Joint Frequency Offset and Channel Estimation for OFDM

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Abstract— **We investigate the problem of joint frequency offset and channel estimation for OFDM systems. The complexity of the joint maximum likelihood (ML) estimation procedure motivates us to propose an adaptive MLE algorithm which iterates between estimating the frequency offset and the channel parameters. Pilot tones are used to obtain the initial estimates and then a decision-directed technique provides an effective estimation technique. The joint modified (averaged) Cramer-Rao lower bounds (MCRB) of the channel coefficients and frequency offset estimates are derived and discussed. It is shown that, for the case of a large number of subcarriers in the OFDM system, there is approximately a 6 dB loss in the frequency offset estimate lower bound due to the lack of knowledge of the channel impulse response (CIR). The degradation of the CIR lower bound is less severe and depends on the channel delay spread. We show both analytically and by simulation, that the channel estimate accuracy is less sensitive to unknown frequency offset than the frequency offset estimation is affected by the unknown CIR. Comprehensive simulations have been carried out to validate the effectiveness of the adaptive joint estimation algorithm.**

I. Introduction

Orthogonal frequency division multiplexing [1] is inherently robust against frequency selective fading, since each subchannel occupies a relatively narrow band, where the channel frequency characteristic is nearly flat. It has already been used in European digital audio broadcasting (DAB), digital video broadcasting (DVB) systems, high performance radio local area network (HIPERLAN) and 802.11a wireless local area networks (WLAN). It has been demonstrated that OFDM is an effective way of increasing data rates and simplifying equalization in wireless communications [1].

Although the carrier frequency is known to the receiver, a frequency drift is not always non-negligible. Another source of frequency offset is the Doppler shift caused by the relative speed between the corresponding transmitter and receiver or the motion of other objects around transceivers. In some cases this deviation is too large for reliable OFDM data transmission. There are two problems affected by the frequency offset: one is the decrease in the amplitude of each sampled value, the other is the introduction of inter-carrier interference (ICI). Both will degrade the performance of an OFDM system in terms of bit error rate (BER). Consequently, the frequency offset must be estimated and compensated for at the receiver to achieve high-quality transmission. In addition, it is not possible to make reliable data decisions unless a good channel estimate is available for coherent demodulation. The Doppler shift causes the channel characteristic to change from time to time. The relative speed between OFDM transceivers may also change from time to time in an outdoor wide-area wireless environment, which causes the frequency offset to vary. The time-varying nature of both the frequency offset and the CIR requires the need for real-time estimation of both. A number of channel estimation algorithms [3]-[5] and frequency offset estimation algorithms [6]-[8] have been proposed in the literature. Usually, perfect frequency synchronization is assumed in deriving channel estimation algorithms. On the other hand, perfect channel estimation or simply, an additive Gaussian channel model is assumed in deriving the frequency offset. As far as we are aware, there have few studies that address such combined estimation problem. In [7], channel estimation is carried out after the frequency offset is compensated and it deals with only a specific frame structure of IEEE 802.11a. Li and Ritcy [8] present a simplified ML estimation algorithm of the frequency offset using only demodulated decisions, and their algorithm does not incorporate channel estimation into consideration.

The main objective of our study is to investigate the use of ML algorithms for joint estimation of the channel frequency offset and the CIR in an OFDM system that is subject to slow time varying frequency selective fading.

II. System Model and Assumptions

The schematic diagram of Figure 1 is a baseband equivalent representation of an OFDM system. The input binary data is first fed into a serial to parallel (S/P) converter. Each data stream then modulates the corresponding subcarrier by MPSK or MQAM. Modulations can vary from one subcarrier to another in order to achieve the maximum capacity or the minimum bit error rate (BER) under various constraints. In this paper we use, for simplicity, only QPSK in all the subcarriers, and M to denote the number of subcarriers in the OFDM system. The modulated data symbols, represented by complex variables $X(0), \cdots, X(M-1)$, are then transformed by the inverse fast Fourier transform (IFFT). The output symbols are

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denoted as $x(0), \cdots, x(M-1)$. In order to avoid inter-frame interference (IFI[1]), cyclic prefix (CP) symbols, which replicate the end part of the IFFT output symbols, are added in front of each frame. The parallel data are then converted back to a serial data stream before being transmitted over the frequency selective channel. The received data $y(0), \cdots, y(M-1)$ corrupted by multipath fading and AWGN are converted back to $Y(0), \cdots, Y(M-1)$ after discarding the prefix, and applying FFT and demodulation.

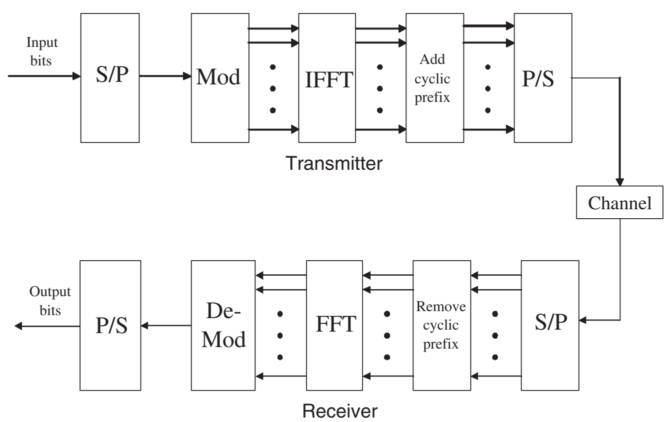

Fig. 1. Baseband OFDM system model

The channel model we adopt in the present paper is a multipath slowly time varying fading channel, which can be described by

$$y(k) = \sum_{l=0}^{L-1} h_l x(k-l) + n(k), \qquad 0 \le k \le M-1, \quad (1)$$

where h_l's $(0 \le l \le L-1)$ are independent complex-valued Rayleigh fading random variables, and n_k's $(0 \le k \le M-1)$ are independent complex-valued Gaussian random variables with zero mean and variance σ^2 for both real and imaginary components. L is the length of the CIR. In the presence of channel frequency offset, the above equation becomes [6]

$$y(k) = e^{j2\pi \frac{k\epsilon}{M}} \sum_{l=0}^{L-1} h_l x(k-l) + n(k), \qquad 0 \le k \le M-1, \quad (2)$$

where ϵ is the channel frequency offset which is normalized by the subcarrier spacing. We assume the frequency acquisition procedure has been completed so that the channel frequency offset is within one half of an interval of the subcarrier spacing, i.e., $|\epsilon| \le \frac{1}{2}$.

If the length of the CP is longer than L, there will be no IFI among OFDM frames. Thus we need to consider only one OFDM frame with M subcarriers in analyzing the system performance. The system model and performance can be easily extended to the case of multiple frames. After discarding the cyclic prefix and performing an FFT at the receiver, we can obtain the received data frame in the frequency domain:

$$\begin{aligned} Y(m) &= \frac{\sin \pi\epsilon}{M \sin \frac{\pi\epsilon}{M}} X(m) H(m) e^{j\pi \frac{(M-1)\epsilon}{M}} \\ &+ ICI(m) + N(m), \end{aligned} \quad (3)$$

where $H(m)$ is the frequency response of the channel at subcarrier m and the set of the transformed noise variables $N(m), 0 \le m \le M-1$ are i.i.d. complex-valued Gaussian variables that have the same distribution as $n(k)$, i.e., with mean zero and variance σ_n^2. The noteworthy term in (3) is the $ICI(m)$, which is given as

$$ICI(m) = \frac{1}{M} \sum_{k=0}^{M-1} \sum_{n \ne m} X(n) H(n) e^{j2\pi \frac{k(n-m+\epsilon)}{M}}, \quad n \ne m. \quad (4)$$

It is not zero if $\epsilon \ne 0$.

Equation (3) shows that the frequency offset degrades the amplitude of the received signal in each suncarrier and introduce inter-carrier interference (ICI). In addition, a common phase shift $\pi \frac{(M-1)\epsilon}{M}$ is introduced to the received signal. That can be used to estimate the frequency offset, as will be discussed in Section V.

In this paper we assume the CIR is constant in each OFDM frame and varies from frame to frame according to the fading rate. Furthermore, we assume the system has perfect timing synchronization.

Notation: We use the standard notations, e.g., $()^T$ denotes the transpose, $()^*$ denotes the complex conjugate operation, $()^H$ denotes the Hermitian, underscore letters stand for column vectors and bold letters stand for matrices.

III. Joint MCRB for Frequency Offset and CIR

In this section we will derive the joint MCRB (JMCRB) for estimates of the frequency offset and the CIR, assuming the transmitted signals are known. First, we write the system model (2) in vector form as

$$\underline{y} = \frac{1}{\sqrt{M}} \mathbf{\Phi} \mathbf{W}^H \mathbf{X} \mathbf{W}_L \underline{h} + \underline{n}, \quad (5)$$

where $\underline{h} = [h_0, \cdots, h_{L-1}]^T$, $\underline{X} = [X(0), \cdots, X(M-1)]^T$, $\underline{y} = [y(0), \cdots, y(M-1)]^T$, $\underline{n} = [n(0), \cdots, n(M-1)]^T$, $\mathbf{\Phi} = [1, e^{j2\pi \frac{\epsilon}{M}}, \cdots, e^{j2\pi \frac{(M-1)\epsilon}{M}}]^T$ and $\underline{H} = \mathbf{W}_L \underline{h}$, $\mathbf{W}_L$ is a $M \times L$ submatrix of $\mathbf{W}$ with $e^{-j2\pi \frac{(i-1)(j-1)}{M}}$ as the element at the i^{th} row and j^{th} column. We also use the notation $\mathbf{X} = \text{diag}(\underline{X})$, which denotes a $M \times M$ matrix with $X(m)$ as its (m, m) entry and zeros elsewhere.

The probability density function of $\underline{y}$ given ϵ, $\underline{X}$ and $\underline{h}$ is

$$\begin{aligned} & f(\underline{y} | \epsilon, \underline{X}, \underline{h}) \\ &= \tfrac{1}{(2\pi\sigma_n^2)^M} \exp\left\{ -\tfrac{1}{2\sigma_n^2} \left\| \underline{y} - \tfrac{1}{\sqrt{M}} \mathbf{\Phi} \mathbf{W}^H \mathbf{X} \mathbf{W}_L \underline{h} \right\|^2 \right\}. \end{aligned} \quad (6)$$

We define the unknown parameters as

$$\underline{\theta} = [\epsilon, \underline{h}_R^T, \underline{h}_I^T]^T, \quad (7)$$

[1]In the literature, the term intersymbol interference (ISI) is used, but we believe inter-frame interference is more appropriate in this paper.

where $\underline{h}_R$ and $\underline{h}_I$ are the real and imaginary parts of the CIR $\underline{h}$. The joint CRLB gives a lower bound for the variance of an unbiased estimate of $\underline{\theta}$. This is

$$CRLB(\theta_i) = \mathbf{I}^{-1}(\underline{\theta})_{ii}, \quad (8)$$

where $\mathbf{I}(\underline{\theta})$ is the Fisher information matrix given by

$$\mathbf{I}(\underline{\theta})_{ij} = E_{\underline{y}}\left\{\frac{\partial}{\partial\theta_i}\log f(\underline{y}|\underline{\theta},\underline{X})\frac{\partial}{\partial\theta_j}\log f(\underline{y}|\underline{\theta},\underline{X})\right\} \quad (9)$$

A detailed derivation of the joint CRLB and MCRB is given in [2]. Here, we only give the results because of limited space:

$$MCRB(\epsilon) = \frac{3M\sigma_n^2}{2\pi^2\sigma_X^2(M-1)(2M-1)\sum_{l=0}^{L-1}|h(l)|^2}, \quad (10)$$

$$JMCRB(\epsilon) = \frac{3M\sigma_n^2}{\pi^2\sigma_X^2(M-1)(M+1)\sum_{l=0}^{L-1}|h(l)|^2}, \quad (11)$$

$$JMCRB(\underline{h}) = \frac{\sigma_n^2}{M\sigma_X^2}\left(2L + \frac{3(M-1)^2}{(M^2-1)}\right). \quad (12)$$

It is easy to find the following relationship between $JMCRB(\epsilon)$ and $MCRB(\epsilon)$:

$$\frac{JMCRB(\epsilon)}{MCRB(\epsilon)} = \frac{2(2M-1)}{(M+1)}, \quad (13)$$

which approaches 4 when M, the number of subcarriers, becomes very large. This implies that there is an approximately 6dB loss of $MCRB(\epsilon)$ when we do not know the CIR.

We also know the MCRB of CIR [3] when the frequency offset is zero or precisely known:

$$MCRB(\underline{h}) = \frac{2L\sigma_n^2}{M\sigma_X^2}. \quad (14)$$

Therefore, we find the following relationship between the MCRB and the joint MCRB of the CIR

$$\frac{JMCRB(\underline{h})}{MCRB(\underline{h})} = 1 + \frac{3(M-1)^2}{2L(M^2-1)}, \quad (15)$$

which depends on M and the channel delay spread L. As M goes to infinity, we have

$$\lim_{M\to\infty}\frac{JMCRB(\underline{h})}{MCRB(\underline{h})} = 1 + \frac{3}{2L}, \quad (16)$$

which means that the larger the channel delay spread, the smaller the relative degradation in the joint MCRB of the CIR in the presence of the channel frequency offset. Comparing this result with the 6dB loss of the frequency offset when the channel is known, the degradation in the CIR estimation is much smaller. Thus, we observe that the channel estimation accuracy is less affected by the presence of unknown frequency offset than the frequency offset estimation accuracy is affected by the unknown CIR. What remains to be done is to develop an algorithm that can achieve these joint lower bounds.

It is important to note that the joint MCRBs are independent of the actual values of frequency offset and CIR.

IV. Direct Joint ML Estimation Algorithm

The joint ML estimates of ϵ and $\underline{h}$ are the values that maximize the probability density function 6, or minimize the distance function $D(\epsilon,\underline{h})$

$$D(\epsilon,\underline{h}) = \left\|\underline{y} - \frac{1}{\sqrt{M}}\mathbf{\Phi}\mathbf{W}^H\mathbf{X}\mathbf{W}_L\underline{h}\right\|^2. \quad (17)$$

Thus, the ML estimates are

$$[\hat{\epsilon},\hat{\underline{h}}] = \arg\min_{\epsilon,\underline{h}} D(\epsilon,\underline{h}). \quad (18)$$

Taking gradients of $D(\epsilon,\underline{h})$ with respect to ϵ and $\underline{h}$ and setting them to zero, we have

$$\frac{\partial}{\partial\epsilon}D(\epsilon,\underline{h}) = \frac{2}{\sqrt{M}}\Im(\underline{y}^H\mathbf{\Psi}\mathbf{\Phi}\mathbf{W}^H\mathbf{X}\mathbf{W}_L\underline{h}) = 0, \quad (19)$$

$$\frac{\partial}{\partial\underline{h}}D(\epsilon,\underline{h}) = \frac{1}{\sqrt{M}}(\mathbf{W}_L^H\mathbf{X}^H\mathbf{W}\mathbf{\Phi}^H\underline{y})^* - \frac{1}{M}(\mathbf{W}_L^H\mathbf{X}^H\mathbf{X}\mathbf{W}_L\underline{h})^* = 0. \quad (20)$$

It is by no means straightforward to solve the above two equations to obtain the solution of the joint ML estimation problem, because we need to solve a set of equations with $2L+1$ unknown parameters. There is obviously no explicit solution for the direct joint minimization which is a nonlinear minimization problem.

V. Adaptive Joint ML Estimation Algorithm

The above minimization problem is actually a highly nonlinear optimization problem with respect to ϵ and $\underline{h}$, which can be solved by a steepest descent algorithm. It contains two steps as stated in the previous section. After finding an initial estimate of ϵ and $\underline{h}$ in the first step, we carry out the following standard steepest descent procedure

$$\epsilon^{p+1} = \epsilon^p - \lambda^p\nabla_\epsilon D(\epsilon^p,\underline{h}^p), \quad (21)$$

$$\underline{h}^{p+1} = \underline{h}^p - \mu^p\nabla_{\underline{h}} D(\epsilon^p,\underline{h}^p), \quad (22)$$

where ϵ^p and $\underline{h}^p$ are the p_{th} estimates of ϵ and $\underline{h}$, λ^p and μ^p are the step sizes and $\nabla_\epsilon D(\epsilon^p,\underline{h}^p)$ and $\nabla_{\underline{h}} D(\epsilon^p,\underline{h}^p)$ are the gradients of $D(\epsilon,\underline{h})$ at ϵ^p and $\underline{h}$.

However, a disadvantage of the steepest descent algorithm is its slow convergence rate. Rather than using the steepest descent algorithm for estimating the CIR in each iteration, we will iterate at each step using a simpler estimation procedure by assuming the frequency offset is known and focusing on the CIR estimation. Then assuming that the CIR is known, we apply the steepest descent algorithm to update the estimate of the frequency offset. This iterative procedure is repeated until convergence. To be more precise, we can obtain $\underline{h}_{k+1}$ from a simpler least-squares (LS) estimate as

$$\underline{h}^{p+1} = \sqrt{M}(\mathbf{W}_L^H\mathbf{X}^H\mathbf{X}\mathbf{W}_L)^{-1}\mathbf{W}_L^H\mathbf{X}^H\mathbf{W}(\mathbf{\Phi}^{p+1})^H\underline{y}, \quad (23)$$

where $\mathbf{\Phi}^{p+1} = [1, e^{j2\pi\frac{\epsilon^{p+1}}{M}}, \cdots, e^{j2\pi\frac{(M-1)\epsilon^{p+1}}{M}}]^T$

The initial estimates of the CIR are obtained by using simple LS algorithm assuming there is no frequency offset, i.e., $\epsilon = 0$.

This assumption is suitable for the case when the receiver has no knowledge about the exact fractional part of the frequency offset. After obtaining the initial estimates of CIR, we use the time domain (TD) estimation algorithm to obtain the initial estimate of ϵ, assuming the estimates of CIR are perfect. At each time instance index k we compute an estimate ϵ_k^0 of ϵ as

$$k\epsilon_k^0 = \frac{M}{2\pi}\text{angle}\left(\frac{y(k)}{\sum_{l=0}^{L-1} h_l x(k-l)}\right), \quad 1 \le k \le M-1. \quad (24)$$

Then, we combine these $M-1$ initial estimates of ϵ to obtain the actual initial estimate of ϵ as

$$\epsilon^0 = \frac{2}{M(M-1)} \sum_{k=1}^{M-1} k\epsilon_k^0 \quad (25)$$

The above joint estimation algorithm is especially desirable when we use OFDM preambles or training frames, which do not have particular structure. However, in a practical OFDM system only some pilot symbols are inserted in the time-frequency grid. In order to apply the adaptive joint estimation algorithm work to this more practical framework, we need to make some modifications in the above algorithm. We have to replace $\mathbf{X}$ by $\mathbf{X}^p$ which is the p^{th} estimates of the transmitted signal. In particular, Equation (23) becomes

$$\underline{h}^{p+1} = \sqrt{M}(\mathbf{W}_L^H(\mathbf{X}^p)^H\mathbf{X}^p\mathbf{W}_L)^{-1}\mathbf{W}_L^H(\mathbf{X}^p)^H\mathbf{W}(\mathbf{\Phi}^{p+1})^H\underline{y},$$

and the signal detection procedure is carried out by using simple division and signal mapping (i.e., hard decision)

$$\underline{X}^{p+1} = \text{Hard Decision}\left\{\frac{\underline{Y}}{\mathbf{W}_L\underline{h}^{p+1}}\right\}, \quad (26)$$

where the division is component-wise division of two vectors.

The following simulation results verify the effectiveness of the above joint ML estimation algorithm. Although the initial estimates of the frequency offset and CIR are very poor, especially when the frequency offset is large, the joint estimation algorithm appears to converge to the correct point for both the frequency offset and the CIR. This is showed in Figure 2 and 3. These two figures also validate our derivation of the joint modified CRLB. Unlike the case when the CIR is known and fixed, the MSE of the frequency offset can not achieve the joint modified CRLB. However, the difference is very small. Furthermore, the performance of different frequency offsets is the same. Unlike the frequency offset, the joint modified CRLB of CIR can always be achieved by the algorithm whether or not the channel is slowly changing during the transmission.

Figure 4, 5 and 6 show the performance of the MSE of frequency offset, MSE of CIR and BER of the system, respectively, when only 8 pilot symbols are known for those OFDM frames with pilots inserted. The performance degrades, but not that much, especially when the SNR is large. In particular, the MSE of the frequency offset has an approximately 2dB loss when $MSE(\epsilon) = 10^{-4}$. However, the performance degradation of $MSE(\underline{h})$ depends on the frequency offset itself. A larger frequency offset leads to more degradation of $MSE(\underline{h})$. The same observation can be made for the BER performance. All the degradation comes from the erroneous detection of the transmitted signals. Note that we simulated an uncoded OFDM system. Inclusion of a channel coding scheme should improve the overall performance.

Figure 7 shows the mean frequency offset, when the joint ML estimation algorithm is adopted. It is clear that the joint ML estimates of the frequency offset are unbiased when all the transmitted symbols are known. However, if only some pilot symbols are known, the estimates of the frequency offset become biased when E_b/N_0 is small, say less than 14dB. Furthermore, the biased estimated values are always smaller (i.e., negative bias) than the actual frequency offset.

VI. Conclusion

We have considered the problem of joint frequency offset and channel estimation for OFDM systems. The joint MCRB is derived for this problem. It is shown that there is approximately a 6dB loss in the MCRB due to lack of knowledge of the channel. The performance loss of the channel estimation depends on the channel delay spread. Since the joint ML estimation is seen to be very complex, an iterative procedure is developed for the joint estimation: a straightforward LS estimate of the CIR is made assuming the frequency offset is known and then the time domain frequency offset estimation algorithm is used assuming the CIR is known. By means of simulation, it is shown that this procedure is effective, converges to the correct points, and comes close to achieving the joint MCRB for the frequency offset and the CIR. Furthermore, the frequency offset estimate is unbiased when the E_b/N_0 is larger than 14dB.

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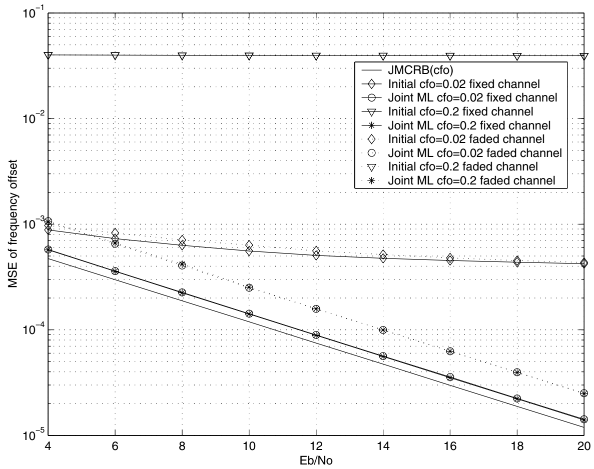

Fig. 2. Mean square error of the joint ML estimation of the frequency offset when the transmitted signals are known

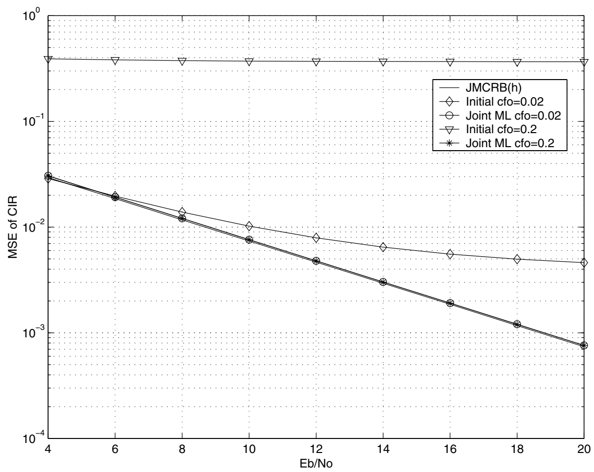

Fig. 3. Mean square error of the joint ML estimation of the CIR when the transmitted signals are known

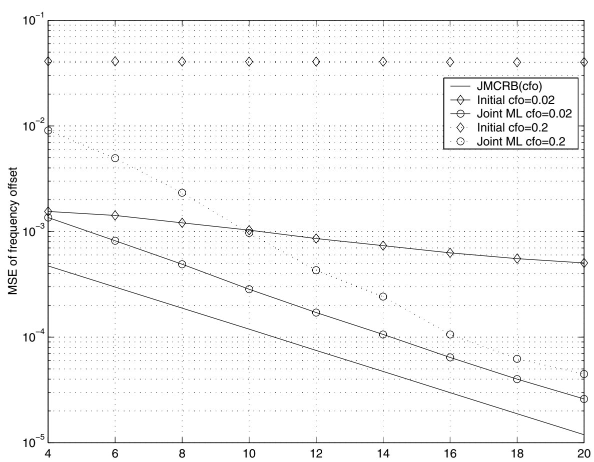

Fig. 4. Mean square error of the joint ML estimation of the frequency offset when only pilot symbols are known

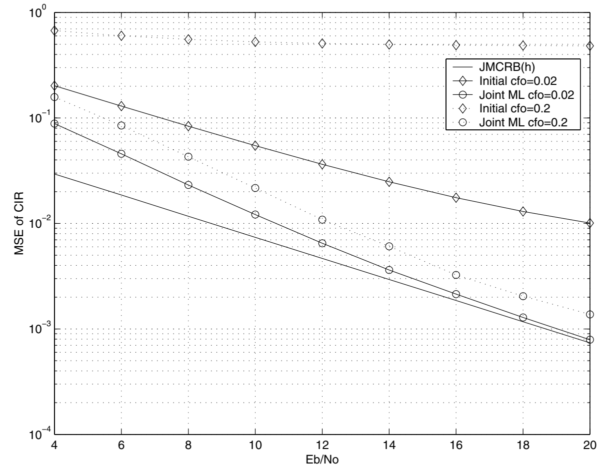

Fig. 5. Mean square error of the joint ML estimation of the CIR when only pilot symbols are known

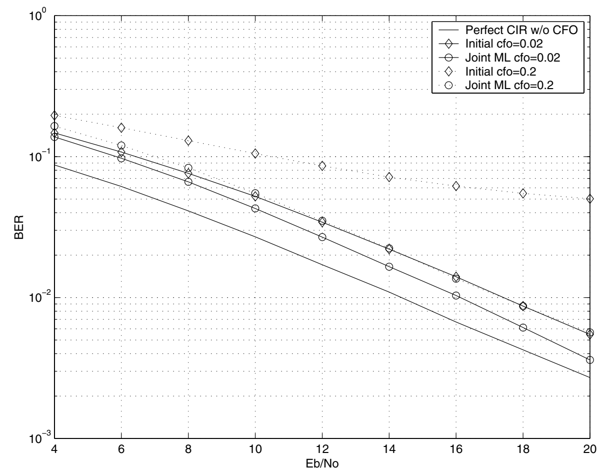

Fig. 6. Bit error rate of the joint ML estimation algorithm when only pilot symbols are known

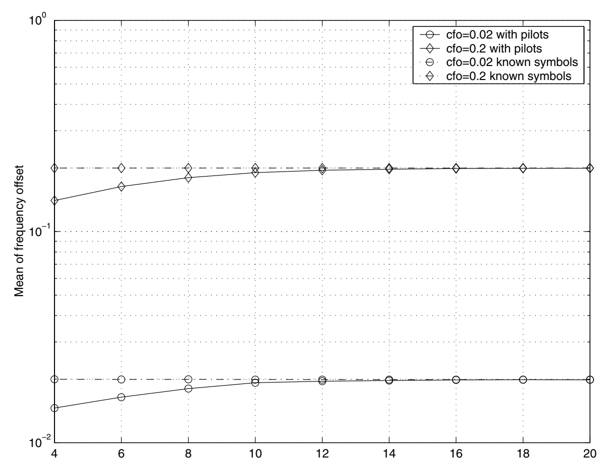

Fig. 7. Mean of the frequency offset both for known $\underline{X}$ and pilot symbols only via the joint ML estimation algorithm