

# Joint Integrated Probabilistic Data Association - JIPDA\*

**Darko Mušicki**

CSSIP

Department of Electrical and Electronics Eng.

The University of Melbourne

Parkville, VIC 3010

Australia

[d.musicki@ee.mu.oz.au](mailto:d.musicki@ee.mu.oz.au)

**Rob Evans**

CSSIP

Department of Electrical and Electronics Eng.

The University of Melbourne

Parkville, VIC 3010

Australia.

[r.evans@ee.mu.oz.au](mailto:r.evans@ee.mu.oz.au)

**Abstract** - *This paper presents a new algorithm for multi-target tracking. In multi-target situations, multiple tracks may share the same measurement(s). Joint events are formed by creating all possible combinations of track-measurement assignments. The probabilities for these joint events are calculated. The expressions for the joint events incorporate the probabilities of track existence of individual tracks, as well as an efficient approximation for the cluster volume and an a-priori probability of the number of clutter measurements in each cluster. From these probabilities the data association and track existence probabilities of individual tracks are obtained. These probabilities will allow track update in the classic PDA fashion, as well as automatic track initiation, maintenance and termination. The JIPDA algorithm is recursive and integrates seamlessly with the IPDA algorithm. Simulations are used to verify the performance of the algorithm and compare it with the performance of the IPDA, IPDA-DLL and IJPDA algorithms in a dense and non-homogenous clutter environment, in crossing target situations.*

**Keywords:** IPDA, Joint IPDA, IPDA-DLL, IJPDA, PDA, Joint PDA, data association, target tracking, estimation.

## 1 Introduction

Data association algorithms deal with situations where there are measurements of uncertain origin. In many radar and sonar applications, for example, measurements (detections) originate not only from targets being tracked, but also from thermal noise as well as from various obstacles such as terrain, clouds etc. Unwanted measurements are often termed clutter. Furthermore, true measurements from the target are present in each scan only with a certain probability of detection. In a multi-target situation, the measurements may have originated from one of various targets.

Automatic track initiation and termination under such conditions require some knowledge about track existence. A track exists if it is based on a target (which

follows a specified dynamic and detection model) measurements, and is not a product of random clutter. If a track follows a target, we shall call it a true track otherwise we shall call it a false track.

One of the most often used algorithms for data association in target tracking is Probabilistic Data Association-PDA [1,2]. PDA uses all measurements in the validation gate (window) of the track being updated and approximates the probability distribution function of the target state after each update with a Gaussian probability distribution. All algorithms mentioned in the text below use the same approach and the same approximation.

Unfortunately, the PDA algorithm is derived conditioned on target existence, which effectively removes the target existence information. The Integrated PDA (IPDA) [3,4,5,6,7,8] algorithm does not assume target existence and provides data association (PDA) formulae together with expressions for probability of target existence in a recursive manner. Data Association coefficients (denoted by  $\beta$  in [1]) are identical for the PDA and IPDA algorithms. A different approach was taken in [9, 10] where it is assumed that a target exists behind each track, and the probability of perceivability of the target is recursively calculated instead of the probability of the existence of the target. In spite of the differences between these algorithms, the authors [10] have elected to reuse the name IPDA for that algorithm as well. To differentiate between the two algorithms, we use the acronym IPDA-DLL for the algorithm presented in [10] (DLL being the authors' surname initials).

IPDA and PDA are derived under the assumption of a single target (single track). Each measurement can be either clutter or a measurement of the target being followed. In real-life situations with multiple targets with crossing trajectories, this is no longer true. It has been shown [11,12] that PDA can get "confused" under these circumstances and start following a different target, or it can diverge altogether and stop following any target. To

---

\* Work supported by CSSIP Melbourne and DSTO

remedy this situation, the Joint PDA (JPDA) [11,12] algorithm has been created.

The JPDA algorithm allows the possibility that a measurement may have originated by one of a number of candidate tracks or by clutter. In each scan JPDA partitions tracks into clusters, where tracks in each cluster have common measurements. It generates all possible joint measurement to track assignments, which are called joint events, and calculates the a-posteriori probability of each joint event. From these probabilities, the data association coefficients of each track are calculated and then used to update the track estimates.

JPDA has the same problem as PDA, since it assumes that the target(s) exist. Tracks are not differentiated according to the probability of target existence, and track maintenance is difficult without the probability of target existence information. JPDA is also rather complex because it creates a joint event for each possible combination of measurement origin. The number of joint events can grow very rapidly in a dense clutter situation. Another problem is that the area of each cluster is assumed to encompass the whole surveillance region.

To improve upon JPDA, the Integrated JPDAF (IJPDA) algorithm [13] has been published. It builds upon IPDA-DLL [10] and also uses the probability of target perceivability to develop recursive expressions for the a-posteriori probability of target perceivability and data association for each track. The number of joint events is much higher than in the case of JPDA. The IJPDA also assumes the area of each cluster to encompass the whole surveillance region.

The Joint IPDA (JIPDA) algorithm (dealing with Joint IPDA tracks), presented in this paper, is developed in a similar fashion to the IPDA algorithm. It uses the probability of target existence and results in recursive expressions for the probability of target existence and data association coefficients. The number of joint events is the same as in the case of JPDA. JIPDA uses an efficient approximation to calculate the volume of each cluster (no longer is the entire surveillance region used), and uses a better approximation for the number of false measurements within the cluster. When a cluster consists of a single track, the JIPDA becomes identical to IPDA. Thus, JIPDA integrates seamlessly with IPDA in the sense that a track can be processed with either (as the circumstances dictate), with no transition effects when switching from one to the other, common thresholds etc.

The original IPDA algorithm and its derivatives [3,4,5,6] have two models of target existence propagation. Markov Chain One recognizes two possibilities: the target either does not exist, or it exists and is visible with a probability of detection. Markov Chain Two recognizes

the possibility of target existing, but not being visible, in addition to the two possibilities of the Markov Chain One. This paper will present only the Markov Chain One version of JIPDA.

Section 2 defines the individual target cluster and the cluster area approximation and the a-priori estimated number of clutter measurements in the cluster. The joint events and associated a-posteriori probabilities are presented in Section 3, together with the a-posteriori probabilities of each track's target existence and data association coefficients. The use of the data association coefficients to update each track's estimation is well covered in [1,2,4,11], as well as other publications, and will not be repeated in this paper. Simulation is used to show the improvements of JIPDA over the IPDA, IPDA-DLL and IJPDA algorithms in crossing targets situations in a dense and non-uniform clutter situation. Simulation results are presented in Section 4, followed by concluding remarks in Section 5.

## 2 Cluster Overview

JIPDA allows the possibility that multiple tracks interfere with each other. This happens when two or more tracks have at least one common measurement in their validation gates (windows) in a particular scan.

In each scan, tracks are grouped (partitioned) into clusters. A cluster is a set of tracks, which share no measurements with any track not belonging to the cluster. Thus, a single, isolated, track is a cluster. A trivial cluster is the set of all tracks, however, as the number of operations grows exponentially with the number of tracks in a given cluster, each cluster should contain a minimal set of tracks fulfilling the definition.

A single cluster of tracks in one scan is examined below. Any track and any measurement mentioned below belong to the same cluster and cluster area respectively.

Let  $T$  denote the number of the tracks in the cluster, let  $m$  and  $m_t$  denote the total number of measurements in the cluster and the number of measurements in the window of track  $t$  respectively and let  $V_t$  denote the window area of track  $t$ . The cluster area, with volume denoted by  $V$ , is a union of individual track windows. The approximate expression for  $V$  used in JIPDA is

$$V_{ap} = \frac{m}{\sum_{t=1}^T m_t} \sum_{t=1}^T V_t \quad (1)$$

$$V = \max(V_{\max}, V_{ap}) \quad (2)$$

where  $V_{\max} = \max(V_t)$  is the maximum window area of individual tracks.

The a-priori estimated number of clutter measurements  $\hat{m}$  is

$$\hat{m} = \sum_{i=1}^m \left( \prod_{t=1}^T \left( 1 - \frac{P_D^t P_W^t P_{k,k-1}^t}{m_t} \right)^{\mu(t,i)} \right) \quad (3)$$

where  $P_D^t$ ,  $P_W^t$  and  $P_{k,k-1}^t$  denote the probability of detection, gating probability and the a-priori probability of track existence respectively for track  $t$ .  $\mu(t,i)$  is one if measurement  $i$  is in the window of track  $t$  and zero otherwise.

### 3 JIPDA Data Association

There typically exist a number of possible assignments of measurements to tracks and we consider each feasible assignment to be a separate joint event. The joint events generated with JIPDA are the same as the joint events generated for the JPDA algorithm [11]. The following constraints must be observed for each joint event,

- Each track can be assigned zero measurements or one of the measurements which falls in the individual window of the track.
- Each measurement can be allocated to zero or one of the existing tracks.

Two joint events are different if assignment of at least one measurement is different. The joint events generated in this manner are mutually exclusive, and they should form a complete set. The joint events can be generated in many ways [14,15] and the process is formally described in [8].

Let  $\chi_i$  denote the joint event  $i$ , and let  $\mathbf{X}$  denote the number of joint events in the cluster. Let  $T0$  and  $T1$  denote the set of tracks allocated no measurements, and the set of tracks allocated one measurement respectively in the joint event. The a-posteriori probability of  $\chi_i$  :

$$P\{\chi_i | Z^k\} = C^{-1} \prod_{t \in T0} (1 - P_D^t P_W^t P_{k,k-1}^t)$$

$$\prod_{t \in T1} \left( P_D^t P_W^t P_{k,k-1}^t \frac{P_i^t V}{\hat{m}} \right) \quad (4)$$

where  $P_i^t$  is the a-priori conditional probability density of the measurement allocated to track  $t$ , conditioned on it appearing within the track window. It is usually approximated with a truncated Gaussian density as detailed in [1].

The joint events form a complete set and the constant  $C$  is calculated using

$$\sum_{j=1}^{\mathbf{X}} P\{\chi_j | Z^k\} = 1 \quad (5)$$

The a-posteriori probabilities of individual track events are obtained by summing the a-posteriori probabilities of all joint events containing the event. Denote with  $\Xi(t,i)$  the set of joint events in which track  $t$  has been allocated measurement  $i$ , with measurement 0 denoting no measurement. Set  $\Xi(t,i)$  may be empty.

The a-posteriori probability of no measurement originating from the track  $t$  is

$$P\{\chi_0^t | Z^k\} = \sum_{\chi_e \in \Xi(t,0)} P\{\chi_e | Z^k\} \quad (6)$$

and the a-posteriori probability that track  $t$  exists and that measurement  $i$  originated from the track  $t$  is

$$P\{\chi^t \chi_i^t | Z^k\} = \sum_{\chi_e \in \Xi(t,i)} P\{\chi_e | Z^k\} \quad (7)$$

The a-posteriori probability that track  $t$  exists and that no measurements have originated from track  $t$  is

$$P\{\chi^t \chi_0^t | Z^k\} = \frac{(1 - P_D^t P_W^t) P_{k,k-1}^t}{1 - P_D^t P_W^t P_{k,k-1}^t} P\{\chi_0^t | Z^k\} \quad (8)$$

The a-posteriori probability of track existence of track  $t$  is

$$P_{k,k}^t = P\{\chi^t \chi_0^t | Z^k\} + \sum_{i \in \{\mu(t,i) > 0\}} P\{\chi^t \chi_i^t | Z^k\} \quad (9)$$

where  $\{\mu(t,i) > 0\}$  denotes the set of measurements falling in the window of track  $t$ .

The  $\beta$  parameters for track  $t$  are

$$\beta_0^t = \frac{P\{\chi^t \chi_o^t | Z^k\}}{P_{k,k}^t} \quad (10)$$

$$\beta_i^t = \frac{P\{\chi^t \chi_i^t | Z^k\}}{P_{k,k}^t}; i \in \{\mu(t,i) > 0\} \quad (11)$$

The  $\beta$  parameters are used for track estimation [1,2,4,10].

When the cluster contains one track only, the JIPDA data association and the probability of track existence expressions become identical to the IPDA data association and the probability of track existence expressions.

## 4 Simulation

The purpose of the simulation below is to compare the JIPDA algorithm with IPDA, IPDA-DLL and IJPDA with respect to the track discrimination and target crossing situation outcome, in a heavy and non-homogenous clutter environment.

Tracks are initiated automatically, using two-point differencing and initial track probability assignment as described in [7,8]. Tracks get confirmed if the probability of target existence exceeds the confirmation threshold and are terminated if the probability falls below the termination threshold. Termination thresholds were kept separate for confirmed and unconfirmed tracks. For reasons of simplicity, the thresholds were kept constant during each simulation experiment, although better results would be obtained if they were made dependent on track 'age'. In this case, JIPDA and IJPDA are computationally not feasible on all tracks; thus they are implemented on confirmed tracks only. In the case of JIPDA, IPDA is used on non-confirmed tracks and in the case of IJPDA, IPDA-DLL is implemented on non-confirmed tracks. In IPDA and IPDA-DLL experiments, IPDA and IPDA-DLL are applied to all tracks after initialization.

The sum of confirmed false track scans was approximately equal for each simulation experiment and in the vicinity of 600 over 24000 scans in each of the simulation experiments.

A two-dimensional surveillance situation was considered. The area under surveillance was 1000 m long and 400 m wide. The false measurements satisfied a

Poisson distribution with density  $1.0 \times 10^{-4}$  /scan /m<sup>2</sup> with two patches with sevenfold clutter density. The high clutter density patches are rectangular with corner coordinates  $(X_{\min}, X_{\max}, Y_{\min}, Y_{\max})$  of (330, 490, 203, 303)m and (718, 840, 100, 200)m.

Each experiment consisted of 1000 runs with each run consisting of 24 scans. In each simulation run one target reappears in scan 1 with an initial state of  $x'(1) = [130m \ 35m/s \ 200m \ 0m/s]$ , and maintains constant speed thereafter. This trajectory just edges the high-clutter areas and this will tend to 'diverge' tracks away from the true trajectory and into the high intensity clutter, thus turning true tracks into false ones. The other target always follows a second straight-line uniform speed trajectory, designed to intersect the first target trajectory in scan 19, with the angle of the two trajectories being 10°. The true track situation is observed on scan 14 and then again on scan 24. False tracks are carried over from one simulation run to the other, in order to simulate long and continuous operation.

The target motion is modelled in Cartesian coordinates as

$$x(k+1) = Fx(k) + v(k) \quad (12)$$

where  $x(k)$  is the target state vector at time  $k$  and consists of the position and velocity in each of the 2 coordinates

$$x' = [x \ \dot{x} \ y \ \dot{y}] \quad (13)$$

with transition matrix

$$F = \begin{bmatrix} F_T & 0 \\ 0 & F_T \end{bmatrix}; F_T = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad (14)$$

where  $T$  is the sampling period of 1s. The plant noise  $v(k)$  is zero-mean white Gaussian noise with known covariance

$$E[v(k)v(j)'] = Q\delta(k, j) \quad (15)$$

where  $\delta(k, j)$  is the Kronecker delta function and

$$Q = q \begin{bmatrix} Q_T & 0 \\ 0 & Q_T \end{bmatrix}; Q_T = \begin{bmatrix} T^4/4 & T^3/2 \\ T^3/2 & T^2 \end{bmatrix} \quad (16)$$

with  $q = 0.75$ .

The detection probability was 0.9 throughout the experiment and the sensor introduced independent errors in x and y coordinates with the root mean square of 5m. The tracking filter was a simple Kalman filter based on the described trajectory and sensor models.

The selection window size for both algorithms was chosen so that the a-priori windowing probability  $P_w$  was 0.9999.

The IPDA and JIPDA algorithms used a Markov Chain One target existence propagation model with parameters

$$\begin{aligned} p_{11} &= 0.98; & p_{21} &= 0; \\ p_{12} &= 0.02; & p_{22} &= 1. \end{aligned} \tag{17}$$

The same parameters are used to model the target perceivability for IPDA-DLL and IJPDA algorithms.

Tracks were declared to be true tracks, if they were:

- Selecting target measurements for at least two consecutive scans in which the target was detected;
- Have not missed target measurement more than one consecutive scan in which the target was detected since;
- The track estimated state was within the predefined boundary of true target state.

The target crossing performance of the JIPDA, IPDA, IPDA-DLL and IJPDA algorithms are shown in Table 1. As IPDA and IPDA-DLL are used on all unconfirmed tracks, and JIPDA becomes identical to IPDA for one-track clusters, only cases where two confirmed tracks were following each of the two targets were considered.

Five possible outcomes were recognized on scan 24:

- a) Both tracks continue to follow their original targets;
- b) Only one track continues to follow the original target. The fate of the other target is irrelevant;
- c) Both tracks switch targets,
- d) One track switches target, the other becomes false or terminated,
- e) Both tracks become false or terminated.

Table 1: Trajectory intersection results

	JIPDA	IJPDA	IPDA	IPDA-DLL
Total	487	280	478	271
(a)	470	271	230	186
(b)	17	6	225	84
(c)	0	2	0	0
(d)	0	1	17	1
(e)	0	0	6	0

The number of successful outcomes (a) shows superior performance of JIPDA over IPDA and IJPDA. It also shows the advantages of using multi-target algorithms (JIPDA and IJPDA) in target-crossing situations over their single-target counterparts (IPDA and IPDA-DLL respectively).

Apparently better relative target-crossing results of IPDA-DLL algorithm over IPDA are the result of comparatively poor track discrimination performance of IPDA-DLL, as shown in a much smaller number of cases (the ‘‘Total’’ row of Table 1). Thus, only tracks with better detection, less clutter and smaller measurement noise are confirmed using IJPDA and IPDA-DLL, resulting in tracks with smaller estimation errors as input to the trajectory intersection situation comparison.

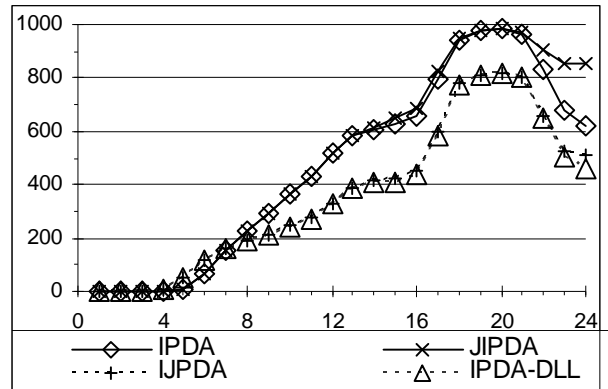


Figure 1 Track Discrimination Comparison – target one

The track discrimination performance of the algorithms is illustrated in Figure 1 for the first target. Corresponding curves for the second target follow the same pattern. Each curve shows the number of scans in which a confirmed track using one of the algorithms was following one of the targets. The horizontal axis depicts the time in scans from the start of the simulation run. The curves are almost identical for both IPDA and JIPDA until

the time of target crossing, after which JIPDA performs better. The outcome is similar for IPDA-DLL and IJPDA.

The JIPDA algorithm clearly improves the IPDA algorithm in track-crossing situations. It also improves the track discrimination performance of IPDA. It was observed that JIPDA, when used in the manner described, adds insignificant time to the simulations. The IJPDA algorithm also improves the IPDA-DLL algorithm in a similar manner; however their respective performances appear to be considerably worse than JIPDA and IPDA respectively in this environment.

## 5 Conclusions

This paper introduces the Joint IPDA algorithm for tracking multiple targets in clutter. JIPDA integrates seamlessly with Integrated Probabilistic Data Association (IPDA). A track can be followed by either of the algorithms as the situation dictates, and changing from one to the other require no change to the target state and incurs no transient effects. Furthermore, applying JIPDA to a single-track cluster results in IPDA.

The simulation experiments presented in this paper indicate that use of JIPDA is beneficial as it enhances both track discrimination and crossing-track performance.

## References

- [1] Y. Bar-Shalom, and E. Tse, *Tracking in a cluttered environment with Probabilistic Data Association*, Automatica, vol. 11, pp 451-460, Sept. 1975.
- [2] Y. Bar-Shalom, *Tracking Methods in a Multitarget Environment*, (1978) IEEE Trans. Automatic Control, Vol.23, No.4.
- [3] D. Mušicki and R. Evans, *Integrated probabilistic data association - finite resolution*, 32nd IEEE Conference on Decision and Control, San Antonio, Texas, Dec.16-18, 1993, pp. 912-917.
- [4] D. Mušicki, R. Evans and S. Stanković, *Integrated probabilistic data association (IPDA)*, IEEE Trans. Automatic Control, AC-39, no. 6, pp 1237-1241, June 1994.
- [5] D. Mušicki, *Automatic tracking of maneuvering targets in clutter using IPDA*, PhD thesis, University of Newcastle, New South Wales, Australia, Sept 1994.
- [6] D. Mušicki and R. Evans, *Integrated probabilistic data association - finite resolution Automatica*, April 1995.
- [7] D. Mušicki and R. Evans, *Data Association using clutter map information*, WOSPA conference, Brisbane, Australia, December 1997.
- [8] D. Mušicki and R. Evans, *Clutter Map Information for Data Association and Track Initialization*, Submitted for publishing, IEEE Trans. Aerospace and Electronic Systems.
- [9] S. B. Colegrove and J.K. Ayliffe *An extension of Probabilistic Data Association to Include Track Initiation and Termination* 20<sup>th</sup> IREE International Convention, Melbourne 1985, pp 853-856.
- [10] J. Dezert, N. Li and X. R. Li *A New Formulation of JPDAF for Tracking in Clutter*, European Control Conference (ECC99), Karlsruhe, Germany, Sept. 1999.
- [11] T.E. Formann, Y.Bar-Shalom and M. Scheffe *Sonar Tracking of Multiple Targets Using Joint Probabilistic Data Association* IEEE Joun. Oceanic Engineering, OE-8, No.3, pp 173-183, July 1983.
- [12] K.C.Chang and Y.Bar-Shalom, *Joint Probabilistic Data Association for Multitarget Tracking with Possibly Unresolved Measurements and Maneuvers* IEEE Trans. Automatic Control, AC-29, pp 585-594, July 1984.
- [13] J. Dezert, N. Li and X. R. Li *Theoretical development of an Integrated JPDAF for multitarget tracking in clutter* Proc. Workshop ISIS-GDR/NUWC, ENST, Paris, 1998.
- [14] I. J. Cox and M.L. Miller, *On Finding Ranked Assignments with Application to Multitarget Tracking and Motion Correspondence*, IEEE Trans AES, vol. AES-31, no. 1, pp. 486-489 January 1995.
- [15] R. Danchick and G.E. Newman, *A Fast Method for Finding the Exact N-best Hypotheses for Multitarget Tracking*, IEEE Trans. AES, vol. AES-29, no. 2, pp. 555-560, April 1993.