Joint Design of Hybrid Beamforming and Reflection Coefficients in RIS-aided mmWave MIMO Systems

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Abstract—This paper considers a reconfigurable intelligent surface (RIS)-aided millimeter wave (mmWave) downlink communication system where hybrid analog-digital beamforming is employed at the base station (BS). We formulate a power minimization problem by jointly optimizing hybrid beamforming at the BS and the response matrix at the RIS, under the signalto-interference-plus-noise ratio (SINR) constraints at all users. The problem is highly challenging to solve due to the nonconvex SINR constraints as well as the unit-modulus phase shift constraints for both the RIS reflection coefficients and the analog beamformer. A two-layer penalty-based algorithm is proposed to decouple variables in SINR constraints, and manifold optimization is adopted to handle the non-convex unitmodulus constraints. We also propose a low-complexity sequential optimization method, which optimizes the RIS reflection coefficients, the analog beamformer, and the digital beamformer sequentially without iteration. Furthermore, the relationship between the power minimization problem and the max-min fairness (MMF) problem is discussed. Simulation results show that the proposed penalty-based algorithm outperforms the stateof-the-art semidefinite relaxation (SDR)-based algorithm. Results also demonstrate that the RIS plays an important role in the power reduction.

Index Terms—Reconfigurable Intelligent Surface (RIS), mmWave, hybrid beamforming, sub-connected structure, manifold optimization.

I. INTRODUCTION

The millimeter wave (mmWave) communication over 30-300 GHz spectrum is a key technology in 5G and beyond wireless networks to provide high data-rate transmission [2]– [4]. Compared with sub-6 GHz, the high directivity at high frequency bands makes mmWave communication much more sensitive to signal blockage. One promising and cost-effective solution to overcome the blockage issue is to deploy Reconfigurable Intelligent Surfaces (RISs). An RIS is an artificial meta-surface consisting of a large number of passive reflection

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elements that can be programmed to electronically control the phase of the incident electromagnetic waves [5], [6]. With the help of a smart controller, RISs can be controlled to enhance the desirable signals via coherent combining, or to suppress the undesirable interference via destructive combining. RISs are spectrum- and energy-efficient since they do not require radio frequency (RF) components or dedicated energy supply. Furthermore, from the implementation perspective, RISs have appealing advantages such as low profile, light-weight, and conformal geometry. Recently, RISs have emerged as a promising technique to enhance the performance of wireless communication systems, especially in mmWave bands [7]–[9].

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As RISs bring a new degree-of-freedom to the optimization of beamforming design, a key issue of interest in RIS-aided wireless communication systems is to jointly design the active beamforming at the multi-antenna base stations (BSs) and the passive reflection coefficients at the RIS. There have been several prior studies investigating this problem under different system setups and assumptions [10]-[15]. Specifically, the work [10] studies the power minimization problem under the signal-to-interference-plus-noise ratio (SINR) constraints and proposes a semidefinite relaxation (SDR) based algorithm for the joint active and passive beamforming design. The work [11] extends [10] to the scenario with multiple RISs and a near-optimal analytical solution is derived. The work [12] aims to maximize the minimum weighted SINR at the users and proposes a low-complexity inexact-alternatingoptimization approach. The work [13] focuses on the energy efficiency problem under individual quality-of-service (QoS) requirements as well as maximum power constraints. Under the maximum transmit power constraints, the work [14] aims to maximize the minimum SINR, and the work [15] aims to maximize the weighted-sum-rate (WSR) of all users. Moreover, RISs have also been studied under other communication setups, such as secure communication [16], [17], unmanned aerial vehicle (UAV) communication [18], [19], and simultaneous wireless information and power transfer (SWIPT) systems [20], [21]. Note that in all these works on joint active-passive beamforming design, the active beamforming at the BS is fully digital as in most of the multiple-input-multiple-output (MIMO) beamforming literature, which requires each antenna to be connected to one RF chain, and hence has a high hardware cost.

Unlike the fully digital beamforming structure, hybrid analog and digital (A/D) beamforming at the BS is more practical in mmWave systems since it employs a reduced number of RF chains [22], [23]. It is therefore desirable to consider hybrid beamforming in RIS-aided mmWave communications as a cost-effective alternative. There are very few works along this line of research except [24]–[26]. In specific, the work [24] considers the individual design of the digital beamformer, the analog beamformer, and the RIS phase shifts to achieve low error rate in a wideband system. The work [25] investigates the WSR maximization in a nonorthogonal multiple access (NOMA) system by jointly designing the power allocation, the RIS phase shifts and the hybrid beamforming vector. Therein, the manifold optimization method is adopted for the design of the phase shifts at both the RIS and the analog beamformer, while the digital beamforming is obtained by the successive convex approximation (SCA) based algorithm. The work [26] focuses on maximizing the spectral efficiency in a singleuser mmWave MIMO system by jointly optimizing the RIS reflection coefficients and the hybrid beamforming vector at the BS. The manifold optimization is adopted to handle the RIS reflection coefficients, and then the digital beamforming is obtained through the singular value decomposition (SVD) of the cascaded channel.

In this work, we consider an RIS-aided multi-user downlink mmWave system, and investigate the joint design of hybrid beamforming at the BS and reflection coefficients at the RIS. Unlike the previous works [24]–[26] which all employ the fully-connected hybrid architecture at the BS with each RF chain connected to all antenna elements, we employ the sub-connected hybrid architecture with each RF chain only connected to a disjoint subset of antenna elements. The sub-connected architecture is more appealing for its further reduced hardware cost and power consumption.

The main contributions and results of this paper are listed as follows.

- We first formulate the so-called QoS problem for minimizing the total transmit power at the BS subject to individual SINR constraints at all users. The problem is highly non-convex due to the deeply coupled variables and the unit-modulus phase shifts constraints. To tackle this problem, we propose a two-layer penalty-based algorithm where the block coordinate descent (BCD) method is adopted in the inner layer to solve a penalized problem and the penalty factor is updated in the outer layer until convergence. The penalty method can decouple the optimization variables and make the problem much easier to handle. In the BCD method, considering the same unitmodulus constraints on both the BS analog beamformer and the RIS response matrix, they can be updated simultaneously by using the manifold optimization method.
- In order to reduce the complexity, we propose a sequential optimization method where the RIS coefficients are obtained by maximizing the channel gain of the user with the worst channel state; the analog beamforming is obtained by minimizing the Euclidean distance between the fully digital beamforming and the hybrid beamforming; and the digital beamforming is optimally obtained by the second-order cone program (SOCP) method.
- We discuss a closely related problem of the QoS problem, which is the max-min fairness (MMF) problem. The MMF problem is more difficult to solve than the QoS problem due to its non-smooth objective function.

However, we can solve the MMF problem by solving a series of QoS problems.

Finally, we conduct comprehensive simulations to validate the performance of the proposed algorithms. It is shown that the proposed penalty-based algorithm outperforms the traditional SDR-based optimization algorithm. Results also demonstrate that the proposed hybrid beamforming at the BS can perform closely to a fully digital beamforming system. In addition, the transmit power at the BS can be greatly reduced by employing a large number of RIS elements on the BS side or the user side. Furthermore, it is sufficient for practical use when both the RIS and the analog beamformer have 3-bit quantizers.

The rest of the paper is organized as follows. Section II introduces the RIS-aided mmWave MIMO system model, and formulates the power minimization problem. A twolayer penalty-based algorithm is proposed to solve the power minimization problem in Section III. A low-complexity sequential optimization method is proposed in Section IV. The relationship between the QoS problem and the MMF problem is studied in Section V. Simulation results are provided in Section VI. Finally, Section VII concludes this paper.

Notations: The imaginary unit is denoted by $j = \sqrt{-1}$. Vectors and matrices are denoted by bold-face lower-case and upper-case letters, respectively. The conjugate, transpose, conjugate transpose and pseudo-inverse of the vector x are denoted by $\mathbf{x}^*, \mathbf{x}^T, \mathbf{x}^H$ and \mathbf{x}^{\dagger} , respectively. Further, we use I and O to denote an identity matrix and all-zero matrix of appropriate dimensions, respectively; we use $\mathbb{C}^{x \times y}$ to denote the space of $x \times y$ complex-valued matrices. The notations $\arg(\cdot)$ and $\operatorname{Re}(\cdot)$ denote the argument and real part of a complex number, respectively. The notations $\mathbb{E}(\cdot)$ and $\operatorname{Tr}(\cdot)$ denote the expectation and trace operation, respectively; \odot represents the Hadamard product; $\|\cdot\|$ represents the Frobenius norm. For a vector \mathbf{x} , diag(\mathbf{x}) denotes a diagonal matrix with each diagonal element being the corresponding element in x. For a vector x, $\nabla f(\mathbf{x}_i)$ denotes the gradient vector of function $f(\mathbf{x})$ at the point \mathbf{x}_i . Finally, The distribution of a circularly symmetric complex Gaussian (CSCG) random vector with mean vector x and covariance matrix Σ is denoted by $\mathcal{CN}(x,\Sigma)$; and ~ stands for "distributed as".

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

As shown in Fig. 1, we consider an RIS-aided downlink mmWave communication system where one BS, equipped with M antennas, communicates with K single-antenna users via the help of one RIS equipped with F unit cells. The BS employs the sub-connected hybrid A/D beamforming structure with N RF chains, each connected to a disjoint subset of D = M/N antennas. Let s_k denote the information signal intended to user k, for $k \in \mathcal{K} \triangleq \{1, 2, \ldots, K\}$. The signals are assumed to be independent of each other and satisfy $\mathbb{E}(|s_k|^2) = 1$. Each of these signals is first weighted by a digital beamforming vector, denoted as $\mathbf{w}_k \in \mathbb{C}^{N \times 1}$. These weighted signal vectors are summed together and each entry is sent to an RF chain, then multiplied by an analog beamforming

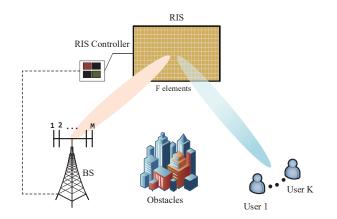


Fig. 1. An RIS-aided downlink mmWave communication system.

vector, denoted as $\mathbf{v}_n \in \mathbb{C}^{D \times 1}$, for $n \in \mathcal{N} \triangleq \{1, 2, \dots, N\}$. Each entry of \mathbf{v}_n , denoted as $v_{n,d}, \forall d \in \mathcal{D} \triangleq \{1, 2, \dots, D\}$ is a phase shifter. Discrete phase shifts are considered. Denote S_a as the set of all possible phase shifts for the analog beamformer, given by

$$\mathcal{S}_a \triangleq \left\{ e^{j\theta} \mid \theta \in \left\{ 0, \frac{2\pi}{2^{Q_1}}, \dots, \frac{2\pi(2^{Q_1}-1)}{2^{Q_1}} \right\} \right\}, \quad (1)$$

where Q_1 is the number of control bits for each analog phase shifter. In the special case when $Q_1 = \infty$, it becomes continuous phase shifts. The overall analog beamforming matrix can be represented as

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{v}_2 & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{v}_N \end{bmatrix}.$$
 (2)

The total transmit power of the BS is given by

$$P_{\text{transmit}} = \sum_{k=1}^{K} \|\mathbf{V}\mathbf{w}_k\|^2 = D \sum_{k=1}^{K} \|\mathbf{w}_k\|^2.$$
 (3)

The RIS is connected to the BS through an RIS control link for transmission and information exchange. Let $\mathcal{F} \triangleq \{1, 2, \ldots, F\}$ denote the set of all RIS unit cells, and define the response matrix at the RIS as

$$\Theta = \operatorname{diag}(b_1, b_2, \dots, b_F), \tag{4}$$

where $b_f = \beta_f e^{j\theta_f}$, $\beta_f \in [0, 1]$ and $\theta_f \in [0, 2\pi)$ are the amplitude reflection coefficient and the phase shift of the *f*-th unit cell, respectively. In this paper, we assume $\beta_f = 1, \forall f \in \mathcal{F}$ to maximize the signal reflection. Denote S_r as the set of all possible phase shifts for the RIS reflection coefficients, given by

$$\mathcal{S}_r \triangleq \left\{ e^{j\theta} \mid \theta \in \left\{ 0, \frac{2\pi}{2^{Q_2}}, \dots, \frac{2\pi(2^{Q_2}-1)}{2^{Q_2}} \right\} \right\}, \quad (5)$$

where Q_2 is the number of control bits for each RIS element. Again the special case of $Q_2 = \infty$ corresponds to the continuous phase shifts.

We assume that the BS-user direct link is blocked, and thus the direct path can be ignored. The signal power reflected two or more times is much lower than that reflected just once due to the high free-space path loss. Thus, we ignore the power of the signals that are reflected by the RIS more than once. In addition, we assume that the channel state information (CSI) of all links involved is perfectly known at the BS and all the channels experience quasi-static flat-fading. How to obtain accurate CSI is an important and challenging issue in the RIS-aided communication system. The CSI can be obtained through uplink pilots due to the channel reciprocity and some early attempts can be found in [27]–[31]. Suppose that $\mathbf{G} \in \mathbb{C}^{F \times M}$ is the channel matrix from the BS to the RIS, $\mathbf{h}_k^H \in \mathbb{C}^{1 \times F}$ is the channel vector from the RIS to user k. Then the received signal of user k can be represented as

$$y_k = \mathbf{h}_k^H \mathbf{\Theta} \mathbf{G} \mathbf{V} \sum_{j=1}^K \mathbf{w}_j s_j + n_k, \forall k \in \mathcal{K},$$
(6)

where $n_k \sim C\mathcal{N}(0, \sigma_k^2)$ is the additive white Gaussian noise at the receiver of user k with zero mean and variance σ_k^2 . The received SINR of user k can be expressed as

$$\operatorname{SINR}_{k} = \frac{|\mathbf{h}_{k}^{H} \Theta \mathbf{G} \mathbf{V} \mathbf{w}_{k}|^{2}}{\sum_{j \neq k} |\mathbf{h}_{k}^{H} \Theta \mathbf{G} \mathbf{V} \mathbf{w}_{j}|^{2} + \sigma_{k}^{2}}, \forall k \in \mathcal{K}.$$
 (7)

B. mmWave Channel Model

We adopt the widely used narrowband clustered channel model [32] for mmWave communications. Specifically, the channel matrix between the BS and the RIS can be written as

$$\mathbf{G} = \sqrt{\frac{MF}{N_{cl_1}N_{ray_1}}} \sum_{i=1}^{N_{cl_1}} \sum_{l=1}^{N_{ray_1}} \alpha_{il} \mathbf{a}_R(\phi_{il}^{Rr}, \delta_{il}^{Rr}) \mathbf{a}_B(\phi_{il}^B, \delta_{il}^B)^H.$$
(8)

Here, N_{cl_1} denotes the number of scattering clusters, N_{ray_1} denotes the number of rays in each cluster, and α_{il} denotes the channel coefficient of the *l*-th ray in the *i*-th propagation cluster. Moreover, $\mathbf{a}_R(\phi_{il}^{Rr}, \delta_{il}^{Rr})$ and $\mathbf{a}_B(\phi_{il}^B, \delta_{il}^B)$ represent the receive array response vectors of the RIS and the transmit array response vectors of the BS respectively, where $\phi_{il}^{Rr}(\phi_{il}^B)$ and $\delta_{il}^{Rr}(\delta_{il}^B)$ represent azimuth and elevation angles of arrival at the RIS (or departing from the BS). The channel vector between the RIS and the *k*-th user can be represented as

$$\mathbf{h}_{k} = \sqrt{\frac{F}{N_{\text{cl}2}N_{\text{ray}2}}} \sum_{i=1}^{N_{\text{cl}2}} \sum_{l=1}^{N_{\text{ray}2}} \beta_{il} \mathbf{a}_{R}(\phi_{il}^{Rt}, \delta_{il}^{Rt}).$$
(9)

Here, N_{cl2} , N_{ray_2} , β_{il} , ϕ_{il}^{Rt} and δ_{il}^{Rt} are defined in the same way as above.

In this paper, we consider the uniform planar array (UPA) structure at both BS and RIS. Consequently, the array response vector can be denoted as

$$\mathbf{a}_{z}(\phi,\delta) = \frac{1}{\sqrt{A_{1}A_{2}}} \left[1, \dots, e^{j\frac{2\pi}{\lambda}d_{1}(o\sin\phi\sin\delta+p\cos\delta)} \\ \dots, e^{j\frac{2\pi}{\lambda}d_{1}((A_{1}-1)\sin\phi\sin\delta)+(A_{2}-1)\cos\delta)} \right]^{T},$$
(10)

where $z \in \{R, B\}$, λ is the signal wavelength, d is the antenna or unit cell spacing which is assumed to be half wavelength

distance, $0 \le o < A_1$ and $0 \le p < A_2$, A_1 and A_2 represent the number of rows and columns of the UPA in the 2D plane, respectively.

C. Problem Formulation

We consider the QoS problem which aims to minimize the transmit power at the BS by jointly optimizing the digital beamforming matrix $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_K] \in \mathbb{C}^{N \times K}$ and the analog beamforming matrix \mathbf{V} at the BS, as well as the overall response matrix $\boldsymbol{\Theta}$ at the RIS, subject to QoS constraints for all users. The problem can be formulated as

$$\mathcal{P}_0: \quad \min_{\{\mathbf{V}, \mathbf{W}, \mathbf{\Theta}\}} \quad D\sum_{k=1}^K \|\mathbf{w}_k\|^2$$
(11a)

s.t.
$$SINR_k \ge \gamma_k, \forall k \in \mathcal{K},$$
 (11b)

$$v_{n,d} \in \mathcal{S}_a, \forall n \in \mathcal{N}, \forall d \in \mathcal{D},$$
 (11c)

$$b_f \in \mathcal{S}_r, \forall f \in \mathcal{F},$$
 (11d)

where $\gamma_k > 0$ is the minimum SINR requirement of user k.

The problem \mathcal{P}_0 is highly non-convex due to the nonconvex SINR constraints (11b) and the unit-modulus phase shifts constraints (11c), (11d), and thus difficult to be optimally solved. A commonly used approach to solve such problem approximately is to apply the BCD technique in conjunction with the SDR method as in [10], [33]. The BCD technique updates just one block of variables while fixing all the others at a time. In particular, at each iteration, the digital beamforming matrix can be solved via SOCP, while both the analog beamforming matrix and the RIS response matrix can be solved via SDR. Note that SDR cannot guarantee the feasibility due to the rank-one constraint and thus an additional randomization procedure is generally needed. Its complexity is high for the large RIS size. In addition, when the number of users is close to the number of RF chains, the above approach may become invalid because the randomization procedure may fail to find a feasible solution even after a large number of randomization. In this work, we propose a two-layer penalty-based algorithm to solve the problem \mathcal{P}_0 as detailed in the next section.

III. PENALTY-BASED JOINT OPTIMIZATION ALGORITHM

In this section, we propose a two-layer penalty-based method by exploiting the penalty method, where the BCD method is adopted in the inner layer to solve a penalized problem and the penalty factor is updated in the outer layer until convergence. Specifically, we firstly introduce auxiliary variables $\{t_{k,j}\}$ to represent $\mathbf{h}_k^H \Theta \mathbf{GV} \mathbf{w}_j$ such that the variables \mathbf{W}, \mathbf{V} and Θ can be decoupled. Then, the non-convex constraints (11b) can be equivalently written as

$$\frac{\left|t_{k,k}\right|^{2}}{\sum_{i,j\neq k}^{K}\left|t_{k,j}\right|^{2}+\sigma_{k}^{2}} \ge \gamma_{k}, \forall k \in \mathcal{K},$$
(12a)

$$t_{k,j} = \mathbf{h}_k^H \boldsymbol{\Theta} \mathbf{G} \mathbf{V} \mathbf{w}_j, \forall k, j \in \mathcal{K}.$$
(12b)

Then, the equality constraints (12b) can be relaxed and added to the objective function as a penalty term. Thereby,

the original problem \mathcal{P}_0 can be converted to the following penalized problem

$$\mathcal{P}_{1}(\rho) : \min_{\mathbf{v}, \mathbf{w}, \Theta, \{t_{k,j}\}} \quad D \sum_{k=1}^{K} \|\mathbf{w}_{k}\|^{2} \\ + \frac{\rho}{2} \sum_{j=1}^{K} \sum_{k=1}^{K} \left|\mathbf{h}_{k}^{H} \Theta \mathbf{G} \mathbf{V} \mathbf{w}_{j} - t_{k,j}\right|^{2}$$
(13a)
s.t. (12a), (11c), (11d), (13b)

where $\rho > 0$ is the penalty factor. Generally, the choice of ρ is crucial to balance the original objective function and the equality constraints. It is seen that the objective function in $\mathcal{P}_1(\rho)$ is dominated by the penalty term when ρ is large enough and consequently the equality constraints (12b) can be well met by the solution. Therefore, we can start with a small value of ρ to get a good starting point, and then by gradually increasing ρ , a high precision solution can be obtained.

There are mainly two different methods to handle the discrete phase shifts. First, the optimal solution can be found by the exhaustive search method [34]. However, its complexity is too high to be practical. The second method is to relax the discrete phases to continuous ones and then apply projection [11], [35]. As such, in the rest of the paper, we adopt the projection method. Specifically, we first relax the discrete phase shifts of analog beamforming and RIS coefficients to continuous ones, then solve the relaxed problem with the proposed algorithms, finally project the obtained continuous solution back to the discrete set.

A. Inner Layer: BCD Algorithm for Solving Problem $\mathcal{P}_1(\rho)$

For any given ρ , though the problem $\mathcal{P}_1(\rho)$ is still nonconvex, all the optimization variables $\{\mathbf{W}, \{\mathbf{\Theta}, \mathbf{V}\}, \{t_{k,j}\}\}\$ are decoupled in the constraints. We therefore adopt the BCD method to optimize each of them alternately.

1) Optimize W: When V, Θ and $\{t_{k,j}\}$ are fixed, problem $\mathcal{P}_1(\rho)$ becomes an unconstrained convex problem. Consequently, the optimal W can be obtained by the first-order optimality condition, i.e.,

$$\mathbf{w}_{k} = \rho \mathbf{A}_{1}^{-1} \sum_{j=1}^{K} \tilde{\mathbf{h}}_{j}^{H} t_{j,k}, \forall k \in \mathcal{K},$$
(14)

where $\tilde{\mathbf{h}}_j = \mathbf{h}_j^H \boldsymbol{\Theta} \mathbf{G} \mathbf{V}$ and $\mathbf{A}_1 = 2D\mathbf{I}_N + \rho \sum_{i=1}^K \tilde{\mathbf{h}}_j^H \tilde{\mathbf{h}}_j$.

2) Optimize $\{\Theta, \mathbf{V}\}$: Let $\mathbf{b} \triangleq [b_1, b_2, \dots, b_F]^H$, $\mathbf{x} \triangleq [\mathbf{v}_1^T, \mathbf{v}_2^T, \dots, \mathbf{v}_N^T]^T \in \mathbb{C}^{M \times 1}$, and $\mathbf{Y}_j \triangleq$ diag $\{w_{j,1}\mathbf{I}_D, \dots, w_{j,N}\mathbf{I}_D\} \in \mathbb{C}^{M \times M}$, where $|x_m| = 1, \forall m \in \mathcal{M} \triangleq \{1, 2, \dots, M\}$ and $w_{j,n}$ denotes the *n*-th entry of \mathbf{w}_j . Then, we can rewrite $\mathbf{V}\mathbf{w}_j = \mathbf{Y}_j\mathbf{x} \in \mathbb{C}^{M \times 1}$ so that the optimization problem is formulated in term of (\mathbf{b}, \mathbf{x}) . When the digital beamforming matrix \mathbf{W} and the auxiliary variables $\{t_{k,j}\}$ are fixed, the problem $\mathcal{P}_1(\rho)$ is reduced to (with constant terms ignored)

$$\min_{\mathbf{b},\mathbf{x}} \quad f(\mathbf{b},\mathbf{x}) = \sum_{j=1}^{K} \sum_{k=1}^{K} \left| \mathbf{b}^{H} \mathbf{c}_{k,j} \mathbf{x} - t_{k,j} \right|^{2}$$
(15a)

s.t.
$$|b(f)| = 1, \forall f \in \mathcal{F},$$
 (15b)
 $|x(m)| = 1, \forall m \in \mathcal{M},$ (15c)

where $\mathbf{c}_{k,j} = \text{diag}(\mathbf{h}_k^H)\mathbf{G}\mathbf{Y}_j \in \mathbb{C}^{F \times M}$. In the following, we would like to adopt three different methods to tackle the problem (15).

Method One: Alternating Optimization The first idea is to alternately optimize one of the variables **b** and **x** while keeping the other fixed. When **x** is fixed, the main obstacles of the problem (15) lie in the unit-modulus phase shifts constraints (15b). Note that they form a complex circle manifold $\mathcal{M} = \{\mathbf{b} \in \mathbb{C}^F : |b_1| = \cdots = |b_F| = 1\}$ [36]. Therefore, the problem (15) can be efficiently solved by the manifold optimization technique. In specific, we adopt the Riemannian conjugate gradient (RCG) algorithm. The RCG algorithm is widely applied in hybrid beamforming design [37] and recently applied in RIS-aided systems as well [38], [15]. In the following we briefly review the general procedure of the RCG algorithm.

Each iteration of the RCG algorithm involves four key steps, namely, to compute the Riemannian gradient, to do the transport, to find the search direction and to do the retraction.

Denote $f(\mathbf{b}) = \sum_{j=1}^{K} \sum_{k=1}^{K} |\mathbf{b}^{H}\mathbf{c}_{k,j}\mathbf{x} - t_{k,j}|^{2}$. For any given point \mathbf{b}_{i} , the Riemannian gradient grad $f(\mathbf{b}_{i})$ is defined as the orthogonal projection of the Euclidean gradient $\nabla f(\mathbf{b}_{i})$ onto the tangent space $T_{\mathbf{b}_{i}}\mathcal{M}$ of the manifold \mathcal{M} at point \mathbf{b}_{i} , which can be expressed as

$$T_{\mathbf{b}_i}\mathcal{M} = \left\{ \mathbf{b} \in \mathbb{C}^F : \operatorname{Re}\left\{ \mathbf{b} \odot \mathbf{b}_i^* \right\} = \mathbf{0}_F \right\}.$$
(16)

The Euclidean gradient at the point \mathbf{b}_i is given by

$$\nabla f(\mathbf{b}_i) = 2\sum_{j=1}^{K} \sum_{k=1}^{K} \mathbf{c}_{k,j} \mathbf{x} (\mathbf{x}^H \mathbf{c}_{k,j}^H \mathbf{b} - t_{k,j}^H).$$
(17)

Then, the Riemannian gradient at the point \mathbf{b}_i is given by

grad
$$f(\mathbf{b}_i) = \nabla f(\mathbf{b}_i) - \operatorname{Re} \{\nabla f(\mathbf{b}_i) \odot \mathbf{b}_i^*\} \odot \mathbf{b}_i.$$
 (18)

With the Riemannian gradient, the optimization technique in the Euclidean space can be extended to the manifold space. Here, we adopt the conjugate gradient method, where the search direction can be updated by

$$\boldsymbol{\eta}_{i+1} = -\operatorname{grad} f(\mathbf{b}_{i+1}) + \lambda_1 \mathcal{T}_{\mathbf{b}_i \to \mathbf{b}_{i+1}} \left(\boldsymbol{\eta}_i \right), \quad (19)$$

where η_i is the search direction at \mathbf{b}_i , λ_1 is the update parameter chosen as the Polak-Ribiere parameter [36], and $\mathcal{T}_{\mathbf{b}_i \to \mathbf{b}_{i+1}}(\eta_i)$ is the transport operation. Note that η_i and η_{i+1} lie in different tangent spaces and they cannot be conducted directly. Therefore, the transport operation $\mathcal{T}_{\mathbf{b}_i \to \mathbf{b}_{i+1}}(\eta_i)$ is needed to map the previous search direction from its original tangent space to the current tangent space at the current point \mathbf{b}_{i+1} . The transport operation is given by

$$\mathcal{T}_{\mathbf{b}_{i}\to\mathbf{b}_{i+1}}(\boldsymbol{\eta}_{i}):T_{\mathbf{b}_{i}}\mathcal{M}\mapsto T_{\mathbf{b}_{i+1}}\mathcal{M}:$$
$$\boldsymbol{\eta}_{i}\mapsto\boldsymbol{\eta}_{i}-\operatorname{Re}\left\{\boldsymbol{\eta}_{i}\odot\mathbf{b}_{i+1}^{*}\right\}\odot\mathbf{b}_{i+1}.$$
(20)

Algorithm 1 RCG Algorithm for solving problem (15) with fixed \mathbf{x}

Input: $\{\mathbf{c}_{k,j}\}, \mathbf{x}, \mathbf{b}_0 \in \mathcal{M}$

- 1: Calculate $\eta_0 = -\operatorname{grad} f(\mathbf{b}_0)$ according to (18) and set i = 0;
- 2: repeat
- 3: Choose the Armijo backtracking line search step size λ_2 ;
- 4: Find the next point \mathbf{b}_{i+1} using retraction according to (21);
- 5: Calculate the Riemannian gradient $\operatorname{grad} f(\mathbf{b}_{i+1})$ according to (18);
- 6: Calculate the transport $\mathcal{T}_{\mathbf{b}_i \to \mathbf{b}_{i+1}}(\boldsymbol{\eta}_i)$ according to (20);
- 7: Choose the Polak-Ribiere parameter λ_1 ;
- 8: Calculate the conjugate direction η_{i+1} according to (19);

9:
$$i \leftarrow i+1$$

1

10: **until** $\| \operatorname{grad} f(\mathbf{b}_i) \|_2 \leq \epsilon_1$.

Since the updated point may leave the previous manifold space, a retraction operation $\operatorname{Retr}_{\mathbf{b}}(\lambda_2 \eta_i)$ is needed to project the point back to the manifold:

$$\operatorname{Retr}_{\mathbf{b}_{i}}(\lambda_{2}\boldsymbol{\eta}_{i}): T_{\mathbf{b}_{i}}\mathcal{M} \mapsto \mathcal{M}:$$
$$\lambda_{2}\boldsymbol{\eta}_{i} \mapsto \frac{(\mathbf{b}_{i} + \lambda_{2}\boldsymbol{\eta}_{i})_{j}}{\left| (\mathbf{b}_{i} + \lambda_{2}\boldsymbol{\eta}_{i})_{j} \right|}, \qquad (21)$$

where λ_2 is the Armijo backtracking line search step size, and $(\mathbf{b}_i + \lambda_2 \boldsymbol{\eta}_i)_j$ denotes the *j*-th entry of $\mathbf{b}_i + \lambda_2 \boldsymbol{\eta}_i$.

The key steps are introduced above, and the consequent algorithm for solving the problem (15) with fixed x is summarized in *Algorithm* 1. *Algorithm* 1 is guaranteed to converge to a stationary point [36].

When b is fixed, x can be also updated similarly by the RCG algorithm.

Method Two: RCG-based Joint Optimization Note that both b and x of the problem (15) are subject to unit-modulus constraints. Thus we can concatenate them and treat as a higher-dimensional vector subject to the same unit-modulus constraints. Specifically, let $\mathbf{z} = [\mathbf{b}^{H}, \mathbf{x}^{H}]^{H} \in \mathbb{C}^{(F+M)\times 1}$, and we can rewrite the problem (15) as follows

$$\min_{\mathbf{z}} \quad f(\mathbf{z}) = \sum_{j=1}^{K} \sum_{k=1}^{K} \left| \mathbf{z}^{H} \mathbf{d}_{k,j} \mathbf{z} - t_{k,j} \right|^{2}$$
(22a)

s.t.
$$|z(i)| = 1, \forall i \in \mathbb{Z},$$
 (22b)

where $\mathbf{d}_{k,j} = \begin{bmatrix} \mathbf{I}_{F \times F} \\ \mathbf{O}_{M \times F} \end{bmatrix} \mathbf{c}_{k,j} [\mathbf{O}_{M \times F} \quad \mathbf{I}_{M \times M}] \in \mathbb{C}^{(M+F) \times (M+F)}$ and $\mathcal{Z} \triangleq \{1, 2, \dots, F+M\}$. The Euclidean gradient of the function $f(\mathbf{z})$ is given by

$$\nabla f(\mathbf{z}) = \begin{bmatrix} 2 \sum_{j=1}^{K} \sum_{k=1}^{K} \mathbf{c}_{k,j} \mathbf{x} \left(\mathbf{x}^{H} \mathbf{c}_{k,j}^{H} \mathbf{b} - t_{k,j}^{H} \right) \\ 2 \sum_{j=1}^{K} \sum_{k=1}^{K} \mathbf{c}_{k,j}^{H} \mathbf{b} \left(\mathbf{b}^{H} \mathbf{c}_{k,j} \mathbf{x} - t_{k,j} \right) \end{bmatrix}. \quad (23)$$

Therefore, the problem (22) can be effectively solved by the RCG algorithm.

Note that the objective function of the problem (15) is convex over **b** or **x**. In the alternating optimization, the subproblem is reduced to an unconstrained convex problem in the manifold space. Therefore, the optimal solution can be obtained for each subproblem by the RCG algorithm. However, the function f(z) is not jointly convex in **b** and **x**. Thus, in the RCG-based joint optimization, only the suboptimal solution can be obtained.

Method Three: SCA-based Joint Optimization The RCG algorithm requires multiple projections. If we directly optimize the phase shifts, the projection procedure is no longer needed. Then the problem (22) becomes an unconstraint non-convex problem, i.e.,

$$\min_{\phi} \quad f(\phi) = \sum_{j=1}^{K} \sum_{k=1}^{K} \left| (e^{j\phi})^{H} \mathbf{d}_{k,j} e^{j\phi} - t_{k,j} \right|^{2}, \quad (24)$$

where $\phi = \angle z$. Though the above problem is still difficult to solve optimally, we only need to solve its surrogate problem by exploiting the SCA technique, and the BCD method will converge to a stationary solution [39]. Specifically, denote the surrogate function for $f(\phi)$ by $g(\phi, \bar{\phi})$. Then, ϕ can be updated by solving the following surrogate problem

$$\boldsymbol{\phi} = \arg\min_{\boldsymbol{\phi} \in \mathbb{R}^{F+M}} g(\boldsymbol{\phi}, \bar{\boldsymbol{\phi}}). \tag{25}$$

The surrogate function $g(\phi, \bar{\phi})$ needs to satisfy following the two constraints [39, Proposition 1]:

$$g(\phi, \phi) = f(\phi), \tag{26a}$$

$$g(\phi, \phi) \ge f(\phi).$$
 (26b)

We can construct the surrogate function by the second order Taylor expansion:

$$g(\phi, \bar{\phi}) = f(\bar{\phi}) + \nabla f(\bar{\phi})^T (\phi - \bar{\phi}) + \frac{1}{2\kappa} \|\phi - \bar{\phi}\|^2, \quad (27)$$

where $\nabla f(\bar{\phi})$ is the gradient, and κ is chosen to satisfy (26b) locally within a bounded feasible set. Then, ϕ is updated by

$$\phi = \bar{\phi} - \kappa \nabla f(\bar{\phi}). \tag{28}$$

In practice, the parameter κ can be determined by the Armijo rule:

$$f(\bar{\phi}) - f(\phi) \ge \zeta \kappa \|\nabla f(\bar{\phi})\|^2, \tag{29}$$

where $0 < \zeta < 0.5$, κ is the largest element in $\{\beta \kappa_0^i\}_{i=0,1,...}$ that makes (29) satisfied, $\beta > 0$ and $0 < \kappa_0 < 1$.

3) Optimize $\{t_{k,j}\}$: With other variables fixed, problem $\mathcal{P}_1(\rho)$ can be reduced to

$$\min_{\{t_{k,j}\}} \quad \sum_{j=1}^{K} \sum_{k=1}^{K} \left| \mathbf{h}_{k}^{H} \boldsymbol{\Theta} \mathbf{G} \mathbf{V} \mathbf{w}_{j} - t_{k,j} \right|^{2}$$
(30a)

s.t.
$$\frac{\left|t_{k,k}\right|^{2}}{\sum_{j\neq k}^{K}\left|t_{k,j}\right|^{2}+\sigma_{k}^{2}} \geq \gamma_{k}, \forall k \in \mathcal{K}.$$
 (30b)

Algorithm 2 Penalty-based Optimization Algorithm

1: Initialize **V**, Θ , ρ and $\{t_{k,j}\}, \forall k, j \in \mathcal{K}$.

- 2: repeat
- 3: repeat4: Upo
 - Update \mathbf{W} by (14);
- 5: Update Θ and V by solving problem (15);
- 6: Update $\{t_{k,j}\}$ by solving problem (30);
- 7: **until** The decrease of the objective value of problem $\mathcal{P}_1(\rho)$ is below threshold $\epsilon_2 > 0$.
- 8: Update ρ by (32).
- 9: **until** The stopping indicator ξ in (33) is below threshold $\epsilon_3 > 0$.
- 10: Project Θ and V onto the discrete sets S_r and S_a , respectively;
- 11: Update W by solving problem (41) with the projected Θ and V.

The objective function is convex over $\{t_{k,j}\}$. Although the constraints (30b) are non-convex, they can be translated to the form of second-order cones as follows,

$$\sqrt{1 + \frac{1}{\gamma_k}} t_{k,j} \ge \left\| \begin{array}{c} \mathbf{A}_2^H \mathbf{e}_k \\ \sigma_k \end{array} \right\|_2, \forall k \in \mathcal{K}, \tag{31}$$

where $\mathbf{A}_2 \in \mathbb{C}^{K \times K}$ denotes a matrix with the entry in its k-th row and j-the column being $t_{k,j}$, i.e., $\mathbf{A}_2[k,j] = t_{k,j}$, and $\mathbf{e}_k \in \mathbb{C}^{K \times 1}$ denotes a vector with the k-th entry being one and others being zeros. Then, the problem (30) can be effectively and optimally solved by the SOCP method [40].

B. Outer Layer: Update Penalty Factor

The penalty factor ρ is initialized to be a small number to find a good starting point, then gradually increased to tighten the penalty. Specifically,

$$\rho := \frac{\rho}{c}, 0 < c < 1, \tag{32}$$

where c is a constant scaling parameter. A larger c may lead to a more precise solution with a longer running time.

C. Algorithm

The overall penalty-based optimization algorithm is summarized in *Algorithm* 2. Define the stopping indicator ξ as follows,

$$\xi \triangleq \max\left\{ |\mathbf{h}_k^H \boldsymbol{\Theta} \mathbf{G} \mathbf{V} \mathbf{w}_j - t_{k,j}|^2, \forall k, j \in \mathcal{K} \right\}.$$
(33)

When ξ is below a pre-defined threshold $\epsilon_3 > 0$, the equality constraints (12b) are considered to be satisfied and the proposed algorithm is terminated. Since we start with a small penalty and gradually increase its value, the objective value of problem $\mathcal{P}_1(\rho)$ is finally determined by the penalty part and the equality constraints are guaranteed to be satisfied. Note that, for any given penalty factor ρ , the objective value of the problem $\mathcal{P}_1(\rho)$ solved through the BCD method is non-increasing over iterations in the inner layer. And the optimal value of the problem $\mathcal{P}_1(\rho)$ is bounded by the SINR constraints. Thereby, based on the Theorem 4.1 of the work [41], the proposed Algorithm 2 is guaranteed to converge.

Let us consider the complexity of the proposed algorithm. Let us first compare the complexities of the three different methods, which are dominated by computing the Euclidean gradient. Thus, the complexity of Alternating Opt is $\mathcal{O}(I_{\mathbf{b}}K^2F + I_{\mathbf{x}}K^2M)$, where $I_{\mathbf{b}}$ and $I_{\mathbf{x}}$ denote the required iteration times of the RCG algorithm to update b and x, respectively. The complexity of RCG-based Joint Opt is $\mathcal{O}(I_{\mathbf{z}}K^2(F+M))$, where $I_{\mathbf{z}}$ denotes the required iteration times of the RCG algorithm to update z. The complexity of SCA-based Joint Opt is $\mathcal{O}(I_a K^2 (F + M))$, where I_a denotes the iteration number of the Armijo search. As will be shown in Section VI-B, the RCG-based joint optimization method outperforms the other two methods. Thus, we adopt the RCGbased joint optimization method and analyze its complexity. It can be shown that the complexity of computing W in (14) is $\mathcal{O}(N^3 + KN^2 + K^2N)$. Besides, the complexity of solving problem (30) is $\mathcal{O}(K^{3.5})$. Thereby, the overall complexity of Algorithm 2 is $\mathcal{O}(I_{out}I_{in}(N^3 + KN^2 + K^2N + I_zK^2(F +$ $M) + K^{3.5})$ where I_{out} and I_{in} denote the outer and inner iteration times required for convergence, respectively.

IV. SEQUENTIAL OPTIMIZATION

To reduce the complexity of solving the problem \mathcal{P}_0 , we develop a sequential optimization approach in this section. Specifically, we first optimize the RIS response matrix Θ , then optimize the analog beamformer V, and finally optimize the digital beamformer W without iteration.

A. RIS Design

Looking at the SINR constraints (11b), and we can get

$$|\mathbf{h}_{k}^{H}\boldsymbol{\Theta}\mathbf{G}\mathbf{V}\mathbf{w}_{k}| - \gamma_{k}\sum_{j\neq k}|\mathbf{h}_{k}^{H}\boldsymbol{\Theta}\mathbf{G}\mathbf{V}\mathbf{w}_{j}| \geq 0, \forall k \in \mathcal{K}.$$
 (34)

For simplicity, let the transmit beamforming vectors at the BS be set based on the maximum-ratio transmission (MRT) principle, i.e., $\mathbf{V}\mathbf{w}_k = (\mathbf{h}_k^H \boldsymbol{\Theta} \mathbf{G})^H$. Note that the transmit beamforming vectors here are only used to extract the optimization of the RIS response matrix. The actually adopted transmit beamforming vectors are designed later. Then, the problem (34) is translated to

$$\|\mathbf{h}_{k}^{H}\boldsymbol{\Theta}\mathbf{G}\|^{2} - \gamma_{k} \sum_{j \neq k} \|\mathbf{h}_{k}^{H}\boldsymbol{\Theta}\mathbf{G}\mathbf{G}^{H}\boldsymbol{\Theta}^{H}\mathbf{h}_{j}\| \geq 0, \forall k \in \mathcal{K}.$$
(35)

The inequality (35) should be satisfied for all users. Therefore, in order to ensure the receive signal quality of each user, we maximize the worst case of the left-hand side of (35) among all users, i.e.,

$$\max_{\Theta} \min_{k \in \mathcal{K}} \|\mathbf{h}_{k}^{H} \Theta \mathbf{G}\|^{2} - \gamma_{k} \sum_{j \neq k} \|\mathbf{h}_{k}^{H} \Theta \mathbf{G} \mathbf{G}^{H} \Theta^{H} \mathbf{h}_{j}\|$$
(36a)
s.t. $b_{f} \in \mathcal{S}_{r}, \forall f \in \mathcal{F}.$ (36b)

The SDR technique can be adopted to solve the above problem. Specifically, let us introduce an auxiliary variable ϖ , and let $\mathbf{B} = \mathbf{b}\mathbf{b}^{H}$. After dropping the rank-one constraint, the problem (36) can be relaxed into

$$\max_{\mathbf{B},\varpi} \quad \varpi \tag{37a}$$

s.t.
$$\operatorname{Tr}(\eta_k \eta_k^H \mathbf{B}) \ge \varpi + \gamma_k \sum_{j \neq k} \|\operatorname{Tr}(\zeta_{k,j} \mathbf{B})\|, \forall k \in \mathcal{K},$$
(37b)

$$\mathbf{B}_{f,f} = 1, \forall f \in \mathcal{F},
 (37c)
 \mathbf{B} \succ 0,
 (37d)$$

$$\mathbf{B} \succeq \mathbf{0},\tag{37d}$$

where $\eta_k = \text{diag}(\mathbf{h}_k^H)\mathbf{G} \in \mathbb{C}^{F \times M}$ and $\zeta_{k,j} = \text{diag}(\mathbf{h}_k^H)\mathbf{G}\mathbf{G}^H \text{diag}(\mathbf{h}_j) \in \mathbb{C}^{F \times F}$. The problem (37) is convex and can be optimally solved by a standard convex solver such as CVX [42]. After solving the problem (37), the optimal B can be obtained. Then, we need to obtain the value of b, which has the direct relationship to **B**. Generally, there is no guarantee that the relaxed problem (37) has a rank-one optimal solution. If $rank(\mathbf{B}) = 1$, then we can obtain the optimal b by taking the eigenvalue decomposition of B. Otherwise, if $rank(\mathbf{B}) > 1$, an additional Gaussian randomization procedure is needed to produce a rank-one solution [10], [43]. Specifically, suppose that the eigenvalue decomposition of B is $\mathbf{B} = \mathbf{U}\Sigma\mathbf{U}^{H}$. Then, let $\overline{\mathbf{b}} = \mathbf{U}\Sigma^{1/2}\mathbf{r}$, where $\mathbf{r} \sim \mathcal{CN}(0, \mathbf{I}_{F})$. Finally, project $\overline{\mathbf{b}}$ to the pre-defined set \mathcal{S}_r , i.e.,

$$b_f = e^{j \angle b_f}, \tag{38}$$

where $\angle b_f = \arg \min_{\angle b_f \in S_r} |\angle b_f - \angle \overline{b}_f|$. With many independently generated r, the one that makes ϖ maximum is taken as the solution.

B. Analog Beamforming Design

We then optimize the analog beamforming after the RIS has been configured. The orthogonal match pursuit (OMP) method is widely adopted to design the analog beamformer [32]. If the BS adopts the fully digital beamforming structure, the optimal digital beamforming \mathbf{W}_{opt} can be obtained by solving the following problem

$$\min_{\mathbf{W}} \quad D\sum_{k=1}^{K} \|\mathbf{w}_k\|^2 \tag{39a}$$

s.t.
$$\frac{|\mathbf{h}_{k}^{H} \boldsymbol{\Theta} \mathbf{G} \mathbf{w}_{k}|^{2}}{\sum_{j \neq k} |\mathbf{h}_{k}^{H} \boldsymbol{\Theta} \mathbf{G} \mathbf{w}_{j}|^{2} + \sigma_{k}^{2}} \geq \gamma_{k}, \forall k \in \mathcal{K}.$$
 (39b)

Note that the above problem can be optimally solved by the SOCP method. We adopt an overlapping codebook A with an overlapping coefficient μ to improve the spatial resolution due to the limited resolution of the conventional DFT codebook [44]. A larger μ represents higher resolution of the codebook. The codebook can be represented as A = $[\mathbf{a}_B(\psi_1,\phi_1),\ldots,\mathbf{a}_B(\psi_1,\phi_{\mu N_z}),\ldots]$

 $\mathbf{a}_B(\psi_{\mu N_y}, \phi_{\mu N_z})]$, where N_y and N_z denote the horizontal and vertical lengths, $\psi_i = \frac{2\pi(i-1)}{\mu N_y}$, $i = 1, 2, \dots, \mu N_y$ and $\phi_j = \frac{2\pi(j-1)}{\mu N_z}, j = 1, 2, \dots, \mu N_z$, respectively. Then, we can use a selection matrix $\mathbf{T} \in \mathbb{R}^{\mu^2 N_y N_z \times N}$ to select proper

columns. Specifically, the analog beamforming problem can under a total power budget P_T , which can be formulated as be formulated as

$$\mathbf{T}^* = \underset{\mathbf{T}, \mathbf{F}_{BB}}{\operatorname{arg\,min}} \quad \left\| \mathbf{W}_{\text{opt}} - \mathbf{A}_t \mathbf{T} \mathbf{F}_{BB} \right\|_F \tag{40a}$$

s.t.
$$\left\| \operatorname{diag} \left(\mathbf{T} \mathbf{T}^H \right) \right\|_0 = N,$$
 (40b)

where $\mathbf{A}_t = \mathbf{e}_t \odot \mathbf{A}, t \in \mathcal{N}$, and \mathbf{e}_t is an $M \times 1$ zerovector with the entry from (t-1)D + 1 to tD being one. Since the structure of analog beamforming is sub-connected, we use e_t to modify the codebook. Then, the OMP method can be applied to obtain the selection matrix T^* . The analog beamforming can be recovered, i.e., $\mathbf{V} = \mathbf{A}_t \mathbf{T}^*$. Finally, the discrete analog beamforming can be obtained by mapping V to the nearest discrete value in S_a .

C. Digital Beamforming Design

After obtaining the RIS phase shifts and the analog beamforming vector, we need to obtain the optimal digital beamforming matrix. The digital beamforming can be obtained by solving following problem

$$\min_{\mathbf{W}} \quad D\sum_{k=1}^{K} \|\mathbf{w}_{k}\|^{2} \tag{41a}$$

s.t.
$$\frac{|\mathbf{h}_{k}^{H} \Theta \mathbf{G} \mathbf{V} \mathbf{w}_{k}|^{2}}{\sum_{j \neq k} |\mathbf{h}_{k}^{H} \Theta \mathbf{G} \mathbf{V} \mathbf{w}_{j}|^{2} + \sigma_{k}^{2}} \ge \gamma_{k}, \forall k \in \mathcal{K}.$$
(41b)

Note that the digital beamforming W_{opt} obtained by solving the problem (39) is only used for the analog beamforming design. The problem (41) is the conventional power minimization problem in the multiple-input-single-output (MISO) system, which can be effectively and optimally solved by the SOCP method [40].

Here, we consider the complexity of the sequential optimization. The complexity of the RIS design is dominated by the SDR technique, which is $\mathcal{O}(F^6)$ [45]. The complexity of the analog beamforming is dominated by the OMP technique, which is $\mathcal{O}(\mu^2 MFN^3)$. The complexity of the digital beamforming design is $\mathcal{O}(N^{3.5}K^{3.5})$ [46]. Thus, the overall computational complexity of the Sequential Optimization is $\mathcal{O}(F^6 + \mu^2 MFN^3 + N^{3.5}K^{3.5})$. The advantage of this algorithm is that it does not need to perform iterative operations.

V. EXTENSION TO THE MAX-MIN FAIRNESS PROBLEM

A closely related problem of the QoS problem \mathcal{P}_0 is the MMF problem, which aims to maximize the performance of the worse-case user under a fixed total transmit power budget. In this section, we discuss the relationship between the OoS problem and the MMF problem, and the extension of the proposed algorithm to solve the MMF problem. In specific, the MMF problem is to maximize the weighted minimum SINR

$$Q_{0}: \max_{\{\mathbf{V}, \mathbf{W}, \mathbf{\Theta}\}} \min_{k \in \mathcal{K}} \quad \frac{1}{\gamma_{k}} \frac{|\mathbf{h}_{k}^{H} \mathbf{\Theta} \mathbf{G} \mathbf{V} \mathbf{w}_{k}|^{2}}{\sum_{j \neq k} |\mathbf{h}_{k}^{H} \mathbf{\Theta} \mathbf{G} \mathbf{V} \mathbf{w}_{j}|^{2} + \sigma_{k}^{2}} \quad (42a)$$

t.
$$D\sum_{k=1}^{K} \|\mathbf{w}_k\|^2 \le P_T,$$
 (42b)

$$(11c), (11d)$$
 (42c)

where $\gamma_k > 0$ denotes the weight parameter of user k. A larger value of γ_k indicates that user k has a higher priority in transmission.

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Let us compare the problem \mathcal{P}_0 and the problem \mathcal{Q}_0 . Let $\boldsymbol{\gamma} \triangleq [\gamma_1, \gamma_2, \dots, \gamma_K]^T$. For a given set of channels and noise powers, \mathcal{P}_0 is parameterized by γ . We use the notation $\mathcal{P}_0(\boldsymbol{\gamma})$ to account for this, and $P_T = \mathcal{P}_0(\boldsymbol{\gamma})$ to denote the associated minimum power. Similarly, Q_0 is parameterized by γ and P_T . Then, $\mathcal{Q}_0(\gamma, P_T)$ and $\xi = \mathcal{Q}_0(\gamma, P_T)$ are used to represent the dependence and the associated maximum worstcase weighted SINR, respectively. Similar to [47], [48], we have the following proposition.

Proposition 1: The QoS problem \mathcal{P}_0 and the MMF problem Q_0 have the following relationship:

$$\xi = \mathcal{Q}_0(\gamma, \mathcal{P}_0(\xi\gamma)), \tag{43a}$$

$$P_T = \mathcal{P}_0(\mathcal{Q}_0(\boldsymbol{\gamma}, P_T)\boldsymbol{\gamma}). \tag{43b}$$

Proof: Contradiction argument is used to prove (43a). For the problem $\mathcal{P}_0(\xi \gamma)$, denote the optimal solution and the associated optimal value as $\{\mathbf{W}^{\mathcal{P}_0}, \mathbf{\Theta}^{\mathcal{P}_0}, \mathbf{V}^{\mathcal{P}_0}\}$ and $P_T^{\mathcal{P}_0}$, respectively. It is observed that the set $\{\mathbf{W}^{\mathcal{P}_0}, \mathbf{\Theta}^{\mathcal{P}_0}, \mathbf{V}^{\mathcal{P}_0}\}$ is also a feasible solution with the objective value ξ to the problem $\mathcal{Q}_0(\boldsymbol{\gamma}, P_T^{\mathcal{P}_0})$. Since $\boldsymbol{\Theta}$ and V have unit-modulus constraints, we can only scale W. Assume there is another solution $\{\mathbf{W}^{\mathcal{Q}_0}, \mathbf{\Theta}^{\mathcal{P}_0}, \mathbf{V}^{\mathcal{P}_0}\}\$ with bigger objective value $\xi^{\mathcal{Q}_0} > \xi$. Then, we can appropriately scale down the digital beamforming with the SINR constraints of the problem $\mathcal{P}_0(\xi \gamma)$ still satisfied. The resulting solution $\{c \mathbf{W}^{\mathcal{Q}_0}, \mathbf{\Theta}^{\mathcal{P}_0}, \mathbf{V}^{\mathcal{P}_0}\} (0 < c < 1)$ has a smaller transmit power than $P_T^{\mathcal{P}_0}$, which contradicts the optimality of $\{\mathbf{W}^{\mathcal{P}_0}, \bar{\mathbf{\Theta}}^{\mathcal{P}_0}, \mathbf{V}^{\mathcal{P}_0}\}$. (43b) can be proved in the similar way and the details are omitted.

Generally, the MMF problem Q_0 is more difficult to solve than the QoS problem \mathcal{P}_0 due to the non-smooth objective function. Based on Proposition 1, we can solve the MMF problem by solving a series of QoS problems. Specifically, let us consider the following problem $\mathcal{P}_2(\varsigma)$, i.e.,

$$\mathcal{P}_2(\varsigma) : \min_{\{\mathbf{V}, \mathbf{W}, \mathbf{\Theta}\}} \quad D\sum_{k=1}^K \|\mathbf{w}_k\|^2$$
(44a)

.t.
$$\operatorname{SINR}_k \ge \varsigma \gamma_k, \forall k \in \mathcal{K},$$
 (44b)

$$(11c), (11d).$$
 (44c)

For a given set of channels, noise powers and γ , \mathcal{P}_2 is parameterized by ς . Note that the problem $\mathcal{P}_2(\varsigma)$ is a linear function over ς . A larger ς leads to a larger objective value of \mathcal{P}_2 . Thus, in order to solve the problem \mathcal{Q}_0 , we can do a bisection search over ς of the problem \mathcal{P}_2 until its objective value is P_T . Then, the corresponding result is the solution to Q_0 with the total power budget being P_T .

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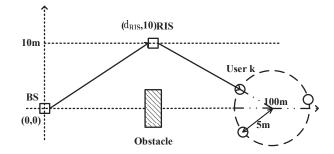


Fig. 2. The simulated RIS-aided communication scenario.

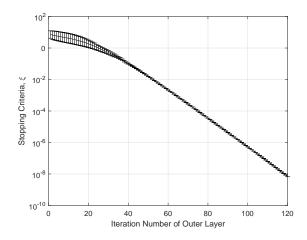


Fig. 3. Stopping indicator of the penalty-based algorithm.

VI. SIMULATION RESULTS

In this section, we evaluate the performance of our proposed algorithms. We consider an RIS-aided multiuser mmWave communication system which operates at 28 GHz with bandwidth 251.1886 MHz. Thus, the noise power is $\sigma_k^2 = -174 +$ $10 \log_{10} B = -90$ dBm. We consider a 6×6 UPA structure at the BS with N = 6 RF chains and a total of M = 36 antennas located at (0 m, 0 m) as shown in Fig. 2. The RIS is located at $(d_{RIS} \text{ m}, 10 \text{ m})$ and equipped with $F_1 \times F_2$ unit cells where $F_1 = 6$ and F_2 can vary. Users are uniformly and randomly distributed in a circle centered at (100 m, 0 m) with radius 5 m. As for the mmWave channel, we set $N_{cl_1} = N_{cl_2} = 5$ clusters, $N_{ray_1} = N_{ray_2} = 10$ rays per cluster; the azimuth and elevation angles of arrival and departure follow the Lapacian distribution with an angle spread of 10 degrees; the complex gain α_{il} and β_{il} follow the complex Gaussian distribution $\mathcal{CN}(0, 10^{-0.1PL(d)})$, and PL(d) can be modeled as [49]:

$$PL(d) = \varphi_a + 10\varphi_b \log_{10}(d) + \varphi_c(dB), \qquad (45)$$

where $\varphi_c \sim \mathcal{N}(0, \sigma^2)$, $\varphi_a = 72.0, \varphi_b = 2.92$ and $\sigma = 8.7$ dB. The auxiliary variables $\{t_{k,j}\}$ are initialized following $\mathcal{CN}(0, 1)$. The penalty factor is initialized as $\rho = 10^{-3}$. Other system parameters are set as follows unless specified otherwise later: $K = 3, F_2 = 6, d_{RIS} = 50, c = 0.9, \epsilon_1 = \epsilon_3 = 10^{-7}, \epsilon_2 = 10^{-4}, \gamma_k = 10$ dB, $\forall k \in \mathcal{K}$. All simulation curves are averaged over 100 independent channel realizations. The simulations are carried out on a computer with Intel i7-7700 CPU at 3.60 GHz and with 16.0 GB RAM.

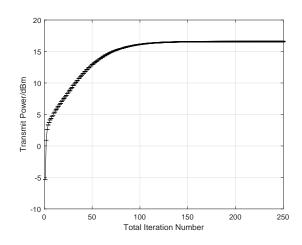


Fig. 4. Convergence of the penalty-based algorithm.

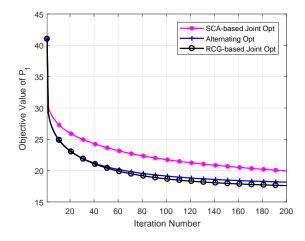


Fig. 5. Convergence comparison with fixed penalty $\rho = 1$ when solving problem (15) by different methods.

A. Convergence Performance of the Penalty-based Algorithm

First, let us look at the convergence performance of the penalty-based algorithm. We show the stopping indicator (33) of the penalty-based algorithm in Fig. 3 and the average convergence of the penalty-based algorithm in Fig. 4 in the case of continuous phase shifts of analog beamformer and RIS coefficients. These curves are plotted with the average plus and minus the standard deviation. Note that the transmit power increases as the total number of iterations increases. This is because that a larger ρ corresponding to a larger penalty for violating the equality restrictions, necessitating a higher transmit power to reduce the penalty term. It is observed that the stopping indicator can always meet the predefined accuracy 10^{-7} after about 110 outer layer iterations in Fig. 3. Thus, the solutions obtained by Algorithm 2 satisfy all SINR constraints. Fig. 4 shows that the proposed algorithm converges after about 200 total iterations, which means that the inner layer runs averagely 2 times.

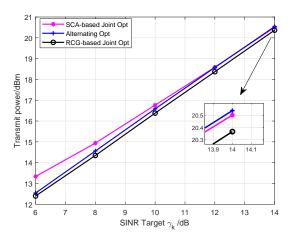


Fig. 6. Transmit power versus SINR targets when solving problem (15) by different methods.

	Running time (s)						
	F=10	F=20	F=40	F=80			
Alternating Opt	142.9865	152.4747	157.7086	163.2711			
RCG-based Joint Opt	134.5921	142.8421	143.1132	148.7838			
SCA-based Joint Opt	103.1461	104.0654	105.5587	110.4485			
TADLE I							

TABLE I COMPUTATIONAL TIME COMPARISON.				
COMPUTATIONAL	TIME	COMPARISON.		

B. Performance and Computational Comparison of Solving Problem (15) by Different Methods

We first compare the performance of different methods of solving problem (15) as described in Section III-A2. Fig. 5 illustrates the objective value of \mathcal{P}_1 versus the iteration number when the penalty factor ρ is fixed to one. Fig. 6 illustrates the transmit power versus SINR targets. Though the optimal solution can be obtained for each subproblem in alternating optimization, it converges to a worse local optimum compared with the RCG-based joint optimization as shown in Fig. 5 and Fig. 6. Though the SCA-based joint optimization does not require projection, it performs worse than the RCG-based joint optimization as shown in Fig. 5 and Fig. 6. It is also seen from Fig. 6 that the gap between the SCA-based method and the RCG-based method decreases as the SINR targets increase. However, we have tested the results when the SINR target is 20dB, the RCG-based method still outperforms the SCA-based method.

We further compare the computational time with fixed penalty $\rho = 1$ when solving problem (15) by different methods in Table I. Here, we set the RIS $F_1 \times F_2$ unit cells where $F_1 = 5$ and F_2 can vary. It is found that the SCA-based Joint Opt runs the fastest, while the Alternating Opt runs the slowest.

Overall, the RCG-based Joint Opt converges to the best point, and the time consumed is somewhere in the middle. Therefore, the RCG-based Joint Opt is a good choice among the three methods. In the following, we adopt the RCG-based joint optimization method.

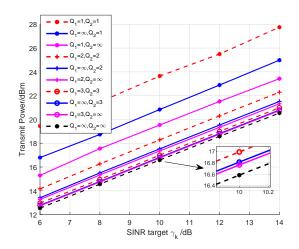


Fig. 7. Influence of discrete phase shifts.

C. Influence of Discrete Phase Shifts

We consider that the number of control bits at the analog beamformer and at the RIS, i.e., Q_1 and Q_2 , can be designed separately, and each can take values from $\{1, 2, 3, \infty\}$, where ∞ corresponds to continuous phase shifts. Fig. 7 shows that when there is only one control bit for both analog beamformer and RIS, i.e. $Q_1 = Q_2 = 1$, the power gap to the ideal case with continuous phase shifts is up to 7 dB; when $Q_1 = Q_2 = 2$ and $Q_1 = Q_2 = 3$, the gap reduces quickly to 1.5 dB and 0.4dB, respectively. This suggests that having 3 bits for the discrete phase shifts is enough in practice. It is also seen from Fig. 7 that the BS is more robust to the discrete phase shifts than the RIS. In specific, the performance at $Q_1 = 1, Q_2 =$ ∞ is about 2 dB better than that at $Q_1 = \infty, Q_2 = 1$. We believe that the analog beamforming at the BS has a larger dimension of regulation than the RIS. Specifically, the analog beamforming contains many RF chains and each RF chain can serve one user, while all users are served by the same RIS. Therefore, the BS is more robust to the discrete phase shifts than the RIS.

D. Performance Comparison with Other Schemes

To demonstrate the efficiency of the proposed algorithms and to reveal some design insights, we compare the performance of the following algorithms when $Q_1 = 3$ and $Q_2 = 3$.

- Penalty-Manifold joint design with hybrid beamforming structure (Penalty-Manifold HB): This is the proposed *Algorithm* 2 for joint design of hybrid beamforming and RIS phase shifts.
- Penalty-Manifold joint design with fully digital beamforming structure (Penalty-Manifold FD): This is the proposed *Algorithm* 2 but changing the hybrid beamforming to the fully digital beamforming at the BS. This is done by setting D = 1.
- Penalty-Manifold joint design with random Θ (Random Θ): The phase shifts at the RIS are randomly selected to be feasible values. Then the hybrid beamforming matrices $\{\mathbf{W}, \mathbf{V}\}$ at the BS are obtained by using the penalty-manifold joint algorithm as in *Algorithm* 2, where the

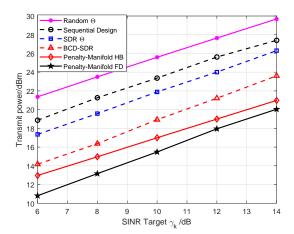


Fig. 8. Transmit power versus SINR targets.

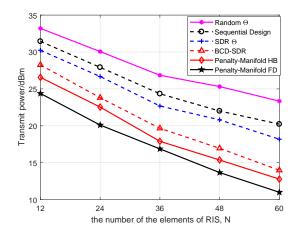


Fig. 9. Transmit power versus the number of the elements of RIS.

update of Θ is skipped. This is to find out the significance of optimizing the phase shifts at the RIS.

- Penalty-Manifold joint design with SDR Θ (SDR Θ): The phase shifts at the RIS are designed by using the SDR approach as stated in Section IV-A. Then the hybrid beamforming matrices $\{W, V\}$ at the BS are obtained by using the penalty-manifold joint algorithm as in *Algorithm* 2, where the udpate of Θ is skipped. This is again to find out the significance of optimizing the phase shifts at the RIS.
- BCD-SDR joint design (BCD-SDR): The conventional BCD method in conjunction with the SDR method, as mentioned in Section II-C.
- Sequential design: the proposed sequential design where RIS phase shifts, analog beamforming, and digital beamforming are optimized sequentially in Section IV. In order to make the sequential optimization method be more effective, we try different overlapping coefficients μ from 1 to 4 and let the best result be the final solution.

Fig. 8 illustrates the transmit power versus SINR targets. We first observe that the Penalty-Manifold joint design outperforms the start-of-the-art BCD-SDR joint design, which verifies the effectiveness of the proposed algorithm. Second,

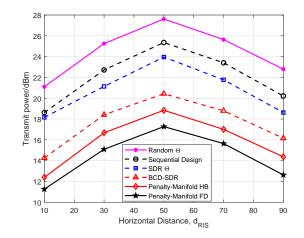


Fig. 10. Transmit power versus the horizontal distance of RIS.

	Running time (s)				
	F=10	F=20	F=40	F=80	
SDR-BCD	54.2175	61.1350	169.0588	461.3819	
Penalty-Manifold FD	96.0028	101.2406	115.0541	116.8831	
Sequential Design	15.3721	17.9422	20.1504	37.0946	

TABLE II COMPUTATIONAL TIME COMPARISON.

it is seen that the Penalty-Manifold joint design with random Θ performs the worst among all the considered schemes. By simply changing the random Θ to the SDR Θ (while keeping the joint design of $\{\mathbf{W}, \mathbf{V}\}$ unchanged), the transmit power consumption can be reduced by 4 dB. If Θ is involved in the Penalty-Manifold joint design, another about 5 dB power reduction can be obtained. These observations indicate that the design of RIS phase shifts plays the crucial role for performance optimization. Third, we observe that the sequential design is about 1dB worse than the joint design with SDR Θ . This suggests that, when the RIS response matrix is designed sequentially, further optimizing the hybrid beamforming at the BS can only bring marginal improvement. Last but not least, we observe that the power consumed by Penalty-Manifold beamforming is about 2dB higher than the power consumed by Penalty-Manifold FD. Note that the hybrid beamforming has a much lower hardware cost since it only employs N = 6RF chains at the BS, while the fully digital beamforming has M = 36 RF chains. This means that the proposed hybrid beamforming is effective.

The influence of the RIS element number is considered in Fig. 9. When the RIS element number increases from 12 to 60, the transmit power decreases about 15dB. Thus, we conclude that the RIS can greatly reduce the transmit power by installing a large number of elements.

Fig. 10 illustrates the transmit power versus the RIS horizontal distance. It is seen that as the RIS horizontal distance d_{RIS} increases, the transmit power increases firstly, and reaches the peak at 50 m, then decreases. This can be explained that the received power through the reflection of the RIS in the far field is proportional to $d_1^{-2}d_2^{-2}$, where d_1 and d_2 denote the distances between the BS-RIS and RIS-user, respectively. It is found that the RIS can be located near the BS or users to save energy.

We further compare in Table II the running time for various values of F. Here, μ is set to be 3. We set the RIS $F_1 \times F_2$ unit cells where $F_1 = 5$ and F_2 can vary. It is observed that the time consumed by the SDR-BCD method increases greatly as F increases. It is interesting that the computational time of Penalty-Manifold FD is insensitive to F. And the time consumed by the Sequential Design is the least among the algorithms, which means that it has the lowest complexity.

VII. CONCLUSION

In this paper, we investigate an RIS-aied downlink MIMO system, with the objective of minimizing the transmit power at the BS by jointly optimizing the hybrid A/D beamforming at the BS, as well as the overall response-coefficient at the RIS, subject to individual minimum SINR constraints. The nonconvex problem is first solved by the penalty-based algorithm with manifold optimization, followed by a low-complexity sequential optimization. In particular, we propose three different methods for optimizing the BS analog beamforming and the RIS response matrix in the penalty-based algorithm. The RCGbased joint optimization is found to outperform the other two methods but it has a slightly higher complexity. Extensive simulation results demonstrate that the proposed algorithm outperforms the state-of-art BCD-SDR algorithm. Our simulation results provide useful insights into the corresponding wireless system design. In particular, the simulation results show that utilizing a large number of RIS units could help reduce the transmit power at the BS greatly. Moreover, 3-bit quantizers of both the RIS and the analog beamformer could approach the performance of continuous phase shifters.

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