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Joint Power and Channel Allocation for Underlay D2D Communications with Proportional Fairness

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***Abstract*—Since D2D (Device-to-Device) communication was proposed in cellular network as a new paradigm for enhancing network performance, many works have been done on resource allocation to improve system throughput and energy efficiency (EE) for underlay D2D communications. However, the system long-term average fairness as one of the system main performance metrics was rarely considered especially when users are moving. In this paper, we formulate the joint power and channel allocation problem aiming at maximizing the system fairness subject to the minimum required SINRs (Signal to Interference and Noise Ratios) and power consumption limits of cellular and active D2D links. To solve the above problem practically, we first decompose our original problem into two sub-problems (power and channel allocation), then solve them sequentially. Simulation results show that our proposed algorithm can dramatically enhance the system fairness and slightly improve the system throughput comparing with existing method.**

***Index Terms*—D2D communications; Power Allocation; Channel Allocation; Proportional Fairness.**

I. INTRODUCTION

D2D (Device-to-Device) communication has been drawn to attention the needs of investigations to enable higher data rate local services in recent years [1] [2]. The most important aspect of D2D communication is that it can greatly improve the network capacity by spectrum reusing between cellular and D2D links [3].

Many works aim to increase the network throughput for D2D underlay communications [4]- [6]. This is done both by works and by proof so that the maximum system throughput can be achieved when at least one of the users transmit at its maximum power [5] [6]. Authors in [5] then transform the channel allocation as a maximum weight bipartite matching problem, which can be solved by the well-known Hungarian algorithm. Authors in [6] formulate the channel allocation by using the graph-based approach, and solved by an iterative rounding algorithm. [7] assumes the cellular transmission powers are fixed, then uses the convex approximation technique to formulate power allocation as a convex optimization problem, which can be solved by standard convex algorithms.

In addition, many works on improving system energy efficiency (EE) for D2D underlay communications have been studied recently. By considering the properties of fractional programming, [8] and [9] transform the original non-convex EE problem into an equivalent optimization problem with subtractive form, which is solved by the proposed efficient iterative scheme. [10] proposes three resource allocation algorithms: dual-based, BnB (Branch-and-Bound) and RBR (Relaxation-Based Rounding) algorithms with different complexity levels.

However, most of above works focus on the static network while completely ignoring the mobility feature of users. For mobile network communications, the system long term fairness should be considered to avoid the scenario that the transmission links with poor channel gain are always forbidden. In general, three well-known schedulers such as round robin (RR), Max-min and proportional fairness (PF) can be applied to improve system fairness. Normally, the increase of fairness will result in decrease of throughput, vice versa, especially in RR and Max-min scheduling. The PF scheduling can offer a trade-off between system throughput and fairness [11].

There are few works on how PF scheduling scheme is applied in D2D underlay communications. Authors in [12] transform the PF scheduling into an assignment problem form by applying *Maclaurin* series expansion without considering the mutual interferences. However, the transmission powers of all users are allocated the same value and authors do not consider the QoS requirements of all links. This will lead to the harmful interferences between cellular and D2D links. [13] assumes that the system is completely fair when allocating the power for both cellular and D2D links, which is unrealistic.

In order to get a fairer and realistic system without sacrificing system throughput, in this paper we allocate the powers and take into account the effect of average data rates of all users during each time slot. We first formulate the PF scheduling scheme for D2D underlay communications aiming at maximizing the system fairness. Since the above problem is MINLP (Mix-Integer Non-Linear Programming), which can not be solved in polynomial time, we then divide it into two sub-problems, and solve them sequentially. The first sub-problem is the optimal power allocation which is transformed to the maximization of the weighted sum of current data rates of all links. Given the above power allocation, the channel allocation as the second sub-problem becomes an ILP (Integer Linear Programming), which is solved by standard LP (Linear programming) effectively. Simulation results show that our proposed algorithm can dramatically enhance the system fairness comparing with existing method without sacrificing the overall throughput .

The rest of the paper is organized as follows. Section II

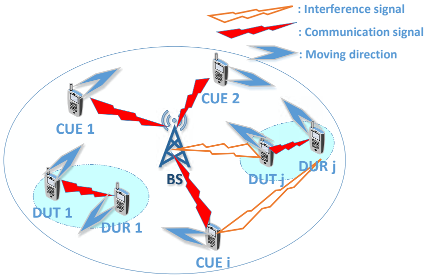

Fig. 1: The system model of dynamic D2D underlay communications.

introduces the system model and the system PF scheduling for D2D underlay communications. Problem formulations and proposed algorithm are shown in section III. Simulation results and analysis are presented in section IV. Section V concludes this paper.

II. System Model and PF scheduling

A. System Model

We consider a dynamic single cell system with a BS (Base Station) in the centre, where K cellular users (CUEs) in the set $\mathcal{K} = \{1, ...i, ...K\}$, and L D2D pairs in the set $\mathcal{L} = \{1, ...j, ...L\}$. Each D2D pair includes a transmitter (DUT) and a receiver (DUR) as shown in Fig.1. Since all users (include D2D pairs) move in every time slot, the locations of all users and the channel state informations (CSIs) are updated in every time slot. We assume that each cellular link has been pre-allocated an orthogonal uplink channel resource, and each D2D link can only reuse no more than one channel resource of cellular link, and each channel resource of cellular link is assigned to at most one D2D link.

When D2D pair j reuses the same channel resource of CUE i, the SINRs (Signal to Interference and Noise Ratios) of cellular link i and D2D pair j at time slot t can be expressed as

$$\gamma^C_{i,j,t} = \frac{p^C_{i,j,t} h_{i,B,t}}{\sigma^2 + p^D_{i,j,t} h_{j,B,t}}, \tag{1}$$

$$\gamma^D_{i,j,t} = \frac{p^D_{i,j,t} h_{j,t}}{\sigma^2 + p^C_{i,j,t} h_{i,j,t}}, \tag{2}$$

in which $p^C_{i,j,t}$ and $p^D_{i,j,t}$ are the transmission powers of CUE i and DUT j in time slot t, respectively. $h_{i,B,t}$ is the channel gain between CUE i and BS in time slot t and $h_{j,B,t}$ is the interfering channel gain from DUT j to BS in time slot t. $h_{j,t}$ is the channel gain between D2D pair j in time slot t. $h_{i,j,t}$ is the interfering channel gain from CUE i to DUR j in time slot t. σ^2 is the noise power.

The data rates in bits per second per hertz (i.e normalized by the channel bandwidth) of cellular link i and D2D link j in time slot t can be expressed as

$$r^C_{i,j,t} = \log_2(1 + \gamma^C_{i,j,t}), \tag{3}$$

$$r^D_{i,j,t} = \log_2(1 + \gamma^D_{i,j,t}). \tag{4}$$

When cellular links (or cellular users) do not experience any co-channel interferences from D2D links (or D2D transmitters), the maximum throughput could be achieved when they transmit with their maximum power (i.e. p^C_{max}). Thus, the data rate of cellular link i without reusing can be expressed as[1]

$$r^C_{i,t} = \log_2(1 + \frac{p^C_{max} h_{i,B,t}}{\sigma^2}). \tag{5}$$

B. PF scheduling

In this paper, a system is fair if it provides the equal average data rate to all links over a long-duration service time and each link is activated only if the minimum SINR requirement is satisfied in every time slot. Here, the PF scheduling is used to achieve the system fairness. As proven in [12], the PF scheduling scheme in D2D underlay communications can be expressed as

$$F = \arg\max_{S} \{ \sum_{i \in \mathcal{K}} \frac{r^S_{i,t}}{R_{i,t-1}} + \sum_{j \in \mathcal{L}} \frac{r^S_{j,t}}{R_{j,t-1}} \}, \tag{6}$$

where $r^S_{i,t}$ and $r^S_{j,t}$ are the current data rates of CUE i and D2D pair j achieved by scheduling S in time slot t, respectively. $R_{i,t-1}$ and $R_{j,t-1}$ are the average date rates of CUE i and D2D pair j during previous time $(t-1)$ slots, respectively.

From (6) we can know, the optimal PF scheduling scheme F is trying to allocate the appropriate resources (power and channel allocation in this paper) for links which have higher current data rate and lower previous average data rate. That is we aim to determine the current data rates for all links at time slot t which can maximize the sum ratios function (PF function) in (6). Meanwhile, the average data rate at time slot t can be iteratively obtained for the next time slot

$$R_{i(j),t} = \frac{(t-1)R_{i,(j),t-1} + r_{i(j),t}}{t}, t \geq 2. \tag{7}$$

In this paper, we will only discuss the PF scheduling when $t \geq 2$, and use the same PF scheduling for $t = 1$ in [13].

III. Problem Formulations and Proposed Algorithm when $t \geq 2$

A. Problem Formulations

According to (6), in order to improve the system fairness, the objective is to maximize the sum of PF functions of all links while guaranteeing the SINRs of all cellular and active D2D links. Therefore, the problem can be mathematically expressed as

$$\begin{aligned}(\boldsymbol{P}^*_t \boldsymbol{\chi}^*_t) = \arg\max_{\boldsymbol{P}_t \boldsymbol{\chi}_t} \{ & \sum_{i=1}^{K} (1 - \sum_{j=1}^{L} \chi_{i,j,t}) \frac{r^C_{i,t}}{R_{i,t-1}} \\ & + \sum_{i=1}^{K} \sum_{j=1}^{L} \chi_{i,j,t} \frac{r^C_{i,j,t}}{R_{i,t-1}} + \sum_{j=1}^{L} \sum_{i=1}^{K} \chi_{i,j,t} \frac{r^D_{i,j,t}}{R_{j,t-1}} \},\end{aligned} \tag{8}$$

---

[1] We assume that without reusing, the cellular links always can meet the minimum SINRs constraints.

s.t.

$$\gamma_{i,j,t}^{C} \geq \gamma_{min}^{C}, 0 \leq p_{i,j,t}^{C} \leq p_{max}^{C}, \forall i \in \mathcal{K}, \tag{8a}$$

$$\gamma_{i,j,t}^{D} \geq \gamma_{min}^{D}, 0 \leq p_{i,j,t}^{D} \leq p_{max}^{D}, \forall j \in \mathcal{L}, \tag{8b}$$

$$\sum_{j=1}^{L} \chi_{i,j,t} \leq 1, \forall i \in \mathcal{K}, \tag{8c}$$

$$\sum_{i=1}^{K} \chi_{i,j,t} \leq 1, \forall j \in \mathcal{L}, \tag{8d}$$

$$\chi_{i,j,t} \in \{0,1\}, \forall i \in \mathcal{K}, \forall j \in \mathcal{L}, \tag{8e}$$

where both $\boldsymbol{P}_t$ and $\boldsymbol{\chi}_t$ are the $(K \times L)$ power and channel allocation matrices at time t, respectively. $P_{i,j,t} = [(p_{i,j,t}^{C^*}, p_{i,j,t}^{D^*})]$ is the power vector when cellular link i and D2D j reuse the same channel in time slot t, where $p_{i,j,t}^{C^*}$ and $p_{i,j,t}^{D^*}$ are the optimal power allocation for cellular user i and D2D link j, respectively. The index $\chi_{i,j,t} = 1$, if cellular link i and D2D pair j reuse the same channel resource, otherwise, $\chi_{i,j,t} = 0$ in time slot t. γ_{min}^{C} and γ_{min}^{D} are the minimum SINR requirements of cellular and D2D links, respectively. p_{max}^{C} and p_{max}^{D} are the maximum transmission powers of cellular and D2D transmitters.

In (8), the first term is the sum PF function of cellular links without reusing, the second term is the sum PF function of cellular links under reusing, and the last term is sum PF function of all D2D links. Constraint (8a) shows that the minimum SINR requirement and the transmission power limit of individual cellular in all transmission intervals are guaranteed, Similarly, constraint (8b) shows that the minimum SINR requirement and the transmission power limit of each active D2D links are guaranteed. Constraint (8c) shows each cellular link can only be shared by no more than one D2D link, and constraint (8d) shows each D2D link can reuse no more than one cellular link's resource. The final constraint (8e) means the value of channel allocation indicator is binary.

The optimization in (8) is a MINLP problem, which is NP-Hard. We now proposed a PF scheduling scheme to address the resource allocation problem for underlay D2D communications. Specifically, we divide the problem in (8) into two subproblems: one problem is to maximize the sum PF function by optimal power allocation while guaranteeing SINR requirements of both D2D and cellular links. Based on the optimal power allocation results, the second problem is to maximize the sum of all users PF functions through channel allocation for multiple CUEs and D2D pairs.

B. Proposed Algorithm

In this subsection, we formulate the two sub-problems: Optimal Power Allocation and Channel Allocation, then solve them sequentially.

1) Optimal Power Allocation: Here, the objective of the power allocation is to optimize the sum PF functions of one D2D pair and one CUE link which share the same channel resource while meeting their minimum SINR requirements. And this procedure will be repeated for all reuse possible between cellular and D2D links.

When D2D pair j shares the same channel resource with cellular links i at time slot t, the power allocation becomes

$$\begin{aligned}(p_{i,j,t}^{C^*}, p_{i,j,t}^{D^*}) &= \arg \max_{(p_{i,j,t}^{C}, p_{i,j,t}^{D})} \left(\frac{r_{i,j,t}^{C}}{R_{i,t-1}} + \frac{r_{i,j,t}^{D}}{R_{j,t-1}}\right) \\ &= \arg \max_{(p_{i,j,t}^{C}, p_{i,j,t}^{D})} \left\{\frac{\log_2(1+\gamma_{i,j,t}^{C})}{R_{i,t-1}} \right. \\ &\qquad \left. + \frac{\log_2(1+\gamma_{i,j,t}^{D})}{R_{j,t-1}}\right\} \\ &= \arg \max_{(p_{i,j,t}^{C}, p_{i,j,t}^{D})} \{(1+\gamma_{i,j,t}^{C})(1+\gamma_{i,j,t}^{D})^{\beta}\}\end{aligned} \tag{9}$$

s.t.

$$\gamma_{i,j,t}^{C} \geq \gamma_{min}^{C}, \gamma_{i,j,t}^{D} \geq \gamma_{min}^{D}, \tag{9a}$$

$$0 \leq p_{i,j,t}^{C} \leq p_{max}^{C}, 0 \leq p_{i,j,t}^{D} \leq p_{max}^{D}. \tag{9b}$$

where $\beta = \frac{R_{i,t-1}}{R_{j,t-1}}$. Note that the values of $R_{i,t-1}$ and $R_{j,t-1}$ are known at time slot t.

Constraint (9a) makes sure the SINRs of cellular and D2D links satisfy the minimum requirements, and (9b) are the transmission power constraint of both links.

As shown in [5] [6] that the optimal system performance will be achieved when at least one of the cellular and D2D links transmit its maximum power. We define $\Omega_{i,j,t}$ is the feasible power allocation solutions set of problem in (9), $\Omega 1_{i,j,t}$ and $\Omega 2_{i,j,t}$ are the feasible sets when cellular and D2D users transmit at its maximum power, respectively. Therefore, we have the following proposition.

Proposition 1. If the problem in (9) is feasible, its optimal power allocation solution belongs to the set $\Omega_{i,j,t} = \Omega 1_{i,j,t} \cup \Omega 2_{i,j,t}$; otherwise, the set is empty $\Omega_{i,j,t} = \phi$.

The set $\Omega_{i,j,t}$ can be obtained as following. We first assume $p_{i,j,t}^{C} = p_{max}^{C}$, the above problem in (9) becomes

$$(p_{max}^{C}, p_{i,j,t}^{D^*}) = \arg \max_{(p_{max}^{C}, p_{i,j,t}^{D})} f(p_{max}^{C}, p_{i,j,t}^{D}), \tag{10}$$

s.t.

$$\frac{p_{max}^{C} h_{i,B,t}}{\sigma^2 + p_{i,j,t}^{D} h_{j,B,t}} \geq \gamma_{min}^{C}, \frac{p_{i,j,t}^{D} h_{j,t}}{\sigma^2 + p_{max}^{C} h_{i,j,t}} \geq \gamma_{min}^{D}, \tag{10a}$$

$$0 \leq p_{i,j,t}^{D} \leq p_{max}^{D}, \tag{10b}$$

where $f(p_{max}^{C}, p_{i,j,t}^{D}) = \{(1 + \frac{p_{max}^{C} h_{i,B,t}}{\sigma^2 + p_{i,j,t}^{D} h_{j,B,t}}) \times (1 + \frac{p_{i,j,t}^{D} h_{j,t}}{\sigma^2 + p_{max}^{C} h_{i,j,t}})^{\beta}\}$. According to constraints (10a)-(10b), we can get the continuous closed and bounded feasible set of $p_{i,j,t}^{D}$, which is $[p_{low,i,j,t}^{D}, p_{up,i,j,t}^{D}]$. The lower and upper bounds $p_{low,i,j,t}^{D}$ and $p_{up,i,j,t}^{D}$ are expressed as

$$\begin{aligned}p_{low,i,j,t}^{D} &= \max\{0, \frac{\gamma_{min}^{D}(\sigma^2 + p_{max}^{C} h_{i,j,t})}{h_{j,t}}\}, \\ p_{up,i,j,t}^{D} &= \min\{p_{max}^{D}, \frac{(p_{max}^{C} h_{i,B,t} - \gamma_{min}^{C} \sigma^2)}{h_{j,B,t} \gamma_{min}^{C}}\},\end{aligned} \tag{11}$$

respectively.

The set $\Omega 1_{i,j,t}$ is feasible only when $p_{low,i,j,t}^{D} \leq p_{up,i,j,t}^{D}$, otherwise $\Omega 1_{i,j,t}$ is empty.

When $\Omega 1_{i,j,t}$ is feasible, the maximum value of $f(p^C_{max}, p^D_{i,j,t})$ can be obtained by solving the following equation

$$f'(p^C_{max}, p^D_{i,j,t}) = \frac{A_{i,j,t}(p^D_{i,j,t})^2 + B_{i,j,t}p^D_{i,j,t} + U_{i,j,t}}{V_{i,j,t}} = 0, \tag{12}$$

where

$$\begin{aligned} A_{i,j,t} =& \beta h_{j,t} h^2_{j,B,t}, \\ B_{i,j,t} =& (\beta - 1) p^C_{max} h_{i,B,t} h_{j,B,t} h_{j,t} \\ & + 2\beta h_{j,t} h_{j,B,t} \sigma^2, \\ U_{i,j,t} =& \beta h_{j,t} \sigma^2 (\sigma^2 + p^C_{max} h_{i,B,t}) \\ & - p^C_{max} h_{i,B,t} h_{j,B,t} (\sigma^2 + p^C_{max} h_{i,j,t}), \\ V_{i,j,t} =& (\sigma^2 + p^C_{max} h_{i,j,t})^\beta (\sigma^2 + p^D_{i,j,t} h_{j,B,t})^2. \end{aligned} \tag{13}$$

If $\Delta_{i,j,t} = B^2_{i,j,t} - 4A_{i,j,t}U_{i,j,t} \geq 0$, then (12) has two solutions:

$$p1^D_{i,j,t} = \frac{-B_{i,j,t} - \sqrt{\Delta_{i,j,t}}}{2A_{i,j,t}}, p2^D_{i,j,t} = \frac{-B_{i,j,t} + \sqrt{\Delta_{i,j,t}}}{2A_{i,j,t}}. \tag{14}$$

Since $A_{i,j,t}$ is always positive, so $p1^D_{i,j,t}$ and $p2^D_{i,j,t}$ correspond to the local maximum and minimum points of function $f(p^C_{max}, p^D_{i,j,t})$, respectively. If $p1^D_{i,j,t} \in [p^D_{low,i,j,t}, p^D_{up,i,j,t}]$, then $p1^D_{i,j,t}$ is the optimal solution of function $f(p^C_{max}, p^D_{i,j,t})$. If not, the bound point $p^D_{low,i,j,t}$ or $p^D_{up,i,j,t}$ is the optimal solution. This is because when $p1^D_{i,j,t} \notin [p^D_{low,i,j,t}, p^D_{up,i,j,t}]$, $f(p^C_{max}, p^D_{i,j,t})$ is a convex function in $[p^D_{low,i,j,t}, p^D_{up,i,j,t}]$. Therefore, the optimal value of $f(p^C_{max}, p^D_{i,j,t})$ can be obtained at either $p^D_{low,i,j,t}$ or $p^D_{up,i,j,t}$.

If $\Delta_{i,j,t} < 0$, it means the values of $f'(p^C_{max}, p^D_{i,j,t})$ are always positive, so $f(p^C_{max}, p^D_{i,j,t})$ will increase monotonically in $[p^D_{low,i,j,t}, p^D_{up,i,j,t}]$. Therefore, $p^D_{up,i,j,t}$ is the optimal solution. In summary, the feasible set $\Omega 1_{i,j,t}$ can be expressed as

$$\Omega 1_{i,j,t} = \begin{cases} \{(p^C_{max}, p1^D_{i,j,t})\}, \\ \quad \text{if } \Delta_{i,j,t} \geq 0, p1^D_{i,j,t} \in [p^D_{low,i,j,t}, p^D_{up,i,j,t}], \\ \{(p^C_{max}, p^D_{low,i,j,t}), (p^C_{max}, p^D_{up,i,j,t})\}, \\ \quad \text{if } \Delta_{i,j,t} \geq 0, p1^D_{i,j,t} \notin [p^D_{low,i,j,t}, p^D_{up,i,j,t}], \\ \{(p^C_{max}, p^D_{up,i,j,t})\}, \\ \quad \text{if } \Delta_{i,j,t} < 0. \end{cases} \tag{15}$$

Since the $\Omega 2_{i,j,t}$ can be obtained in the similar way, the deviation of $\Omega 2_{i,j,t}$ is omitted due to the space limitation.

Then, the $\Omega_{i,j,t}$ can be obtained according to $\Omega_{i,j,t} = \Omega 1_{i,j,t} \cup \Omega 2_{i,j,t}$. After that, optimal power allocation $(p^{C^*}_{i,j,t}, p^{D^*}_{i,j,t})$ can be obtained by comparing all feasible power allocation solutions in $\Omega_{i,j,t}$, which can bring the maximum value of (9). Thus, the optimal data rates of cellular link i $(r^{C^*}_{i,j,t})$ and D2D link j $(r^{D^*}_{i,j,t})$ can be calculated:

$$\begin{aligned} r^{C^*}_{i,j,t} =& \log_2(1 + \frac{p^{C^*}_{i,j,t} h_{i,B,t}}{\sigma^2 + p^{D^*}_{i,j,t} h_{j,B,t}}), \\ r^{D^*}_{i,j,t} =& \log_2(1 + \frac{p^{D^*}_{i,j,t} h_{j,t}}{\sigma^2 + p^{C^*}_{i,j,t} h_{i,j,t}}). \end{aligned} \tag{16}$$

When $\Omega_{i,j,t}$ is empty, we set $r^{C^*}_{i,j,t} = r^{D^*}_{i,j,t} = Q$, where Q is a sufficiently small value meaning that D2D link j and cellular link i can not reuse the same channel resource in this time slot.

2) Channel Allocation: After the power allocation considering all the reuse possibles, the channel allocation can be modelled

$$\begin{aligned} \boldsymbol{\chi}^*_t = \arg\max_{\boldsymbol{\chi}_t} \{ & \sum_{i=1}^{K} (1 - \sum_{j=1}^{L} \chi_{i,j,t}) \frac{r^C_{i,t}}{R_{i,t-1}} \\ & + \sum_{i=1}^{K} \sum_{j=1}^{L} \chi_{i,j,t} \frac{r^{C^*}_{i,j,t}}{R_{i,t-1}} + \sum_{j=1}^{L} \sum_{i=1}^{K} \chi_{i,j,t} \frac{r^{D^*}_{i,j,t}}{R_{j,t-1}} \}, \end{aligned} \tag{17}$$

s.t. constraints ($8c$)-($8e$).

As the binary variables $\chi_{i,j,t}$ are only unknown variables, problem in (17) is an ILP, which can be effectively solved by the standard LP methods (such as Gomorys cutting plane method, simplex method and Balas method) [14].

Algorithm 1 presents the operational procedure of the proposed joint power and channel allocation algorithm, where T is the total scheduling time. We set $T = 20$, as commonly used for PF scheduling in practical systems [15]. Note that all the following results are presented and analysed at the time slot $t = 20$ if is not be specified.

In Algorithm 1, to initialize our system, we obtain the average data rates of all links for first time slot according to the method in [13]. We then conduct the joint optimal power and channel allocation for each subsequent time slot from second to a chosen T time slots. Moreover, in each subsequent time slot, we decompose the problem in (8) into two subproblems: optimal power and channel allocation, and solve them sequentially. After that, the current data rates of all cellular and D2D users ($r_{i,t}$ and $r_{j,t}$) can be obtained as shown in Step *16-22* at time slot t. It means when D2D link j reuses the same channel with cellular link i in time slot t, the current data rates of cellular and D2D links can be obtained directly. Otherwise, it means the cellular link does not share its channel with any D2D links.

Finally, the average data rates of all users can be calculated in time slot. Meanwhile, the Jain's fairness index J_t which is used to measure the long-term fairness between different users can be obtained

$$J_t = \frac{|\sum_{i=1}^{K} R_{i,t} + \sum_{j=1}^{L} R_{j,t}|^2}{(K + L)(\sum_{i=1}^{K} R^2_{i,t} + \sum_{j=1}^{L} R^2_{j,t})}. \tag{18}$$

J_t can take the values between 0 and 1: 1, means completely fair at time t (all average data rates are equal); 0, means absolutely unfair at time t (the divergence of all average data rates is very large). The decrease in divergence of all average data rates results in the increase of fairness index J_t.

Algorithm 1 : Joint power and channel allocation algorithm.

```
Initialization: Get $R_{i(j),1}, \forall i \in \mathcal{K}, \forall j \in \mathcal{L}$ for $t = 1$ according to method in [13].
for all t=2:T do
    Optimal Power Allocation:
    for all $i \in \mathcal{K}, j \in \mathcal{L}$ do
        Obtain $\Omega_{i,j,t}$ according to Proposition 1
        if $\Omega_{i,j,t} = \phi$ then
            $r^{C*}_{i,j,t} = r^{D*}_{i,j,t} = Q$
        else
            $(p^{C*}_{i,j,t}, p^{D*}_{i,j,t}) = \arg \max_{(p^{C}_{i,j,t}, p^{D}_{i,j,t}) \in \Omega_{i,j,t}} (\frac{r^{C}_{i,j,t}}{R_{i,t-1}} + \frac{r^{D}_{i,j,t}}{R_{j,t-1}})$, Obtain $r^{C*}_{i,j,t}$ and $r^{D*}_{i,j,t}$ according to (16)
        end if
    end for
    $r^{C}_{i,t}$ can be obtained by (5) directly
    Channel Allocation:
    $\boldsymbol{\chi}^*_t$ is obtained by solving problem in (17) through standard LP methods.
    for all $i \in \mathcal{K}, j \in \mathcal{L}$ do
        if $\chi_{i,j,t} = 1$ then
            $r_{i,t} = r^{C*}_{i,j,t}$, $r_{j,t} = r^{D*}_{i,j,t}$
        else
            $r_{i,t} = r^{C}_{i,t}$, $r_{j,t} = Q$
        end if
    end for
    Calculate $R_{i(j),t}, \forall i \in \mathcal{K}, \forall j \in \mathcal{L}$ according to (7) and $J_t$ according to (18)
end for
```

IV. SIMULATION RESULTS AND ANALYSIS

In this section, Monte Carlo simulation is used to evaluate the performance of our proposed algorithm. We consider a single cellular network with a radius of 500m. The BS is located in the centre of the cell, cellular users and D2D transmitters are distributed uniformly in the cell. The D2D receivers are distributed uniformly in a disk centred by the corresponding D2D transmitters, and with a radius of d_{max}. Since we consider the mobility, all users will move in every time slot following the random-walk model, where they choose their speeds and directions in the rage [0,100] (m/s) and $[0, 2\pi]$, respectively. The channel gain in our proposed model is modelled as $h_{a,b} = d_{a,b}^{-\alpha}\kappa$ for all communication links, where $d_{a,b}$ is the distance between node a and b, α is the pathloss exponent, κ represents the Rayleigh fading. Our simulation parameters are summarized in TABLE I.

We compare our *Proposed Algorithm 1* with the existing method in [13] which is referred to *Existing Method*. As discussed above, authors in [13] allocate the transmission powers without considering the effect of average data rates. This will lead to a lower system fairness. Also, we define the probability of success for D2D communications as the ratio of the number of D2D pairs which meet the minimum SINR requirements to the total number of D2D pairs L.

TABLE I: Simulation Parameters

Maximum distance between D2D pairs d_{max} (m)	(20,...,300)
Number of cellular users K	20
Number of D2D pairs L ($L \leq K$)	(1,..,20)
Maximum cellular transmission power p^{C}_{max}(W)	0.5
Maximum D2D transmission power p^{D}_{max} (W)	0.5
SINR requirements of cellular links γ^{C}_{min} (dB)	5
SINR requirements of D2D links γ^{D}_{min} (dB)	15
Noise power σ^2 (dB)	-110
Pathloss exponent for all communications α	3

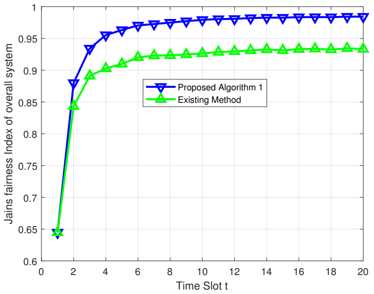

Fig. 2: J_t of overall system with *Proposed Algorithm 1* and the *Existing Method* for different time slot t when $d_{max} = 50m$, $L = 10$.

Fig.2 shows the J_t of the *Proposed Algorithm 1* comparing with the *Existing Method* for different time slots. We can see that as the time slot t increases, the J_t of overall system increases when time slot t is small ($t \leq 10$). That is because during the first few time slots the users with low average data rates have more chance to improve their current data rates so that the divergence of all users' average data rates is reduced. This increase slows down and converges when $t = 18$.

Fig.3 shows the comparison of J_t between the *Proposed Algorithm 1* and the *Existing Method* for various d_{max}. The J_t of overall system first increases and then decreases with the increase of d_{max}. This is because when d_{max} is small, the current data rates of D2D links are larger than that of cellular links due to short transmission distance. However, the current data rates of D2D links decrease as the increase of d_{max}. At the point $d_{max} = 120m$, the current data rates of D2D and cellular links are close to each other. Therefore, the J_t of overall system reaches the peak value at this point. With the continuous increase of d_{max}, the current data rates of D2D links become smaller than that of cellular links. Thus, the differences between current data rates of cellular and D2D links become larger again, leading to the decrease of J_t of overall system. In any cases, our *Proposed Algorithm 1* has higher J_t than that of the *Existing Method.*

Fig.4 shows the probability of success for D2D communications in *Proposed Algorithm 1* for different d_{max} comparing with the *Existing Method*. From Fig.4 we can see the probability of success decreases with the increase of d_{max}. That

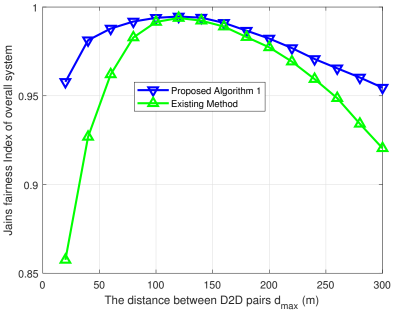

Fig. 3: J_t of overall system with *Proposed Algorithm 1* and the *Existing Method* for different d_{max} when $L = 10$.

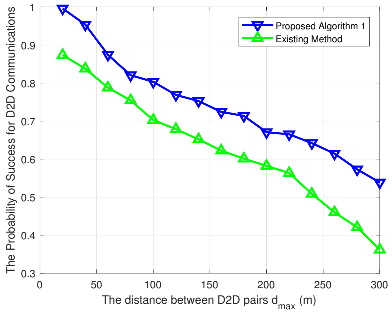

Fig. 4: The probability of success for D2D communications with *Proposed Algorithm 1* and the *Existing Method* for different d_{max} when $L = 10$.

is because the channel gains of D2D pairs become worse as the increase of d_{max}, leading to fewer successful D2D links. However, the proposed algorithm provides much better successful rate than the *Existing Method*.

Fig.5 shows the system sum rates of the *Proposed Algorithm 1* and the *Existing Method* for different L with various d_{max}. We can see that as the increase of L, the system sum data rates increase dramatically when $d_{max} = 40m$, and slightly when $d_{max} = 120$, and keep stable when $d = 240m$. Observing this results together with Fig.4, the system sum data rate increases with the increase of L when d_{max} is small because of the higher probability of success. However, as the decrease of successful probability, the increment of system sum data rate due to lager L is decreasing. Thus, the system sum data rate only increases slightly with the increase of L when d_{max} is large. In any cases, our *Proposed Algorithm 1* has higher system data rate than that of the *Existing Method*. The main reason is that our *Proposed Algorithm 1* has larger number of successful D2D links than that of the *Existing Method* as shown in Fig.4. This leads to greater system sum data rate.

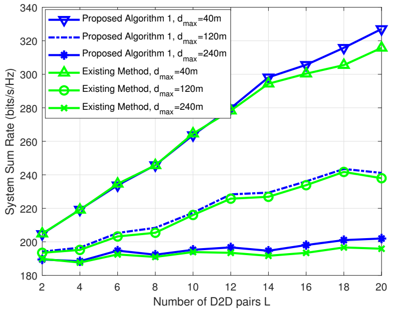

Fig. 5: The system sum rates of *Proposed Algorithm 1* and the *Existing Method* for different L with various d_{max}.

V. CONCLUSION

In this paper, we first formulated the joint power and channel allocation for underlay D2D communications which aims to maximize the system fairness subjects to the minimum SINR requirements and the power consumption limits of cellular and active D2D links in every time slot. In order to solve the above optimization effectively, we decompose it into two subproblems, then solve them subsequently. Simulation results show that our proposed algorithm achieves better PF than the existing algorithm with improved overall throughput.

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