap-and-trade policy with parameters $C > 0$ and $c_p > 0$. We have $TC_3(Q_3^*, G_3^*) \leq TC_1(Q_1^*, G_1^*)$ for any value of c_p. Furthermore, given a value of the common parameter C, there exists a positive value of c_p such that $E(Q_3^*, G_3^*) \leq E(Q_1^*, G_1^*)$.*

Proof: By definition of (Q_1^*, G_1^*), we know that $E(Q_1^*, G_1^*) \leq C$. Since $X = C - E(Q_1^*, G_1^*) \geq 0$, it follows from the expressions for $TC_1(Q, G)$ and $TC_3(Q, G)$ that $TC_3(Q_1^*, G_1^*) \leq TC_1(Q_1^*, G_1^*)$ for $c_p > 0$. Combining this with the fact that $TC_3(Q_3^*, G_3^*) \leq TC_3(Q_1^*, G_1^*)$, we have $TC_3(Q_3^*, G_3^*) \leq TC_1(Q_1^*, G_1^*)$.

For the second part of the proof, let us consider Expression (3.2). This expression, independent of the emission regulation type, assumes a minimum value of $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta}$ when $Q = Q^e$ units are ordered and $G = \frac{\alpha}{2\beta}$ monetary units are invested. Therefore, $E(Q_1^*, G_1^*) \geq \sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta}$. Furthermore, at very large values of c_p, (Q_3^*, G_3^*) approaches $\left(Q^e, \frac{\alpha}{2\beta}\right)$ and $E(Q_3^*, G_3^*)$ approaches $\sqrt{2\hat{A}\hat{h}D} + \hat{c}D - \frac{\alpha^2}{4\beta}$. Therefore, a large enough value of c_p can be chosen such that $E(Q_1^*, G_1^*) \geq E(Q_3^*, G_3^*)$. ■

Lemma 11 implies that corresponding to every cap policy, there exists a cap-and-trade policy with lower carbon emissions and lower costs per unit time for the retailer if the value of the carbon trading price is right. Lemmas 10 and 11 together imply that given a tax policy it is possible to have

$$TC_3\left(Q_3^*, G_3^*\right) \leq TC_1\left(Q_1^*, G_1^*\right) \leq TC_2\left(Q_2^*, G_2^*\right)$$

with appropriate values of C and c_p.

Chapter 4

Numerical Analysis

In this chapter, we present the results of a numerical study to further investigate how the retailer's annual costs and emissions change with respect to the policy parameters, and how the investment option and its parameters affect the annual costs and emissions under each policy. In addition to $TC_i\left(Q_i^*, G_i^*\right)$ and $E_i\left(Q_i^*, G_i^*\right)$, we define a new measure to assess the increase in costs relative to the decrease in emissions. We refer to this measure as *cost of unit emission reduction* and we define it as follows for policy i

$$\frac{TC_i\left(Q_i^*, G_i^*\right) - TC\left(Q^0, 0\right)}{E\left(Q^0, 0\right) - E\left(Q_i^*, G_i^*\right)}.$$

It is important to note that some of our analytical results in Chapter 3 provide general explanations to the issues that are brought up in this section more explicitly. Our numerical analysis complements these findings, particularly where only limited analytical results were possible. Because the solution under the cap policy as given in Theorem 1 is more complex than those under the tax and the cap-and-trade policies, it has been possible to obtain more analytical results involving the latter two policies. Therefore, it is no coincidence that more of the numerical results in this chapter concern the cap policy.

Our analysis in Chapter 3 reveals that how $\frac{A}{h}$ compares to $\frac{\hat{A}}{\hat{h}}$ is an important characteristic of the setting that affects the solutions under all three policies.

Therefore, our analysis considers two sets of instances: one with $A = 100$, $h = 3$, $\hat{A} = 4$, and $\hat{h} = 3$, and the other with $A = 10$, $h = 4$, $\hat{A} = 100$, and $\hat{h} = 8$. Here, we have $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ in the first set of instances and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ in the second set of instances. In all instances, we take $D = 500$, $c = 6$, and $\hat{c} = 2$. In what follows, we first present our results for the cap policy, then we proceed with our findings on the tax and cap-and-trade policies.

4.1 Numerical Study for Cap Policy

In this section, we present the results of our numerical study on cap policy with two main objectives: first, to characterize how the annual costs, savings achieved by investment, and the cost of unit emission reduction change under different values of the policy parameter C, and secondly, to gain insights on how the retailer makes a choice between two investment options with different parameters.

Figure 4.1(a) shows an illustration of how $TC_1(Q_1^*, G_1^*)$ changes with respect to varying values of C for the case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$. Figure 4.1(b) is a similar plot for the case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$. The resulting annual cost and emission levels for some specific instances under three scenarios (i.e., cap policy, cap policy without investment, no-policy) are also presented in Table 4.1. It can be observed from Figures 4.1(a) and 4.1(b) that starting from the smallest possible values of C (based on Expression (3.6)), $TC_1(Q_1^*, G_1^*)$ first exhibits a strictly decreasing pattern with respect to increasing values of C, and then, the costs level in both figures. The value of C after which annual costs become constant coincides with $E(Q^0, 0)$. If $C \geq E(Q^0, 0)$, then the cap is no longer restrictive, and the solution to the retailer's problem under no emission policy optimizes his/her costs under the cap policy as well. As a result, in both figures, $TC_1(Q_1^*, G_1^*)$ ranges from $TC_1\left(Q^e, \frac{\alpha}{2\beta}\right)$ to $TC_1(Q^0, 0)$. It can also be observed from both figures that a one-unit decrease in the cap is more costly to the retailer at its already small values.

Table 4.1 reports some instances to illustrate the possible different solution types to the retailer's problem under the cap policy, as given in Theorem 1 (see

Figure 4.1: Behavior of $TC_1\left(Q_1^*, G_1^*\right)$ for Varying Values of C Under a Cap Policy

(a) An Illustration for the Case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ (b) An Illustration for the Case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$

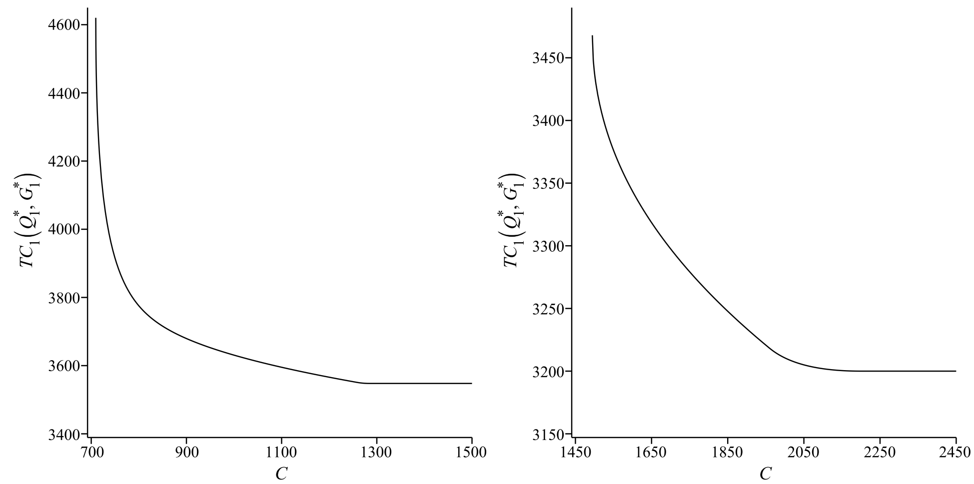


Table A.1 and Table A.2 for more illustrative examples). In the first set of instances, characterized by $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$, $Q_1^* = Q^0$ and $G_1^* = 0$ for $C \geq 1284.816$. Similarly, in the second set of instances, $Q_1^* = Q^0$ and $G_1^* = 0$ for $C \geq 2200$. For those values of C that are large enough (i.e., $C \geq 1284.816$ and $C \geq 2200$ in the first and second sets, respectively), having a cap policy does not change the solution in comparison to a no-policy case because the cap amount is not restrictive. Therefore, we have $TC_1\left(Q_1^*, G_1^*\right) = TC_1\left(Q_1^*(0), 0\right) = TC\left(Q^0, 0\right)$ in such instances. In the third instances of each set ($C = 1270$ and $C = 2110$ in the first and the second sets, respectively), we have $TC\left(Q^0, 0\right) < TC_1\left(Q_1^*, G_1^*\right) = TC_1\left(Q_1^*(0), 0\right)$ and $E\left(Q^0, 0\right) > E\left(Q_1^*, G_1^*\right) = E\left(Q_1^*(0), 0\right)$. Here, the cap policy helps to decrease emissions at the expense of increased costs, and the retailer does not invest in new technology to further reduce emissions even if such an option exists. In the second instances of each set ($C = 1170$ and $C = 1910$ in the first and the second sets, respectively), we have $TC\left(Q^0, 0\right) < TC_1\left(Q_1^*, G_1^*\right) < TC_1\left(Q_1^*(0), 0\right)$ and $E\left(Q^0, 0\right) > E\left(Q_1^*, G_1^*\right) = E\left(Q_1^*(0), 0\right)$. Again, the cap policy reduces annual emissions and increases annual costs, but different than the third instances, the investment option helps to achieve the same emissions at lower costs in comparison to no investment opportunity. Finally, the first instances of each set are illustrative of situations in which it is not possible to be within the

Table 4.1: Varying Numerical Examples Under the Cap Policy for Some Values of the Cap Given $\alpha = 4$ and $\beta = 0.01$

Instances with $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ $(Q^0 = 182.574,\ Q^e = 36.515,\ Q^\alpha = 164.114,\ E(Q^0, 0) = 1284.816,\ TC(Q^0, 0) = 3547.723)$									
C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
1070	–	–	–	158.904	51.994	1070	3605.005	–	–
1170	100	13.333	100	162.127	22.666	1170	3574.257	1170	3650
1270	172.26	7.74	172.26	172.26	0	1270	3548.649	1270	3548.649
1370	241.137	5.529	182.574	182.574	0	1284.816	3547.723	1284.816	3547.723
Instances with $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ $(Q^0 = 50,\ Q^e = 111.803,\ Q^\alpha = 76.376,\ E(Q^0, 0) = 2200,\ TC(Q^0, 0) = 3200)$									
C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
1710	–	–	–	82.556	68.043	1710	3293.72	–	–
1910	134.704	92.796	92.796	77.283	11.879	1910	3231.142	1910	3239.474
2110	220.918	56.582	56.582	56.582	0	2110	3201.531	2110	3201.531
2310	283.391	44.109	50	50	0	2200	3200	2200	3200

allowed emission limits without making an investment.

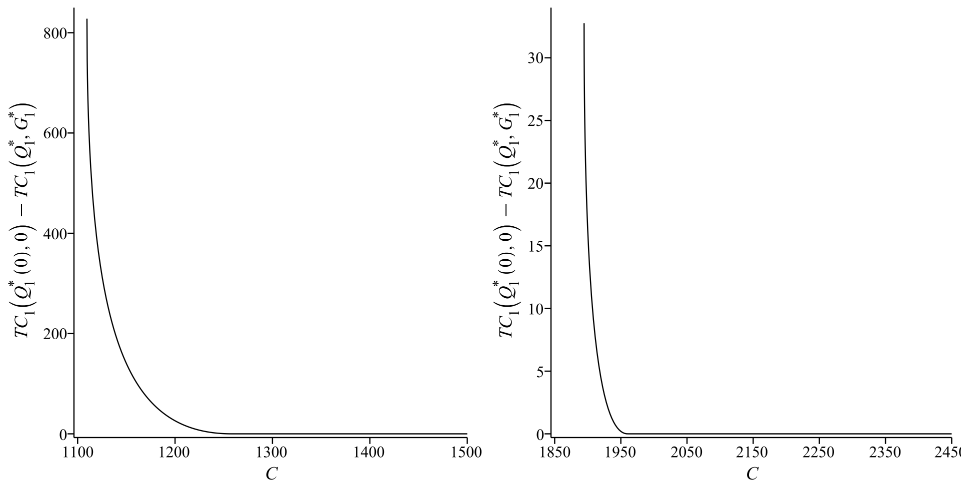
In Lemma 1, we have shown that $TC_1(Q_1^*(0), 0) - TC_1(Q_1^*, G_1^*) \geq 0$. The exact value of $TC_1(Q_1^*(0), 0) - TC_1(Q_1^*, G_1^*)$ is a measure of the savings due to the investment opportunity under the cap policy. Figure 4.2 illustrates how this difference changes with respect to C for the cases of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$. In both cases, values of C for which $Q_1^*(0)$ exists are considered. As a result, we have $C \geq 1109.545$ in Figure 4.2(a) and $C \geq 1894.427$ in Figure 4.2(b). Observe also that the savings due to the investment opportunity are more significant at tight values of the cap. Furthermore, the retailer no longer uses the investment opportunity (i.e., $G_1^* = 0$) if C is greater than or equal to $E(Q^\alpha, 0)$.

Figures 4.3(a) and 4.3(b) illustrate how the cost of unit emission reduction changes for varying values of the cap in cases of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, respectively. We know from Lemma 2 that $E(Q_1^*, G_1^*) \leq E(Q^0, 0)$. Both figures are plotted for those values of C at which $E(Q_1^*, G_1^*) < E(Q^0, 0)$. Mainly, Figure 4.3(a) considers values of C up to 1284.816 and Figure 4.3(b) considers values of C up to 2200. Observe that in both cases, reducing the annual emission level by one unit is more costly at small values of C. Furthermore, in case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$, the cost of a one-unit emission increases more rapidly as C gets smaller in comparison to the case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$.

Figure 4.4 shows the effect of α on total average annual cost in cap policy.

Figure 4.2: Savings due to an Investment Opportunity for Varying Values of the Cap Under a Cap Policy

(a) An Illustration for the Case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ (b) An Illustration for the Case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$

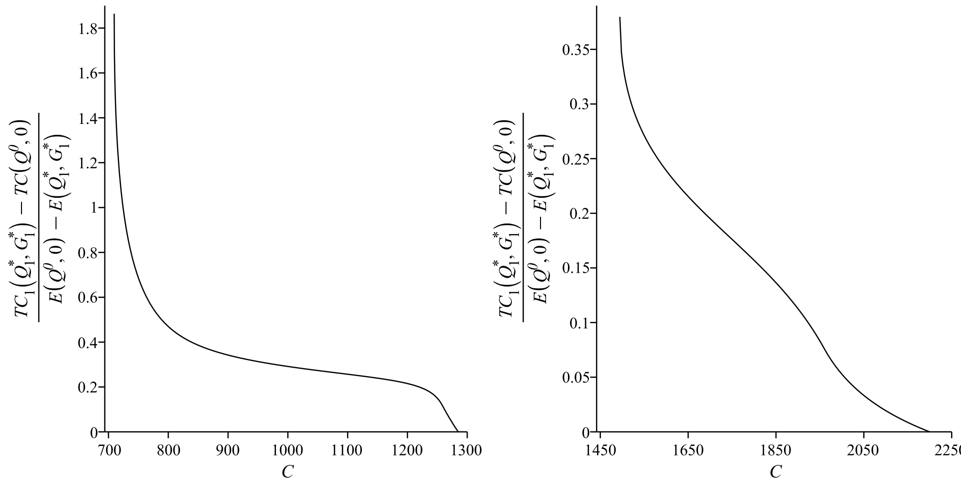


The α values are chosen to satisfy (A1) and (A4) simultaneously in both graphs (see Table A.3 and Table A.4 for detailed solutions of the underlying instances). Total cost is bounded below by $TC_1(Q^0, 0)$ and above by $TC_1(Q^e, \frac{\alpha}{2\beta})$. It can be observed from the plots that the retailer's costs are lowered if he/she chooses the investment option with higher value of α for a given β. The relation between β and total average cost for different $\frac{A}{h}$ and $\frac{\hat{A}}{\hat{h}}$ values is depicted in Figure 4.5. Again, β values are chosen to satisfy assumptions (A1) and (A4) simultaneously. The total costs are bounded below by $TC_1(Q^0, 0)$ and above by $TC_1(Q^e, \frac{\alpha}{2\beta})$ similar to Figure 4.4. We can observe from the graphs that the retailer prefers the investment option with a smaller β among those with the same value of α.

In Lemma 3, we have shown that among two investment options with different parameters, the retailer should choose the one with higher α and smaller β. In Figure 4.6, we show over numerical examples that if the investment option with higher α does not have smaller β, whether it is a better investment option or not depends on how high the α value is. Specifically, in Figure 4.6(a), for the case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$, setting $C = 840$, $\alpha_1 = 9.4$, $\beta_1 = 0.02$, and $\beta_2 = 0.02$, we change the value of α_2 and track the difference between the minimum annual costs resulting from the two investment options. $TC_1\left(Q_1^*, G_1^* | \alpha_1 = 9.4, \beta_1 = 0.02\right)$ refers to the

Figure 4.3: Cost of Unit Emission Reduction for Varying Values of the Cap Under a Cap Policy

(a) An Illustration for the Case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ (b) An Illustration for the Case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$



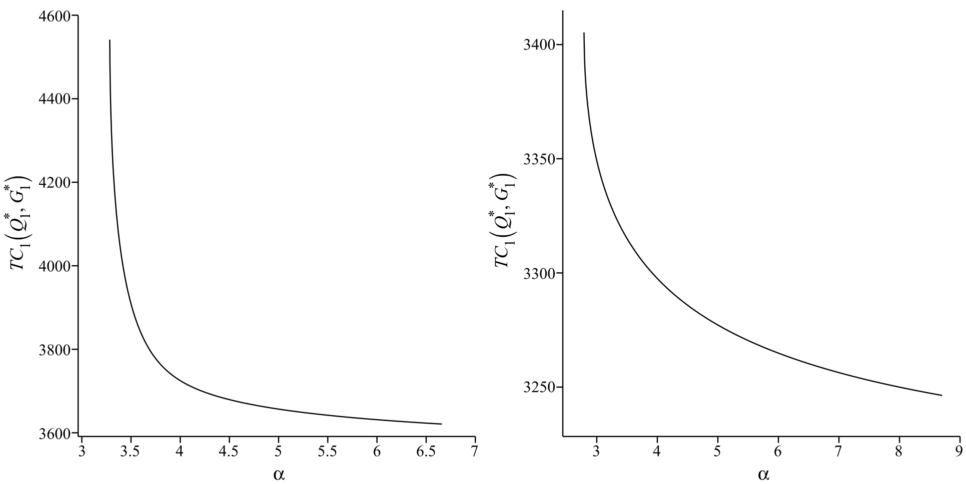
minimum costs, given that the first investment option has parameters $\alpha_1 = 9.4$ and $\beta_1 = 0.02$. Similarly, $TC_1\left(Q_1^*, G_1^* | \alpha_2, \beta_2 = 0.025\right)$ denotes the minimum costs if the second investment option has a value of α_2 as given on the x-axis, and $\beta_2 = 0.025$. Figure 4.6(a) shows that for all values of $\alpha_2 < 9.656$, the first investment option has lower costs. As α_2 increases beyond this value, the second investment option becomes more preferable. Figure 4.6(b) illustrates a similar result for the case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, setting $C = 1700$, $\alpha_1 = 12.3$, and $\beta_1 = 0.02$, $\beta_2 = 0.025$. The second investment option becomes better as α_2 is increased beyond 12.445. Notice that for values of α_2 between 12.3 and 12.445, the second investment option still has higher α and higher β, yet the first investment option leads to lower annual costs.

Figure 4.7 presents the retailer's total cost indifference curves between the efficiency parameter α and the decreasing return parameter β for the two general cases. Figure 4.7a illustrates the case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ in a setting where $C = 840$. Here, α takes values between 3.219 and 6, and β ranges from 0.005 to 0.026. Any values of α and β paired on this curve lead to the same total average annual cost (i.e., 3724.965). In Figure 4.7b, we consider the case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ in a setting where $C = 1700$. Any values of α and β paired on this curve lead to 3297.559 as the

Figure 4.4: Behavior of $TC_1\left(Q_1^*, G_1^*\right)$ for Varying Values of α

(a) An Illustration for the Case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ and C=840

(b) An Illustration for the Case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ and C=1700



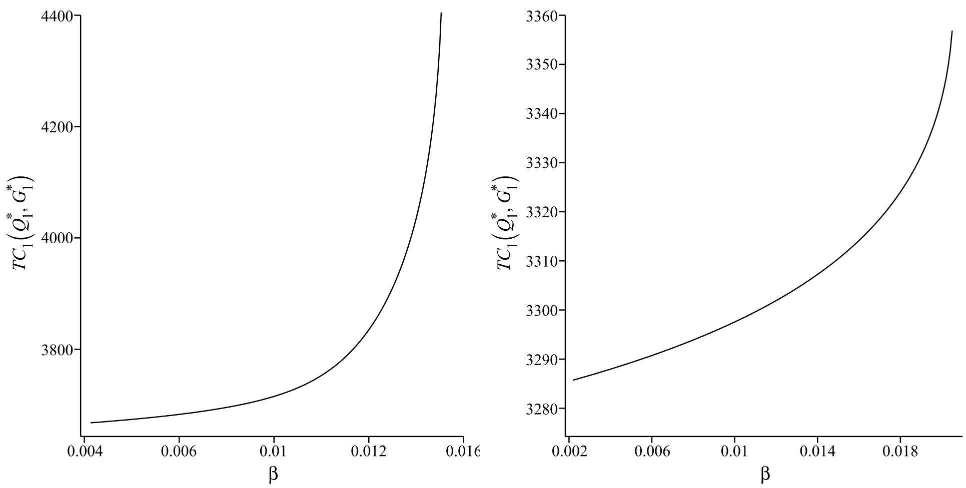
retailer's total average annual cost. In this figure, α values are between 3.636 and 6, and β ranges from 0.005 to 0.041. We can see that total cost indifference curves have nonlinear shapes in both figures. However, they can be approximated by linear lines over α and β. This shows that any amount of increase in the value of the diminishing return parameter β can be compensated by almost the same amount of increase in efficiency parameter α irrespective of the current absolute values of α and β.

4.2 Numerical Study for Tax Policy and Cap-and-Trade Policy

Corollary 2 and Lemma 4 provide analytical results for $TC_2\left(Q_2^*, G_2^*\right)$ and $TC_2\left(Q_2^*(0), 0\right) - TC_2\left(Q_2^*, G_2^*\right)$, which imply that both measures are increasing in p. In our numerical analysis for the tax policy, then, we proceed with investigating the effect of policy parameter p on the cost of unit emission reduction (i.e., $\frac{TC_2\left(Q_2^*,G_2^*\right)-TC\left(Q^0,0\right)}{E(Q^0,0)-E\left(Q_2^*,G_2^*\right)}$). In Figure 4.8(a), which pertains to the case of

Figure 4.5: Behavior of $TC_1\left(Q_1^*, G_1^*\right)$ for Varying Values of β

(a) An Illustration for the Case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ and C=840 (b) An Illustration for the Case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ and C=1700



$\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$, the cost of unit emission reduction is strictly convex in p, with a minimum at $p = 0.463$. In our numerical experimentation with various instances having $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, we observe that $\frac{TC_2\left(Q_2^*, G_2^*\right) - TC\left(Q^0, 0\right)}{E(Q^0, 0) - E\left(Q_2^*, G_2^*\right)}$ assumes a shape similar to the one in Figure 4.8(a). In Figure 4.8(b), for the case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, we change the value of $\hat{A}$ to 1000 to illustrate an extreme situation where the cost of unit emission reduction increases almost linearly with increasing p over all its possible values.

As in the case of the tax policy, our numerical analysis for the cap-and-trade policy focuses on investigating how the cost of unit emission reduction changes with respect to policy parameters. Corollary 4 and Lemma 8 provide analytical results for $TC_3\left(Q_3^*, G_3^*\right)$ and $TC_3\left(Q_3^*(0), 0\right) - TC_3\left(Q_3^*, G_3^*\right)$. Figure 4.9 presents three different illustrations of how the cost of unit emission reduction behaves with changing values of c_p. In the examples underlying Figures 4.9(a) and 4.9(c), there exist values of c_p ($c_p \geq 0.9754$ in Figure 4.9(a) and $c_p \geq 1.148$ in Figure 4.9(c)) at which the retailer sells his/her cap. In both of these examples, as c_p increases beyond these values, $TC_3\left(Q_3^*, G_3^*\right)$ gets smaller and smaller due to the revenue earned from selling permits. $TC_3\left(Q_3^*, G_3^*\right)$ falls below $TC\left(Q^0, 0\right)$ when $c_p \geq 4.061$ and when $c_p \geq 9.75$ in the examples of Figure 4.9(a) and

Figure 4.6: Comparison of Costs under Two Different Investment Options in Case of a Cap Policy

(a) An Illustration for the Case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ and $C = 840$

(b) An Illustration for the Case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ and $= 1700$

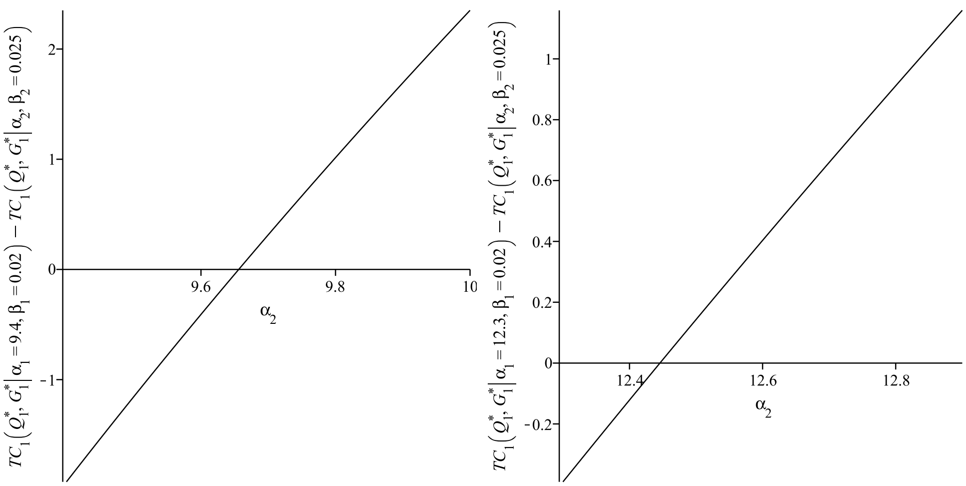


Figure 4.9(c), respectively. Figure 4.9(b) illustrates an example to Corollary 3. Because the retailer does not sell any carbon permits, regardless of the value of c_p, $TC_3\left(Q_3^*, G_3^*\right)$ is always greater than $TC_3\left(Q_3^*(0), 0\right)$. Furthermore, as c_p increases, the cost of unit emission reduction increases.

4.3 Numerical Comparison of the Three Policies

In Section 3.2.4, we proved that for any tax policy, there exists a cap policy with lower annual costs and no greater annual emissions. Similarly, for any cap policy, there exists a cap-and-trade policy with no greater annual costs and no greater annual emissions. In this subsection, we investigate how the differences between the annual costs and the annual emissions of any two policies change with respect to the problem parameters.

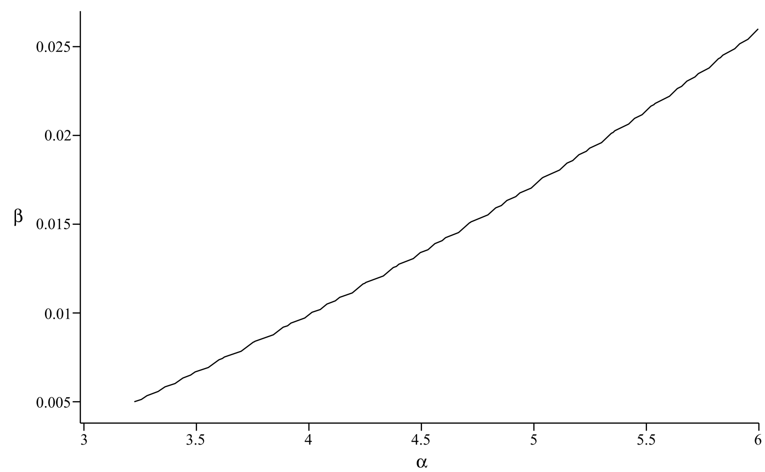
In Figure 4.10, we present two illustrations for the comparison of the cap and tax policies in a setting with parameters $A = 100$, $h = 3$, $\hat{A} = 4$, $\hat{h} = 3$, $\alpha = 4$, and $\beta = 0.01$. Figure 4.10(a) shows a plot of how $TC_1\left(Q_1^*, G_1^*\right) - TC_2\left(Q_2^*, G_2^*\right)$

and $E(Q_1^*, G_1^*) - E(Q_2^*, G_2^*)$ simultaneously change for varying values of C, given that the tax policy has $p = 0.26$. For values of C lower than 758.832, the tax policy is better in terms of annual costs. As C increases beyond this value, the cap policy becomes better in terms of annual costs and annual emissions up until $C = 1227.296$. For C values larger than 1227.296, the cap policy is more advantageous because of its resulting costs, however the tax policy is better because of its resulting emissions. Figure 4.10(b) presents a similar plot, given that tax policy has $p = 1.26$. At all values of C, the cap policy is more advantageous for the retailer because of its resulting costs. However, the tax policy leads to lower annual emissions for the retailer in comparison to any cap policy with parameter $C \geq 818.520$. Observe from both Figure 4.10(a) and Figure 4.10(b) that there is no value of C at which the tax policy is better for both its costs and its emissions, as also implied by Lemma 10.

For the same setting underlying Figure 4.10, we next compare the cap policy to the cap-and-trade policy. We consider two different values of c_p for the latter: 0.26 and 1.26. Figure 4.11(a) shows how $TC_1(Q_1^*, G_1^*) - TC_2(Q_2^*, G_2^*)$ and $E(Q_1^*, G_1^*) - E(Q_2^*, G_2^*)$ simultaneously change with varying values of C when $c_p = 0.26$. At all values of C, the cap-and-trade policy leads to lower annual costs, however, the cap policy results in lower annual emissions than the cap-and-trade does if $C < 1227.296$. Otherwise, the cap-and-trade policy is also better in terms of annual emissions. Similarly, Figure 4.11(b) shows that cap-and-trade policy is more advantageous for the retailer because of its resulting costs at all values of C, however, the dominance of one policy over another in terms of annual emissions changes depending on the value of C. Specifically, if $C \geq 818.520$, then the cap-and-trade policy dominates in terms of both measures, otherwise, the cap policy leads to lower annual emissions for the retailer.

Figure 4.7: Total Cost Indifference Curves Between α and β Under a Cap Policy

(a) An Illustration for the Case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ and $TC_1(Q_1^*, G_1^*) = 3724.965$



(b) An Illustration for the Case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ and $TC_1(Q_1^*, G_1^*) = 3297.559$

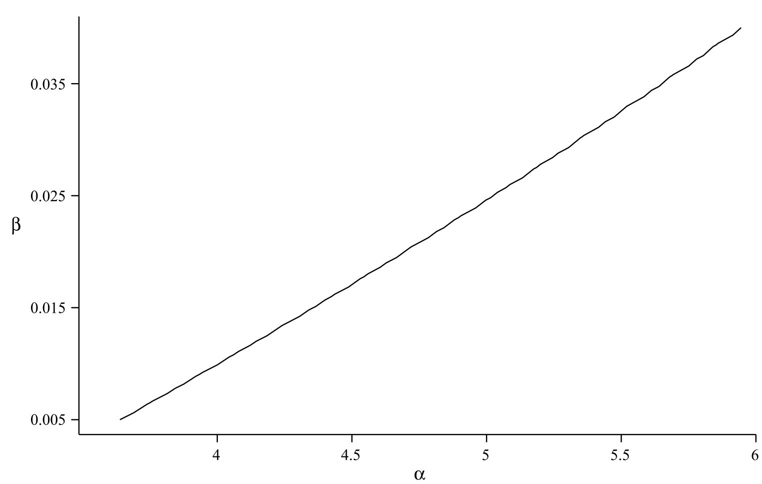


Figure 4.8: Cost of Unit Emission Reduction for Varying Values of Tax Under a Tax Policy

(a) An Illustration for the Case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ (b) An Illustration for the Case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$

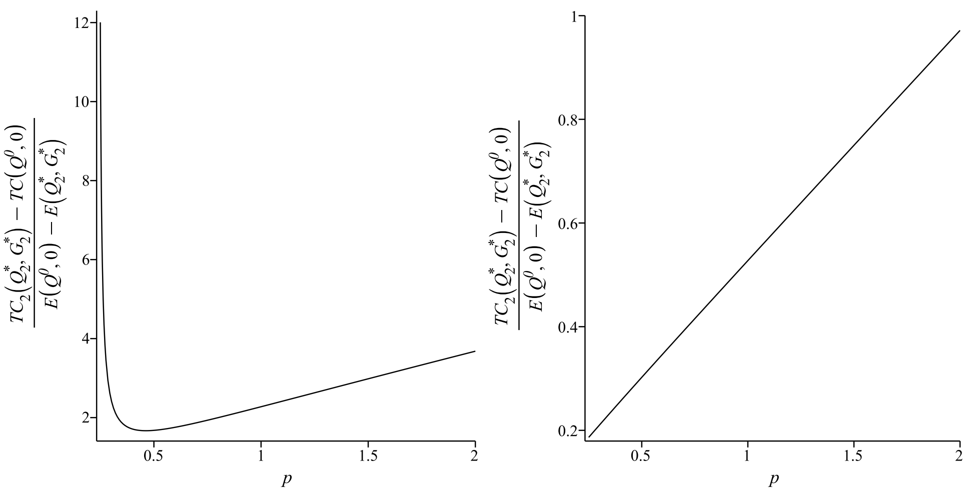
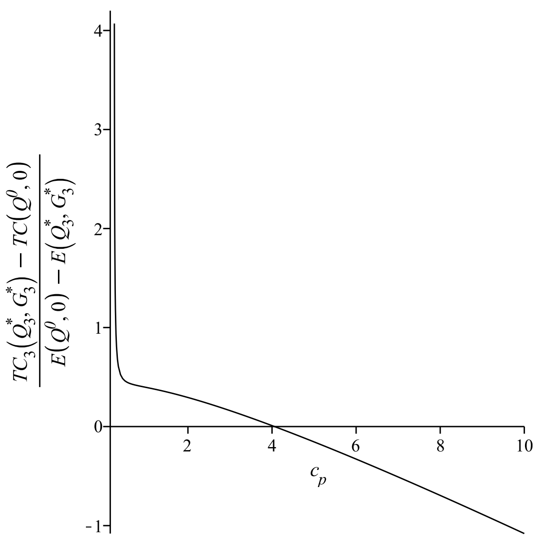
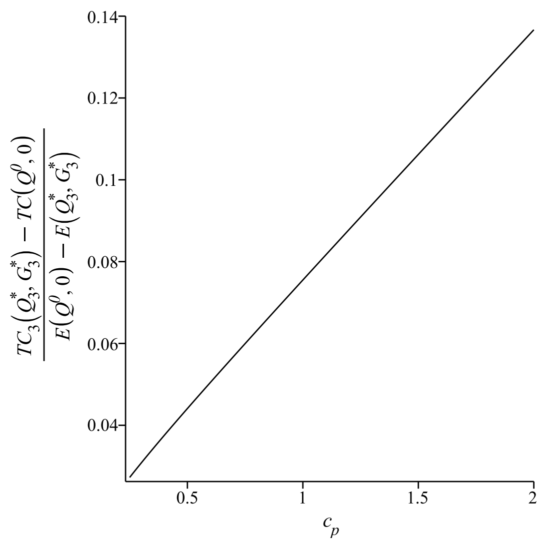


Figure 4.9: Cost of Unit Emission Reduction for Varying Values of the Trading Price Under a Cap-and-Trade Policy

(a) An Illustration for the Case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ ($\hat{A} = 4$, $\hat{h} = 3$)



(b) An Illustration for the Case of $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ ($\hat{A} = 20$, $\hat{h} = 8000$)



(c) An Illustration for the Case of $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ ($\hat{A} = 200$, $\hat{h} = 8$)

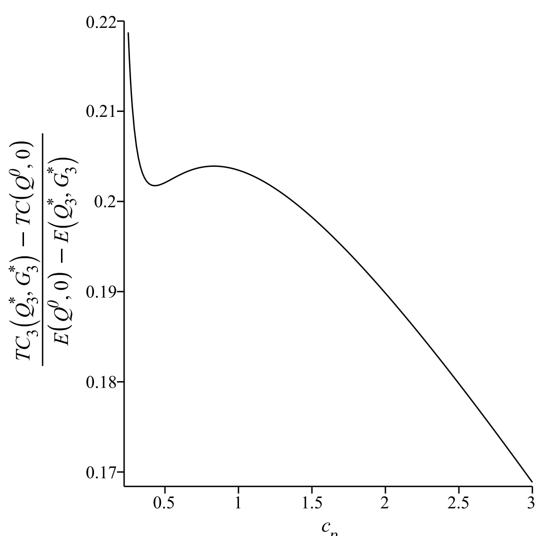
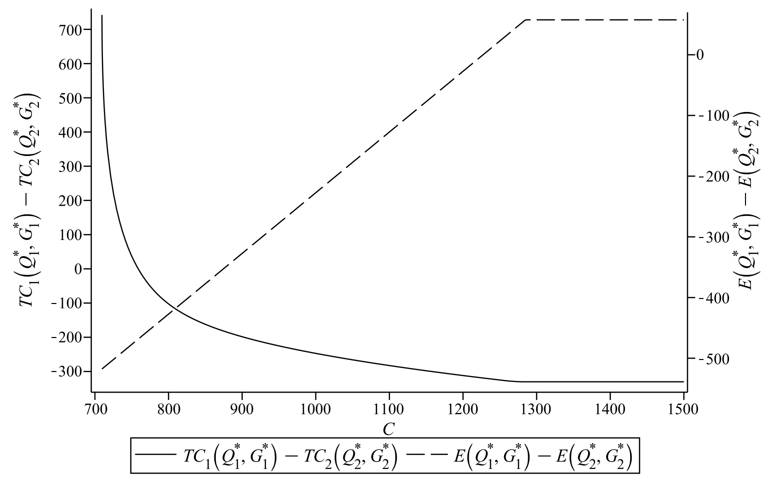


Figure 4.10: Comparison of Tax Policy to Cap Policy for Annual Costs and Annual Emissions

(a) An Illustration if Tax Policy has $p = 0.26$



(b) An Illustration if Tax Policy has $p = 1.26$

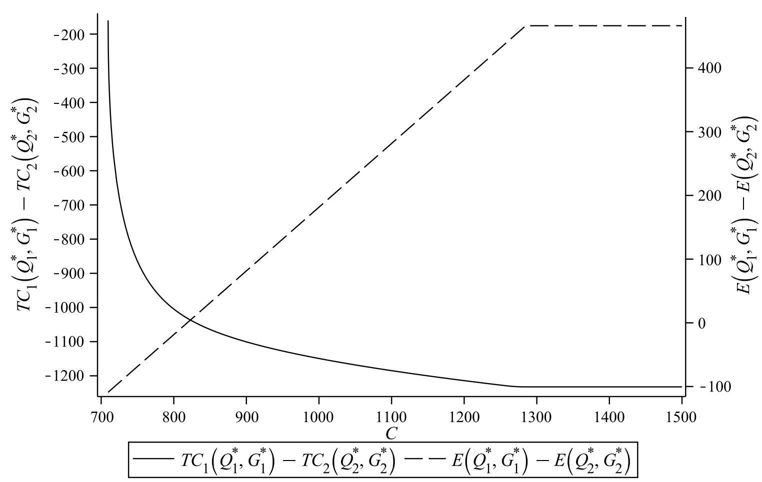
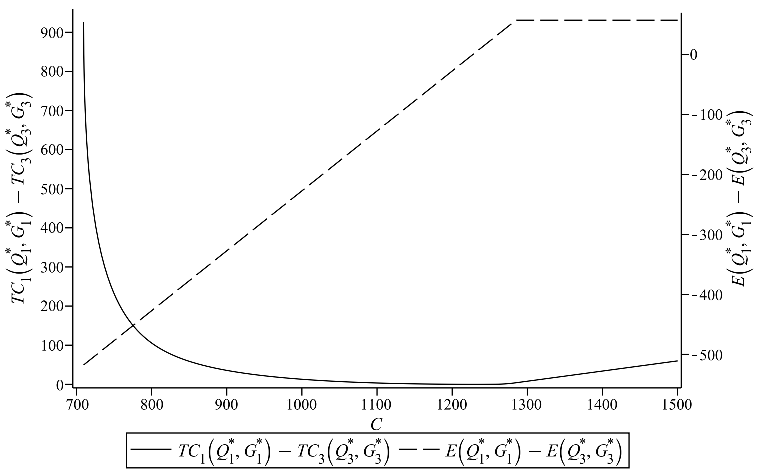
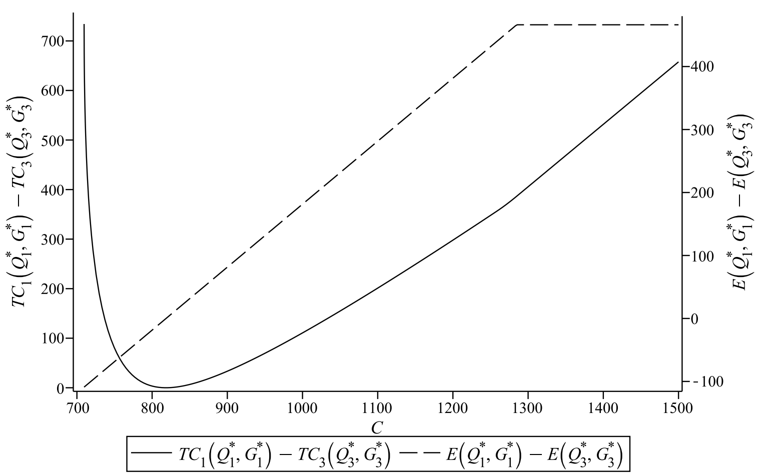


Figure 4.11: Comparison of Cap Policy to Cap-and-Trade Policy for Annual Costs and Annual Emissions

(a) An Illustration if Cap-and-Trade Policy has $c_p = 0.26$



(b) An Illustration if Cap-and-Trade Policy has $c_p = 1.26$



Chapter 5

An Extension to the Newsvendor Problem

In this chapter, we extend our analysis to the newsvendor problem under a cap-and-trade policy with an opportunity to invest in green technologies. In this setting, the retailer sells each unit of a single product type at $\$r$. The procurement cost of the product is $\$c$ per unit and leftover products can be salvaged at a price $\$v$/unit. The retailer incurs $\$b$ as loss of goodwill cost per unit of unsatisfied demand. The retailer's carbon emission associated with replenishment is linearly proportional to the replenishment quantity by a factor of e (i.e., carbon emission quantity is $e \times Q$ units if Q units are ordered). As in the case of EOQ model studied in earlier chapters, carbon emission can be reduced in an amount of $\alpha G - \beta G^2$ when $\$G$ are invested in green technologies.

Demand during the single selling period, $D(A, h(G))$, is composed of two elements; a random component A and a deterministic component $h(G)$ (i.e., $D(A, h(G)) = A + h(G)$. $f(.)$ and $F(.)$ are corresponding the p.d.f and c.d.f. of random component A. In this setting, there is a pool of customers who are environmentally sensitive. If $\$G$ are invested in green technologies, not only the carbon emission is reduced, but also demand increases in an amount of $h(G)$ due to the existence of environmentally sensitive customers.

Effect of environmental effort on customer demand is observed more in case of clean production technologies rather than in case of end-of-pipe emission reduction technologies. Clean production technologies may increase the customers' valuation of the product by changing product design or improving manufacturing process (e.g., customers' willingness to pay a premium for hybrid-electric car Toyota Pirius, or increase in demand of electric vehicles as reported in Tesla Motors' web site [41]). As mentioned in Chapter 2, a product's environmental effect on its demand has been studied before. Letmathe and Balakrishnan [28] model product demand as a decreasing function of emission amount. Krass et al. [38] put an emphasis on premium demand of different emission reduction technologies. Raz et al. [36] construct a price dependent demand model in which price elasticity decreases (i.e., demand increases) with innovation effort shown by the firm. Different than these studies, we model explicitly the existence of an emission regulation policy. Also, in our problem setting, investment in green technologies reduces emissions and increases demand simultaneously. Finally, although we later study the special cases, our general analysis does not assume any special functional forms for the dependency between demand and environmental efforts.

In this setting, although there is a significant number of environmentally sensitive customers, we assume that there exists an upper bound on the total number that can be attracted by investing in green technologies. Specifically, we assume that $h(G) \leq \bar{A}$. Furthermore, we take $h(G)$ as a nondecreasing and concave function of the investment amount (i.e., $h^{'}(G) \geq 0$ and $h^{''}(G) \leq 0$ where $h(0) = 0$). We also study some special cases of this function; those are $h(G) = \delta G$ and $h(G) = \delta(\alpha G - \beta G^2)$. The first function reflects a setting where environmentally sensitive customers are kind of myopic in the sense that they are affected by the information on company's investment amount, but they are short-sighted about how much actual reduction in emissions will be achieved. The second function reflects a setting where environmentally sensitive customers make more informative decisions caring about the actual emission reduction rather than the monetary value of the investment amount.

Other characteristics of this system are:

(A1) The ordinal relationship between the product's retail price, procurement cost and salvage value as in the classical Newsboy model, also holds in this setting. That is,

$$r > c > v. \tag{5.1}$$

(A2) Unit underage cost of the product is more than the profit that would be gained by selling carbon equivalent permit of one unit production at the market price. That is,

$$r + b - c > c_p e. \tag{5.2}$$

(A3) Let Q^{NVCT} be the solution of the equation $F(Q) = \frac{r+b-c-c_p e}{r+b-v}$, which is the solution of newsvendor problem under cap-and-trade policy without investment option. We have

$$eQ^{NVCT} - \frac{\alpha^2}{4\beta} > 0. \tag{5.3}$$

Due to (A1), the retailer's marginal profit from the sales of a unit item is positive and there is no motivation for the retailer to order an infinite amount. (A2) guarantees that the retailer would not be better off by not doing business in the current season and selling all his/her carbon allowance in the market instead. Finally, (A3) is equivalent to saying that emission due to ordering the optimal replenishment quantity of the retailer under a cap-and-trade system without investment opportunity cannot be totally eliminated even if the maximum reduction of emission is achieved by investment. Note that, under this assumption, we also have $eQ - \frac{\alpha^2}{4\beta} > 0$ for all $Q > Q^{NVCT}$. It is important to emphasize that the retailer's optimal order quantity without the cap-and-trade regulation, say Q^{NV} (i.e., the Newsvendor solution), is greater than Q^{NVCT} as $F\left(Q^{NV}\right) = \frac{r+b-c}{r+b-v}$. This, jointly with (A3), implies that $eQ^{NV} - \frac{\alpha^2}{4\beta} > 0$.

In the next subsection, we formulate and analyze the retailer's problem under a cap-and-trade policy and assuming the existence of an investment opportunity as described above.

5.1 General Analysis

The retailer is subject to a carbon cap of C units under a cap-and-trade policy. The unit carbon trading price is $\$c_p$. The timeline of events is as follows: cap amount C is determined by policy makers and the retailer decides the production quantity Q and investment amount G at the beginning of the period. Then, demand is realized and the retailer sells or buys carbon allowance depending on C at the end of the period.

The problem can be formulated as follows:

$$\begin{aligned} \max \quad & \pi^{CT}(Q,G) \\ \text{s.t.} \quad & eQ - \alpha G + \beta G^2 + X = C, \\ & Q \geq 0, \\ & G \geq 0, \\ & h(G) \leq \bar{A}. \end{aligned}$$

where $\pi^{CT}(Q,G) = rE[\min\{A, Q-h(G)\}] + h(G)r - cQ + c_pX + vE[Q - A - h(G)]^+ - bE[A+h(G)-Q]^+ - G$. Note that an alternative expression for $\pi^{CT}(Q,G)$ is given by

$$\begin{aligned} \pi^{CT}(Q,G) &= rE[A] + rh(G) + vQ - vE[A] - vh(G) - cQ \\ &\quad + (r+b-v)\int_{Q-h(G)}^{\infty} (Q - h(G) - a)f(a)da + c_pX - G. \end{aligned}$$

Here, $E[A]$ refers to the expected value of the random component of demand.

In the next theorem, we characterize the optimal solution of the retailer under the cap-and-trade policy and availability of investment option. We refer to the optimal pair of order quantity and investment amount as (Q^*, G^*).

Theorem 4 *Let G_1 and G_2 be the investment values such that $G_1 = \frac{h^{'}(G_1)(r-c-c_pe)+\alpha c_p-1}{2\beta c_p}$ and $h(G_2) = \bar{A}$, respectively. Define Q_1 and Q_2 as the order* *quantities such that* $F(Q_1 - h(G_1)) = \frac{r+b-c-c_pe}{r+b-v}$, $F(Q_2 - h(G_2)) = \frac{r+b-c-c_pe}{r+b-v}$. *Optimal solution of the retailer's problem is as follows:*

$$(Q^*, G^*) = \arg \max_{(Q,G)\in S} \pi^{CT}(Q, G)$$

where $S = \{(Q, G) \in \widetilde{S}$ *s.t.* $Q \geq 0,\ G \geq 0,\ h(G) \leq \bar{A}\}$. *Here,* $\widetilde{S} = \{(Q_1, G_1), (Q_2, G_2), (Q^{NVCT}, 0)\}$.

Proof: Plugging $C - eQ + \alpha G - \beta G^2$ in place of X, the objective function can be rewritten as

$$\begin{aligned} &rE[\min\{A, Q - h(G)\}] + h(G)r - cQ + c_p(C - eQ + \alpha G - \beta G^2) \\ &+ vE[Q - A - h(G)]^+ - bE[A + h(G) - Q]^+ - G. \end{aligned} \tag{5.4}$$

The above expression is continuous and differentiable, and its Hessian matrix is

$$\begin{pmatrix} -(r+b-v)f(Q-h(G)) & (r+b-v)h^{'}(G)f(Q-h(G)) \\ (r+b-v)h^{'}(G)f(Q-h(G)) & (r+b-v)h^{''}(G)F(Q-h(G))-(r+b-v)(h^{'}(G))^2f(Q-h(G))-2\beta c_p-h^{''}(G)b \end{pmatrix}.$$

This matrix is not necessarily negative semidefinite, therefore the objective function, depending on the form of $h(G)$, may not be jointly concave with respect to Q and G. Hence, KKT conditions are necessary but not sufficient. The necessary conditions for optimality are as follows:

$$r + b - c - c_pe - (r + b - v)F(Q - h(G)) + \mu_1 = 0, \tag{5.5}$$

$$c_p(\alpha - 2\beta G) + h^{'}(G)(r + b - v)F(Q - h(G)) - 1 - bh^{'}(G) - \lambda_1 h^{'}(G) + \mu_2 = 0, \tag{5.6}$$

$$\lambda_1(\bar{A} - h(G)) = 0 \tag{5.7}$$

$$\mu_1 Q = 0 \tag{5.8}$$

$$\mu_2 G = 0 \tag{5.9}$$

$$\lambda_1 \geq 0, \quad \mu_1 \geq 0, \quad \mu_2 \geq 0 \tag{5.10}$$

$$Q \geq 0, G \geq 0. \tag{5.11}$$

There are 8 possible scenarios depending on the values of KKT multipliers. However, only 3 of them may lead to feasible solutions.

Case 1: $\lambda_1 = 0$, $\mu_1 = 0$, $\mu_2 = 0$

Expressions (5.7), (5.8), and (5.9) are satisfied since all multipliers are zero. Expression (5.5) leads to

$$(r + b - v)F\left(Q - h(G)\right) = r + b - c - ec_p. \tag{5.12}$$

Utilizing this in Expression (5.6), we obtain

$$c_p(\alpha - 2\beta G) + h^{'}(G)(r + b - c - ec_p) - 1 - bh^{'}(G) = 0,$$

which further leads to

$$G = \frac{h^{'}(G)(r - c - c_p e) + \alpha c_p - 1}{2\beta c_p}. \tag{5.13}$$

Referring Q_1 and G_1 as the order quantity and the investment amount that simultaneously satisfy Expression (5.12) and Expression (5.13), Q_1 and G_1 may be optimal if they also satisfy the feasibility conditions (those are $Q \geq 0$, $G \geq 0$, and $h(G) \leq \bar{A}$).

Case 2: $\lambda_1 \neq 0$, $\mu_1 = 0$, $\mu_2 = 0$

Since $\lambda_1 \neq 0$, Expression (5.7) implies

$$h(G) = \bar{A}. \tag{5.14}$$

Using the above expression and the fact that $\mu_1 = 0$, Expression (5.5) can be rewritten as

$$r + b - c - ec_p - (r + b - v)F(Q - \bar{A}) = 0. \tag{5.15}$$

Let Q_2 and G_2 be the order quantity and the investment amount that simultaneously satisfy Expression (5.14) and (5.15). For this pair to be a feasible solution, λ_1 has to be greater than zero. Expression (5.6), which involves λ_1, now reduces to

$$c_p(\alpha - 2\beta G) + h^{'}(G)(r + b - c - ec_p) - 1 - bh^{'}(G) - \lambda_1 h^{'}(G) = 0,$$

which leads to

$$\lambda_1 = (r - c - ec_p) + \frac{c_p(\alpha - 2\beta G) - 1}{h^{'}(G)}.$$

$\lambda_1 > 0$ should be satisfied for (Q_2, G_2) to be considered.

Case 3: $\lambda_1 = 0$, $\mu_1 = 0$, $\mu_2 \neq 0$

Expression (5.9) jointly with the fact $\mu_2 \neq 0$ implies $G = 0$. Expression (5.5) then reduces to

$$r + b - c - ec_p - (r + b - v)F(Q) = 0. \tag{5.16}$$

Note that, the above expression has a unique solution, which is Q^{NVCT}. The feasibility of of this solution necessitates $\mu_2 > 0$. Expression (5.6), which involves μ_2, can now be rewritten as

$$\alpha c_p + h^{'}(0)(r + b - c - ec_p) - 1 - bh^{'}(0) + \mu_2 = 0.$$

Therefore, we should have $1 - \alpha c_p - h^{'}(0)(r - c - c_p e) > 0$. ■

Corollary 6 *Let the random component of customer demand have exponential distribution with parameter θ. Then, the optimal solution is the pair among $(Q_1, G_1), (Q_2, G_2), (Q^{NVCT}, 0)$ that maximizes the retailer's expected profits subject to feasibility conditions. Here,*

$$G_1 = \frac{h^{'}(G_1)(r - c - c_p e) + \alpha c_p - 1}{2\beta c_p}, \quad Q_1 = h(G_1) - \frac{ln(\frac{c + c_p e - v}{r + b - v})}{\theta},$$

$$h(G_2) = \bar{A}, \quad Q_2 = \bar{A} - \frac{ln(\frac{c + c_p e - v}{r + b - v})}{\theta},$$

and

$$Q^{NVCT} = -\frac{ln(\frac{c + c_p e - v}{r + b - v})}{\theta}.$$

In the remaining part of this chapter, we will analyze the special cases of the general problem where $h(G)$ assumes specific forms.

5.1.1 Analysis of Special Case I: $h(G) = \delta G$

In this case, deterministic part of demand that reflects the behavior of the environmentally sensitive customers, exhibits a linearly increasing pattern with respect to the investment amount. The optimal solution presented in the next corollary, follows from Theorem 4. Furthermore, when $h(G)$ assumes this special form, proof of Theorem 4 implies that the retailer's objective function is concave with respect to Q and G, and hence, KKT conditions are sufficient for optimality.

Corollary 7 *Let Q_1 and Q_2 be the order quantities such that*

$$F\left(Q_1 - \delta\left(\frac{\alpha}{2\beta} + \frac{\delta(r-c-c_pe)-1}{2\beta c_p}\right)\right) = \frac{r+b-c-c_pe}{r+b-v},$$

and

$$F(Q_2 - \bar{A}) = \frac{r+b-c-c_pe}{r+b-v}.$$

Then, optimal pair of retailer's order quantity and his/her investment amount can be obtained as follows:

$$(Q^*, G^*)) = \begin{cases} (Q^{NVCT}, 0) & \text{if} \quad \delta(r-c-c_pe) + \alpha c_p - 1 < 0, \\ (Q_2, \frac{\bar{A}}{\delta}) & \text{if} \quad \frac{\alpha}{2\beta} + \frac{\delta(r-c-c_pe)-1}{2\beta c_p} > \frac{\bar{A}}{\delta}, \\ \left(Q_1, \frac{\alpha}{2\beta} + \frac{\delta(r-c-c_pe)-1}{2\beta c_p}\right) & o.w. \end{cases}$$

5.1.2 Analysis of Special Case II: $h(G) = \delta(\alpha G - \beta G^2)$

In this case, deterministic part of demand that reflects the behavior of the environmentally sensitive customers, exhibits a linearly increasing pattern with respect to the actual emission reduction amount with the use of green technologies. The optimal solution presented in the next corollary, again follows from Theorem 4.

Corollary 8 *Let $G_1 = \frac{\alpha}{2\beta} - \frac{1}{2\beta(\delta(r-c-c_pe)+c_p)}$, $G_2 = \frac{\alpha}{2\beta} - \frac{\sqrt{\alpha^2 - \frac{4\beta\bar{A}}{\delta}}}{2\beta}$ and $G_3 = \frac{\alpha}{2\beta} + \frac{\sqrt{\alpha^2 - \frac{4\beta\bar{A}}{\delta}}}{2\beta}$ be the investment values. Define Q_1 and Q_2 as the order quantities such that $F\left(Q_1 - \delta\left(\frac{\alpha^2}{4\beta} - \frac{1}{4\beta(\delta(r-c-c_pe)+c_p)^2}\right)\right) = \frac{r+b-c-c_pe}{r+b-v}$ and $F\left(Q_2 - \bar{A}\right) =$*

$\frac{r+b-c-c_p e}{r+b-v}$, *respectively. Then, optimal pair of retailer's order quantity and investment amount to his/her maximization problem can be obtained as follows:*

$$(Q^*, G^*) = \arg \max_{(Q,G)\in S} \pi^{CT}(Q, G)$$

where $S = \{(Q, G) \in \widetilde{S}$ *s.t.* $Q \geq 0,\ G \geq 0,\ \delta\,(\alpha G - \beta G^2) \leq \bar{A}\}$. *Here,* $\widetilde{S} = \{(Q_1, G_1), (Q_2, G_2), (Q_2, G_3), (Q^{NVCT}, 0)\}$.

Chapter 6

Conclusion

In this thesis, we mainly study a retailer's joint decisions on inventory replenishment and emission reduction investment operating under the conditions of the classic EOQ model. We consider three emission regulation policies; cap, tax, and cap-and-trade. Our results provide guidelines and insights about five issues: (i) how much the retailer should order at each replenishment and how much he/she should invest in emission reduction to minimize long-run average costs, (ii) what the impact of having an investment option is on the retailer's annual costs and emissions, (iii) how the retailer's annual costs and emissions under an emission regulation policy compare to those when no regulation is in place, (iv) how the retailer should choose among different investment options available, and (v) how the different regulation policies compare in terms of the retailer's annual emissions and costs.

Analytical expressions for the optimal replenishment quantity and investment amount for the cap policy, tax policy, and cap-and-trade policy are presented in Theorems 1, 2, and 3, respectively. Our findings imply that an investment option may help the retailer to reduce his/her costs significantly under all policies; however, the retailer's annual emissions level does not decrease due to investing in case of the cap policy. Under the tax policy and the cap-and-trade policy, the retailer always takes advantage of the investment opportunity to further reduce his/her emissions, which implies that there is better motivation for governments to make investment opportunities available under the tax or cap-and-trade policy.

When carefully designed, all three regulation policies are effective in reducing carbon emissions. The cap and tax policies always lead to higher annual costs for the retailer compared to when no regulation policy is in place. On the other hand, under a cap-and-trade policy, the retailer may reduce his/her costs by selling permits equivalent to his/her excess carbon capacity. For the retailer not to profit solely from selling permits, there must exist an upper bound on the maximum annual carbon emission (see Corollary 5).

The investment function considered in this study has a nonlinear form characterized by two parameters. Lemmas 3, 6, and 9 provide guidelines in terms of those parameters on how the retailer should choose among different investment options. Our results imply that in case of the cap policy, the right choice of investment opportunity may help the retailer further reduce his/her annual costs, but it does not have an impact on annual emissions. We show that a better investment opportunity for reducing costs may lead to more annual emissions in some cases under a tax policy or a cap-and-trade policy. We also characterize the cases in which it is possible to reduce both the annual costs and the annual emissions by the right choice of investment opportunity.

We also show that for a given cap or cap-and-trade policy, it is not possible to design a tax policy that leads to both lower costs and lower emissions. On the other hand, for a given tax policy, a better cap policy can be designed by the appropriate choice of cap value. Further, for a given cap policy, there may exist a cap-and-trade policy that is better for both the resulting costs and the resulting emissions.

In our numerical analysis, we have defined a measure that we refer to as "cost of unit emission reduction". This measure is the ratio of the cost increase to the savings in emissions, and its value for a certain policy can be considered as the *social cost* of that policy. We have observed that the social cost becomes very high as the policy parameters are tightened in case of cap and tax policies (i.e., annual carbon emission cap is decreased in cap policy, or tax paid for one unit emission in increased in tax policy). In fact, the increase in social cost is more emphasized when the company's ratio of fixed cost of replenishment to his/her inventory holding cost rate is very high (i.e. $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ in terms of problem parameters). This suggests, in an inventory setting under a cap or a tax policy, reducing the company's ordering costs along with green technologies may decrease the social cost. However, we believe further research is needed to explore what kind of production/inventory related parameters are of significance in reducing cost of compliance to emission regulations. Our numerical analysis (see Figure 4.9) shows that a cap-and-trade policy, considering the measure of cost of unit emission reduction, may sometimes be rewarding (other times costly) depending on whether the company is able to generate excess carbon allowance to sell or not.

The use of a quadratic emission reduction function has made it possible to obtain analytical results that lead to the implications as discussed above. An important characteristic of this function, which we have utilized extensively in our analysis, is that it is a concave, increasing function until a certain value of investment (i.e., $\frac{\alpha}{(2\beta)}$). The increasing behavior of the function upto a certain point shows that investment in green technology is efficient in reducing emissions, but there is a maximum potential of abatement. The concavity implies that it becomes more costly to reduce emissions as emissions are decreased (the low hanging fruit has been picked). The analytical expressions we have derived, naturally depend on the parameters of this function, however, we believe our general conclusions still hold in case of other investment functions which exhibit these characteristics.

A similar model has also been developed for a retailer operating under the conditions of the newsboy model assuming a cap-and-trade policy is in place. Different than the EOQ model, the existence of environmentally sensitive customers is taken into account. That is, an investment in green technology not only reduces carbon emissions but also helps to attract more customers. A preliminary analysis has been done for this model, however, its implications have to be further investigated.

Our models assume a single item. An immediate extension would be to study the joint decisions for replenishment and allocation of limited investment budget (for emission reduction) among multiple items to maximize the profits. The core of our study considers a retailer operating under the conditions of the EOQ model, which is one of the fundamental models of inventory theory. The questions raised in this thesis can also be investigated for settings with different inventory replenishment policies.

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Appendix A

Tables

Table A.1: Numerical Illustrations Under the Cap Policy for Varying Values of the Cap Given $\alpha = 4$, $\beta = 0.01$ and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$

Instances with $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$ $\left(Q^0 = 182.574,\ Q^e = 36.515,\ Q^\alpha = 164.114,\ E\left(Q^0, 0\right) = 1284.816,\ TC\left(Q^0, 0\right) = 3547.723\right)$									
C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E\left(Q_1^*, G_1^*\right)$	$TC_1\left(Q_1^*, G_1^*\right)$	$E\left(Q_1^*(0), 0\right)$	$TC_1\left(Q_1^*(0), 0\right)$
1495	–	–	0	109	194.14	1495	3458.485	–	–
1500	–	–	0	104	181.219	1500	3438.056	–	–
1505	–	–	0	102	173.777	1505	3426.934	–	–
1510	–	–	0	100	167.857	1510	3418.364	–	–
1515	–	–	0	99	162.76	1515	3411.155	–	–
1520	–	–	0	98	158.2	1520	3404.829	–	–
1525	–	–	0	97	154.027	1525	3399.131	–	–
1530	–	–	0	96	150.153	1530	3393.913	–	–
1535	–	–	0	95	146.516	1535	3389.074	–	–
1540	–	–	0	94	143.075	1540	3384.545	–	–
1545	–	–	0	93	139.8	1545	3380.276	–	–
1550	–	–	0	93	136.667	1550	3376.228	–	–
1555	–	–	0	92	133.659	1555	3372.372	–	–
1560	–	–	0	92	130.76	1560	3368.684	–	–
1565	–	–	0	91	127.958	1565	3365.145	–	–
1570	–	–	0	91	125.245	1570	3361.739	–	–
1575	–	–	0	90	122.61	1575	3358.453	–	–
1580	–	–	0	90	120.049	1580	3355.275	–	–

Table A.1 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
1585	–	–	0	89	117.554	1585	3352.196	–	–
1590	–	–	0	89	115.121	1590	3349.208	–	–
1595	–	–	0	89	112.744	1595	3346.303	–	–
1600	–	–	0	88	110.42	1600	3343.475	–	–
1605	–	–	0	88	108.145	1605	3340.72	–	–
1610	–	–	0	87	105.917	1610	3338.03	–	–
1615	–	–	0	87	103.732	1615	3335.404	–	–
1620	–	–	0	87	101.587	1620	3332.835	–	–
1625	–	–	0	87	99.481	1625	3330.322	–	–
1630	–	–	0	86	97.412	1630	3327.86	–	–
1635	–	–	0	86	95.377	1635	3325.447	–	–
1640	–	–	0	86	93.374	1640	3323.08	–	–
1645	–	–	0	85	91.403	1645	3320.757	–	–
1650	–	–	0	85	89.462	1650	3318.475	–	–
1655	–	–	0	85	87.548	1655	3316.233	–	–
1660	–	–	0	85	85.662	1660	3314.028	–	–
1665	–	–	0	84	83.802	1665	3311.86	–	–
1670	–	–	0	84	81.967	1670	3309.725	–	–
1675	–	–	0	84	80.155	1675	3307.623	–	–
1680	–	–	0	84	78.367	1680	3305.552	–	–
1685	–	–	0	84	76.601	1685	3303.512	–	–
1690	–	–	0	83	74.855	1690	3301.5	–	–
1695	–	–	0	83	73.131	1695	3299.516	–	–

Table A.1 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
1700	–	–	0	83	71.426	1700	3297.559	–	–
1705	–	–	0	83	69.74	1705	3295.627	–	–
1710	–	–	0	83	68.072	1710	3293.72	–	–
1715	–	–	0	82	66.422	1715	3291.837	–	–
1720	–	–	0	82	64.79	1720	3289.977	–	–
1725	–	–	0	82	63.174	1725	3288.139	–	–
1730	–	–	0	82	61.575	1730	3286.322	–	–
1735	–	–	0	82	59.991	1735	3284.526	–	–
1740	–	–	0	81	58.422	1740	3282.751	–	–
1745	–	–	0	81	56.868	1745	3280.994	–	–
1750	–	–	0	81	55.329	1750	3279.257	–	–
1755	–	–	0	81	53.804	1755	3277.538	–	–
1760	–	–	0	81	52.292	1760	3275.837	–	–
1765	–	–	0	81	50.793	1765	3274.153	–	–
1770	–	–	0	81	49.307	1770	3272.486	–	–
1775	–	–	0	80	47.834	1775	3270.835	–	–
1780	–	–	0	80	46.373	1780	3269.2	–	–
1785	–	–	0	80	44.924	1785	3267.58	–	–
1790	–	–	0	80	43.487	1790	3265.975	–	–
1795	–	–	0	80	42.061	1795	3264.385	–	–
1800	–	–	0	80	40.646	1800	3262.809	–	–
1805	–	–	0	80	39.242	1805	3261.247	–	–
1810	–	–	0	79	37.849	1810	3259.699	–	–

Table A.1 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
1815	–	–	0	79	36.466	1815	3258.164	–	–
1820	–	–	0	79	35.093	1820	3256.641	–	–
1825	–	–	0	79	33.73	1825	3255.132	–	–
1830	–	–	0	79	32.376	1830	3253.634	–	–
1835	–	–	0	79	31.033	1835	3252.149	–	–
1840	–	–	0	79	29.698	1840	3250.675	–	–
1845	–	–	0	79	28.372	1845	3249.213	–	–
1850	–	–	0	79	27.056	1850	3247.762	–	–
1855	–	–	0	78	25.748	1855	3246.321	–	–
1860	–	–	0	78	24.449	1860	3244.892	–	–
1865	–	–	0	78	23.158	1865	3243.473	–	–
1870	–	–	0	78	21.875	1870	3242.065	–	–
1875	–	–	0	78	20.6	1875	3240.666	–	–
1880	–	–	0	78	19.334	1880	3239.277	–	–
1885	–	–	0	78	18.075	1885	3237.898	–	–
1890	–	–	0	78	16.824	1890	3236.529	–	–
1895	115.877	107.873	107.873	78	15.58	1895	3235.169	1895	3262.097
1900	125	100	100	77	14.343	1900	3233.818	1900	3250
1905	130.366	95.884	95.884	77	13.114	1905	3232.476	1905	3243.914
1910	134.704	92.796	92.796	77	11.892	1910	3231.142	1910	3239.474
1915	138.492	90.258	90.258	77	10.677	1915	3229.818	1915	3235.912
1920	141.926	88.074	88.074	77	9.469	1920	3228.501	1920	3232.919
1925	145.106	86.144	86.144	77	8.267	1925	3227.193	1925	3230.33

Table A.1 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
1930	148.094	84.406	84.406	77	7.072	1930	3225.893	1930	3228.049
1935	150.93	82.82	82.82	77	5.884	1935	3224.602	1935	3226.011
1940	153.642	81.358	81.358	77	4.702	1940	3223.318	1940	3224.173
1945	156.25	80	80	77	3.527	1945	3222.041	1945	3222.5
1950	158.77	78.73	78.73	77	2.357	1950	3220.773	1950	3220.969
1955	161.213	77.537	77.537	76	1.194	1955	3219.511	1955	3219.56
1960	163.589	76.411	76.411	76	0.037	1960	3218.258	1960	3218.258
1965	165.906	75.344	75.344	75	0	1965	3217.05	1965	3217.05
1970	168.171	74.329	74.329	74	0	1970	3215.927	1970	3215.927
1975	170.388	73.362	73.362	73	0	1975	3214.879	1975	3214.879
1980	172.562	72.438	72.438	72	0	1980	3213.9	1980	3213.9
1985	174.698	71.552	71.552	72	0	1985	3212.983	1985	3212.983
1990	176.798	70.702	70.702	71	0	1990	3212.124	1990	3212.124
1995	178.865	69.885	69.885	70	0	1995	3211.316	1995	3211.316
2000	180.902	69.098	69.098	69	0	2000	3210.557	2000	3210.557
2005	182.911	68.339	68.339	68	0	2005	3209.843	2005	3209.843
2010	184.894	67.606	67.606	68	0	2010	3209.17	2010	3209.17
2015	186.852	66.898	66.898	67	0	2015	3208.536	2015	3208.536
2020	188.788	66.212	66.212	66	0	2020	3207.939	2020	3207.939
2025	190.703	65.547	65.547	66	0	2025	3207.375	2025	3207.375
2030	192.598	64.902	64.902	65	0	2030	3206.843	2030	3206.843
2035	194.474	64.276	64.276	64	0	2035	3206.341	2035	3206.341
2040	196.332	63.668	63.668	64	0	2040	3205.868	2040	3205.868

Table A.1 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
2045	198.174	63.076	63.076	63	0	2045	3205.421	2045	3205.421
2050	200	62.5	62.5	63	0	2050	3205	2050	3205
2055	201.811	61.939	61.939	62	0	2055	3204.603	2055	3204.603
2060	203.607	61.393	61.393	61	0	2060	3204.228	2060	3204.228
2065	205.39	60.86	60.86	61	0	2065	3203.876	2065	3203.876
2070	207.16	60.34	60.34	60	0	2070	3203.544	2070	3203.544
2075	208.918	59.832	59.832	60	0	2075	3203.231	2075	3203.231
2080	210.664	59.336	59.336	59	0	2080	3202.938	2080	3202.938
2085	212.398	58.852	58.852	59	0	2085	3202.663	2085	3202.663
2090	214.122	58.378	58.378	58	0	2090	3202.405	2090	3202.405
2095	215.836	57.914	57.914	58	0	2095	3202.163	2095	3202.163
2100	217.539	57.461	57.461	57	0	2100	3201.938	2100	3201.938
2105	219.233	57.017	57.017	57	0	2105	3201.727	2105	3201.727
2110	220.918	56.582	56.582	57	0	2110	3201.531	2110	3201.531
2115	222.594	56.156	56.156	56	0	2115	3201.35	2115	3201.35
2120	224.261	55.739	55.739	56	0	2120	3201.182	2120	3201.182
2125	225.921	55.329	55.329	55	0	2125	3201.027	2125	3201.027
2130	227.572	54.928	54.928	55	0	2130	3200.884	2130	3200.884
2135	229.216	54.534	54.534	55	0	2135	3200.754	2135	3200.754
2140	230.853	54.147	54.147	54	0	2140	3200.635	2140	3200.635
2145	232.483	53.767	53.767	54	0	2145	3200.528	2145	3200.528
2150	234.105	53.395	53.395	53	0	2150	3200.432	2150	3200.432
2155	235.721	53.029	53.029	53	0	2155	3200.346	2155	3200.346

Table A.1 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
2160	237.331	52.669	52.669	53	0	2160	3200.271	2160	3200.271
2165	238.934	52.316	52.316	52	0	2165	3200.205	2165	3200.205
2170	240.532	51.968	51.968	52	0	2170	3200.149	2170	3200.149
2175	242.123	51.627	51.627	52	0	2175	3200.102	2175	3200.102
2180	243.709	51.291	51.291	51	0	2180	3200.065	2180	3200.065
2185	245.29	50.96	50.96	51	0	2185	3200.036	2185	3200.036
2190	246.865	50.635	50.635	51	0	2190	3200.016	2190	3200.016
2195	248.435	50.315	50.315	50	0	2195	3200.004	2195	3200.004
2200	250	50	50	50	0	2200	3200	2200	3200
2205	251.56	49.69	50	50	0	2200	3200	2200	3200
2210	253.115	49.385	50	50	0	2200	3200	2200	3200
2215	254.666	49.084	50	50	0	2200	3200	2200	3200
2220	256.212	48.788	50	50	0	2200	3200	2200	3200
2225	257.754	48.496	50	50	0	2200	3200	2200	3200
2230	259.292	48.208	50	50	0	2200	3200	2200	3200
2235	260.825	47.925	50	50	0	2200	3200	2200	3200
2240	262.355	47.645	50	50	0	2200	3200	2200	3200
2245	263.88	47.37	50	50	0	2200	3200	2200	3200

Table A.2: Numerical Illustrations Under the Cap Policy for Varying Values of the Cap Given $\alpha = 4$, $\beta = 0.01$ and $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$

Instances with $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ $\left(Q^0 = 182.574,\ Q^e = 36.515,\ Q^\alpha = 164.114,\ E\left(Q^0, 0\right) = 1284.816,\ TC\left(Q^0, 0\right) = 3547.723\right)$									
C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E\left(Q_1^*, G_1^*\right)$	$TC_1\left(Q_1^*, G_1^*\right)$	$E\left(Q_1^*(0), 0\right)$	$TC_1\left(Q_1^*(0), 0\right)$
710	39.993	199.581	-7.162	-186.171	4509.79	710	–	–	0
715	49.949	198.117	-7.298	-182.702	4274.056	715	–	–	0
720	56.224	196.971	-7.439	-179.227	4170.603	720	–	–	0
725	61.559	195.843	-7.587	-175.747	4100.416	725	–	–	0
730	66.397	194.684	-7.74	-172.26	4047.325	730	–	–	0
735	70.914	193.472	-7.9	-168.766	4004.925	735	–	–	0
740	75.196	192.194	-8.068	-165.266	3969.92	740	–	–	0
745	79.292	190.842	-8.243	-161.757	3940.36	745	–	–	0
750	83.233	189.407	-8.426	-158.241	3914.982	750	–	–	0
755	87.035	187.884	-8.618	-154.715	3892.917	755	–	–	0
760	90.711	186.267	-8.819	-151.181	3873.536	760	–	–	0
765	94.265	184.554	-9.031	-147.635	3856.371	765	–	–	0
770	97.7	182.739	-9.254	-144.079	3841.06	770	–	–	0
775	101.016	180.823	-9.489	-140.511	3827.319	775	–	–	0
780	104.211	178.805	-9.737	-136.929	3814.919	780	–	–	0
785	107.281	176.686	-10	-133.333	3803.672	785	–	–	0
790	110.225	174.47	-10.278	-129.722	3793.425	790	–	–	0
795	113.038	172.163	-10.574	-126.092	3784.048	795	–	–	0

Table A.2 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
800	115.719	169.77	-10.889	-122.444	3775.43	800	–	–	0
805	118.264	167.3	-11.226	-118.774	3767.479	805	–	–	0
810	120.673	164.763	-11.586	-115.081	3760.115	810	–	–	0
815	122.948	162.17	-11.973	-111.36	3753.268	815	–	–	0
820	125.09	159.532	-12.39	-107.61	3746.879	820	–	–	0
825	127.102	156.859	-12.842	-103.824	3740.897	825	–	–	0
830	128.99	154.163	-13.333	-100	3735.275	830	–	–	0
835	130.757	151.453	-13.87	-96.13	3729.976	835	–	–	0
840	132.411	148.738	-14.46	-92.206	3724.966	840	–	–	0
845	133.959	146.026	-15.114	-88.22	3720.213	845	–	–	0
850	135.405	143.324	-15.843	-84.157	3715.694	850	–	–	0
855	136.759	140.639	-16.667	-80	3711.384	855	–	–	0
860	138.025	137.974	-17.607	-75.726	3707.265	860	–	–	0
865	139.211	135.334	-18.7	-71.3	3703.318	865	–	–	0
870	140.323	132.722	-20	-66.667	3699.528	870	–	–	0
875	141.366	130.141	-21.597	-61.736	3695.881	875	–	–	0
880	142.346	127.591	-23.67	-56.33	3692.367	880	–	–	0
885	143.268	125.075	-26.667	-50	3688.973	885	–	–	0
890	144.136	122.592	-33.333	-40	3685.69	890	–	–	0
895	144.955	120.144	–	–	3682.511	895	–	–	0
900	145.729	117.73	–	–	3679.427	900	–	–	0
905	146.46	115.351	–	–	3676.431	905	–	–	0
910	147.153	113.006	–	–	3673.518	910	–	–	0

Table A.2 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
915	147.81	110.695	–	–	3670.682	915	–	–	0
920	148.434	108.417	–	–	3667.918	920	–	–	0
925	149.027	106.171	–	–	3665.221	925	–	–	0
930	149.592	103.957	–	–	3662.588	930	–	–	0
935	150.13	101.774	–	–	3660.014	935	–	–	0
940	150.644	99.622	–	–	3657.496	940	–	–	0
945	151.134	97.499	–	–	3655.032	945	–	–	0
950	151.604	95.405	–	–	3652.617	950	–	–	0
955	152.053	93.339	–	–	3650.251	955	–	–	0
960	152.484	91.3	–	–	3647.929	960	–	–	0
965	152.898	89.287	–	–	3645.65	965	–	–	0
970	153.295	87.301	–	–	3643.412	970	–	–	0
975	153.676	85.339	–	–	3641.213	975	–	–	0
980	154.043	83.402	–	–	3639.051	980	–	–	0
985	154.397	81.488	–	–	3636.924	985	–	–	0
990	154.738	79.597	–	–	3634.831	990	–	–	0
995	155.067	77.729	–	–	3632.771	995	–	–	0
1000	155.384	75.882	–	–	3630.742	1000	–	–	0
1005	155.69	74.056	–	–	3628.742	1005	–	–	0
1010	155.987	72.251	–	–	3626.771	1010	–	–	0
1015	156.273	70.466	–	–	3624.828	1015	–	–	0
1020	156.551	68.7	–	–	3622.911	1020	–	–	0
1025	156.82	66.952	–	–	3621.019	1025	–	–	0

Table A.2 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
1030	157.08	65.223	–	–	3619.152	1030	–	–	0
1035	157.333	63.512	–	–	3617.309	1035	–	–	0
1040	157.578	61.819	–	–	3615.489	1040	–	–	0
1045	157.816	60.142	–	–	3613.691	1045	–	–	0
1050	158.048	58.482	–	–	3611.914	1050	–	–	0
1055	158.272	56.838	–	–	3610.157	1055	–	–	0
1060	158.491	55.209	–	–	3608.421	1060	–	–	0
1065	158.704	53.596	–	–	3606.704	1065	–	–	0
1070	158.911	51.997	–	–	3605.005	1070	–	–	0
1075	159.112	50.413	–	–	3603.325	1075	–	–	0
1080	159.309	48.844	–	–	3601.663	1080	–	–	0
1085	159.5	47.288	–	–	3600.017	1085	–	–	0
1090	159.687	45.745	–	–	3598.388	1090	–	–	0
1095	159.869	44.216	–	–	3596.776	1095	–	–	0
1100	160.047	42.7	–	–	3595.179	1100	–	–	0
1105	160.221	41.196	–	–	3593.597	1105	–	–	0
1110	160.39	39.705	40	33.333	3592.03	1110	4310	1110	40
1115	160.556	38.226	50	26.667	3590.478	1115	4075	1115	50
1120	160.718	36.758	56.33	23.67	3588.939	1120	3972.122	1120	56.33
1125	160.876	35.302	61.736	21.597	3587.414	1125	3902.504	1125	61.736
1130	161.03	33.857	66.667	20	3585.903	1130	3850	1130	66.667
1135	161.182	32.423	71.3	18.7	3584.405	1135	3808.216	1135	71.3
1140	161.33	31.001	75.726	17.607	3582.919	1140	3773.864	1140	75.726

Table A.2 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
1145	161.475	29.588	80	16.667	3581.446	1145	3745	1145	80
1150	161.617	28.186	84.157	15.843	3579.985	1150	3720.366	1150	84.157
1155	161.756	26.794	88.22	15.114	3578.536	1155	3699.097	1155	88.22
1160	161.892	25.413	92.206	14.46	3577.098	1160	3680.572	1160	92.206
1165	162.026	24.041	96.13	13.87	3575.672	1165	3664.324	1165	96.13
1170	162.157	22.678	100	13.333	3574.257	1170	3650	1170	100
1175	162.285	21.325	103.824	12.842	3572.852	1175	3637.319	1175	103.824
1180	162.411	19.981	107.61	12.39	3571.458	1180	3626.057	1180	107.61
1185	162.535	18.646	111.36	11.973	3570.075	1185	3616.034	1185	111.36
1190	162.656	17.32	115.081	11.586	3568.701	1190	3607.099	1190	115.081
1195	162.775	16.002	118.774	11.226	3567.338	1195	3599.128	1195	118.774
1200	162.892	14.694	122.444	10.889	3565.984	1200	3592.016	1200	122.444
1205	163.006	13.393	126.092	10.574	3564.639	1205	3585.673	1205	126.092
1210	163.119	12.101	129.722	10.278	3563.304	1210	3580.023	1210	129.722
1215	163.23	10.817	133.333	10	3561.978	1215	3575	1215	133.333
1220	163.338	9.541	136.929	9.737	3560.661	1220	3570.546	1220	136.929
1225	163.445	8.272	140.511	9.489	3559.353	1225	3566.611	1225	140.511
1230	163.55	7.011	144.079	9.254	3558.053	1230	3563.15	1230	144.079
1235	163.653	5.758	147.635	9.031	3556.762	1235	3560.125	1235	147.635
1240	163.755	4.512	151.181	8.819	3555.479	1240	3557.501	1240	151.181
1245	163.855	3.274	154.715	8.618	3554.204	1245	3555.247	1245	154.715
1250	163.953	2.042	158.241	8.426	3552.937	1250	3553.335	1250	158.241
1255	164.05	0.818	161.757	8.243	3551.678	1255	3551.741	1255	161.757

Table A.2 – continued from previous page

C	Q_1	Q_2	$Q_1^*(0)$	Q_1^*	G_1^*	$E(Q_1^*, G_1^*)$	$TC_1(Q_1^*, G_1^*)$	$E(Q_1^*(0), 0)$	$TC_1(Q_1^*(0), 0)$
1260	165.266	0	165.266	8.068	3550.442	1260	3550.442	1260	165.266
1265	168.766	0	168.766	7.9	3549.417	1265	3549.417	1265	168.766
1270	172.26	0	172.26	7.74	3548.649	1270	3548.649	1270	172.26
1275	175.747	0	175.747	7.587	3548.12	1275	3548.12	1275	175.747
1280	179.227	0	179.227	7.439	3547.816	1280	3547.816	1280	179.227
1285	182.574	0	182.702	7.298	3547.723	1284.816	3547.723	1284.816	182.574
1290	182.574	0	186.171	7.162	3547.723	1284.816	3547.723	1284.816	182.574
1295	182.574	0	189.636	7.031	3547.723	1284.816	3547.723	1284.816	182.574

Table A.3: Behavior of $TC_1\left(Q_1^*, G_1^*\right)$ for given β and varying values of α when $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$

Instances with $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$, and $C = 840$											
$\beta = 0.001$		$\beta = 0.005$		$\beta = 0.01$		$\beta = 0.015$		$\beta = 0.02$		$\beta = 0.025$	
α	$TC_1\left(Q_1^*, G_1^*\right)$	α	$TC_1\left(Q_1^*, G_1^*\right)$	α	$TC_1\left(Q_1^*, G_1^*\right)$	α	$TC_1\left(Q_1^*, G_1^*\right)$	α	$TC_1\left(Q_1^*, G_1^*\right)$	α	$TC_1\left(Q_1^*, G_1^*\right)$
1.1	4787.638	2.4	4104.611	3.3	4329.701	4.1	4105.047	4.7	4164.155	5.2	4374.475
1.2	4167.901	2.5	3948.001	3.4	4026.946	4.2	3953.783	4.8	3988.236	5.3	4063.193
1.3	4034.937	2.6	3869.92	3.5	3908.663	4.3	3868.881	4.9	3893.591	5.4	3942.506
1.4	3962.513	2.7	3823.102	3.6	3839.646	4.4	3813.258	5	3831.44	5.5	3867.11
1.5	3913.912	2.8	3791.931	3.7	3794.725	4.5	3774.374	5.1	3787.395	5.6	3814.344
1.6	3877.9	2.9	3769.454	3.8	3763.789	4.6	3746.213	5.2	3754.937	5.7	3775.398
1.7	3849.639	3	3752.223	3.9	3741.571	4.7	3725.317	5.3	3730.467	5.8	3745.791
1.8	3826.612	3.1	3738.403	4	3724.966	4.8	3709.468	5.4	3711.735	5.9	3722.885
1.9	3807.346	3.2	3726.944	4.1	3712.063	4.9	3697.152	5.5	3697.204	6	3704.963
2	3790.905	3.3	3717.203	4.2	3701.679	5	3687.328	5.6	3685.766	6.1	3690.82
2.1	3776.655	3.4	3708.765	4.3	3693.068	5.1	3679.282	5.7	3676.598	6.2	3679.56
–	–	3.5	3701.346	4.4	3685.755	5.2	3672.533	5.8	3669.098	6.3	3670.494
–	–	3.6	3694.745	4.5	3679.423	5.3	3666.754	5.9	3662.833	6.4	3663.086
–	–	3.7	3688.814	4.6	3673.856	5.4	3661.719	6	3657.497	6.5	3656.93
–	–	3.8	3683.441	4.7	3668.9	5.5	3657.271	6.1	3652.872	6.6	3651.723
–	–	3.9	3678.542	4.8	3664.442	5.6	3653.294	6.2	3648.804	6.7	3647.244
–	–	4	3674.048	4.9	3660.398	5.7	3649.704	6.3	3645.181	6.8	3643.33
–	–	4.1	3669.904	5	3656.704	5.8	3646.436	6.4	3641.921	6.9	3639.867
–	–	4.2	3666.067	5.1	3653.307	5.9	3643.441	6.5	3638.962	7	3636.766

Table A.3 – continued from previous page

$\beta = 0.001$		$\beta = 0.005$		$\beta = 0.01$		$\beta = 0.015$		$\beta = 0.02$		$\beta = 0.025$	
α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$
–	–	4.3	3662.498	5.2	3650.169	6	3640.68	6.6	3636.255	7.1	3633.964
–	–	4.4	3659.169	5.3	3647.255	6.1	3638.122	6.7	3633.764	7.2	3631.411
–	–	4.5	3656.053	5.4	3644.539	6.2	3635.74	6.8	3631.458	7.3	3629.068
–	–	4.6	3653.129	5.5	3641.998	6.3	3633.515	6.9	3629.314	7.4	3626.905
–	–	4.7	3650.376	5.6	3639.614	6.4	3631.428	7	3627.312	7.5	3624.898
–	–	–	–	5.7	3637.37	6.5	3629.465	7.1	3625.435	7.6	3623.027
–	–	–	–	5.8	3635.253	6.6	3627.613	7.2	3623.671	7.7	3621.276
–	–	–	–	5.9	3633.25	6.7	3625.862	7.3	3622.006	7.8	3619.632
–	–	–	–	6	3631.352	6.8	3624.202	7.4	3620.433	7.9	3618.082
–	–	–	–	6.1	3629.55	6.9	3622.626	7.5	3618.941	8	3616.618
–	–	–	–	6.2	3627.835	7	3621.125	7.6	3617.524	8.1	3615.232
–	–	–	–	6.3	3626.201	7.1	3619.695	7.7	3616.175	8.2	3613.915
–	–	–	–	6.4	3624.641	7.2	3618.33	7.8	3614.889	8.3	3612.663
–	–	–	–	6.5	3623.151	7.3	3617.024	7.9	3613.661	8.4	3611.469
–	–	–	–	6.6	3621.724	7.4	3615.774	8	3612.486	8.5	3610.329
–	–	–	–	–	–	7.5	3614.575	8.1	3611.36	8.6	3609.239
–	–	–	–	–	–	7.6	3613.424	8.2	3610.28	8.7	3608.194
–	–	–	–	–	–	7.7	3612.318	8.3	3609.244	8.8	3607.193
–	–	–	–	–	–	7.8	3611.254	8.4	3608.246	8.9	3606.231
–	–	–	–	–	–	7.9	3610.229	8.5	3607.287	9	3605.306
–	–	–	–	–	–	8	3609.241	8.6	3606.362	9.1	3604.415
–	–	–	–	–	–	8.1	3608.288	8.7	3605.47	9.2	3603.557

Table A.3 – continued from previous page

$\beta = 0.001$		$\beta = 0.005$		$\beta = 0.01$		$\beta = 0.015$		$\beta = 0.02$		$\beta = 0.025$	
α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$
–	–	–	–	–	–	–	–	8.8	3604.609	9.3	3602.729
–	–	–	–	–	–	–	–	8.9	3603.777	9.4	3601.93
–	–	–	–	–	–	–	–	9	3602.973	9.5	3601.158
–	–	–	–	–	–	–	–	9.1	3602.195	9.6	3600.411
–	–	–	–	–	–	–	–	9.2	3601.441	9.7	3599.688
–	–	–	–	–	–	–	–	9.3	3600.711	9.8	3598.988
–	–	–	–	–	–	–	–	9.4	3600.003	9.9	3598.31
–	–	–	–	–	–	–	–	–	–	10	3597.652
–	–	–	–	–	–	–	–	–	–	10.1	3597.013
–	–	–	–	–	–	–	–	–	–	10.2	3596.393
–	–	–	–	–	–	–	–	–	–	10.3	3595.79
–	–	–	–	–	–	–	–	–	–	10.4	3595.204
–	–	–	–	–	–	–	–	–	–	10.5	3594.634

Table A.4: Behavior of $TC_1(Q_1^*, G_1^*)$ for given β and varying values of α when $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$

Instances with $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, and $C = 1700$											
$\beta = 0.001$		$\beta = 0.005$		$\beta = 0.01$		$\beta = 0.015$		$\beta = 0.02$		$\beta = 0.025$	
α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$
0.9	3625.474	2	3429.838	2.8	3392.092	3.5	3349.68	4	3343.221	4.5	3329.756
1	3525.276	2.1	3395.96	2.9	3362.977	3.6	3336.633	4.1	3329.618	4.6	3319.507
1.1	3479.833	2.2	3377.754	3	3349.132	3.7	3327.768	4.2	3321.001	4.7	3312.4
1.2	3449.781	2.3	3364.757	3.1	3339.326	3.8	3320.899	4.3	3314.472	4.8	3306.836
1.3	3427.518	2.4	3354.572	3.2	3331.622	3.9	3315.249	4.4	3309.156	4.9	3302.223
1.4	3410.012	2.5	3346.187	3.3	3325.247	4	3310.434	4.5	3304.652	5	3298.268
1.5	3395.714	2.6	3339.067	3.4	3319.799	4.1	3306.233	4.6	3300.736	5.1	3294.8
1.6	3383.723	2.7	3332.888	3.5	3315.042	4.2	3302.506	4.7	3297.268	5.2	3291.709
1.7	3373.464	2.8	3327.44	3.6	3310.821	4.3	3299.156	4.8	3294.156	5.3	3288.921
1.8	3364.551	2.9	3322.577	3.7	3307.03	4.4	3296.116	4.9	3291.333	5.4	3286.382
1.9	3356.709	3	3318.192	3.8	3303.592	4.5	3293.334	5	3288.752	5.5	3284.05
2	3349.739	3.1	3314.206	3.9	3300.45	4.6	3290.771	5.1	3286.373	5.6	3281.896
2.1	3343.489	3.2	3310.559	4	3297.559	4.7	3288.397	5.2	3284.17	5.7	3279.895
2.2	3337.844	3.3	3307.201	4.1	3294.885	4.8	3286.187	5.3	3282.119	5.8	3278.027
2.3	3332.712	3.4	3304.095	4.2	3292.398	4.9	3284.121	5.4	3280.201	5.9	3276.277
2.4	3328.02	3.5	3301.208	4.3	3290.077	5	3282.183	5.5	3278.402	6	3274.632
2.5	3323.709	3.6	3298.516	4.4	3287.903	5.1	3280.359	5.6	3276.707	6.1	3273.079
2.6	3319.731	3.7	3295.996	4.5	3285.86	5.2	3278.638	5.7	3275.107	6.2	3271.611
2.7	3316.045	3.8	3293.63	4.6	3283.933	5.3	3277.008	5.8	3273.592	6.3	3270.218

Table A.4 – continued from previous page

$\beta = 0.001$		$\beta = 0.005$		$\beta = 0.01$		$\beta = 0.015$		$\beta = 0.02$		$\beta = 0.025$	
α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$
–	–	3.9	3291.403	4.7	3282.113	5.4	3275.463	5.9	3272.154	6.4	3268.895
–	–	4	3289.301	4.8	3280.388	5.5	3273.994	6	3270.786	6.5	3267.635
–	–	4.1	3287.312	4.9	3278.751	5.6	3272.595	6.1	3269.483	6.6	3266.432
–	–	4.2	3285.427	5	3277.194	5.7	3271.26	6.2	3268.24	6.7	3265.283
–	–	4.3	3283.636	5.1	3275.71	5.8	3269.985	6.3	3267.051	6.8	3264.183
–	–	4.4	3281.932	5.2	3274.294	5.9	3268.764	6.4	3265.912	6.9	3263.129
–	–	4.5	3280.308	5.3	3272.94	6	3267.594	6.5	3264.82	7	3262.117
–	–	4.6	3278.758	5.4	3271.644	6.1	3266.471	6.6	3263.772	7.1	3261.144
–	–	4.7	3277.277	5.5	3270.402	6.2	3265.393	6.7	3262.764	7.2	3260.208
–	–	4.8	3275.859	5.6	3269.209	6.3	3264.355	6.8	3261.794	7.3	3259.307
–	–	4.9	3274.5	5.7	3268.064	6.4	3263.356	6.9	3260.86	7.4	3258.438
–	–	5	3273.196	5.8	3266.962	6.5	3262.393	7	3259.959	7.5	3257.599
–	–	5.1	3271.943	5.9	3265.901	6.6	3261.464	7.1	3259.089	7.6	3256.788
–	–	5.2	3270.739	6	3264.879	6.7	3260.567	7.2	3258.249	7.7	3256.005
–	–	5.3	3269.579	6.1	3263.892	6.8	3259.7	7.3	3257.437	7.8	3255.247
–	–	5.4	3268.462	6.2	3262.94	6.9	3258.862	7.4	3256.651	7.9	3254.513
–	–	5.5	3267.386	6.3	3262.02	7	3258.05	7.5	3255.89	8	3253.801
–	–	5.6	3266.346	6.4	3261.13	7.1	3257.264	7.6	3255.152	8.1	3253.111
–	–	5.7	3265.343	6.5	3260.269	7.2	3256.502	7.7	3254.437	8.2	3252.442
–	–	5.8	3264.372	6.6	3259.435	7.3	3255.763	7.8	3253.743	8.3	3251.792
–	–	5.9	3263.434	6.7	3258.626	7.4	3255.046	7.9	3253.069	8.4	3251.161
–	–	6	3262.526	6.8	3257.843	7.5	3254.35	8	3252.414	8.5	3250.547

Table A.4 – continued from previous page

$\beta = 0.001$		$\beta = 0.005$		$\beta = 0.01$		$\beta = 0.015$		$\beta = 0.02$		$\beta = 0.025$	
α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$
–	–	6.1	3261.646	6.9	3257.082	7.6	3253.673	8.1	3251.778	8.6	3249.95
–	–	–	–	7	3256.344	7.7	3253.015	8.2	3251.159	8.7	3249.369
–	–	–	–	7.1	3255.627	7.8	3252.375	8.3	3250.557	8.8	3248.803
–	–	–	–	7.2	3254.93	7.9	3251.753	8.4	3249.97	8.9	3248.252
–	–	–	–	7.3	3254.253	8	3251.147	8.5	3249.399	9	3247.716
–	–	–	–	7.4	3253.593	8.1	3250.556	8.6	3248.843	9.1	3247.192
–	–	–	–	7.5	3252.952	8.2	3249.981	8.7	3248.301	9.2	3246.682
–	–	–	–	7.6	3252.327	8.3	3249.42	8.8	3247.772	9.3	3246.184
–	–	–	–	7.7	3251.718	8.4	3248.873	8.9	3247.256	9.4	3245.697
–	–	–	–	7.8	3251.125	8.5	3248.34	9	3246.752	9.5	3245.223
–	–	–	–	7.9	3250.547	8.6	3247.819	9.1	3246.26	9.6	3244.759
–	–	–	–	8	3249.982	8.7	3247.31	9.2	3245.78	9.7	3244.306
–	–	–	–	8.1	3249.432	8.8	3246.813	9.3	3245.31	9.8	3243.863
–	–	–	–	8.2	3248.895	8.9	3246.328	9.4	3244.851	9.9	3243.43
–	–	–	–	8.3	3248.37	9	3245.854	9.5	3244.403	10	3243.006
–	–	–	–	8.4	3247.857	9.1	3245.39	9.6	3243.964	10.1	3242.591
–	–	–	–	8.5	3247.356	9.2	3244.936	9.7	3243.535	10.2	3242.185
–	–	–	–	8.6	3246.867	9.3	3244.492	9.8	3243.115	10.3	3241.788
–	–	–	–	8.7	3246.388	9.4	3244.058	9.9	3242.703	10.4	3241.399
–	–	–	–	–	–	9.5	3243.633	10	3242.301	10.5	3241.018
–	–	–	–	–	–	9.6	3243.217	10.1	3241.906	10.6	3240.645
–	–	–	–	–	–	9.7	3242.809	10.2	3241.52	10.7	3240.279

Table A.4 – continued from previous page

$\beta = 0.001$		$\beta = 0.005$		$\beta = 0.01$		$\beta = 0.015$		$\beta = 0.02$		$\beta = 0.025$	
α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$
–	–	–	–	–	–	9.8	3242.41	10.3	3241.141	10.8	3239.92
–	–	–	–	–	–	9.9	3242.018	10.4	3240.77	10.9	3239.568
–	–	–	–	–	–	10	3241.635	10.5	3240.406	11	3239.223
–	–	–	–	–	–	10.1	3241.259	10.6	3240.049	11.1	3238.884
–	–	–	–	–	–	10.2	3240.89	10.7	3239.699	11.2	3238.552
–	–	–	–	–	–	10.3	3240.528	10.8	3239.355	11.3	3238.226
–	–	–	–	–	–	10.4	3240.173	10.9	3239.018	11.4	3237.906
–	–	–	–	–	–	10.5	3239.825	11	3238.687	11.5	3237.592
–	–	–	–	–	–	10.6	3239.483	11.1	3238.362	11.6	3237.283
–	–	–	–	–	–	–	–	11.2	3238.043	11.7	3236.98
–	–	–	–	–	–	–	–	11.3	3237.73	11.8	3236.682
–	–	–	–	–	–	–	–	11.4	3237.422	11.9	3236.389
–	–	–	–	–	–	–	–	11.5	3237.12	12	3236.102
–	–	–	–	–	–	–	–	11.6	3236.823	12.1	3235.819
–	–	–	–	–	–	–	–	11.7	3236.53	12.2	3235.541
–	–	–	–	–	–	–	–	11.8	3236.243	12.3	3235.267
–	–	–	–	–	–	–	–	11.9	3235.961	12.4	3234.999
–	–	–	–	–	–	–	–	12	3235.683	12.5	3234.734
–	–	–	–	–	–	–	–	12.1	3235.41	12.6	3234.474
–	–	–	–	–	–	–	–	12.2	3235.141	12.7	3234.218
–	–	–	–	–	–	–	–	12.3	3234.877	12.8	3233.966
–	–	–	–	–	–	–	–	–	–	12.9	3233.718

Table A.4 – continued from previous page

$\beta = 0.001$		$\beta = 0.005$		$\beta = 0.01$		$\beta = 0.015$		$\beta = 0.02$		$\beta = 0.025$	
α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$	α	$TC_1(Q_1^*, G_1^*)$
–	–	–	–	–	–	–	–	–	–	13	3233.473
–	–	–	–	–	–	–	–	–	–	13.1	3233.233
–	–	–	–	–	–	–	–	–	–	13.2	3232.996
–	–	–	–	–	–	–	–	–	–	13.3	3232.763
–	–	–	–	–	–	–	–	–	–	13.4	3232.533
–	–	–	–	–	–	–	–	–	–	13.5	3232.307
–	–	–	–	–	–	–	–	–	–	13.6	3232.084
–	–	–	–	–	–	–	–	–	–	13.7	3231.865