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JOINT MIMO CHANNEL TRACKING AND SYMBOL DECODING FOR ORTHOGONAL SPACE-TIME BLOCK CODES

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ABSTRACT

In this paper, we consider the problem of channel tracking for a MIMO communication system which uses orthogonal space-time block codes (OSTBCs) as the underlying space-time coding scheme. We propose a two-step tracking algorithm. As the first step, Kalman filtering is used to obtain an initial channel estimate for the current block based on the channel estimates obtained for previous blocks. Then, in the second step, the so-obtained initial channel estimate is refined using a decision-directed iterative method. We show that due to specific properties of OSTBCs, both the Kalman filter and the decision-directed algorithm can be significantly simplified. Simulation results show that the proposed tracking method can provide accurate enough channel estimates.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) communications and space-time coding have been the focus of extensive research efforts. Among different space-time coding schemes presented in the literature, orthogonal space-time block codes (OSTBCs) [1], [2] are of particular interest because they achieve full diversity at an affordable receiver complexity. Indeed, given the MIMO channel, the maximum likelihood (ML) optimal receiver for OSTBCs consists of a linear receiver followed by a symbol-by-symbol decoder. Also, it has recently been shown in [3] that for majority of OSTBCs, the MIMO channel is blindly identifiable. However, this interesting property of OSTBCs is based on the assumption that the channel is fixed during a long enough time interval. In practice, the channel may however be time-varying due to the mobility of the transmitter and/or receiver, as well as due to the carrier frequency mismatch between the transmitter and receiver. Therefore, channel tracking is an essential processing in these cases.

Kalman filtering based channel tracking has been studied in [4] in application to channel tracking for MIMO communication systems which use Alamouti scheme as the underlying OSTBC. In this paper, we extend the result of [4] for any type of OSTBCs and show that Kalman filtering can be significantly simplified due to the specific structure of OSTBCs. Unlike [4], we assume that the channel is fixed during transmission of each block of data, and it can only change between blocks. Based on such an assumption, we develop a two-step channel tracking algorithm. In the first step, Kalman filtering is used at the beginning of each block to track the channel. In the second step, to improve the quality of the channel estimate obtained by Kalman filtering, we propose a simple iterative channel estimation technique. This iterative method is in fact a decision-directed algorithm and it consists of sequential use of a linear receiver and a linear channel estimator.

2. BACKGROUND

Consider a MIMO system with N transmitter and M receiver antennas. In a time-varying flat-fading channel scenario, the received signal, at time t, can be written as

$$\mathbf{y}(t) = \mathbf{x}(t)\mathbf{H}(t) + \mathbf{v}(t) \tag{1}$$

where $\mathbf{H}(t)$ is the $N \times M$ channel matrix with its (i, j) element equal to the *time-varying* channel coefficient between the ith transmit antenna and the jth received antenna, $\mathbf{y}(t)$ is the $1 \times M$ vector of received data, and $\mathbf{x}(t)$ is the $1 \times N$ vector of transmit data.

We consider a block transmission scheme and assume that within the block period T, the channel is fixed, i.e., the channel is assumed to be *quasi-static*. However, between different blocks the channel can change. Based on such an assumption, the nth received block can be written as

$$\mathbf{Y}(n) = \mathbf{X}(n)\mathbf{H}(n) + \mathbf{V}(n) \tag{2}$$

where

$$\mathbf{Y}(n) \triangleq [\mathbf{y}^T(nT - T + 1) \cdots \mathbf{y}^T(nT)]^T \tag{3}$$
$$\mathbf{X}(n) \triangleq [\mathbf{x}^T(nT - T + 1) \cdots \mathbf{x}^T(nT)]^T \tag{4}$$
$$\mathbf{V}(n) \triangleq [\mathbf{v}^T(nT - T + 1) \cdots \mathbf{v}^T(nT)]^T \tag{5}$$

are the nth block of the received signals, transmitted signals, and noise, respectively, and $(\cdot)^T$ denotes the transpose operator. The noise is assumed to be zero-mean complex Gaussian and spatio-temporally white with variance $\sigma_v^2/2$ per real dimension.

In space-time block coding, the matrix $\mathbf{X}(n)$ is a mapping which transforms a block of complex symbols to a $T \times N$ complex matrix. Hence, we hereafter replace $\mathbf{X}(n)$ with $\mathbf{X}(\mathbf{s}(n))$ where $\mathbf{s}(n)$ is the nth symbol vector of length K. Let us denote $\mathbf{s}(n)$ as $\mathbf{s}(n) = [s_1(n)\ s_2(n)\ \cdots\ s_K(n)]^T$. The $T \times N$ matrix $\mathbf{X}(\mathbf{s}(n))$ is called an OSTBC [1], [2] if all elements of $\mathbf{X}(\mathbf{s}(n))$ are linear functions of the K complex variables $\{s_k(n)\}_{k=1}^K$ and their complex conjugates, and if, for any arbitrary $\mathbf{s}(n)$, $\mathbf{X}(\mathbf{s}(n))$ satisfies:

$$\mathbf{X}^H(\mathbf{s}(n))\mathbf{X}(\mathbf{s}(n)) = \|\mathbf{s}(n)\|^2\mathbf{I}_N \tag{6}$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix, $\|\cdot\|$ is the Euclidean norm, and $(\cdot)^H$ denotes Hermitian transpose. It follows from the definition of OSTBCs that the matrix $\mathbf{X}(\mathbf{s}(n))$ can be written as

$$\mathbf{X}(\mathbf{s}(n)) = \sum_{k=1}^{K} (\mathbf{C}_k \mathrm{Re}\{s_k(n)\} + \mathbf{D}_k \mathrm{Im}\{s_k(n)\}) \tag{7}$$

where $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote the real and imaginary parts, and $\mathbf{C}_k$ and $\mathbf{D}_k$ matrices are defined as

$$\mathbf{C}_k = \mathbf{X}(\mathbf{u}_k) \quad \text{and} \quad \mathbf{D}_k = \mathbf{X}(j\mathbf{u}_k) \tag{8}$$

where $\mathbf{u}_k$ is the kth column of the identity matrix $\mathbf{I}_K$ and $j = \sqrt{-1}$. Let us define the "underline" operator for a matrix $\mathbf{P}$ as $\underline{\mathbf{P}} \triangleq \left[\text{vec}^T\{\text{Re}(\mathbf{P})\}\ \text{vec}^T\{\text{Im}(\mathbf{P})\}\right]^T$ where $\text{vec}\{\cdot\}$ refers to the vectorization operator stacking all the columns of a matrix on top of each other. Using (7), we rewrite (2) as

$$\tilde{\mathbf{y}}(n) \triangleq \underline{\mathbf{Y}(n)} = \mathbf{A}(\mathbf{H}(n))\,\tilde{\mathbf{s}}_n + \tilde{\mathbf{v}}_n \tag{9}$$

where the following definitions $\tilde{\mathbf{s}}_n \triangleq \underline{\mathbf{s}(n)}$ and $\tilde{\mathbf{v}}_n \triangleq \underline{\mathbf{V}(n)}$ are used and the $2MT \times 2K$ *real* matrix $\mathbf{A}(\mathbf{H}(n))$ is given by

$$\mathbf{A}(\mathbf{H}(n)) = [\underline{\mathbf{C}_1\mathbf{H}(n)} \ldots \underline{\mathbf{C}_K\mathbf{H}(n)}\ \underline{\mathbf{D}_1\mathbf{H}(n)} \ldots \underline{\mathbf{D}_K\mathbf{H}(n)}].$$

It has been shown that for any channel matrix $\mathbf{H}(n)$, the matrix $\mathbf{A}(\mathbf{H}(n))$ satisfies the so-called *decoupling* property, i.e., its columns are orthogonal to each other and they have identical norms [7]. More specifically, it satisfies:

$$\mathbf{A}^T(\mathbf{H}(n))\,\mathbf{A}(\mathbf{H}(n)) = \|\mathbf{H}(n)\|_F^2\mathbf{I}_{2K} \tag{10}$$

where $\|.\|_F$ denotes the Frobenius norm. Let us define the $2MN \times 1$ *time-varying channel vector* $\mathbf{h}(n)$ as $\mathbf{h}(n) \triangleq \underline{\mathbf{H}(n)}$. With a small abuse of notation, we hereafter replace $\mathbf{A}(\mathbf{H}(n))$ with $\mathbf{A}(\mathbf{h}(n))$. Therefore, we rewrite (10) as

$$\mathbf{A}^T(\mathbf{h}(n))\,\mathbf{A}(\mathbf{h}(n)) = \|\mathbf{h}(n)\|^2\mathbf{I}_{2K}\,. \tag{11}$$

Since $\mathbf{A}(\mathbf{h}(n))$ is linear in $\mathbf{h}(n)$, we can write $\text{vec}\{\mathbf{A}(\mathbf{h}(n))\} = \mathbf{\Phi}\mathbf{h}(n)$ where $\mathbf{\Phi}$ is a unique $4KMT \times 2MN$ matrix whose kth column, $[\mathbf{\Phi}]_k$ is given by $[\mathbf{\Phi}]_k = \text{vec}\{\mathbf{A}(\mathbf{e_k})\}$ and $\mathbf{e}_k$ is the kth column of the identity matrix $\mathbf{I}_{2MN}$.

Note that the matrix $\mathbf{\Phi}$ can be written as $\mathbf{\Phi} = [\mathbf{\Phi}_1^T \cdots \mathbf{\Phi}_{2K}^T]^T$ where each sub-matrix $\mathbf{\Phi}_k$ $(k = 1, \ldots 2K)$ describes the linear relationship between the kth column of $\mathbf{A}(\mathbf{h}(n))$ and $\mathbf{h}(n)$, i.e.,

$$[\mathbf{A}(\mathbf{h}(n))]_k = \mathbf{\Phi}_k\mathbf{h}(n) \tag{12}$$

where $[\cdot]_k$ denotes the kth column of a matrix.

Given the channel vector $\mathbf{h}(n)$, the optimal ML decoder for OSTBCs consists of a linear receiver followed by symbol-by-symbol decoder [8]. Indeed, the linear receiver computes $\hat{\tilde{\mathbf{s}}}_n$, the estimate of $\tilde{\mathbf{s}}_n$, as

$$\hat{\tilde{\mathbf{s}}}_n = \frac{1}{\|\mathbf{h}(n)\|^2}\mathbf{A}^T(\mathbf{h}(n))\,\tilde{\mathbf{y}}_n. \tag{13}$$

The symbol-by-symbol decoder then builds $\hat{\mathbf{s}}(n)$, the estimate of vector $\mathbf{s}(n)$, as $\hat{\mathbf{s}}(n) = [\mathbf{I}_K\ j\mathbf{I}_K]\hat{\tilde{\mathbf{s}}}_n$. The kth element of $\hat{\mathbf{s}}(n)$ is compared with all the points in the constellation corresponding to $s_k(n)$ and the closest point to the kth element of $\hat{\mathbf{s}}(n)$ is accepted as the kth decoded symbol.

Note however that implementation of the ML decoder requires the knowledge of the time-varying channel. If the channel is fixed, one can use training to estimate the channel. However, in practice, the channel is time-varying, and hence tracking of the MIMO channel is required.

Without assuming any model for the MIMO channel, joint channel tracking and symbol detection is an ill-posed problem. Fortunately, in many practical scenarios, the wireless channels can be modeled with a few parameters. It has been shown in [5] that the first-order autoregressive (AR) model can be used as a sufficiently precise method to describe the time-varying behavior of wireless channels. Based on this model, we assume that the channel variation between adjacent blocks is modeled as a first order AR model, i.e.,

$$\mathbf{H}(n) = \alpha\mathbf{H}(n-1) + \mathbf{W}(n) \tag{14}$$

where $\mathbf{W}(n)$ is an $N \times M$ noise matrix which is assumed to be zero-mean complex Gaussian with independent entries and variance of $\sigma_w^2/2$ per real dimension. The parameter α is a complex scalar that can be estimated using the method of [6], and hence, it is herein assumed to be known. The noise variance σ_w^2 and α are related as $\sigma_w^2 = \sigma_h^2(1-|\alpha|^2)$ where σ_h^2 is the variance of each element of $\mathbf{H}(n)$ and $|\cdot|$ denotes the amplitude of a complex number.

3. KALMAN FILTER BASED CHANNEL TRACKING

In this Section, we study the problem of channel tracking via Kalman filtering. We propose a two-step channel tracking algorithm. In the first step of this algorithm, Kalman filtering is used to obtain an initial channel estimate for the current block based on the channel estimates obtained for the previous blocks. This initial channel estimate is then refined by using an iterative decision-directed technique which involves a linear ML channel estimator based on the decoded transmitted symbols. In fact, the linearity of such an ML channel estimator follows from the specific properties of OSTBCs. We will also show that due to the specific structure of OSTBCs, Kalman filtering based channel tracking can be significantly simplified. To mathematically derive the two-step channel tracking algorithm, we rewrite (9) as

$$\tilde{\mathbf{y}}_n = \mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{h}(n) + \tilde{\mathbf{v}}_n \tag{15}$$

where the $2MT \times 2MN$ real matrix $\mathbf{B}(\tilde{\mathbf{s}}_n)$ is defined as

$$\mathbf{B}(\tilde{\mathbf{s}}_n) \triangleq [\mathbf{A}(\mathbf{e}_1)\tilde{\mathbf{s}}_n \quad \mathbf{A}(\mathbf{e}_2)\tilde{\mathbf{s}}_n\ \ldots\ \mathbf{A}(\mathbf{e}_{2MN})\tilde{\mathbf{s}}_n]. \tag{16}$$

The following Lemma plays an important role in simplifying the forthcoming Kalman filtering algorithm.

Lemma 1: The matrix $\mathbf{B}(\tilde{\mathbf{s}}_n)$ *has orthogonal columns and the norm of each column is equal to* $\|\mathbf{s}(n)\|^2$*, i.e., it satisfies:*

$$\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{B}(\tilde{\mathbf{s}}_n) = \|\mathbf{s}(n)\|^2\mathbf{I}_{2MN}\,. \tag{17}$$

Proof: See [10]. ■

It follows from (15) and (17) that given $\tilde{\mathbf{s}}_n$, the ML estimate of the channel vector $\mathbf{h}(n)$ can be obtained as

$$\hat{\mathbf{h}}_{\text{ML}}(n) = \frac{1}{\|\mathbf{s}(n)\|^2}\mathbf{B}^T(\tilde{\mathbf{s}}_n)\tilde{\mathbf{y}}_n. \tag{18}$$

This is an important observation because it implies that if the information symbols were available, the optimal ML channel estimation would involve a linear estimator as in (18). However, in practice, the information symbols are not available and need to be estimated. To overcome this problem, one can use a decision-directed channel estimation and symbol detection algorithm. Given an initial channel estimate $\hat{\mathbf{h}}^{(0)}(n)$, one can replace $\mathbf{h}(n)$ in (13) with $\hat{\mathbf{h}}^{(0)}(n)$ and obtain an estimate for $\tilde{\mathbf{s}}_n$, say $\hat{\mathbf{s}}_n^{(0)}$. This estimate of $\tilde{\mathbf{s}}(n)$ will, in turn, be used in (18) instead of $\tilde{\mathbf{s}}(n)$ to obtain a new estimate for $\mathbf{h}(n)$, say $\hat{\mathbf{h}}^{(1)}(n)$. This new channel estimate will be used in (13) instead of $\mathbf{h}(n)$ to obtain a new estimate of $\mathbf{s}(n)$. This procedure is repeated until the normalized difference between two consecutive channel estimates is negligible. This iterative procedure can be viewed as a decision-directed channel estimation scheme. The accuracy of this scheme depends on the availability of a precise enough initial channel vector estimate $\hat{\mathbf{h}}^{(0)}(n)$. We propose to use Kalman filtering to obtain such a precise initial channel estimate, $\hat{\mathbf{h}}^{(0)}(n)$, based on the channel estimates obtained for the previous blocks.

In what follows, we discuss the details of the Kalman filtering technique when applied to our MIMO channel tracking problem. We show that using Lemma 1, the Kalman filter can be simplified significantly. To show this, we use (15) as the observation model of the Kalman filter [9]. Note that the data model in (15) is real-valued. To obtain a real-valued state transition equation, we can rewrite (14) as

$$\mathbf{h}(n) = \mathbf{F}\mathbf{h}(n-1) + \mathbf{w}(n) \tag{19}$$

where

$$\mathbf{F} \triangleq \begin{bmatrix} \mathrm{Re}(\alpha)\mathbf{I}_{MN} & -\mathrm{Im}(\alpha)\mathbf{I}_{MN} \\ \mathrm{Im}(\alpha)\mathbf{I}_{MN} & \mathrm{Re}(\alpha)\mathbf{I}_{MN} \end{bmatrix} \tag{20}$$

and $\mathbf{w}(n) = \underline{\mathbf{W}(n)}$ is the real-valued process noise with covariance matrix $\mathbf{Q} = (\sigma_w^2/2)\mathbf{I}_{2MN}$. We can now use (19) as the real-valued state transition equation required for the Kalman filter.

The Kalman filtering problem for channel tracking in OSTBC-based MIMO communication can now be formally stated as it follows: Given the measurement-to-state matrix $\mathbf{B}(\tilde{\mathbf{s}}_n)$, use the observed data $\tilde{\mathbf{y}}_n$ to find the minimum mean squared error (MMSE) estimate of the state vector $\mathbf{h}(n)$ for each $n \geq 1$.

Given the estimate of the state at time $n-1$, $\mathbf{h}(n-1|n-1)$ and the associated error covariance matrix $\mathbf{P}(n-1|n-1)$, the Kalman filter [9] is used to obtain the estimate of the state at time n, i.e., $\mathbf{h}(n|n)$ and the associated error covariance matrix $\mathbf{P}(n|n)$. The Kalman filtering algorithm can be summarized as it follows:

$$\mathbf{h}(n|n-1) = \mathbf{F}\mathbf{h}(n-1|n-1) \tag{21}$$

$$\mathbf{P}(n|n-1) = \mathbf{F}\mathbf{P}(n-1|n-1)\mathbf{F}^T + \mathbf{Q} \tag{22}$$

$$\hat{\tilde{\mathbf{y}}}_n = \mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{h}(n|n-1) \tag{23}$$

$$\nu(n) = \tilde{\mathbf{y}}_n - \hat{\tilde{\mathbf{y}}}_n \tag{24}$$

$$\mathbf{P}_\nu(n) = \mathbf{R} + \mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{P}(n|n-1)\mathbf{B}^T(\tilde{\mathbf{s}}_n) \tag{25}$$

$$\mathbf{G}(n) = \mathbf{P}(n|n-1)\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{P}_\nu^{-1}(n) \tag{26}$$

$$\mathbf{h}(n|n) = \mathbf{h}(n|n-1) + \mathbf{G}(n)\nu(n) \tag{27}$$

$$\mathbf{P}(n|n) = \mathbf{P}(n|n-1) - \mathbf{G}(n)\mathbf{P}_\nu(n)\mathbf{G}^T(n) \tag{28}$$

where $\mathbf{h}(n|n-1)$ is the predicted state, $\mathbf{P}(n|n-1)$ is the covariance matrix of the predicted state, $\hat{\tilde{\mathbf{y}}}_n$ is the predicted observation, $\nu(n)$ is the innovation process, $\mathbf{P}_\nu(n)$ is the innovation covariance matrix, $\mathbf{G}(n)$ is the Kalman gain [9], and $\mathbf{R} = E\{\tilde{\mathbf{v}}_n\tilde{\mathbf{v}}_n^T\}$ is the covariance matrix of the measurement noise $\tilde{\mathbf{v}}_n$. As we assumed that the measurement noise is spatio-temporally white with a variance of $\sigma_v^2/2$ per real dimension, therefore $\mathbf{R} = (\sigma_v^2/2)\mathbf{I}_{2MT}$ holds true.

The following Lemma uses the result of Lemma 1 to reduce the computational complexity of finding $\mathbf{P}_\nu^{-1}(n)$ in (26).

Lemma 2: If $\mathbf{P}(n-1|n-1)$ *is a diagonal matrix, then,* $\mathbf{P}(n|n-1)$ *in (22) and* $\mathbf{P}(n|n)$ *in (28) are also diagonal, i.e., if*

$$\mathbf{P}(n-1|n-1) = \delta_{n-1}I_{2MN} \tag{29}$$

then

$$\mathbf{P}(n|n-1) = \beta_n\mathbf{I}_{2MN}$$

and

$$\mathbf{P}(n|n) = \delta_n\mathbf{I}_{2MN}$$

where

$$\beta_n = \delta_{n-1}|\alpha|^2 + (\sigma_w^2/2)$$

and

$$\delta_n = \sigma_v^2\beta_n/(2\|s(n)\|^2\beta_n + \sigma_v^2).$$

Proof: Substituting (29) into the predicted state covariance in (22), we can rewrite it as

$$\begin{aligned}\mathbf{P}(n|n-1) &= \delta_{n-1}\mathbf{F}\mathbf{F}^T + \mathbf{Q} = |\alpha|^2\delta_{n-1}\mathbf{I}_{2MN} + \mathbf{Q} \\ &= \left(|\alpha|\delta_{n-1}^2 + \frac{\sigma_w^2}{2}\right)\mathbf{I}_{2MN}.\end{aligned} \tag{30}$$

Inserting (30) into (25) and using matrix inversion lemma, $\mathbf{P}_\nu^{-1}(n)$ can be written as

$$\begin{aligned}\mathbf{P}_\nu^{-1}(n) &= \mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbf{B}(\tilde{\mathbf{s}}_n) \\ &\quad \left(\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1}\mathbf{B}(\tilde{\mathbf{s}}_n) + \mathbf{P}^{-1}(n|n-1)\right)^{-1}\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{R}^{-1} \\ &= \frac{2}{\sigma_v^2}\mathbf{I}_{2MT} - \frac{4}{\sigma_v^4}\mathbf{B}(\tilde{\mathbf{s}}_n) \\ &\quad \left(\frac{2}{\sigma_v^2}\mathbf{B}^T(\tilde{\mathbf{s}}_n)\mathbf{B}(\tilde{\mathbf{s}}_n) + \frac{1}{\beta_n}\mathbf{I}_{2MN}\right)^{-1}\mathbf{B}^T(\tilde{\mathbf{s}}_n) \\ &= \frac{2}{\sigma_v^2}\mathbf{I}_{2MT} - \left(\frac{4\beta_n}{2\|\mathbf{s}(n)\|^2\beta_n\sigma_v^2 + \sigma_v^4}\right)\mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{B}^T(\tilde{\mathbf{s}}_n)\end{aligned} \tag{31}$$

where we have used (17). Inserting (31) into (26), we obtain that

$$\mathbf{P}(n|n) = \left(\frac{\sigma_v^2\beta_n}{2\|s(n)\|^2\beta_n + \sigma_v^2}\right)\mathbf{I}_{2MN}. \tag{32}$$

The proof is complete. ■

Based on Lemma 2, if $\mathbf{P}(0|0)$ is initialized as a diagonal matrix, $\mathbf{P}(n|n-1)$, $\mathbf{P}_\nu^{-1}$, and $\mathbf{P}(n|n)$ always take the form of (30), (31), and (32), respectively.

It is also noteworthy that using (30) and (31), the Kalman filter gain $\mathbf{G}(n)$ in (26) can be simplified as

$$\mathbf{G}(n) = \underbrace{\beta_n\left(\frac{2}{\sigma_v^2} - \frac{4\beta_n\|\mathbf{s}(n)\|^2}{2\|\mathbf{s}(n)\|^2\beta_n\sigma_v^2 + \sigma_v^4}\right)}_{\triangleq \mu_n}\mathbf{B}^T(\tilde{\mathbf{s}}_n). \tag{33}$$

Using (23), (24) and (33), we can simplify (27) as

$$\begin{aligned}\mathbf{h}(n|n) &= \mathbf{h}(n|n-1) + \mu_n\mathbf{B}^T(\tilde{\mathbf{s}}_n)\left(\tilde{\mathbf{y}}(n) - \mathbf{B}(\tilde{\mathbf{s}}_n)\mathbf{h}(n|n-1)\right) \\ &= (1-\mu_n\|\mathbf{s}(n)\|^2)\mathbf{h}(n|n-1) + \mu_n\mathbf{B}^T(\tilde{\mathbf{s}}_n)\tilde{\mathbf{y}}(n)\end{aligned} \tag{34}$$

Therefore, the Kalman filtering algorithm can be simplified as it follows:

$$\mathbf{h}(n|n-1) = \mathbf{F}\mathbf{h}(n-1|n-1) \quad (35)$$

$$\beta_n = \delta_{n-1}\|\alpha\|^2 + \frac{\sigma_w^2}{2} \quad (36)$$

$$\mu_n = \beta_n \left(\frac{2}{\sigma_v^2} - \frac{4\beta_n \|\mathbf{s}(n)\|^2}{2\|\mathbf{s}(n)\|^2 \beta_n \sigma_v^2 + \sigma_v^4} \right) \quad (37)$$

$$\hat{\mathbf{h}}^{(0)}(n) \triangleq \mathbf{h}(n|n) =$$

$$(1-\mu_n\|\mathbf{s}(n)\|^2)\mathbf{h}(n|n-1) + \mu_n \mathbf{B}^T(\tilde{\mathbf{s}}_n)\tilde{\mathbf{y}}(n) \quad (38)$$

$$\delta_n = \sigma_v^2 \beta_n / (2\|s(n)\|^2 \beta_n + \sigma_v^2)\,. \quad (39)$$

The so-obtained $\hat{\mathbf{h}}^{(0)}(n)$ is then used in the aforementioned decision-directed iterative procedure to obtain a more accurate channel estimate.

Remark 1: Note that the simplified Kalman filter requires the knowledge of the symbol vector $\tilde{\mathbf{s}}_n$ (or $\mathbf{s}(n)$). However, the primary objective is to decode $\mathbf{s}(n)$. To overcome this issue, we propose to replace $\tilde{\mathbf{s}}_n$ in Kalman filter equations (35)-(39) by its estimate which is obtained as

$$\hat{\tilde{\mathbf{s}}}_n = \frac{1}{\|\mathbf{h}(n|n-1)\|^2}\mathbf{A}^T(\mathbf{h}(n|n-1))\tilde{\mathbf{y}}_n \quad (40)$$

Remark 2: To initiate the whole process, we also need to obtain an accurate enough channel estimate $\hat{\mathbf{h}}(0)$. To obtain such an initial channel estimate, one can use a training block $\mathbf{s}(0)$ which is known to the receiver. At the beginning of the tracking process, the receiver can then use (18) to obtain the ML estimate of $\mathbf{h}(0)$ as

$$\hat{\mathbf{h}}(0) = \frac{1}{\|\mathbf{s}(0)\|^2}\mathbf{B}^T(\tilde{\mathbf{s}}_0)\tilde{\mathbf{y}}_0\,. \quad (41)$$

Remark 3: To avoid error propagation, we need to repeat training once in a while. The training repetition period (TRP) determines the bandwidth efficiency of system and it is defined as the number of blocks between two consecutive training blocks.

4. SIMULATION RESULTS

We consider a MIMO system with the full-rate real OSTBC of Tarokh (i.e., eqn. (27) of [2] with real BPSK symbols) with $K = M = T = 4$ and $N = 3$. The signal-to-noise ratio (SNR) is defined as σ_h^2/σ_v^2. In each simulation run, the elements of $\mathbf{H}(n)$ are generated based on (14), where $\alpha = 0.998e^{j\frac{\pi}{36}}$ is chosen and σ_w^2 is obtained from $\sigma_w^2 = \sigma_h^2(1-|\alpha|^2)$. We compare our Kalman filtering based method with the coherent ML receiver (which has the perfect knowledge of the time-varying channel), as well as with a channel estimation method which uses the predicted channel estimate as the initial channel estimate for the iterative decision directed method. To compare the performance of these two techniques in terms of channel estimation accuracy, we use normalized mean squared error (NMSE) of the channel estimates defined as

$$\mathrm{NMSE} = E\left\{ \frac{\|\mathbf{H}(n) - \hat{\mathbf{H}}(n)\|^2}{\|\mathbf{H}(n)\|^2} \right\}.$$

Figure 1 shows the real part of the true and the estimated channel coefficients versus block index n. In this figure, SNR = 10 dB and TRP = 10 are chosen. Figure 2 shows the NMSE of channel estimates versus block index n, for different methods and for two different values of SNR. In this figure TRP = 10 blocks is chosen. Figure 3 shows the symbol error rates (SERs) of different methods, versus SNR, for different values of TRP. In this figure, we have also plotted the SER for the (clairvoyant) coherent ML receiver which is aware of the time-varying channel. It is noteworthy that the latter receiver does not correspond to any practical application and it is considered here only for the sake of comparison.

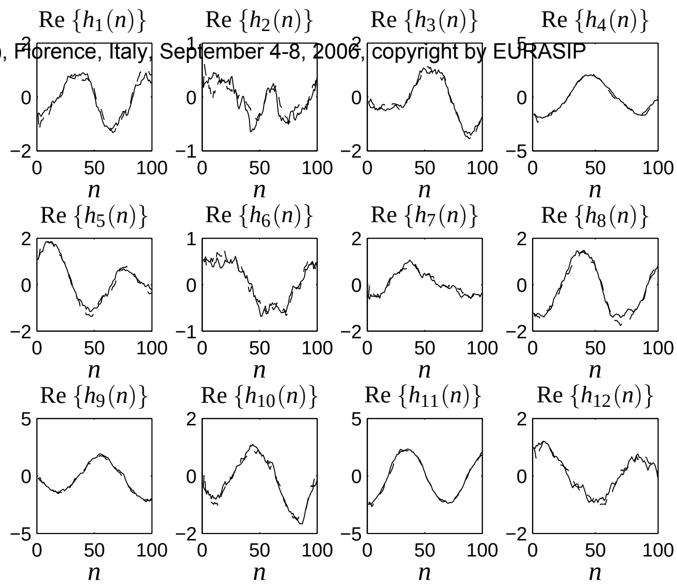

Figure 1: The real parts of true channel coefficients (solid lines) and the estimated channel coefficients (dashed lines), obtained from our Kalman filtering based method, versus block index n for SNR = 10 dB and TRP = 10; second example.

As can be seen from Figure 1, our Kalman filtering based channel tracking technique provides channel estimates that are quite close to the true channel coefficients. Figure 2 shows that our proposed channel tracking algorithm significantly outperforms the prediction based method. Interestingly enough, the NMSE of channel estimates for the prediction based method is not improved as the number of received blocks increases while the NMSE of channel estimates for our proposed technique shows a continuous improvement as the number of received blocks is increased.

In terms of SER, Figure 3 shows that the performance of our channel tracking method can be quite close to that of the coherent ML receiver. Indeed for a TRP = 10, the SER of our technique can be within 1 dB from the coherent ML receiver.

5. CONCLUSIONS

In this paper, we studied the problem of MIMO channel estimation for communication systems that use orthogonal space-time block codes as the underlying space-time coding scheme. For such systems, we proposed a two-step channel tracking algorithm. As the first step, Kalman filtering is used to obtain an initial channel estimate, for each block, based on the channel estimates obtained for the previous blocks. In the second step, an iterative decision-directed method is used to refine the initial channel estimate obtained in the first step. We have shown due to specific structure of OSTBC, both

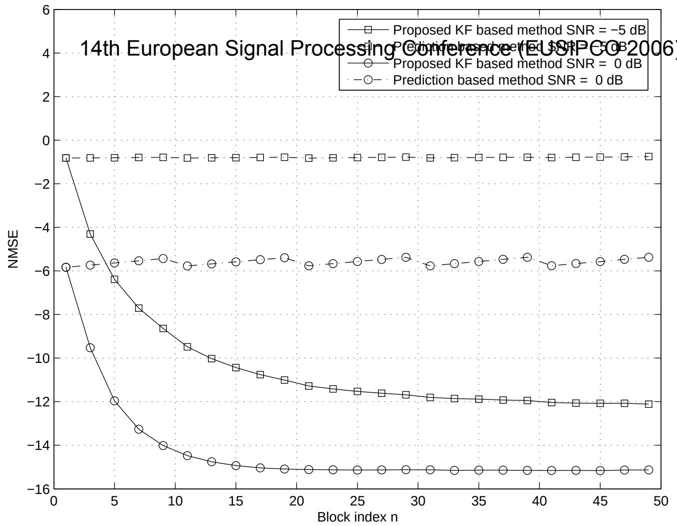

Figure 2: The NMSE of channel estimates obtained from different methods with TRP = 10 blocks and for two different values of SNR, versus block index *n*; second example.

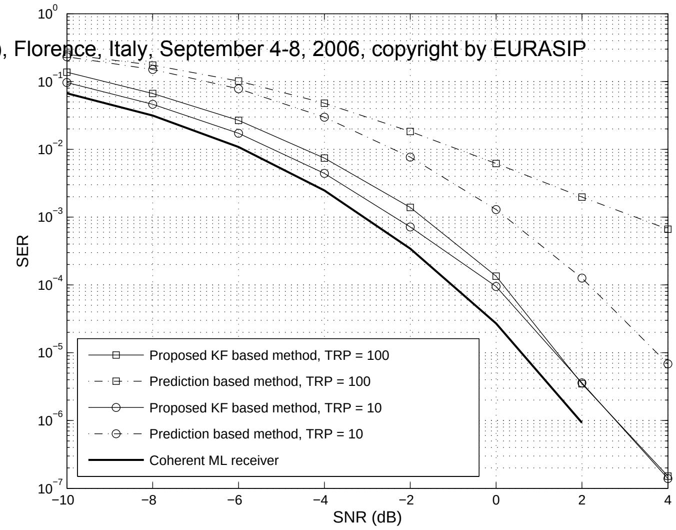

Figure 3: The SERs of different methods, versus SNR, for two different values of TRP; second example.

steps can be significantly simplified. To initiate the tracking process, a training block is required. Also, to avoid error propagation, the training process needs to be repeated once in a while. Simulation results show that, for a training repetition period of 10 blocks, this algorithm can have a performance within 1 dB, in terms of SNR, from the coherent ML receiver which has the perfect knowledge of the channel.

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