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Weibo Hao
Harbin Engineering University
Fang Ye (yefang0923@126.com)
Harbin Engineering University
Qian Sun
Harbin Engineering University

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Joint Resource Allocation of UAV Aided Communication System Based on Multi-Coding Artificial Bee Colony Algorithm

Weibo Hao[1,2], Fang Ye[1,2*] and Qian Sun[1,2]

[1]Key Laboratory of Advanced Marine Communication and Information Technology, Ministry of Industry and Information.
[2]College of Information and Communication Engineering, Harbin Engineering University, Nantong Street, Harbin, 150001, China.

*Corresponding author(s). E-mail(s): yefang0923@126.com;
Contributing authors: haoweibo666@163.com;
yefang0923@126.com;

Abstract

With the advantages of flexible deployment and configuration, fast response, and strong environmental adaptability, UAV aided communication has important applications in temporary hotspot areas, edge areas, and emergency communication scenarios. Compared with traditional wired backhaul links from ground base stations, considering the high mobility of UAV base stations, this paper considers a UAV aided communication system using in-band backhaul technology to maximize the minimum reachable rate of ground users by jointly optimizing the hovering position, bandwidth allocation, transmit power and user association of UAVs. To address this problem, an multi-coding artificial bee colony algorithm is proposed in this paper to solve the proposed joint optimization problem, extending the dimensionality of the search performed on the solution space. Simulation results show that the proposed multivariate coded artificial bee colony algorithm can achieve significant rate gain and improve the minimum reachable rate of ground users in UAV aided communication systems within a limited number of iterations compared to the block coordinate descent method.

Keywords: UAV aided commnication system, Jointly resource allocation, Artificial bee colony, Metaheuristics

1 Introduction

With the development of Unmanned Aerial Vehicle(UAV) technology, UAV-aided communication systems have received considerable attention [1–3]. UAVs can be divided into two main types by wing type: fixed-wing and rotary-wing. Fixed-wing UAVs have greater range and carrying capacity and can take off outside the domain and fly into service areas to provide communication services to users. Rotor-wing UAVs are smaller and have the ability to hover, making them more flexible when deployed [4].

The existing research on UAV-assisted communication mainly focuses on improving the system performance in the following aspects: energy efficiency, capacity, safety and coverage through the allocation of system resources and the optimization of UAV position. It can be found that, in UAV-aided cellular networks, research is more focused on system capacity [5–12]. And in UAV-aided relay tasks, research is focused on capacity [13–15], decoding error probability [16] and outage probability [17]. More attention is payed to energy efficiency in the scenario where UAVs collect sensor data [18–22]. While the Loss link improves channel gain, it also makes it easier for non-cooperative devices to monitor. Therefore the security of UAV-assisted communication systems was studied [23–26]. In some scenarios that do not require high communication quality, such as temporary communication in disaster areas, coverage has become the focus of research, aiming to deploy as few UAVs as possible to cover enough scattered users [27–29]. They also contribute to the integrity of research in the field. The work done in this paper is concerned with the reachable rate of the system, which can be interpreted as the throughput at smaller time scales.

In some scenarios, users inside the service area only need to communicate with each other, when the UAV acts as a central node to provide communication services for them. In systems that use multiple UAV air base stations to serve a set of ground users, the multi-user communication scheduling and association is optimized jointly with UAV trajectory and power control to improve system throughput [5]. In a dynamic scenario where a single UAV simultaneously provides wireless services to multiple ground nodes, trajectory and transmit power of UAVs are jointly optimized to maximize the minimum average throughput [6]. These studies focus on solving the problem of optimal resource allocation for forward transmission links in UAV-assisted communication systems and provide meaningful insights. In other scenarios, users in the service area need to communicate with the ground control center outside the service area or access the core network, which requires backhaul links in the UAV aided communication system. In the literature [7], tethered UAVs was used to temporarily supplement traffic for hotspots, developing a user association strategy and deriving the overall coverage probability of the system. The scheme uses cables to provide wired backhaul links to the UAV aerial base stations, sacrificing the flexibility and mobility of the UAVs. To better exploit the flexibility and mobility of UAV deployment, UAV airborne base stations need wireless backhaul links for communication. Wireless backhaul links will occupy certain bandwidth resources, and some studies [8] have used out-of-band backhaul techniques that allocate dedicated bandwidth resources for backhaul links, which are simple and easy to deploy. Since the system capacity depends on the smaller values of the forward and backhaul links, and the out-of-band backhaul technique cannot flexibly adjust the occupied bandwidth, it is not conducive to the full utilization of spectrum resources. The literature [9] employs content caching techniques to cope with the limited capacity of the wireless backhaul link, and simulation experiments show that the system achievable rate is significantly dependent on the cache capacity at the UAV, and that the scheme is not suitable for latency-sensitive applications. The literature [10] used an in-band backhaul scheme that jointly optimized bandwidth allocation, user association, and UAV location to maximize equivalent spectral efficiency and obtained good results. Due to the non-convex and mixed-integer nature of the joint resource optimization problem, literature [10] decomposes it into three subproblems and relaxes the integer constraint on user association to obtain an approximate solution. This method of decomposing a joint optimization problem into multiple subproblems and performing alternate optimization is called the block coordinate descent method, and is widely used for its ability to simplify complex joint optimization problems. In [11], an architecture of UAV-assisted V2N communication is proposed, in which the UAV-vehicle scheduling scheme and the flight trajectory of UAV are jointly optimized to maximize the minimum rate among vehicles. In the scenario where multiple UAVs collect ground user data and transmit it back to the base station, the sub-channel allocation and UAV speed optimization problems are solved [12]. Like other relative studies [13, 14], they adopt the block coordinate descent method to solve the joint optimization problem of system resources. The joint resource optimization problem with complex coupling relations is difficult to solve due to its nonconvexity and mixed integer nature. The block coordinate descent method reduces the difficulty of solution, but it can only optimize one variable at a time and cannot improve the system achievable rate by coordinated changes of various system resources, which leads to the low system reachable rate that can be achieved by the obtained resource allocation results. This motivates us to find a new method to solve complex joint optimization problems.

Metaheuristics are widely recognized as efficient approaches for many hard optimization problems, which is a generic or higher-level heuristic that is more general in problem solving [30, 31]. As a meta-heuristic algorithm, artificial bee colony algorithm (ABC) along with its modifications has been applied to solve many engineering problems involving communication system due to its excellent performance in continuous, combinatorial, constrained, multi-objective, and large-scale optimization problems [32, 33]. However, the standard ABC algorithm does not have the ability to optimized different types of variables with different value ranges and neighborhood search methods jointly. To the best of our knowledge, no modification that can solve such problem has been developed.

Therefore, this paper improves the standard ABC algorithm and proposes a multi-coding artificial bee colony algorithm(MCABC). In the coding stage, the variables to be optimized in the system are separately coded, initialized, and then combined into a set of tuples. The neighborhood of each variable is searched independently and combined into new tuples by the operation of multidimensional neighborhood search in each iteration, and then the optimal combination is selected based on the greedy law to update the current food source. The proposed algorithm allows multiple variables to be changed simultaneously to enhance the reachable rate instead of changing them sequentially. In addition, the proposed algorithm addresses the problem of large dimensional differences and different convergence rates among different variables in the joint resource allocation problem of UAV aided communication system, and optimizes the neighborhood search operation of the user association matrix accordingly to ensure the collaborative convergence of variables and avoid falling into local optimum.

In this paper, we consider a scenario in which multiple UAVs are used to provide downlink data services to multiple terrestrial users in the service area due to the unavailability of terrestrial base stations in the service area, and terrestrial base stations outside the service area use an in-band backhaul scheme to provide backhaul links to the UAVs. Our objective is to maximize the minimum value of the ground user transmission rate by jointly optimizing the deployment location, bandwidth allocation, transmit power, and user association of the UAVs. The problem is non-convex and mixed-integer in nature and is difficult to solve. To address this problem, an multi-coding artificial bee colony algorithm is proposed in this paper to solve the proposed joint optimization problem, which extends the search dimension of the solution space and obtains a higher minimum achievable rate. The contributions of this paper are as follows:

- In this paper, we consider the in-band backhaul link scheme in UAV aided communication systems and propose a joint resource allocation problem for multiple UAV airborne base stations that integrates the in-band backhaul link resource allocation. Compared with the wired backhaul link of the conventional base station, the UAV airborne base station must use a wireless backhaul link. Due to the low spectrum utilization of the out-of-band backhaul link scheme, this paper adopts the in-band backhaul scheme and flexibly adjusts the bandwidth and channel conditions of the backhaul link through the joint optimization of bandwidth resource allocation and UAV location deployment to balance the backhaul link and the forward link to maximize the minimum reachable rate of users.
- Aiming at the problem that block coordinate descent algorithm is easy to fall into local optimum when solving joint optimization problems, the MCABC algorithm is proposed to solve the joint optimization problem of UAV auxiliary communication system resources with multiple coupling variables. The multidimensional neighborhood search operation is proposed to expand the search dimension of the knowledge space. The simulation results show that,

compared with the block coordinate descent method, the solution obtained by this method can achieve a higher minimum achievable rate.

- To address the problem of large dimensional differences and different search efficiency between variables, the ground users with low reachable rates are changed preferentially when performing neighborhood search on user associations. Due to the large dimensionality of user association variables, it is difficult to obtain high-quality solutions in a single search, which affects the effectiveness of multidimensional neighborhood search. Therefore, in order to improve the search efficiency of user association variables, probabilities are generated based on the reachability rates of ground users, and the ground users are randomly selected to be changed according to the probability. Such improvement can coordinate the convergence rate of different variables and avoid falling into local optimum.

The rest of this paper is organized as follows. In Section 2, the model of the multi-UAV air base station system considering the backhaul link and the formulated problem of maximizing the minimum achievable rate are presented. Section 3 is devoted to the proposed multi-coding artificial bee colony algorithm. The numerical results and discussion are provided in Section 4. The conclusions are presented in Section 5.

2 System model and problem formulation

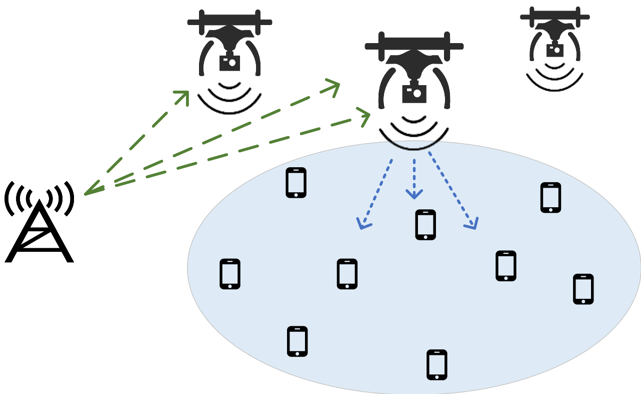


Fig. 1: system model

In this paper, a communication system consisting of one ground base station, N UAVs, and M ground users are considered. As shown in Figure 1, there are M users in the service area, and the ground base station outside the service area cannot provide services for a large number of users effectively due to the long distance and the strong fading of the ground link. So, N rotary-wing UAVs are hovered and deployed over the service area to provide access services for ground users, and the ground base stations outside the service area provide wireless backhaul links for the UAVs. The set of UAVs is donoted as $\mathbf{U} = \{U_1, U_2, ..., U_N\}$, where U_i denotes the ith UAV. The set of terrestrial users is denoted as $\mathbf{GU} = \{GU_1, GU_2, ..., GU_M\}$, where the jth terrestrial user is denoted as GU_j. The ground base station is denoted as BS. It is assumed that the positions of ground base stations and ground users are fixed. Without loss of generality, a Cartesian coordinate system is considered where the BS is located at $(-L, 0, 0)$, and the users are randomly distributed in a circular area with $O\,(R, 0, 0)$ as the center and R as the radius. The coordinates of U_i and GU_j are denoted as $\mathbf{q}_i^U = \left\{x_i^U, y_i^U, z_i^U\right\}$ and $\mathbf{q}_j^{GU} = \left\{x_j^{GU}, y_j^{GU}, 0\right\}$. In particular, $\mathbf{q}_0^{GU}$ is specified to represent the coordinates of the terrestrial base station BS.

Define a binary variable a_{ij}, which indicates that GU_j is served by U_i if $a_{ij} = 1$; otherwise, $a_{ij} = 0$. Each user is served by one and only one UAV.

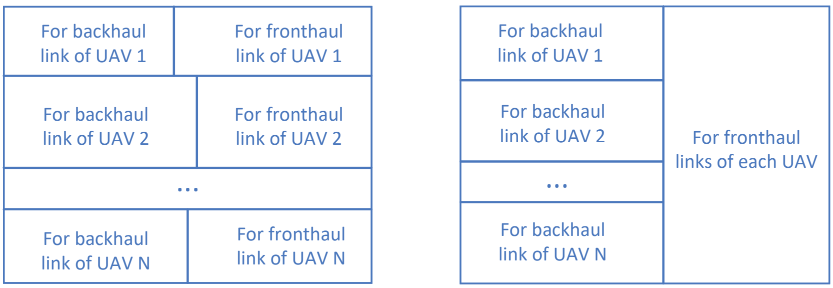
$$a_{ij} \in \{0, 1\}\,, i = 1, ..., N, j = 1, ..., M \tag{1}$$

$$\sum_{i=1}^{N} a_{ij} = 1, j = 1, ..., M \tag{2}$$

All ground users served by UAVs can be represented as the set $\mathbf{GU}_{U_i}$

$$\mathbf{GU}_{U_i} = \{GU_j | a_{ij} = 1, i = 1, ..., N\} \tag{3}$$

As shown in Figure 2, two schemes for bandwidth allocation are considered in this paper: i) allocate frequencies on-demand to the UAVs' fronthaul and backhaul links to avoid co-channel interference; ii) divide frequencies into fronthaul and backhaul links, and in the fronthaul link part, the UAVs use the same frequency band to serve ground users they associate.



(a) Scheme 1 (b) Scheme 2

Fig. 2: two schemes for bandwidth allocation

Air-to-Ground (A2G) and Ground-to-air (G2A) channels are assumed to be dominated by line-of-sight link propagation. According to the free-space path loss model, therefore, the channel power gain between the UAVs and the ground equipment can be denoted by

$$h_{ij} = \frac{\beta_0}{\left\| \mathbf{q}_i^U - \mathbf{q}_j^{GU} \right\|^2} \tag{4}$$

where, β_0 represents the reference channel power gain at the distance $d_0 = 1m$.

In the backhaul link, the ground base station sends the information to the UAVs. Without loss of generality, since this paper considers the situation of insufficient capacity in service areas, it is assumed that the communication demand exceeds the channel capacity, that is, all users need to always communicate. According to the Shannon theorem, the achievable rate from the BS to the UAV can be expressed as

$$C_i^b = \omega_{i0} \log \left(1 + \frac{p_{BS} h_{i0}}{\omega_{i0} \sigma^2} \right) \tag{5}$$

where, ω_{i0} represents the bandwidth allocated to the backhaul link of U_i, p_{BS} represents the transmit power of BS, h_{i0} represents the channel power gain between U_i and BS, σ^2 represents the noise power density.

Scheme 1: Frequencies are allocated on-demand to the UAV's backhaul links and fronthaul links, without co-channel interference. The reachable rate from U_i to GU_j can be expressed as

$$C_{ij} = \omega_{ij} \log \left(1 + \frac{p_{ij} h_{ij}}{\omega_{ij} \sigma^2} \right) \tag{6}$$

where, ω_{ij} represents the bandwidth allocated to the link between U_i to GU_j, p_{ij} represents the transmit power allocated to the link between U_i to GU_j.

Scheme 2: The UAVs' forward links use the same frequency band, and ground users suffer from interference from non-associated UAVs, but at the same time share more spectrum resources. The interference experienced by ground user GU_j associated with UAV U_i can be expressed as:

$$I = \sum_{k \neq i} \frac{p_k}{N_i} h_{kj} \tag{7}$$

where $\frac{p_k}{N_i}$ is the approximation of the interference power of UAV U_k to each ground user in $\mathbf{GU}_{U_i}$, and the relevant assumptions will be presented later. According to the Shannon formula, the reachable rate from UAV U_i to ground user GU_j can be expressed as:

$$C_{ij} = \omega_{ij} \log \left(1 + \frac{p_{ij} h_{ij}}{I + \omega_{ij} \sigma^2} \right) \tag{8}$$

For each set $\mathbf{GU}_{U_i}$, the allocation of its internal system resources is not the focus of this paper. Assuming that fairness is achieved among users in $\mathbf{GU}_{U_i}$, this paper uses a simple method to approximate the reachable rate of users in $\mathbf{GU}_{U_i}$. In this paper, the system resources inside $\mathbf{GU}_{U_i}$ are distributed evenly

$$\omega_{ij} = \frac{\omega_i - \omega_{i0}}{N_i} \tag{9}$$

$$p_{ij} = \frac{p_i}{N_i} \tag{10}$$

where, ω_i represents the bandwidth allocated to U_i, p_i represents the transmit power of U_i, $N_i = \sum_{j=1}^{M} a_{ij}$ represents the number of users served by U_i. The sum of the bandwidth occupied by all UAVs is the system bandwidth ω_{sum}, which is a constant.

$$\sum_{i=1}^{N} \omega_i = \omega_{sum} \tag{11}$$

The transmit power of the UAVs should be less than the maximum transmit power $p_{\max}$.

$$p_i \leq p_{\max}, i = 1, ..., N \tag{12}$$

The sum rate of the fronthaul link for the users served by

$$U_i$$

is

$$C_i^f = \sum_{j=1}^{M} a_{ij} C_{ij} \tag{13}$$

For each UAV base station, the achievable rate that users can obtain is determined by the minimum achievable rates of the backhaul link and the fronthaul link.

$$C_i^U = \min\left(C_i^b, C_i^f\right) \tag{14}$$

Then the reachable rates of users in $\mathbf{GU}_{U_i}$ are averaged to approximate the overall level of user rates in $\mathbf{GU}_{U_i}$

$$C_i^{GU} = \frac{C_i^U}{N_i} \tag{15}$$

The goal of this paper is to maximize the minimum reachable rate for ground users by jointly optimizing UAV locations $\mathbf{Q} = \left\{\mathbf{q}_i^U, i = 1, ..., N\right\}$, bandwidth resource allocation $\omega = \{\omega_i, \omega_{i0}, i = 1, ..., N\}$, transmit power $\mathbf{p} = \{p_i, i = 1, ..., N\}$, and user association $\mathbf{A} = \{a_{ij}, i = 1, ..., N, j = 1, ..., M\}$. Therefore, the optimization problem can be formulated as follows

$$\max_{\mathbf{Q},\omega,\mathbf{p},\mathbf{A}} \min_i C_i^{GU} \tag{16}$$

$$s.t.\mathbf{q}_i^U \in hotspotarea, i = 1, ..., N \tag{16a}$$

$$\sum_{i=1}^{N} \omega_i = \omega_{sum} \tag{16b}$$

$$0 \leq p_i \leq p_{\max}, i = 1, ..., N \tag{16c}$$

$$a_{ij} \in \{0, 1\} \tag{16d}$$

$$\sum_{i=1}^{N} a_{ij} = 1, j = 1, ..., M \tag{16e}$$

(16a) states that UAVs can only operate over service area. (16b) states that the sum of the bandwidth occupied by all UAVs is the total bandwidth of the system. (16c) restricts the UAV transmit power. (16e) states that each ground user is served by one and only one UAV.

Problem (16) is challenging to solve due to the following two main reasons. First, the optimization variable A for user association is binary and thus (16d) involve integer constraints. Second, the objective function is non-convex. Thus, the problem (16) is a mixed-integer non-convex problem, which is difficult to be optimally solved in general.

3 Proposed multi-coding artificial bee colony algorithm

3.1 standard artificial bee colony

ABC is proposed by Karaboga in 2005 [34]. The colony of artificial bees in ABC contains three groups of bees: employed bees, onlooker bees and scout bees. They imitate the minimal model of bees searching for food in nature: employed bees correspond to food sources one-to-one, the onlooker bees receive the information transmitted by the employed bees through dance and select food sources for honey collection, and the scout bees randomly search for new food sources. In ABC, the position of a food source represents a possible solution to the problem and the food amount of a food source corresponds to the quality (fitness) of the associated solution.

The equation of creating a group of food sources (feasible solutions) in the given search space:

$$\mathbf{x}_i = \mathbf{x}_i^{\min} + \mathbf{r}_i \left(\mathbf{x}_i^{\max} - \mathbf{x}_i^{\min}\right) \tag{17}$$

The variation equation of employed bees and onlooker bees is:

$$v_{ij} = x_{ij} + \varphi_{ij} \left(x_{ij} - x_{nj}\right), n \neq i \tag{18}$$

where, $\mathbf{x}_n$ is a different food source from $\mathbf{x}_i$, regarded as one of the neighbor of $\mathbf{x}_i$.

3.2 Multi-coding artificial bee colony

For the joint optimization problem of UAV-assisted communication system resource allocation, a multi-coding artificial bee colony algorithm is proposed to solve it. This section mainly introduces the improved part of MCABC that is different from the standard ABC. First, multiple variables are coded in different ways and initialized according to the coding methods. Second, multiple variables are combined into tuples, each tuple representing a food source (feasible solution). Third, a multi-dimensional neighborhood search operation is performed in the stages of employed bees and onlooker bees. Specifically, a neighborhood search is performed for each variable separately, and then they are combined into a set of new tuples, and a greedy strategy is used to select the best solution to update the current solution.

1. *Food sources initialization:* In MCABC, each food source $\mathcal{F}_k = \{\mathbf{Q}_k, \mathbf{B}_k, \mathbf{p}_k, \mathbf{D}_k\}$ contains four variables of UAV locations, bandwidth allocation, UAV transmit power and user allocation. UAV locations are

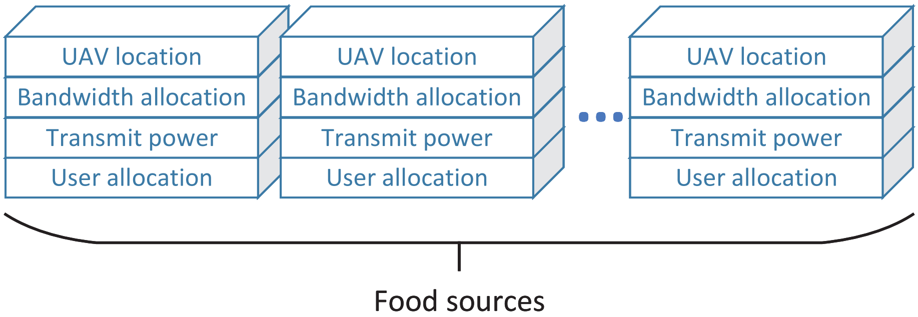


Fig. 3: Multi-coding food sources

represented as:

$$\mathbf{Q}_k = \begin{bmatrix} \mathbf{q}_k^{U_1} \\ \mathbf{q}_k^{U_2} \\ \cdots \\ \mathbf{q}_k^{U_N} \end{bmatrix} = \begin{bmatrix} x_{k1} & y_{k1} & z_{k1} \\ x_{k2} & y_{k2} & z_{k2} \\ & \cdots & \\ x_{kN} & y_{kN} & z_{kN} \end{bmatrix} \tag{19}$$

Bandwidth allocation in scheme 1 is represented as:

$$\mathbf{B}_k = \begin{bmatrix} b_{k11} & b_{k12} \\ b_{k21} & b_{k22} \\ & \cdots \\ b_{kN1} & b_{kN2} \end{bmatrix} \tag{20}$$

where $b_{ki1} = \frac{\omega_{i0}}{B}$ represent the ratio of the bandwidth occupied by the backhaul link of U_i to the total bandwidth, and $b_{ki2} = \frac{\omega_i - \omega_{i0}}{B}$ represent the bandwidth occupied by the fronthaul link of U_i to the total bandwidth. In scheme 2, $\mathbf{B}_k$ represents the proportion of the backhaul link to the total bandwidth, which is a scalar. UAV transmit power is represented as:

$$\mathbf{p}_k = \begin{bmatrix} p_{k1} \\ p_{k2} \\ \cdots \\ p_{kN} \end{bmatrix} \tag{21}$$

where p_{k1} represent the transmit power of U_i. User allocation is represented as:

$$\mathbf{D}_k = \begin{bmatrix} d_{k1} & d_{k2} & \ldots & d_{kM} \end{bmatrix} \tag{22}$$

where $d_{kj} = \{1, 2, ..., N\}$ represent the UAV associated with GU_j. According to the different encoding method, they are initialized as follows:

$$\mathbf{Q}_k = \mathbf{Q}_{\min} + \mathbf{r}_k (\mathbf{Q}_{\max} - \mathbf{Q}_{\min}) \tag{23}$$

$$\mathbf{B}_k = \frac{\mathbf{r}_k}{\sum \mathbf{r}_k} \tag{24}$$

$$\mathbf{p}_k = \mathbf{p}_{\min} + \mathbf{r}_k (\mathbf{p}_{\max} - \mathbf{p}_{\min}) \tag{25}$$

$$\mathbf{D}_k = \text{round} [\mathbf{D}_{\min} + \mathbf{r}_k (\mathbf{D}_{\max} - \mathbf{D}_{\min})] \tag{26}$$

where $\mathbf{r}_k$ represent a matrix of real numbers randomly generated between 0 and 1. (24) represent that a matrix of the same size as $\mathbf{B}_k$ is randomly generated, and divide each element by the sum of all elements to ensure that the sum of the elements in $\mathbf{B}_k$ is 1. round $[\bullet]$ represents the rounding operation.

The multi-coding method allows different variables to adopt different encoding methods, initialization methods and neighborhood search methods according to their special properties, but at the same time they act together on the fitness function.

2. *Employed bee stage:* As shown in fig[1], a multi-dimensional neighborhood search operation is implemented in the employed bee stage to update the current food source. Firstly, neighborhood searches are performed separately to the optimization quantities. Secondly, list all possible permutations. Thirdly, according to the greedy strategy, the permutation with the largest fitness is selected to update the current solution. The pseudocode is shown in Algorithm 1.

   The variation equation for UAV locations is:

$$\mathbf{Q}'_{k,i,j} = \mathbf{Q}_{k,i,j} + r (\mathbf{Q}_{k,i,j} - \mathbf{Q}_{n,i,j}) \tag{27}$$

$$\mathbf{Q}'_{k,i,j} = \min \left(\mathbf{Q}'_{k,i,j}, \mathbf{Q}_{\min,i,j} \right) \tag{28}$$

$$\mathbf{Q}'_{k,i,j} = \max \left(\mathbf{Q}'_{k,i,j}, \mathbf{Q}_{\max,i,j} \right) \tag{29}$$

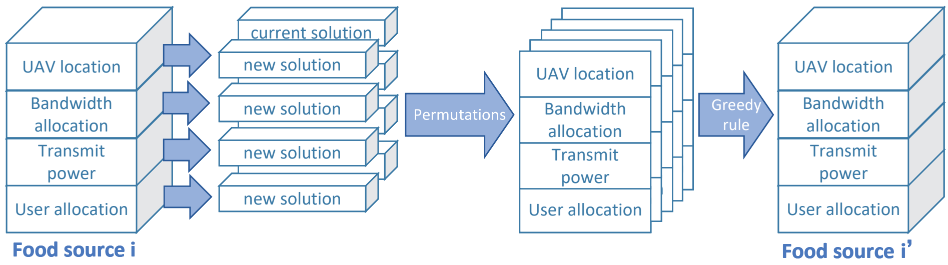


Fig. 4: Multi-dimensional neighborhood search operation

Algorithm 1 Multi-dimensional Neighborhood Search Operation

Input: Food source $\mathbf{x}_k$: $\mathbf{Q}_k$, $\mathbf{B}_k$, $\mathbf{p}_k$, $\mathbf{D}_k$
Output: New food source $\mathbf{x}'_k$: $\mathbf{Q}'_k$, $\mathbf{B}'_k$, $\mathbf{p}'_k$, $\mathbf{D}'_k$
1: randomly initialize n_e food sources
2: bee k searches around $\mathbf{Q}_k$ based on the given rules and produces $\mathbf{Q}'_k$
3: bee k searches around $\mathbf{B}_k$ based on the given rules and produces $\mathbf{B}'_k$
4: bee k searches around $\mathbf{p}_k$ based on the given rules and produces $\mathbf{p}'_k$
5: bee k searches around $\mathbf{D}_k$ based on the given rules and produces $\mathbf{D}'_k$
6: calculate the fitness of all the permutations of $\mathbf{Q}_k$, $\mathbf{Q}'_k$, $\mathbf{B}_k$, $\mathbf{B}'_k$, $\mathbf{p}_k$, $\mathbf{p}'_k$, $\mathbf{D}_k$, $\mathbf{D}'_k$
7: greedy selection among all the permutations
8: replace food source $\mathbf{x}'_k$ by the permutation with the best fitness

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where $\mathbf{Q}_n$ is a neighbor selected randomly, $\mathbf{Q}_{k,i,j}$ is the one element selected randomly in $\mathbf{Q}_k$ to be varied, $r \in [-1, 1]$ is a random number. The variation for bandwidth allocation is:

$$\mathbf{B}'_{k,i,j} = \mathbf{B}_{k,i,j} + r\left(\mathbf{B}_{k,i,j} - \mathbf{B}_{n,i,j}\right) \tag{30}$$

$$\mathbf{B}'_{k,i,j} = \begin{cases} \mathbf{B}'_{k,i,j}, \mathbf{B}'_{k,i,j} \geq 0 \\ -\mathbf{B}'_{k,i,j}, \mathbf{B}'_{k,i,j} < 0 \end{cases} \tag{31}$$

$$\mathbf{B}_k = \frac{\mathbf{B}_k}{\sum \mathbf{B}_k} \tag{32}$$

In scheme 2, equation (32) means that each element in $\mathbf{B}_k$ is divided by the sum of all elements in $\mathbf{B}_k$. In scheme 1, only $\mathbf{B}_k \in [0, 1]$ is required to be guaranteed. The variation equation for transmit power is:

$$\mathbf{p}'_{k,i} = \mathbf{p}_{k,i} + r\left(\mathbf{p}_{k,i} - \mathbf{p}_{n,i}\right) \tag{33}$$

$$\mathbf{p}'_{k,i} = \min\left(\mathbf{p}'_{k,i}, \mathbf{p}_{\min,i}\right) \tag{34}$$

$$\mathbf{p}'_{k,i} = \max\left(\mathbf{p}'_{k,i}, \mathbf{p}_{\max,i}\right) \tag{35}$$

And in the variation equation for user allocation, $N_{ex} = M/10$ elements are varied in each neighborhood search, which are represented as a set EX. The elements in the set EX are selected from all the elements of $\mathbf{D}_k$ according to probability $prob_D$. The probability $prob_D$ is calculated as:

$$\mathbf{C}_{norm} = \frac{1}{\mathbf{C}^{GU} - \min\left(\mathbf{C}^{GU}\right) + 0.5} \tag{36}$$

$$prob_D = \frac{\mathbf{C}_{norm}}{\sum \mathbf{C}_{norm}} \tag{37}$$

where $\mathbf{C}^{GU}$ represents the achievable rate of the fronthaul link between each ground user and the UAV. In this paper, the associations of terrestrial users with low achievable rates are adjusted with greater probability. First, the data distribution of $\mathbf{C}^{GU}$ is adjusted, and then the reciprocal of $\mathbf{C}^{GU}$ is taken to get $\mathbf{C}_{norm}$. $prob_D$ is gotten by dividing each element in $\mathbf{C}_{norm}$ by the sum of all elements. For the selected elements in $\mathbf{D}_k$, the variation equation is:

$$\mathbf{D}_{k,i} = \mathbf{D}_{n,i}, i \in EX \tag{38}$$

In the multi-dimensional neighborhood search operation, variables are searched separately and then combined together. In essence, it expands the search dimension of the solution space, which allows some situations where multiple variables can be changed at the same time to improve the fitness value. For example, it allows ground users to cluster unevenly, while compensating by allocating more bandwidth resources to UAVs serving more ground users. This attempt can help find a more reasonable solution. At the same time, the expansion of the search dimension also increases the amount of calculation. Each multidimensional neighborhood search operation needs to calculate k times of fitness value. In contrast, only four fitness values need to be calculated for each of the four variables to perform a standard ABC neighborhood search.

In addition, for $\mathbf{D}_k$ with large dimension, an adaptive element-to-change selection mechanism is designed, which improves the pertinence of the search and accelerates the convergence speed.

3. *Onlooker bee stage:*Onlooker bee start searching in the neighborhood of the candidate food sources according to a probabilistic model. For food source $\mathcal{F}_i$, the probability of being chosen by an onlooker bee is given by [34]:

$$prob_i = \frac{0.9 * f_i}{\max_i f_i} + 0.1 \tag{39}$$

where f_i represent the fitness value of $\mathcal{F}_i$, equals to $\min C_i$ in this paper, $\max_i f_i$ represent the greatest fitness value of the food sources. Every onlooker bee generates a random real number $rando \in (0, 1)$. If $rando < prob_i$, then the onlooker bees perform multi-dimensional neighborhood search operations on the food sources according to Algorithm 1.

4. *Scout bee stage:* For food sources that cannot be improved by successive neighborhood searches, they are discarded in the scout bee stage and the scout bees search elsewhere. Specifically, when the fitness of all new permutations generated in the multi-dimensional neighborhood search operation is smaller than the current solution, the search times counter is incremented by one, otherwise, the counter is cleared. If the counter has reached the limit of search times, re-initialize the food source.

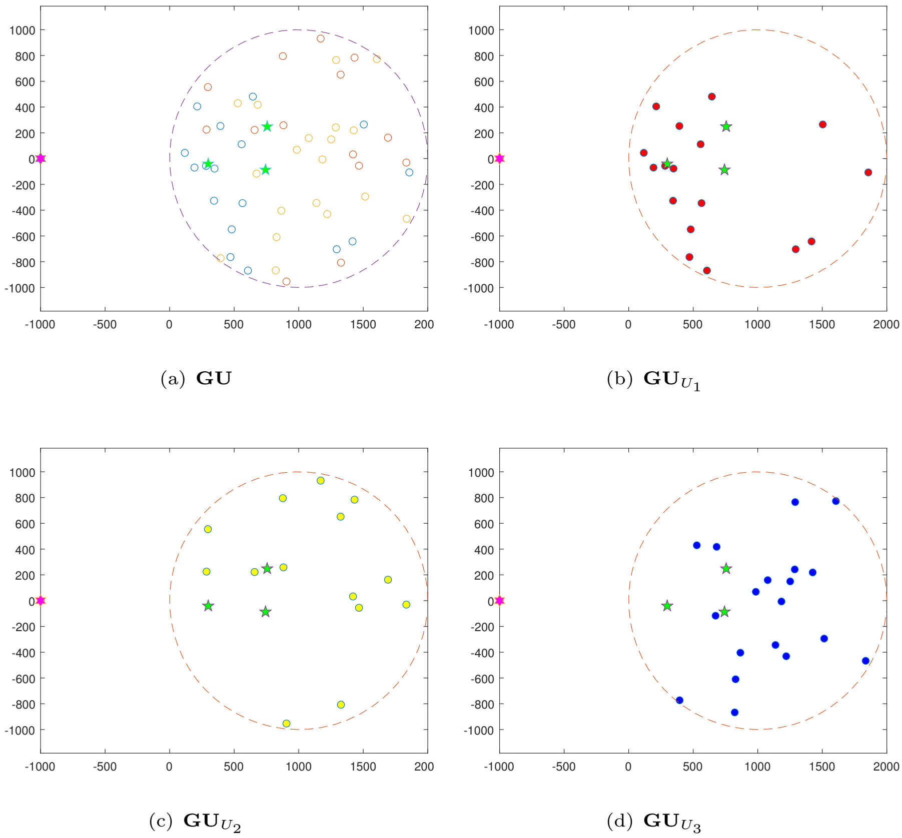
The employed bee stage, the onlooker bee stage and the scout bee stage constitute the main loop of the MCABC algorithm. The pseudocode of the MCABC algorithm is shown in Algorithm 2.

Algorithm 2 MCABC Algorithm

```
Input: food number n_e,maximum iteration number, limit
  randomly initialize n_e food sources
  calculate fitness values of the initialized food sources
  while maximum iteration number is not reached do
      for i ← 1 to n_e do
          employed bee i searches around food source x_i and produces x'_i with
  Algorithm 2
      end for
      for i ← 1 to n_e do
          onlooker bee i chooses a food source based on the fitness of food
  sources
          onlooker bee i searches around the choosen food source x_k and
  produces x'_k with Algorithm 2
      end for
      for i ← 1 to n_e do
          if food source x_i has not been improved in the last limit iterations
  then
              a scout bee flies out and randomly explores the search space to
  produce x'_i
              replace food source x_i by x'_i
          end if
      end for
  end while
```

4 Simulation results and discussion

In this section, numerical simulations are performed to validate the efficiencies and performance of the proposed algorithm for solving the joint resource allocation problem in the UAV-assisted wireless communication system.



(a) **GU**

(b) $\mathbf{GU}_{U_1}$

(c) $\mathbf{GU}_{U_2}$

(d) $\mathbf{GU}_{U_3}$

Fig. 5: UAV locations and user allocation result of scheme 1

In our simulations, we consider the position of the ground base station at $(-1000, 0, 0)$m, and its transmit power is 20W. The terrestrial users are randomly distributed in a circle with $O\,(1000, 0, 0)$m as center and $R = 1000$m as radius. The locations of UAVs are limited to the air above the distribution area of terrestrial users, their heights are limited to $[50, 500]$m, and their transmit power is limited to $[0, 2]$W. The available bandwidth resource of the whole system is 20MHz. The noise power spectrum density $\sigma_0^2 = 3.98 \times 10^{-21}$W/Hz, and $\beta_0 = 60$dB.

To begin with, we consider a scenario where 3 UAVs serve 50 terrestrial users to demonstrate the effectiveness of the algorithm in typical scenarios. Figure 5 is a schematic diagram of UAV locations and user association in the reuse-3 scheme, and Table 1 is the specific numerical results. In Figure 5, the six-pointed star represents the location of the ground base station, the five-pointed star represents the locations of the UAVs, and the ground users are represented by circles. It is noted that UAVs are at the minimum altitudes for

Table 1: Solution of scheme 1

UAV location	$\begin{bmatrix} 298.96 & -40.77 & 50 \\ 755.65 & 248.09 & 50 \\ 742.70 & -87.16 & 50 \end{bmatrix}$
Bandwidth allocation	$\begin{bmatrix} 0.1670 & 01675 \\ 0.1400 & 0.1410 \\ 0.1922 & 0.1923 \end{bmatrix}$
Transmit power	$\begin{bmatrix} 1.93 & 2.00 & 2.00 \end{bmatrix}$
User allocation	As Figure 5

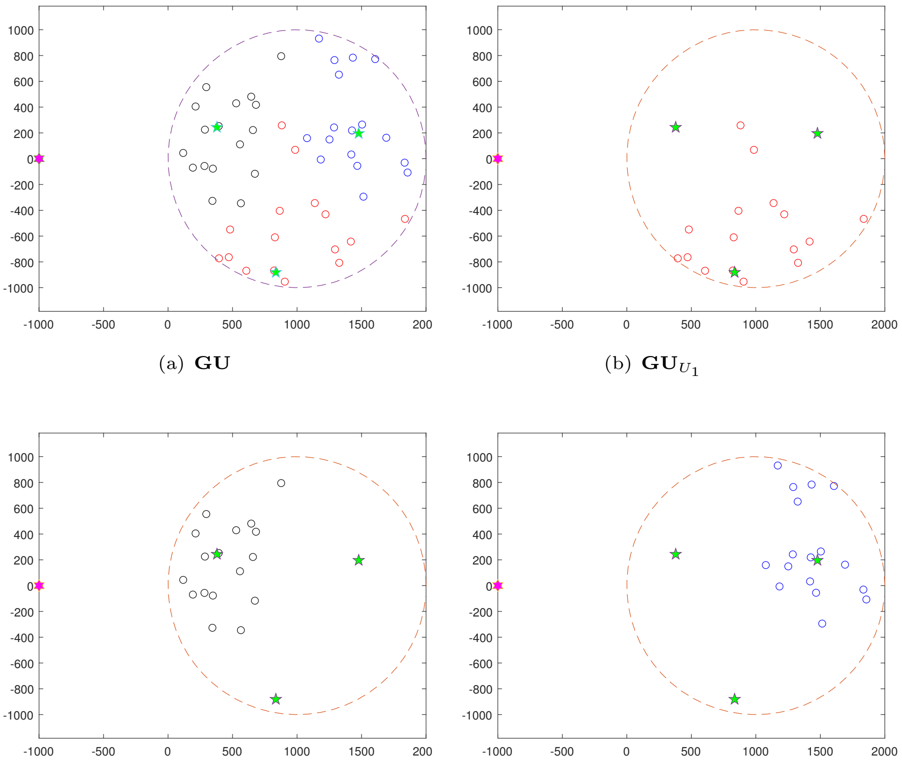
Table 2: Solution of scheme 2

UAV location	$\begin{bmatrix} 834.75 & -881.21 & 50.52 \\ 378.98 & 243.40 & 50 \\ 1477.83 & 196.40 & 50.74 \end{bmatrix}$
Bandwidth allocation	0.1919
Transmit power	$\begin{bmatrix} 0.97 & 0.66 & 0.64 \end{bmatrix}$
User allocation	As Figure 6

better channel conditions and are offset to the respective terrestrial user clusters they serve. As shown in Figure 5(b)5(c)5(d), the terrestrial users show a clustering trend, but there are still a small number of terrestrial users far away from the associated UAVs. Increasing the number of iterations of the algorithm can improve this problem, but in fact the individual terrestrial user assignment results have very little impact on the optimization objective. Since U_3 is associated with more terrestrial users, it obtains more bandwidth resources. Since there is no co-channel interference in this scheme, the transmit power of the UAV is close to the maximum transmit power. The power of U_1 does not reach the maximum because the reachable rate of the users it serves has exceeded the users in the other two clusters, and higher power will not improve the fitness value.

Figure 6 is a schematic diagram of UAV locations and user association in the reuse-1 scheme， in which Figure 6(b)6(c)6(d) shows the terrestrial user clusters served by the three UAVs, and Table 2 is the specific numerical results. It can be seen from the results that in order to reduce the interference, the distances between the UAVs are increased, and the transmit power of the UAVs is correspondingly lower. In order to make up for the rate drop of the fronthaul link due to interference, 80.81% of the bandwidth is allocated to the fronthaul link. In addition, interference exacerbates the impact of user association on the achievable rate, so in scheme 2 the user clusters are closer.

Figure 7 shows the change in achievable rate as the number of UAVs increases in the two schemes. It can be seen from the figure that in the reuse-1 scheme, compared with a single UAV, when the number of UAVs exceeds one, the achievable rate is significantly reduced due to the existence of co-channel interference. But when the number of drones exceeds one, the reachable rate still increases with the number of drones.



(a) **GU** (b) $\mathbf{GU}_{U_1}$

(c) $\mathbf{GU}_{U_2}$ (d) $\mathbf{GU}_{U_3}$

Fig. 6: UAV locations and user allocation result of scheme 2

In order to observe the convergence of the proposed algorithm, in the scenario of 3 UAVs with 50 ground users, we repeatedly run the MCABC algorithm 200 times and take the average value to obtain the convergence curve shown in Figure 8, adopting the reuse-3 scheme. It can be seen that MCABC quickly finds a solution with a high fitness value, and makes minor adjustments to the solution in subsequent iterations. The baseline is the block coordinate descent method, an iterative method that uses standard ABC to optimize each of the variables one by one in each iteration, and fixes other variables unchanged when optimizing a variable. We run the two methods for 200 times respectively, and Figure 9 shows the distribution of fitness values for their given schemes. It can be seen that MCABC algorithm is more stable than the block coordinate descent method, and the obtained solution quality is higher. This is because the multi-dimensional neighborhood search adopted by MCABC expands the search dimension of the solution space, allowing multiple variables to cooperate to improve the quality of the solution, which helps the algorithm to jump out of the local optimum.

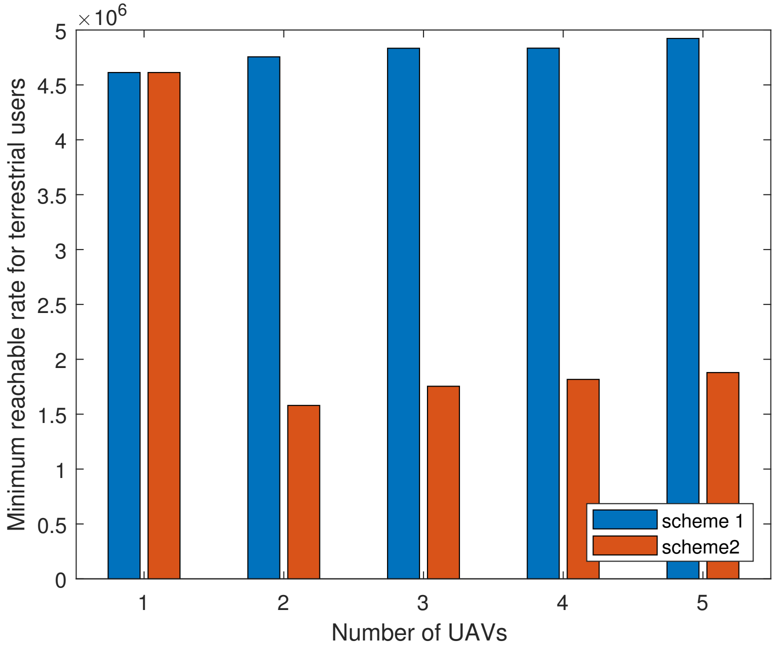


Fig. 7: Minimum reachable rate versus number of UAVs

Figures 10 show the performance of the MCABC algorithms under different numbers of UAVs and terrestrial users in scheme 1. It can be seen from the figures that the MCABC algorithm still performs well when the problem scale becomes larger. Figure 10(a) shows the variation of the minimum reachable rate of terrestrial users when the number of UAVs increases under the condition that the number of terrestrial users is 50. It can be seen from Figure 10(a) that increasing the number of UAVs can also improve the reachable rate of ground users without increasing system resources. This is because more UAVs can make the ground user cluster more concentrated, which means that the distance between the UAV and the farthest user in the cluster becomes shorter, which improves the channel quality. Figure 10(b) shows the change of the minimum reachable rate of ground users when the number of ground users increases under the condition that the number of UAVs is 3. An increase in the number of terrestrial users reduces the reachable rate per terrestrial user. For ease of comparison, we multiply the number of users as a factor by the minimum reachable rate to obtain the "system reachable rate" to observe the performance of the algorithm as the problem size increases. It can be seen from Figure 10(b) that the increase of the problem size has a certain impact on the quality of the solution, because the solution space of the user-allocation domain expands exponentially with the increase of the number of users. However, the quality of the solution did not decline sharply with the size of the problem.

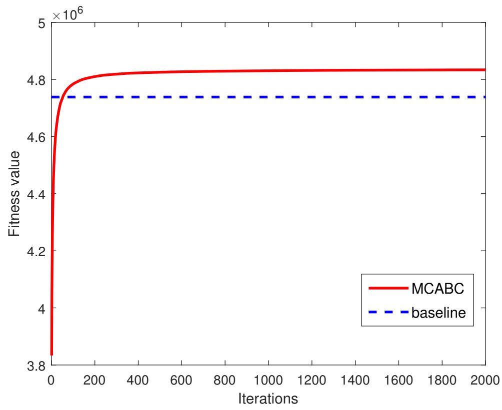


Fig. 8: Convergence of MCABC

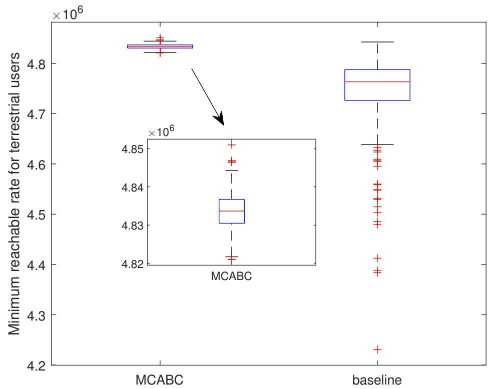
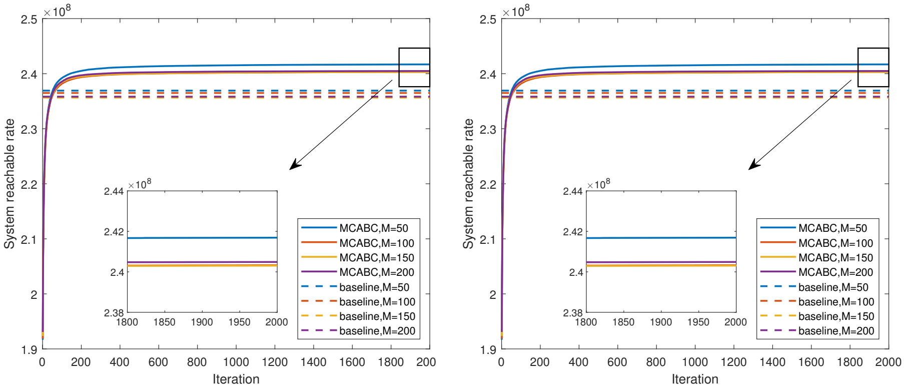


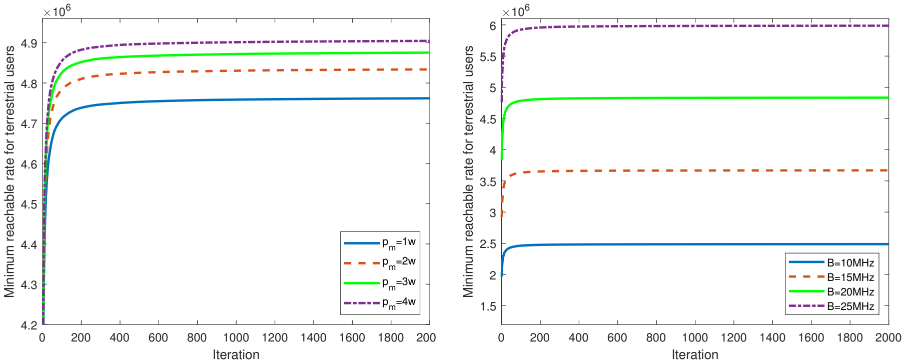
Fig. 9: Distribution of fitness values

This paper further studies the variation of the achievable rate with other parameters of the system in the scheme one. Figure 11(a) shows the variation of the reachable rate with the power of the UAVs. It can be seen from the figure that the reachable rate increases with the power of the UAVs, but the increment continues to decrease. Figure 11(b) shows the achievable rate as a function of the total system bandwidth. It can be seen from the figure that the achievable rate increases significantly with the total system bandwidth. However, bandwidth resources are very limited in practical applications, so it is necessary to introduce new technologies into the system to improve spectrum utilization.



(a) Minimum reachable rate versus N (b) System reachable rate versus M

Fig. 10: MCABC performence when problem size increases



(a) Minimum reachable rate versus UAV transmit power (b) Minimum reachable rate versus system bandwidth

Fig. 11: Minimum reachable rate versus system resource

5 Conclusion

In this paper, the joint resource allocation problem for UAV-assisted communication systems is investigated. In the scenario considered in this paper, the ground base station first sends information to the UAV, which in turn forwards the information to the user. Based on this system model, a joint resource allocation scheme is proposed. The objective of this scheme is to maximize the minimum value of reachable rate for ground users by jointly optimizing the UAV location, occupied bandwidth, transmit power and user association. To address the problem that the joint resource allocation problem is difficult to solve and the existing block coordinate descent method is prone to fall into local optimum, a multi-coding artificial bee colony algorithm is proposed to solve the problem. Simulation results demonstrate that the proposed algorithm can converge to a better result within a limited number of iterations, and the minimum user reachable rate achieved is better than the solution given by the block coordinate descent method.

The simulation results also pointed out that increasing the number of UAVs can improve the reachable rate of ground users, but the effect is not as good as directly increasing the spectrum resources. However, due to the line-of-sight link characteristics of the air-ground channel, the repeated use of limited spectrum resources within the total system bandwidth will cause strong co-channel interference. We plan to introduce new technologies to reduce interference in future work to improve the frequency band utilization of the system.

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Declarations

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Conflict of interest/Competing interests

The authors have no relevant financial or non-financial interests to disclose.

Ethics approval

Not applicable.

Consent to participate

All the authors are informed consent to participate in this study.

Consent for publication

All the authors are informed consent to publicate this study.

Availability of data and materials

Not applicable.

Code availability

Code can be provided if reviewers/editors need it.

Authors' contributions

All authors contributed to the study conception and design. Data collection and analysis were performed by [Weibo Hao], [Fang Ye] and [Qian Sun]. The first draft of the manuscript was written by [Weibo Hao] and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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