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# Joint Sink Mobility and Routing to Maximize the Lifetime of Wireless Sensor Networks: The Case of Constrained Mobility

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**Abstract**—The longevity of wireless sensor networks (WSNs) is a major issue that impacts the application of such networks. While communication protocols are striving to save energy by acting on sensor nodes, recent results show that network lifetime can be prolonged by further involving sink mobility. As most proposals give their evidence of lifetime improvement through either (small-scale) field tests or numerical simulations on rather arbitrary cases, a theoretical understanding of the reason for this improvement and the tractability of the joint optimization problem is still missing. In this paper, we build a framework for investigating the joint sink mobility and routing problem by constraining the sink to a finite number of locations. We formally prove the NP-hardness of the problem. We also investigate the induced subproblems. In particular, we develop an efficient primal-dual algorithm to solve the subproblem involving a single sink, then we generalize this algorithm to approximate the original problem involving multiple sinks. Finally, we apply the algorithm to a set of typical topological graphs; the results demonstrate the benefit of involving sink mobility, and they also suggest the desirable moving traces of a sink.

**Index Terms**—Lifetime, routing, sink mobility, wireless sensor networks (WSNs).

## I. INTRODUCTION

WIRELESS sensor networks (WSNs) are fast emerging as a new networking and sensing paradigm based on a large number of tiny sensor nodes. These networks can be deployed close to or inside the phenomenon under surveillance and, thus, have the potential of providing diverse information to numerous applications. However, the small size of the sensor nodes (hence their capacity-limited power sources) is posing a great challenge: The longevity of WSNs under energy constraints should be addressed before we can benefit from their advantages. Communication protocols that strive to save energy

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in WSNs (e.g., [1]–[12]) mainly focus on the *sensor nodes*,<sup>1</sup> whereas a recent trend indicates a focus shift to the behavior of *sinks*<sup>2</sup> [13]–[20], which can be exploited to further improve the lifetime of WSNs.

There are two approaches, *fast mobility* and *slow mobility*, for exploiting *sink mobility* to improve network lifetime. They are distinguished by the relationship between the moving *speed* of a sink and the tolerable *delay* of the data delivery. On one hand, a sink can “transport” data with its movements if its speed is **high** enough to produce a tolerable data delivery delay [13]–[15] and, hence, spare nodes from the traffic-forwarding load. This is the fast mobility approach, as the sink should move sufficiently fast. On the other hand, moving the sink, even very **infrequently** (say once a week), may still benefit the network lifetime because it can lead to a global load balancing in the entire network [16]–[20]. This is the slow mobility approach because the mobility cannot be used to transport data within a tolerable delay (but it barely affects the delay due to the way it is used). The main reason for the improvement brought by the slow mobility approach is the typical many-to-one traffic pattern in WSNs. Such a pattern imposes a heavy forwarding load on the nodes close to sinks. While no energy-conserving protocol alleviates such a load, moving sinks can distribute the role of bottleneck nodes over time and thus even out the load.

The general reason that sink mobility, no matter if fast or slow, can improve network lifetime lies in the fact that mobility increases the **dimension** (thus the degree of freedom) of the problem. This follows the principle that optimizing an objective in a high-dimension space always leads to a result no worse than what can be achieved in a subspace of reduced dimension. However, solving problems in high-dimension space incurs a higher complexity. Existing approaches either directly consider the practical implementation issues before developing a theoretical understanding [15] or solve simplified subproblems using contemporary software without paying attention to the tractability of the problem in general [17], [18], [20]. This prevents us from getting deeper insight on how and why sink mobility brings lifetime improvement.

In this paper, we investigate the problem of maximum lifetime data collection in WSNs by jointly considering sink mobility and routing. We consider a type of continuously monitoring WSN whose data generation rates of sensors can be

<sup>1</sup>In this paper, the words *sensor*, *sensor node*, and *node* are used interchangeably.

<sup>2</sup>These are the devices that collect data from WSNs; sometimes they are also termed *base stations*.

estimated accurately. We focus on the slow mobility approach and constrain the sink to a finite number of possible locations. We build a unified framework to cover most of the joint sink mobility and routing strategies. Our investigation of the *maximizing network lifetime* (MNL) problem is based on a graph model. We show that the MNL problem involving multiple mobile sinks is NP-hard, but that certain induced subproblems having a practical significance are tractable. Moreover, we show that the MNL problem involving only a single mobile sink can be solved by an efficient primal-dual algorithm; we further generalize this algorithm to approximate the general MNL problem. Finally, we illustrate the benefit of using a mobile sink by applying our algorithm to a set of typical topological graphs.

Our main contributions are the following:

- We provide a constructive proof of the NP-hardness of the MNL problem involving multiple mobile sinks.
- We identify the subproblem that has a potential to guide routing protocol designs in practice.
- We develop an efficient primal-dual algorithm for the subproblem involving a single sink; it is then generalized to approximate the general MNL problem.
- We formally prove the superiority of moving the sinks over keeping them static in the case that the sinks are constrained to where the nodes are.

The rest of this paper is organized as follows. Section II surveys related work. Sections III and IV state our assumptions and formulate the MNL problem. Section V proves the NP-hardness of MNL. Section VI discusses induced subproblems of MNL, and, in particular, Section VII investigates in detail the subproblem that involves only a single mobile sink. Section VIII extends our investigations to the general MNL problem. Section IX reports the numerical experiment results on the algorithm we developed in Section VII. Finally, Section X concludes the paper.

## II. RELATED WORK

We first present the recent work consisting of improving network lifetime with mobile sinks. We then briefly discuss a few topics related to energy conservation in WSNs. Proposals related to sink mobility are also described in [21]–[25], but we will not discuss them because these proposals are either about coping with sink mobility (rather than exploiting it) [21]–[23] or about preventing buffer overflow (rather than extending lifetime) [24], [25]. We are aware that there have been significant efforts in designing online mobility control algorithms (e.g., [26] and [27]), but our offline approach does serve as a benchmark. More importantly, our offline solution is applicable and more efficient provided that the data rates can be accurately estimated. Last but not least, we refer to [28] for a theoretical investigation on load-balancing (including using mobile sinks) in WSNs detecting bursty events.

### A. Moving Sinks to Improve Network Lifetime

If a sink moves fast enough to deliver data with a tolerable delay, WSNs may take advantage of *mobility capacity* [29]. In this *mobile relay* approach [13], [14], the mobile sink “picks up” data from nodes (through one-hop transmissions) and transports

the data with mechanical movements. This approach trades data delivery latency for the reduction of energy consumption of nodes. While both [13] and [14] leverage only on uncontrollable mobility, [15] investigates the controllable mobility. This proposal is a compromise between the mobile relay approach and the *mobile sink* approach [16]–[20]: The sink relays data with its movements, and nodes transmit data (through a multihop routing if necessary) when the sink moves to the closest point to them. A field study is reported in [15], and a simple theoretical analysis on this hybrid approach is presented later in [30]. In our paper, we will briefly investigate the tractability of the lifetime optimization problem using a controllable mobile relay (see Section V); we will not cover the approaches involving uncontrollable sink mobility [13], [14].

If a sink moves infrequently, its average speed is not high enough to produce tolerable data delivery delay. In fact, the sink mobility may take a **discrete** form: The movement trace consists of several *anchor points* between which sinks move and at which they pause. Consequently, data packets have to be carried from their origin to the sinks through multihop routing. However, it has recently been observed that sink mobility still offers benefits in terms of network lifetime [16], [17] thanks to a consequent load-balancing effect. Unfortunately, the formulations in [16] and [17] are only concerned with load balancing within each pause time, which might not lead to load balancing in the whole lifetime. In another contribution [18], Wang *et al.* make a different formulation where the routing paths are predetermined and the pause times become the variables of the lifetime optimization problem. This differs from our approach in that we jointly consider routing and sink mobility. Luo and Hubaux [19] take a continuum model and obtain some forms of optimality by exploiting the symmetry of the assumed circular networks; it is difficult to apply these results to more general network topologies. Papadimitriou and Georgiadis [20] extend the formulation of [18] by jointly considering sink mobility and routing. The full-scale problem is, however, not addressed because of its prohibitive complexity; the sink is hence confined to a limited number of positions in the numerical experiments. An extension of [18] is presented recently in [31], where a more comprehensive formulation is taken and a greedy online algorithm is reported. Note that, apart from [17], the authors of [16], [18]–[20], and [31] consider only a single mobile sink. Most importantly, the hardness of the joint sink mobility and routing for lifetime optimization is not evaluated in [16]–[18], [20], and [31]. The general framework we propose in this paper encompasses all the formulations in [17], [18], [20], and [31].

Recently, Wang *et al.* [32] presented a *mobile node*<sup>3</sup> approach. The idea is that a few powerful mobile nodes are deployed to replace heavily loaded (static) nodes such that these static nodes can shut down for energy saving. The same authors have further investigated the tradeoff between using mobile nodes and deploying dense network [33]. Whereas our framework will not cover this problem due to the fundamental difference between moving nodes and sinks, our analysis still sheds light on understanding this approach from a different

<sup>3</sup>The approach is termed “mobile relay” in [32], but we give it another name in order to be consistent with our terminology (where “mobile relay” is given to another approach [13], [14]).

perspective (see Section VIII-B for details). We also note that, as proved in [32], the mobile node approach can achieve the same order of lifetime as the mobile sink approach only if a sufficient number of mobile nodes ( $O(\sqrt{n})$  for an  $n$ -nodes network) are deployed.

### B. Energy Conservation Protocols

The mobile sink approach is closely related to existing energy-conserving routing (e.g., [1]–[3]). These protocols aim at balancing the energy consumption instead of minimizing the absolute consumed power. The mobile sink approach, by further involving sink mobility, increases the dimension of such optimization problems. Topology control (including transmission power control, e.g., [4]–[6], and node scheduling, e.g., [7]–[9]), clustering (e.g., [10]–[12]), sensor–sink coordination (e.g., [34]), and exploiting limited infrastructures (e.g., [35]) are, among others, important ways of reducing energy consumption. Although the mobile sink approach is orthogonal to these proposals, it serves as a potential complement.

## III. SYSTEM MODEL

We model a WSN as a digraph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , where  $\mathcal{V}$  represents  $|\mathcal{V}| = n$  sensor nodes. There is a cost assignment  $c: \mathcal{V} \times \mathcal{V} \rightarrow \mathbb{R}^+$ , such that  $\exists(i, j) \in \mathcal{E}$  if and only if the transmission energy (defined later) of  $i \in \mathcal{V}$  is no less than  $c(i, j)$ . Apart from the sensor nodes, there is a set  $\mathcal{S}$  ( $|\mathcal{S}| = m < n$ ) of sinks that harvest data from the WSN. The following properties are specified for the network:

- A) Sensor nodes are stationary, but the sinks change their positions from time to time with a negligible traveling time between two positions.
- B) The sink locations can only be chosen within a finite set  $\Lambda \supseteq \mathcal{V}$ . This includes the case where the location of each sink coincides with the location of one of the nodes, which is denoted by  $\Lambda = \mathcal{V}$ .
- C) Each sink, on one hand, behaves like a common node for receiving data. In particular, if a sink is colocated with a node, it inherits the data collection function of that node. On the other hand, a sink has long-range (wireless) communication facilities to transmit data out of the considered WSN.
- D) The data traffic originates from each node  $i \in \mathcal{V}$  (i.e., all nodes are *sources*) and flows to one of the *sinks*  $s \in \mathcal{S}$  (through multihop relaying if no direct connection exists between  $i$  and  $s$ ), and the control traffic involved (e.g., in a routing protocol) is not considered because it has the same effect to all nodes.
- E) Data transmission and reception are the dominating factors for the energy consumption of a node.

We denote by *on-graph* mobility the case  $\Lambda = \mathcal{V}$ , and by *off-graph* mobility the case  $\Lambda \supset \mathcal{V}$ . For the off-graph mobility, we extend the network graph to  $\hat{\mathcal{G}} = (\Lambda, \hat{\mathcal{E}})$ :  $\hat{\mathcal{G}}$  differs from  $\mathcal{G}$  in that: 1) a node  $i \in \Lambda \setminus \mathcal{V}$  has **no** outgoing links; and 2) such a node, as well as any link incident to it, becomes active **only if** a sink is located at that node. Although we focus on  $\mathcal{G}$  in order to facilitate notation, it is straightforward to see that most of our results, in particular the algorithm that solves the MNL

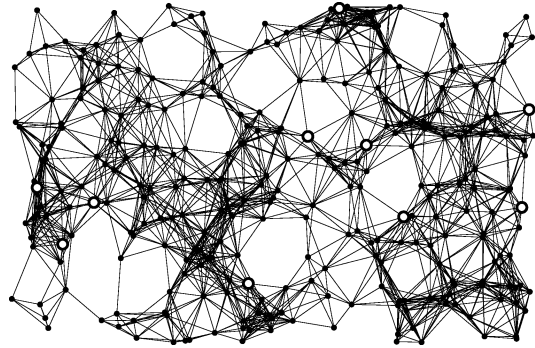


Fig. 1. Network graph model. The WSN has 300 nodes (black points) and 10 sinks (white points). Nodes are uniformly distributed in a rectangular area. Here, we take a cost assignment based on the Euclidean distance and a uniform transmission energy assignment.

problem, also apply to  $\hat{\mathcal{G}}$  (as demonstrated in Section IX). We will emphasize those results that cannot be extended to  $\hat{\mathcal{G}}$  and discuss their implications specifically.

In addition, there are attributes associated with a node  $i \in \mathcal{V}$ :

- F) a value  $E_i$  (Joules) representing the initial energy reserve of the node;
- G) two values  $e_i^{\text{tx}}$  and  $e^{\text{rx}}$  representing the energies for the node to transmit and receive a unit of data (e.g., Joules/byte);
- H) a quantity  $R_{ij}$  that upper-bounds the transmission rate (e.g., bytes/second) of link  $(i, j) \in \mathcal{E}$ ;
- I) a rate  $\lambda_i$  of the information generation.

As shown in [36], most sensor radios have a constant  $e^{\text{rx}}$  regardless of the transmission power of a sender. Fig. 1 shows the graph representation of a WSN.

*Remarks:* First, the assumption A) of a negligible traveling time for a sink comes from the fact that the time during which the sink pauses at a certain location can be long enough to amortize the routing overhead introduced by the sink mobility. We have recently validated this assumption by showing that a significant lifetime improvement can still be achieved when taking the routing overhead into account [37].

Second, a global lifetime maximization could be achieved if no constraints were put on the sink locations. However, this would require us to solve a hard nonconvex optimization problem. The mobility constraints imposed by assumption B) make the problem tractable; the same simplification was applied in [20] and [31]. However, [20] and [31] only consider the case where  $\Lambda \cap \mathcal{V} = \emptyset$ . It is straightforward to see that having  $\Lambda \supseteq \mathcal{V}$  may further improve the lifetime due to a higher degree of freedom and the *substitution effect* (see Sections VIII-B and IX-A for details) that comes as a result of assumption C). In addition, examples in Sections IX-A and C show that the maximum lifetime can sometimes be achieved with only on-graph mobility.

Third, one significant difference between our model and those in [1], [3], and [20] is that we assume the energy consumption of data transmission  $e_i^{\text{tx}}$  is associated with a node instead of a link, as defined by assumption G). We consider this model to be more realistic because, although nodes have the flexibility to tune their own transmission power, it is not cost-effective to dynamically

tune the power for destinations at different distances. In addition, tuning transmission power according to transmission distances is not always feasible either, as a node might not know the distances. Therefore, a reasonable scenario, in our opinion, is that each node sets up a transmission power according to certain topology control mechanisms [4]–[6] at the network initialization phase, and this power is fixed until some topology changes happen.

#### IV. PROBLEM FORMULATION

We begin with the definition of network lifetime for WSNs, and then formulate the problem of *maximizing network lifetime* (MNL) under an optimization framework.

##### A. Network Lifetime

The *network lifetime* can be defined in various ways; these definitions focus on either individual [1] or collective [38] behaviors of nodes. Because the individuality has an implication on the collectiveness (e.g., the death of a node is soon followed by the death of all nodes one-hop away [17]), we define the network lifetime as the time period for the first node to run out of its energy reserve [1].

##### B. The Optimization Problem

We denote the lifetime by  $T$  and use  $t_k$  to indicate the time span for the  $k$ th *epoch*: A new epoch begins when some sinks change their locations. We also define  $q_{ij}^k$  as the total information flow from node  $i$  to node  $j$  during  $t_k$ . By assumption G), the total energy consumed by node  $i$  during  $t_k$  is given by  $e_i^{\text{tx}} \sum_j q_{ij}^k + e^{\text{rx}} \sum_j q_{ji}^k$ , where the sum is over all adjacent vertices of  $i$  (the adjacency is implied by the cost assignment  $c$  and the transmission energies of node  $i$  and its neighbors). In order to indicate the location of a sink during  $t_k$ , we use a binary variable  $\delta_{is}^k$  to represent the relation between the location of a sink  $s \in \mathcal{S}$  and that of node  $i \in \mathcal{V}$ , such that  $\delta_{is}^k = 1$  if sink  $s$  is collocated with node  $i$ , and  $\delta_{is}^k = 0$  otherwise. We also associate with node  $i$  an outgoing flow  $F_i^k$  for epoch  $t_k$ ; it becomes positive only if  $\exists s \in \mathcal{S} : \delta_{is}^k = 1$ . Hence, the mixed-integer nonlinear programming for MNL is as follows:

$$\text{Maximize } T = \sum_k t_k \quad (1)$$

$$\sum_j q_{ij}^k - \sum_j q_{ji}^k - \lambda_i t_k + F_i^k = 0 \quad \forall i, k \quad (2)$$

$$F_i^k - \sum_j \lambda_j \cdot t_k \sum_s \delta_{is}^k \leq 0 \quad \forall i, k \quad (3)$$

$$\sum_s \delta_{is}^k \leq 1, \quad \sum_s \sum_i \delta_{is}^k - m = 0 \quad \forall k \quad (4)$$

$$q_{ij}^k - R_{ij} t_k \leq 0 \quad \forall (i, j), k \quad (5)$$

$$\sum_{k: \delta_{is}^k \neq 1} \left( e_i^{\text{tx}} \sum_j q_{ij}^k + e^{\text{rx}} \sum_j q_{ji}^k \right) - E_i \leq 0 \quad \forall i \quad (6)$$

$$q_{ij}^k, t_k, F_i^k \geq 0 \quad \forall i, j, k \quad (7)$$

$$\delta_{is}^k \in \{0, 1\} \quad \forall i, k, s \quad (8)$$

where  $\sum_i$  means a sum over all possible  $i$  (this **notation** is used throughout the paper). We explain each constraint:

- FLOW CONSERVATION (2)–(4): The outgoing flow exceeds the incoming flow by an amount of  $\lambda_i t_k$ , by assumption I), if  $\sum_s \delta_{is}^k = 0$  (i.e., no sink is collocated with  $i$ ; by (3),  $F_i^k = 0$  if  $\sum_s \delta_{is}^k = 0$ ); otherwise, the flow  $F_i^k$  should be taken into account (again by (3), this quantity is bounded from above by the total flow generated within the  $k$ th epoch  $\sum_j \lambda_j \cdot t_k$ ). Constraint (4) states that no sink is collocated with another sink and the number of sinks is exactly  $m$ .
- RATE CONSTRAINT (5): The total information flow going through a link  $(i, j)$  during  $t_k$  should not exceed the link capacity  $R_{ij} t_k$  due to our assumption H).
- ENERGY CONSTRAINT (6): The energy spent by node  $i$  to transmit and receive data during the whole network lifetime is upper-bounded by the initial energy reserve  $E_i$  due to our assumptions F) and G). Note that the summation is not performed for the epoch during which a sink is collocated with  $i$  due to the assumption C).

In this paper, we ask the question of what is the maximum lifetime, and we consider only WSNs that do not demand high throughput. For such networks, constraint (5) is inactive if the rate set  $\{\lambda_i\}$  is scaled properly. Also, the interferences among links become negligible if the rate set  $\{\lambda_i\}$  is scaled to a very low level. Therefore, we do not take interference into account. However, we could also ask what is the maximum amount of data that can be collected, considering  $\lambda_i$ ,  $i = 1, 2, \dots, n$ , as variables. In such a case, the objective function becomes  $T \sum_i \lambda_i$  and (5) limits the potential choices of  $\lambda_i$ . This requires us to find a good tradeoff between lifetime and throughput. For a detailed treatment of the latter problem, we refer to our recent work [39].

#### V. HARDNESS OF THE PROBLEM

The potential number of sink layouts for the MNL problem is  $\binom{n}{m}$ , which, by Stirling's approximation, is exponential in  $n$  for an arbitrary  $m$ . Given the exponential (in  $n$ ) number of variables in the programming, it is not difficult to believe that the MNL problem is "very hard." In order to formally evaluate its hardness, we consider the following decision problem that is derived as a restricted case for the original MNL, which we term *mobile sink positioning* (MSP):

INSTANCE: A set of nodes  $\mathcal{V}$ , a cost assignment  $c : c(i, j) = e, \forall i, j \in \mathcal{V}$ , a set  $\mathcal{S}$  of sinks with  $|\mathcal{S}| < |\mathcal{V}|$ , and for each  $i \in \mathcal{V}$ , a transmission energy  $e^{\text{tx}}$ , an energy reserve  $E$ , a rate  $\lambda$ , and a positive real  $t$ .

QUESTION: Is there a *sink layout schedule*  $\{(sl_k, t_k)\}$  ( $sl_k$  is a vector of  $[\delta_{is}^k]$ , where  $\delta_{is}^k : \mathcal{V} \times \mathcal{S} \rightarrow \{0, 1\}$  and  $\sum_i \sum_s \delta_{is}^k = |\mathcal{S}|$ ) such that the lifetime  $T = \sum_k t_k$  is at least  $t$ ?

*Proposition 1:* The MSP problem is NP-hard.

*Proof:* The NP-hardness of the MSP problem can be shown by giving a polynomial-time reduction from the DOMINATING SET [40] on a *unit disk graph* to a special case of MSP where a schedule consists of only one element  $(sl, T)$ . For a given network, let  $e = 1$ ,  $e^{\text{tx}} = 1$ ,  $E = 1$ , and  $\lambda = 1$ . It is trivial to see that the graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  induced by  $e$  and  $e^{\text{tx}}$  is a unit disk graph. Now, if  $\mathcal{G}$  admits a lifetime  $T = 1$ , the sink layout  $sl$  suggests the dominating set in  $\mathcal{G}$  because every node has to have a direct link with one of the sinks in order to admit

a lifetime  $T = 1$ . Conversely, if  $\mathcal{G}$  has a dominating set of size  $|\mathcal{S}|$ , taking the dominating set as  $sl$  allows  $\mathcal{G}$  to admit  $T = 1$ : Every node simply transmits data to the sink that dominates it.<sup>4</sup> In other words, answering the decision problem of whether  $\mathcal{G}$  has a dominating set of size  $|\mathcal{S}|$  is equivalent to the decision problem of whether  $\mathcal{G}$  admits a lifetime  $T = 1$ . Since the dominating set on a unit disk graph is shown to be NP-complete, MSP is NP-hard. ■

*Remarks:* The above proof leaves several problems open. First, the proof is not constructive because it would not suggest an algorithm to solve the problem provided that there were an algorithm for the DOMINATING SET problem (a common weakness of applying *restriction* [40] for proving NP-hardness). Second, it fails to expose the structure of the problem: for example, whether the complexity lies in the selection of  $sl_k$  or of  $t_k$ . Finally, it does not show if the problem is tractable with a single sink; the DOMINATING SET problem is tractable if the set contains only one element. Therefore, we provide a constructive proof in Appendix A, and we answer other questions in the upcoming sections.

At this point, it is worth noting that the lifetime optimization problem of the joint (fast) mobility and routing strategy proposed in [15] is also intractable because the corresponding decision problem that we term *single mobile relay positioning* (SMRP) is also NP-hard:

INSTANCE: A set of nodes  $\mathcal{V}$ , a cost assignment  $c : c(i, j) = e, \forall i, j \in \mathcal{V}$ , a set  $\mathcal{S}$  of **sink positions** with  $|\mathcal{S}| < |\mathcal{V}|$ , and for each  $i \in \mathcal{V}$ , a transmission energy  $e^{\text{tx}}$ , an energy reserve  $E$ , a rate  $\lambda$ , a constraint that  $i$  sends data to only one  $s \in \mathcal{S}$ , and a positive real  $t$ .

QUESTION: Is there a sink layout schedule  $\{(\delta_{is}, T)\}$ , where  $\delta_{is} : \mathcal{V} \times \mathcal{S} \rightarrow \{0, 1\}$ ,  $\sum_i \delta_{is} = 1$  (only one mobile sink is allowed) and  $\sum_i \sum_s \delta_{is} = |\mathcal{S}|$ , such that the lifetime  $T$  is at least  $t$ ?

A simple proof can be achieved by again applying the reduction from the DOMINATING SET.

## VI. INDUCED SUBPROBLEMS

As we have shown in Section V, the MSP problem is hard, although it is a simplified version of the original MNL by considering homogenous (in terms of, e.g., energy reserve and consumption) nodes. Therefore, we turn to investigate several relaxed versions of the MNL problem by decomposing the two variables  $sl_k$  and  $t_k$  in the schedule. We note that a common approach of relaxing the integer constraints, i.e., (8), does not work for the MNL problem because this relaxation renders every node a sink and thus leads to a trivial solution.

### A. Uniform Epoch

If we assume a uniform epoch span  $\tau$ , the question asked by the resulting *mobile sink positioning with uniform epoch* (MSP-UE) problem becomes:

QUESTION: Is there a sink layout schedule  $\{(sl_k, \tau)\}$  of length  $l$  such that the lifetime  $T = l\tau$  is at least  $t$ ?

<sup>4</sup>A similar reduction has been shown by Bogdanov *et al.* [41] for proving the hardness of the *base station positioning* (BSP) problem. While their problem intends to find the maximum rate admitted by a WSN, we try to maximize the network lifetime for a given rate.

*Proposition 2:* The MSP-UE problem is NP-hard.

*Proof:* Similar to the proof for *Proposition 1*, the NP-hardness of MSP-UE can be shown by the reduction from the DOMINATING SET to the MSP-UE with single element schedule, i.e.,  $l = 1$ . ■

*Remarks:* The above result shows that relaxing the constraint on the time schedule does not significantly simplify the MSP (and thus the MNL) problem. It hints that the complexity lies in the selection of sink layouts instead of the time schedule. In fact, a further relaxation was described in [17]: It consists in solving a sequence of MSP-UEs with  $l = 1$ . Such a problem is still NP-hard because the hardness of MSP-UE problem does not depend on  $l$ , as shown by the proof of *Proposition 2*.

### B. Predefined Flow Schedule

Given all possible sink layouts  $sl_k: k \leq \binom{n}{m}$ , it is always possible to come up with a flow schedule (feasible but not necessarily optimal<sup>5</sup>)  $fs_k$  (where a  $fs_k$  is a vector of  $[\tilde{q}_{ij}^k] : \mathcal{E} \rightarrow \mathbb{R}_0^+$ ) for each  $sl_k$ . Note that here a flow  $\tilde{q}_{ij}^k$  on a link  $(i, j)$  is defined as the information **rate** (as opposed to the total information  $q_{ij}^k$  defined in Section IV). Now, the MNL problem becomes a search for a time schedule  $\{t_k\}$  such that  $T = \sum_k t_k$  is maximized; we formulate the problem as the following linear program:

$$\text{Maximize } \sum_k t_k \quad (9)$$

$$\sum_{k:i \in fs_k} p_i^k t_k \leq E_i \quad \forall i \quad (10)$$

$$p_i^k, t_k \geq 0 \quad \forall i, j, k \quad (11)$$

where  $p_i^k = e_i^{\text{tx}} \sum_j \tilde{q}_{ij}^k + e^{\text{rx}} \sum_j \tilde{q}_{ji}^k$  is the **power** consumption of node  $i$  during the  $k$ th epoch. This formulation has also been taken by [18] and [19].

Although the above linear program could (potentially) involve an exponential number of variables  $t_k$ , the vector  $[t_k]$  for the optimal solution contains no more than  $n$  nonzero elements. As the matrix  $\mathbf{P} = [p_i^k]$  has a row rank of at most  $n$ , it cannot have more than  $n$  linearly independent columns. Consequently, each basic solution  $[\hat{t}_k]$  contains at most  $n$  nonzero elements. The problem involving only a single sink leads to  $n$  potential layouts, for which the solution becomes straightforward. If multiple sinks are involved, the problem might have a very high complexity due to the possibly exponential number of variables. We can apply a primal-dual algorithm in this case. It is easy to see that the above linear program is a packing LP (a linear program in the form  $\max\{c^T x | Ax \leq b, x \geq 0\}$ ); its  $(1 + \epsilon)$ -approximation algorithm is proposed by Garg and Könemann [42].

We note that the formulation of (9)–(11) also has a significance in practice. In several contemporary implementations, the flow in a practical network is shaped by a certain routing protocol instead of being defined by the solutions of some optimization problems. Therefore, once we introduce a set of sink layouts schedule  $sl_k$  into a WSN, the network itself will figure out a flow schedule  $fs_k$ . This “predefined” schedule can then be

<sup>5</sup>It becomes a Greedy approximation if optimal flows are required. However, Greedy does not necessarily perform better than an arbitrary (feasible) flow because what matters is the complementarity among flows (which leads to the overall optimality) rather than the individual optimality.

taken as the input to the above program to obtain an optimal time schedule. In a companion work [37], we show how this relaxation can be applied when designing a practical routing protocol for supporting mobile sinks.

## VII. MAXIMIZING NETWORK LIFETIME FOR A SINGLE MOBILE SINK (MNL-SMS)

Although the MNL-SMS problem is again a subproblem of MNL, we discuss it in a separate section due to its relevance. The problem is polynomially solvable in its original form because it can be formulated as a linear program with  $O(n^3)$  variables and  $O(n^2)$  constraints. Nevertheless, we propose a primal-dual algorithm that solves the problem efficiently. More importantly, this algorithm will be further generalized to solve the original MNL problem in Section VIII-A.

### A. The MNL-SMS Problem

Based on (1)–(8), the Arc-Flow form of MNL-SMS problem can be formulated as the following linear program:

$$\text{Maximize } \sum_k t_k \quad (12)$$

$$\sum_j q_{ij}^k - \sum_j q_{ji}^k - \lambda_i t_k = 0 \quad \forall k, i \neq k \quad (13)$$

$$\sum_{k:k \neq i} \left( e_i^{\text{tx}} \sum_j q_{ij}^k + e^{\text{rx}} \sum_j q_{ji}^k \right) - E_i \leq 0 \quad \forall i \quad (14)$$

$$q_{ij}^k, t_k \geq 0 \quad \forall i, j, k. \quad (15)$$

We simplify the formulation by assuming that the sink is collocated with node  $k$  during the  $k$ th epoch.<sup>6</sup> We also deliberately drop the rate constraint (5) because we can scale the set  $\{\lambda_i\}$  anyway to meet this constraint. It can be easily seen that the number of constraints is bounded by  $|\mathcal{V}|^2 + |\mathcal{V}|$  (with (13) introducing the first term and (14) accounting for the second) and the number of variables is bounded by  $|\mathcal{V}|^3 + |\mathcal{V}|$  (corresponding to  $q_{ij}^k$  and  $t_k$ , respectively); the problem is hence polynomial in  $n$ . Although this linear program is polynomially solvable (by, for example, the *ellipsoid* algorithm [43]), directly solving it is practically ineffective on all but very small-scale problems (similar to the case of concurrent flow problem [44]). In addition, common techniques such as the interior point or simplex algorithms cannot be extended to address the original MNL problem. In Section VII-B, we will discuss a primal-dual algorithm that solves the problem efficiently. Moreover, we will extend the algorithm to approximate the solution of the original MNL in Section VIII-A.

### B. The Primal-Dual Algorithm

Let us reformulate the MNL-SMS problem into a Path-Flow form

$$\text{Maximize } \sum_k t_k \quad (16)$$

<sup>6</sup>Note that this procedure does not change the problem. It only requires the numbering of the nodes to coincide with that of the epochs.

$$\sum_{p \in P_{ik}} f(p) - \lambda_i t_k = 0 \quad \forall i, k \quad (17)$$

$$\sum_{k:k \neq i} \sum_{p \in P_i^k} f(p)(e_i^{\text{tx}} + 1_{p \notin P_{ik}} \cdot e^{\text{rx}}) - E_i \leq 0 \quad \forall i \quad (18)$$

$$f(p), t_k \geq 0 \quad \forall p, k \quad (19)$$

where  $p$  refers to a certain path,  $f(p)$  is the flow that goes through  $p$ , and  $1_A$  is the indicator function of event  $A$ . Furthermore,  $P_{ik}$  stands for the set of paths between node  $i$  and the sink location during the  $k$ th epoch<sup>7</sup> excluding  $k$ , and  $P_i^k$  is the set of paths that go through node  $i$  in the  $k$ th epoch.

The dual problem is given by

$$\text{Minimize } \sum_i E_i w(i) \quad (20)$$

$$\sum_i \lambda_i W(i, k) \geq 1 \quad \forall k \quad (21)$$

$$\sum_{j \in p, p \in P_{ik}} w(j)(e_j^{\text{tx}} + 1_{j \neq i} \cdot e^{\text{rx}}) - W(i, k) \geq 0 \quad \forall i, k \quad (22)$$

$$w(j) \geq 0 \quad \forall j \quad (23)$$

where  $W(i, k)$  is the weight assigned to a “commodity” (data flow injected at a node) from node  $i$  to the sink location during the  $k$ th epoch and  $w(j)$  is the weight assigned to a node  $j$ . The weight of a node  $w(j)$  represents the marginal “cost” of using an additional unit of energy of the node, and the weight of a commodity  $W(i, k)$  represents the marginal “cost” of rejecting a unit of demand of the commodity. Provided that the maximum lifetime is achieved:

- Equation (21) says that the sum of  $\lambda_i$  multiplied by weights  $W(i, k)$  for all  $n$  commodities in any epoch  $k$  is at least 1. This means that the “cost” of increasing the lifetime by one time unit without admitting yet another  $\sum_i \lambda_i$  units of demand exceeds or balances the “revenue.”
- Equation (22) states that the shortest path between an arbitrary node pair  $i$  and  $k$  (the “cost” of routing a unit of demand) is no less than  $W(i, k)$  (the “cost” of rejecting a unit of demand from  $i$  to  $k$ ).

Otherwise, a longer lifetime could have been “profitable” either by rejecting or by admitting (thus routing) more demands. Here, the length of a path is computed as the sum (over all nodes along the path) of the product of node weight  $w(i)$  and the node energy consumption  $e_i^{\text{tx}} + e^{\text{rx}}$ . Note that we will, with a harmless abuse of notation, use  $e_i^{\text{tx}} + e^{\text{rx}}$  (without the indicator function) for later derivation in order to simplify notation. The actual implementation of the algorithm and the results produced in Section IX do take the indicator function into account.

Usually, the flow maximization problem involving multiple  $s$ - $t$  flows can be solved by one of the algorithms proposed by Garg and Könemann [42]. However, MNL-SMS is a combination of two problems, namely a *maximum concurrent flow* problem and a *maximum multicommodity flow* problem. MNL-SMS is, on one hand, a maximum concurrent flow problem because each node has a demand  $\lambda_i$  and the objective is to find a maximum multiplier  $T$  for all nodes. On the other hand, if the time schedule  $\{t_k\}$  is considered as a set of “commodities,” the objective is to maximize  $\sum_k t_k$  without caring

<sup>7</sup>It is indeed node  $k$  due to the specific numbering taken in Section VII-A.

about any demand (note that some “commodities” can be zero), which indicates a maximum multicommodity problem. Therefore, we need to develop new algorithms to solve MNL-SMS.

Let us denote the objective of the dual problem by  $G(w) = \sum_i E_i w(i)$ . In order to minimize  $G(w)$ ,  $w(i)$  should be as small as possible, but it is bounded from below by  $W(i, k)$  through (21) and (22). Taking an arbitrary assignment  $w$  and  $W(i, k) = \sum_{j \in \min\{p|p \in P_{ik}\}} w(j)(e_j^{\text{tx}} + e^{\text{rx}})$  (i.e., the length of the shortest path from  $i$  to  $k$ ), we meet (22). Then, (21) becomes the following constraints:

$$\sum_i \lambda_i \left[ \sum_{j \in p, p \in P_{ik}} w(j)(e_j^{\text{tx}} + e^{\text{rx}}) \right] \geq 1 \quad \forall k.$$

This assignment is not necessarily feasible because it might violate the above constraints. However, it can be made feasible by finding the most violated constraint and scale the assignment accordingly. In other words, if there is an *oracle* that identifies  $\min_k \rho_k(w)$ :  $\rho_k(w) = \sum_i \lambda_i W(i, k) < 1$ , we can scale all assignments  $w(j)$ ,  $W(i, k)$ ,  $\forall i, j, k$  by  $[\min_k \rho_k(w)]^{-1}$  and make a feasible assignment. Therefore, the dual problem is equivalent to finding a weight assignment  $w: \mathcal{V} \rightarrow \mathbb{R}_0^+$  such that  $G(w)/\rho(w)$ :  $\rho(w) \equiv \min_k \rho_k(w)$  is minimized. We denote  $\min_w [G(w)/\rho(w)]$  by  $\beta$ . Note that this interpretation of the dual problem already suggests a duality theorem analogous to the *max-flow min-distance ratio* theorem [44] (which is in turn analogous to the *max-flow min-cut* theorem of Ford and Fulkerson [45] for single  $s$ - $t$  flow). We will discuss this point more in detail in Section VIII-B.

The algorithm proceeds in iterations. Let  $w_{i-1}$ ,  $W_{i-1}$  be the weight assignment at the beginning of the  $i$ th iteration, and let  $\{t_{k,i-1}\}$  be the time schedule after iterations  $1, \dots, i-1$ . In the  $i$ th iteration, we route  $\sum_l \lambda_l$  units of commodity along the paths (and thus to the corresponding sink location) given by an oracle (we will specify it later) that computes  $\min_k \rho_k(w)$  and let  $t_{k,i} = t_{k,i-1} + 1$ . Let  $f_i(l)$  be the flow through node  $l$  and  $p_{l,k}$ ,  $\forall l$  be the paths suggested by  $\min_k \rho_k(w)$  in this iteration. The new weight assignment to a node  $l$  is given by  $w_i(l) = w_{i-1}(l)(1 + \epsilon f_i(l)(e_l^{\text{tx}} + e^{\text{rx}})/E_l)$ , and the new weight assignments to a commodity are computed as  $W_i(l, k) = \sum_{j \in p_{l,k}} w_i(j)(e_j^{\text{tx}} + e^{\text{rx}})$ . Note that  $p_{l,k}$  is indeed the shortest path from  $l$  to  $k$  because it is suggested by the oracle that computes  $\min_k \rho_k(w)$ . Now, the dual objective is updated as

$$\begin{aligned} G(w_i) &= \sum_l E_l w_i(l) \\ &= G(w_{i-1}) + \epsilon \sum_l w_{i-1}(l) f_i(l) (e_l^{\text{tx}} + e^{\text{rx}}) \\ &= G(w_{i-1}) + \epsilon \cdot \rho(w_{i-1}). \end{aligned} \quad (24)$$

Initially, the weight assignment to a node  $l$  is  $w_0(l) = \delta/E_l$ . The iteration stops when  $G(w_i) \geq 1$  for the first time. We refer to Appendix B for details of setting parameters  $\epsilon$  and  $\delta$ .

The oracle that computes  $\min_k \rho_k(w)$  is simply an extension of the Floyd–Warshall algorithm [46] that computes all-pairs shortest path with a time complexity of  $\Theta(n^3)$ . We organize the results of the Floyd–Warshall algorithm into “clusters”; each

cluster includes paths that have a common end. Then, we run a search algorithm in order to find the best “median”  $k$  that achieves  $\min_k \rho_k(w)$ . This oracle has a time complexity of  $\Theta(n^3)$  (because the later clustering and searching both have a time complexity at least one order lower than that of the Floyd–Warshall algorithm). Combining the oracle with the iteration procedure, we have the following proposition.

*Proposition 3:* Given  $\sum_i \lambda_i \leq E_l/(e_l^{\text{tx}} + e^{\text{rx}}), \forall l$ ,<sup>8</sup> there is an algorithm that computes a  $(1 - \epsilon)^{-2}$ -approximation to the MNL-SMS problem in time  $\Theta(n \log n) \cdot T_{\text{oracle}}$ , where  $T_{\text{oracle}} = \Theta(n^3)$  is the time complexity for the oracle to compute  $\min_k \rho_k(w)$ .

*Proof:* See Appendix B. ■

## VIII. MAXIMIZING NETWORK LIFETIME FOR MULTIPLE MOBILE SINKS

We are now ready to investigate the original MNL problem that may involve multiple mobile sinks. Although it is shown to be NP-hard by *Proposition 1*, we are able to approximate the solution based on the primal-dual algorithm described in Section VII-B. Moreover, we are able to solve the following crucial decision problem:

**TO MOVE OR NOT TO MOVE (TMNTM):** Is there a sink layout schedule  $\{(sl_k, t_k)\}$  such that the lifetime  $T = \sum_k t_k$  is longer than what is achieved by any fixed layout  $sl$ ?

This was never fully addressed in the previous work [17]–[20].

### A. The Approximation Algorithm

The Path-Flow form and its dual of the original MNL problem is the same as those of the MNL-SMS problem (16)–(19) and (20)–(23), apart from the fact that  $P_{ik}$  now stands for the set of paths between node  $i$  and one of the  $m$  sinks during the  $k$ th epoch and  $W(i, k)$  becomes the weight assigned to the “commodity” from node  $i$  to that sink during the  $k$ th epoch. Such a formulation hides the complexity of the problem behind a seemingly simple formulation, as the size of a set  $P_{ik}$  can be enormous, leading to an exponential number variables. A formal evaluation of the complexity is given in Appendix A, where we also point out that, if we had an oracle that is able to solve the  $p$ -median problem below, then we would be able to solve the original MNL problem.

**INSTANCE:** A graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , a weight assignment  $\omega: \mathcal{V} \rightarrow \mathbb{R}_0^+$ , a length assignment<sup>9</sup>  $l: \mathcal{V} \rightarrow \mathbb{R}_0^+$ , positive integer  $K \leq |\mathcal{V}|$ , and positive rational number  $B$ .

**QUESTION:** Is there a set  $\mathcal{P}$  of  $K$  “points on  $\mathcal{G}$ ” such that, if  $d(v)$  is the length of the shortest path (i.e., the sum of all length assignments along the path) from  $v$  to the closest point in  $\mathcal{P}$ , then  $\sum_{v \in \mathcal{V}} \omega(v) \cdot d(v) \leq B$ ?

<sup>8</sup>This assumption is reasonable because each sensor node should be equipped with an energy source that is at least enough for the node to forward data for all nodes in one time unit. Otherwise, if a node  $l$ :  $E_l/(e_l^{\text{tx}} + e^{\text{rx}}) < \sum_i \lambda_i$  is deployed close to a static sink (assuming a randomly deployed WSN), the network lifetime can be even less than one time unit. In addition, it can be proved that an approximation ratio of  $(1 - \epsilon)^{-3}$  is still achievable without this assumption.

<sup>9</sup>Usually, a length assignment is associated with links. However, we can always convert our node-capacity-based problem to a link-capacity-based version by replacing a node with two nodes and a link having the same capacity.



However, the p-median problem is NP-complete [40]. Yet, the following proposition provides us with an approximation algorithm for the original MNL problem.

*Proposition 4:* If the p-median oracle can be approximated within a ratio of  $\alpha > 1$  (i.e., the oracle has an  $\alpha$ -approximation), then the primal-dual algorithm given in Section VII-B along with this oracle provides an  $\alpha \cdot (1 - \epsilon)^{-2}$ -approximation to the original MNL problem.

*Proof:* See Appendix C. ■

*Remarks:* In fact, Arya *et al.* [47] gave a  $(3 + \omega)$ -approximation algorithm for the p-median problem. Therefore, we have an algorithm to approximate the original MNL problem with a factor of  $(3 + \omega)(1 - \epsilon)^{-2}$ .

### B. Duality Theory for MNL and the Answer to TMNTM

There is another benefit coming with the primal-dual interpretation provided in Section VII-B: It helps us to build the related duality theory and allows us to easily address the TMNTM decision problem. We recapitulate the observation that we make on the dual problem of MNL-SMS in the following theorem.

*Theorem 1:* [MAX-LIFETIME MIN-POTENTIAL RATIO THEOREM] Given the maximization lifetime problem formulated in (16)–(19), the optimal lifetime  $T$  is such that

$$T = \min_w \left[ \frac{G(w)}{\rho(w)} \right]$$

where  $G(w) = \sum_i E_i w(i)$  is a linear combination of the energy reserves of all nodes with coefficients  $w(i)$ , and

$$\begin{aligned} \rho(w) &\equiv \min_k \rho_k(w) \\ &= \min_k \left( \sum_i \lambda_i \sum_{j \in \min\{p|p \in P_{ik}\}} w(j)(e_j^{\text{tx}} + e^{\text{rx}}) \right) \end{aligned}$$

is the minimum “potential” (computed as the sum of the minimum “cost,” given  $w(i)$ , to route  $\lambda_i$  from node  $i$  to one of the  $m$  centers) achieved among all possible center layouts (or sink layouts).

We omit the detailed proof of this theorem; see Section VII-B for a sketch of the proof. We also quote the theorem given in [44] and improved in [42] as follows.

*Theorem 2:* [MAX-FLOW MIN-DISTANCE RATIO THEOREM] Given the maximization lifetime problem formulated in (16)–(19) but with a fixed schedule consisting of only one element  $(sl, t)$ , the optimal lifetime  $\hat{T}_{sl}$  is such that

$$\hat{T}_{sl} = \min_w \left[ \frac{G(w)}{\rho_k(w)} \right]$$

where  $G(w)$  and  $\rho_k(w)$  are defined in the previous theorem, and the center layout is defined by  $sl$ .

*Proposition 5:*  $T > \hat{T}^*$ , where  $\hat{T}^* = \max_{sl} \hat{T}_{sl}$ . Literally, the answer to the TMNTM decision problem is positive.

*Proof:* Assume that  $\hat{T} > 0$  is the optimal solution for a certain  $sl$ , and  $\{\hat{w}\}$  is the corresponding weight assignment. By plugging  $\hat{w}$  into the dual problem of MNL (20)–(23), we can always identify a violated constraint with the oracle that computes  $\min_k \rho_k(w)$ . For instance, assume that the current sink

location is  $i$  and its most loaded neighbor is  $j$ . We know that (18) is active for  $j$ ; otherwise, it contradicts the optimality of  $\hat{T}$ . Applying complementary slackness, we have  $\rho_i(\hat{w}) = 1$  (by  $\hat{T} > 0$ ),  $\hat{w}(i) = 0$  (by the fact that (18) is inactive for  $i$  due to assumption C), and  $\hat{w}(j) > 0$  (by the fact that (18) is active for  $j$  and  $j$  is the bottleneck of all the paths passing through it). The potential  $\rho_j(\hat{w})$  is bound to be less than 1 because by moving the sink from  $i$  to  $j$ , we shorten the length of some paths by  $\hat{w}(j)$  without increasing the length of other paths going through  $i$ . Therefore, we identify that  $\{\hat{w}\}$ , as the dual solution, is infeasible. Consequently, according to the principle of *certificate of optimality*, we know that  $\hat{T}$ , as the primal solution, is not optimal, and thus  $T > \hat{T}$ . Let  $\hat{T}^* = \max_{sl} \hat{T}_{sl}$ ; we also have  $T > \hat{T}^*$ . ■

*Remarks:* The proof implicitly assumes that  $\min_k \rho_k(w)$  and  $\hat{T}^*$  are computable. As we show in Appendix A, the oracle that computes the minimum “potential”  $\min_k \rho_k(w)$  is NP-complete. At the same time, results in [41] suggest that computing  $\hat{T}^*$  is NP-hard. Therefore, *Proposition 5* serves only for a pure theoretical purpose. Nevertheless, in any practical implementation, the load-balancing effect almost always makes mobile sinks more advantageous (in terms of lifetime) than static sinks.

Another interesting point is that besides the load-balancing effect that we discuss in Section I, there is another “hidden” benefit from moving the sink: It inherits the data-forwarding load from the colocated node (assumption C) and thus saves the energy consumption of that node. We call this *substitution effect*. The mobile node approach [32] has indeed exploited this effect to improve lifetime. While the load-balancing effect is the driving force behind a significant lifetime improvement (as we will show in Section IX-C), the substitution effect, as presented in the above proof, makes moving sinks superior to keeping them static if the sinks are constrained to be on-graph. As we will explain in Section IX-A, the substitution effect is the only reason that leads to a lifetime improvement in certain (albeit not quite realistic) scenarios.

Last but not least, the results stated in *Proposition 5*, unlike other results we presented in the paper, **cannot** be extended to  $\hat{\mathcal{G}}$  that includes also those off-graph sink locations as its nodes. We give two examples in Fig. 2. Fortunately, we might not have such pathological scenarios in practice. Even if such a case happens, the optimal (off-graph) sink location might not be available (we refer to [37] for a practical example we have experienced). All the examples we give in Section IX confirm that moving the sink, no matter on-graph or off-graph, is always superior to keeping it static.

## IX. NUMERICAL EXPERIMENTS

In this section, we test our primal-dual algorithm by positioning a single mobile sink in several WSNs of typical topologies. We always assign a homogeneous  $\lambda$ ,  $e^{\text{tx}}$ ,  $e^{\text{rx}}$  and  $E$  to all nodes in order to facilitate the interpretation of the results. Without loss of generality, we assume  $\lambda = 1$ ,  $e^{\text{tx}} = e^{\text{rx}} = 0.5$ , and  $E = |\mathcal{V}| = n$ . We set  $\epsilon = 0.01$ . We only investigate two metrics, namely lifetime and pause time distribution, in this section and refer to [37] for the evaluation of other metrics. In the first two subsections, we only consider on-graph sink mobility,

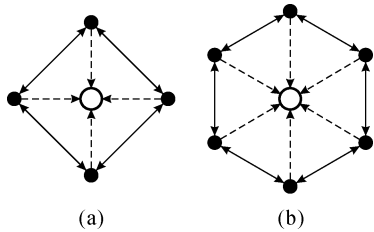


Fig. 2. Two examples to show that Proposition 5 might not hold if sink locations can be off-graph: (a) 4-node ring, (b) 6-node ring. The solid lines represent the original links of the ring networks, and the dash lines represent the new links introduced by an off-graph sink (located at the optimal position). It is straightforward to see that sink mobility does not help in these cases.

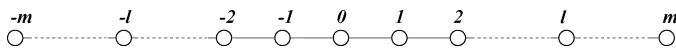


Fig. 3. A line network with  $n = 2m + 1$  nodes. Each node (except nodes  $m$  and  $-m$  that only have one link) has two links with its left and right neighbors.

TABLE I  
COMPARING THE ACHIEVABLE LIFETIME BETWEEN USING A MOBILE SINK AND A STATIC SINK (AT ITS OPTIMAL POSITION) IN LINE NETWORKS

$\mathcal{V}$	Network Lifetime $T$		
	mobile sink	static sink (optimal)	improvement (%)
11	2.765	2.200	25.67
21	2.578	2.100	22.77
41	2.408	2.050	17.47
81	2.285	2.025	12.82

whereas both on-graph and off-graph sink mobilities are considered and compared in the last two subsections. All these problems are solved using the primal-dual algorithm presented in Section VII-B.

### A. Line Network

For the line network shown in Fig. 3, it is easy to see that the best (static) sink location is at node 0, which achieves a lifetime of  $2 + 1/m$ . Note that, according to assumption C), the sink inherits the data collection function of the colocated node. Otherwise, if the sink used the colocated node as its gateway to the network, the lifetime would not change with different sink locations because the colocated node would always take the forwarding load from all the  $2m + 1$  nodes and would thus always “die” first. Now, we run our algorithm to show how a mobile sink should be positioned.

As we show in Table I, using a mobile sink can always achieve a longer lifetime than using a static one. However, the relative improvement decreases with the size of a network. The reason comes from the fact that the substitution effect (see Section VIII-B) is the only cause of the lifetime improvement. In a line network, moving a sink does not lead to load balancing because it can be easily seen that moving the sink only results in an increase of load for some nodes without lightening others’. This is mainly due to the lack of alternative routing paths between an  $s$ - $t$  pair. Therefore, the lifetime improvement is only brought in by the substitution effect, whose absolute quantity grows only sublinearly with the network size. This experiment further supports our statement in Section VIII-B: In scenarios where the number of alternative paths between an  $s$ - $t$  pair can

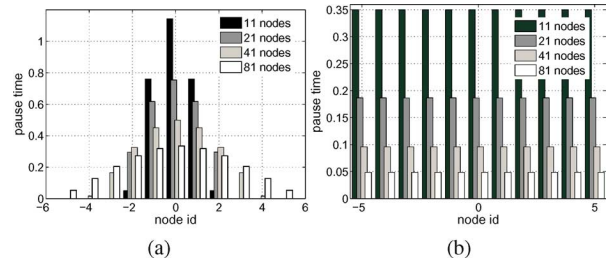


Fig. 4. Pause time distribution of a mobile sink in line and ring networks. (a) Line networks in which only nonzero values are shown. (b) Ring networks, where we zoom to the section between nodes  $-5$  and  $5$ .

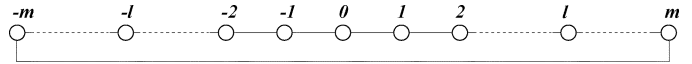


Fig. 5. A ring network with  $n = 2m + 1$  nodes. Each node has two links with its left and right neighbors.

TABLE II  
COMPARING THE ACHIEVABLE LIFETIME BETWEEN USING A MOBILE SINK AND A STATIC SINK IN RING NETWORKS

$\mathcal{V}$	Network Lifetime $T$		
	mobile sink	static sink	improvement (%)
11	3.851	2.200	75.05
21	3.920	2.100	86.66
41	3.940	2.050	92.18
81	3.945	2.025	94.81

be small, it is the substitution effect that makes moving sinks universally superior to keeping them static.

In Fig. 4(a), we also show the trace of the mobile sink. It can be immediately seen that the larger the network size is, the shorter the pause times near node 0 (and thus the longer the pause times far from node 0). The reason is that when the network grows in size, it appears (to nodes close to the center) more and more like a ring. For ring networks (Section IX-B), the sink pauses at every node for the same amount of time. Therefore, a larger line network tends to have a more “spread” pause time distribution.

### B. Ring Network

For the ring network shown in Fig. 5, the achievable lifetime by a static sink is again  $2 + 1/m$ , but it can be obtained by putting the sink at any node due to the symmetry of such a network. The relative improvement is converging to 100% with an increasing network size (Table II). There is no surprise here because the traffic load is fully averaged among all nodes. This averaging effect can be also seen in Fig. 4(b) (where the pause time distribution is illustrated); the sink pauses at every node for the same amount of time  $t = T/(2m + 1)$ .

### C. Grid Network

For grid networks on  $\sqrt{n} \times \sqrt{n}$  lattices, the maximum achievable lifetime by a static sink is  $n/(\lceil(n-5)/4\rceil + 1)$  because the lifetime is maximized if the forwarding load is balanced among the four neighbors of the sink. This lifetime can be obtained by putting the sink at the network center (if  $\sqrt{n}$  is odd) or at any of the four nodes close to the center (if  $\sqrt{n}$  is even). While this lifetime is converging to 4 when  $n \rightarrow \infty$ , the lifetime achieved

TABLE III  
COMPARING THE ACHIEVABLE LIFETIME BETWEEN USING A MOBILE SINK  
AND A STATIC SINK IN GRID NETWORKS

$ \mathcal{V} $	Network Lifetime $T$		
	mobile sink	static sink (optimal)	improvement (%)
9	5.331	4.500	18.47
16	6.509	4.000	62.72
25	8.146	4.167	95.51
49	11.09	4.084	171.7
81	14.08	4.050	247.6
121	17.07	4.033	323.2
144	18.71	4.000	376.8
169	20.26	4.024	403.6
196	21.75	4.000	443.6
225	23.29	4.018	479.7
289	26.33	4.014	555.9

by a mobile sink increases dramatically with the network size (Table III). For small-size networks (e.g.,  $|\mathcal{V}| = 9$  in Table III), the substitution effect dominates the load-balancing effect, so the relative improvement is small. With an increasing network size, the number of alternative paths between an  $s$ - $t$  pair is also increasing. Consequently, the load-balancing effect becomes increasingly remarkable and thus produces significant improvements on the lifetime.

Compared to the results of [18, Table I], our results in Table III exhibit a further increase of 10% to 75% in the lifetime.<sup>10</sup> A straightforward comparison to [20] is not possible because arbitrary networks on a square lattice are considered there. However, it might make sense to note the significant difference between the achievable improvements: The approach in [20] achieves an improvement of 24.5% for  $|\mathcal{V}| = 100$ , whereas ours already achieves 10 times of that value for  $|\mathcal{V}| = 81$ . These comparisons confirm our statement in Section II that, in both [18] and [20], further optimizations are still possible.

We illustrate the pause time distribution in four networks in Fig. 6. Our observation is that the sink tends to move toward the periphery of a network with an increasing  $n$ . The intuition is that, for a 3D grid on a sphere, the sink should pause everywhere with same time period (analogous to a ring in 2D). Therefore, the pause times spread out when the network grows in size and thus appears to the nodes that are close to the center more and more like a sphere grid (analogous to a line in 1D). This observation also corroborates the result in [19]: The network periphery, as a sink moving trace, is asymptotically optimal. Note that we investigate in [19] the asymptotical case where the node density is large enough to make the necessary radio ranges infinitely small. In that case, the shortest paths between any  $s$ - $t$  pair happen to be straight lines.

We also consider the off-graph sink mobility, where the sink can also move to the vertices of another grid that is complementary to the original network, as shown in Fig. 7(a). The results show that, for all the networks shown in Table III, off-graph mobility **does not** further improve the lifetime compared to on-graph mobility. In fact, even the pause time distribution

<sup>10</sup>The only counterexample is when  $|\mathcal{V}| = 16$ . However, this seems to be an outlier case in [18], as it is not compliant with the monotonic increase in the lifetime improvements, which can be easily observed in Table III and [18, Table I].

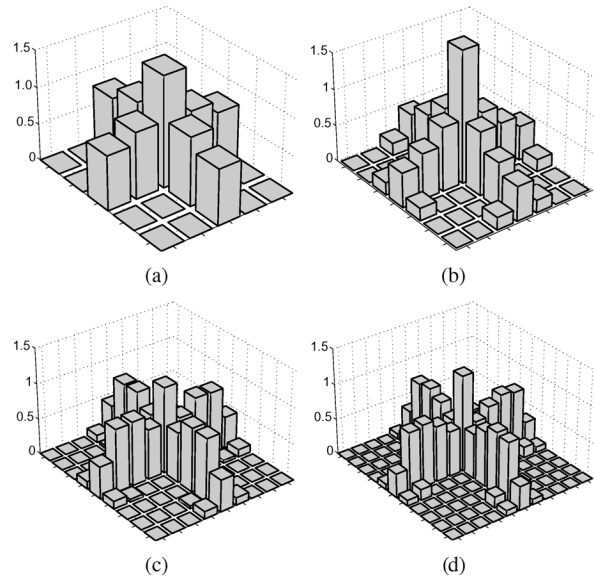


Fig. 6. Pause time distribution of a mobile sink in grid networks. The  $z$ -axis represents the pause time. (a) 25 nodes. (b) 49 nodes. (c) 81 nodes. (d) 121 nodes.

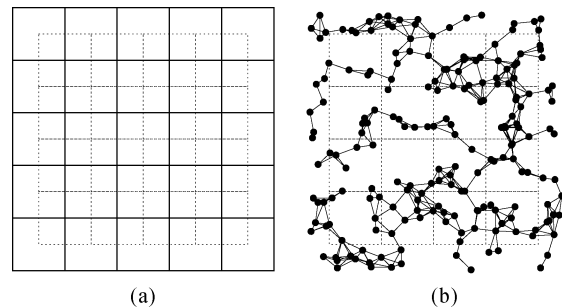


Fig. 7. Illustrations of off-graph sink mobility. The original network is represented by (a) the solid grid and (b) the black nodes. The sink, in addition to be able to move on-graph, may also move to locations represented by the vertices of the dash grid. (a) Grid Network. (b) Arbitrary Network.

remains to be the same after relaxing the on-graph constraint on the sink mobility. This interesting observation shows that, for networks that are well connected, on-graph sink mobility is sufficient to achieve the maximum lifetime.

#### D. Arbitrary Network

We also perform experiments on arbitrary networks (nodes uniformly distributed within a square). Fig. 7(b) shows such a network and the possible off-graph sink locations (represented by the dash grid). We consider both 100-node and 200-node networks with a  $10 \times 10$  off-graph grid, and each with 30 trials. In Fig. 8, we compare the maximum lifetime achieved in four cases—namely, static on-graph sink, mobile on-graph sink, static off-graph sink, and mobile off-graph sink. We use the boxplot to summarize the results we have obtained, in which each case is depicted by five quantities: lower quartile (25%), median, upper quartile (75%), and the two extreme observations. It is immediate to see that moving the sink always improves the lifetime compared to fixing it, no matter whether the mobility is on-graph or off-graph. Also, it is not a surprise to observe that allowing off-graph sink locations (for both mobile and static sinks) outperforms constraining

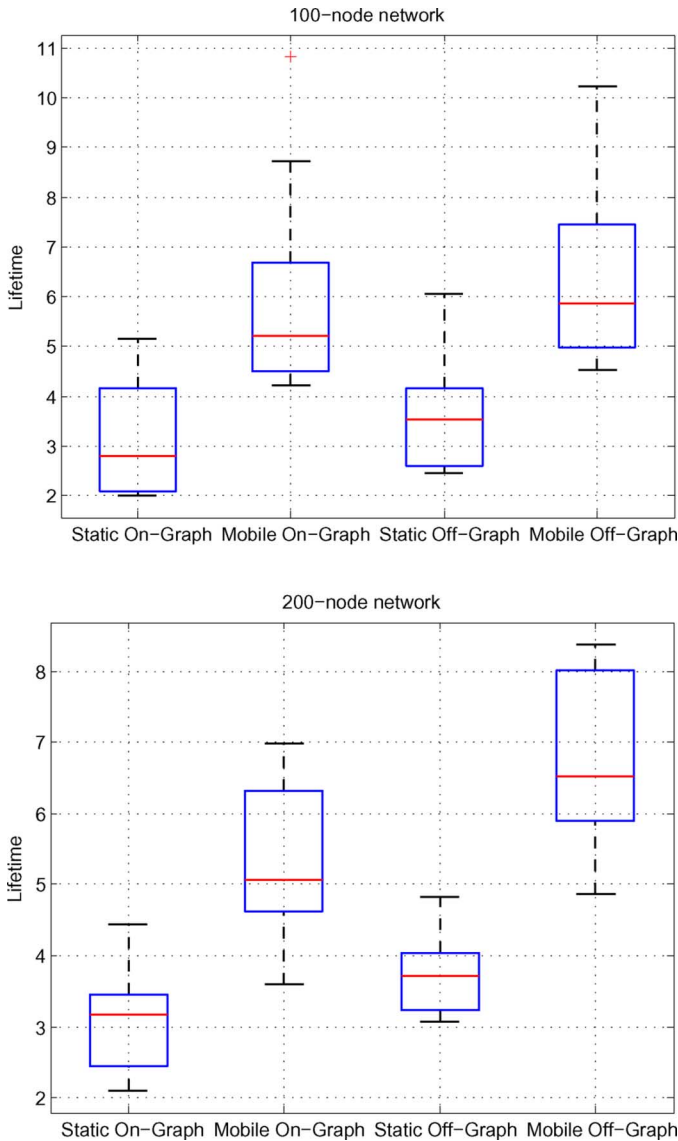


Fig. 8. Comparing different sink behaviors in arbitrary networks with 100 and 200 nodes.

those locations on-graph, this is, of course, at a cost of higher complexity in solving the problem. Fortunately, our algorithm handles this complexity very well given a reasonable number of the off-graph locations.

It is also interesting to look at the pause time distribution. Fig. 9 illustrates one such case (other cases exhibit the same trend). A direct observation is that the sink tends to pause at the nodes whose degrees are high (for on-graph locations) and at the off-graph locations around which the node density is high. This is intuitive because the more neighbors a node or a location has, the more balanced load can be achieved by colocating the sink with it. A slightly surprising observation is that not many locations are chosen by the optimal sink mobility: only five positions for on-graph mobility and 10 positions for off-graph mobility. This is quite different from the grid network. In fact, most arbitrarily deployed networks have a topology close to a tree rather than a mesh. It is quite intuitive to see that the sink mobility will concentrate around the root of a balanced tree.

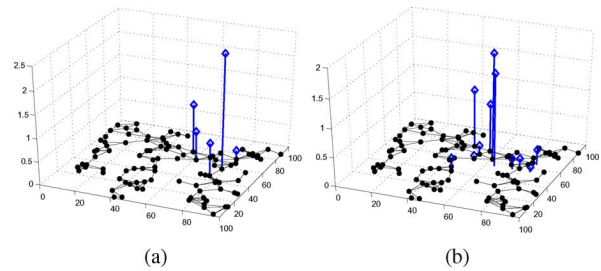


Fig. 9. Pause time distribution of a mobile sink in an arbitrary network. The  $z$ -axis represents the pause time. (a) On-graph mobility. (b) Off-graph mobility.

## X. CONCLUSION

In this paper, we have built a unified framework to analyze the *maximizing network lifetime* (MNL) problem in WSNs. Our investigation, based on a graph model, jointly considers sink mobility and routing for lifetime maximization. We have formally proved the NP-hardness of the MNL involving multiple mobile sinks. We have then identified the subproblem that has a potential to guide routing protocol designs in practice. In particular, we have developed an efficient algorithm to solve the MNL problem involving only a single mobile sink; we have further generalized the algorithm to approximate the general MNL problem. In addition, using the duality theory, we have proved that, for on-graph mobility, moving the sinks is always better than keeping them static. Finally, we have illustrated the benefit of using a mobile sink by applying our algorithm to a set of typical topological graphs.

As for future directions, we are in the process of engineering the routing protocol that we proposed to support sink mobility [37] in order to approach the upper bound characterized in this paper. We are also working on an online algorithm, derived from the approximation algorithm, to guide sink mobility in the face of network dynamics.

## APPENDIX A

### AN ALTERNATIVE PROOF OF PROPOSITION 1

Our proof of *Proposition 1* is based on the principle of *equivalence of separation and optimization* [48]. [48, Theorem 3.3] states that a linear programming problem is solvable in polynomial time (by the *ellipsoid* algorithm [43]) if and only if there exists a *separation oracle* that has a polynomial complexity. The oracle identifies a violated constraint or verifies that there is no such a constraint. The theorem also implies that the linear programming problem is NP-hard if the separation oracle is NP-complete because there is a polynomial-time reduction of the separation oracle to the linear programming problem.

Let us reformulate the MNL problem into a Path-Flow form by dropping the nonessential constraint (5)

$$\text{Maximize } \sum_k t_k \quad (25)$$

$$\sum_{p \in P_{ik}} f(p) - \lambda_i t_k = 0 \quad \forall k, i \notin \mathcal{V}_k \quad (26)$$

$$\sum_{k: i \notin \mathcal{V}_k} \sum_{p \in P_{ik}^k} f(p) (e_i^{tx} + 1_{p \notin P_{ik}} \cdot e^{rx}) \leq E_i \quad \forall i \quad (27)$$

$$f(p), t_k \geq 0 \quad \forall p, k \quad (28)$$

where  $p$  refers to a certain path,  $f(p)$  is the flow that goes through  $p$ ,  $\mathcal{V}_k \subset \mathcal{V}$  is the set of nodes where the sinks are located during  $t_k$ , and  $1_A$  is the indicator function of event  $A$ . Furthermore,  $P_{ik}$  stands for the set of paths between node  $i$  and one of the  $m$  sinks during the  $k$ th epoch, and  $P_i^k$  represents the set of paths that go through node  $i$  in the  $k$ th epoch.

The dual problem is given by

$$\text{Minimize } \sum_i E_i w(i) \quad (29)$$

$$\sum_i \lambda_i W(i, k) \geq 1 \quad \forall k \quad (30)$$

$$\sum_{j \in p, p \in P_{ik}} w(j)(e_j^{\text{tx}} + 1_{j \neq i} \cdot e^{\text{rx}}) - W(i, k) \geq 0 \quad \forall i, k \quad (31)$$

$$w(j) \geq 0 \quad \forall j \quad (32)$$

where  $W(i, k)$  is the weight assigned to a ‘‘commodity’’ (data flow injected at a node in our case) from node  $i$  to one of the  $m$  sinks during the  $k$ th epoch, and  $w(j)$  is the weight assigned to a node  $j$ .

The separation oracle for the dual problem checks if the constraints in (30) and (31) are violated. It is equivalent to verify if the following constraint is violated:

$$\min_k \left[ \sum_i \lambda_i \sum_{j \in \min\{p | p \in P_{ik}\}} w(j)(e_j^{\text{tx}} + e^{\text{rx}}) \right] \geq 1.$$

For simplicity, we abuse the notation by omitting the indicator function. Now, by taking  $\omega(i) = \lambda_i$ ,  $l(i) = w(i)(e_i^{\text{tx}} + e^{\text{rx}})$ ,  $K = |\mathcal{S}|$ , and  $d(i) = \sum_{j \in \min\{p | p \in P_{ik}\}} w(j)(e_j^{\text{tx}} + e^{\text{rx}})$ , the minimization problem leads to the following decision problem:

INSTANCE: A graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , a weight assignment  $\omega(i): \mathcal{V} \rightarrow \mathbb{R}_0^+$ , a length assignment  $l(i): \mathcal{V} \rightarrow \mathbb{R}_0^+$ , positive integer  $K \leq |\mathcal{V}|$ , and positive rational number  $B$ .

QUESTION: Is there a set  $\mathcal{P}$  of  $K$  ‘‘points on  $\mathcal{G}$ ’’ such that  $\sum_{i \in \mathcal{V}} \omega(i) \cdot d(i) \leq B$ ?

The problem, which is known as the  $p$ -median problem [40], is NP-complete. Therefore, our arguments at the beginning of this section suggest that MNL is NP-hard. Q.E.D.

Since we explicitly show a polynomial reduction of the  $p$ -median problem to our MNL problem in the above proof, any solution (should it exist) to the  $p$ -median problem could be directly applied to solve MNL.

## APPENDIX B

### PROOF OF PROPOSITION 3

Our proof of *Proposition 3* is based on the proof of Garg and Könemann [42]. We have the following relation from (24):

$$G(w_i) = G(w_{i-1}) + \epsilon \cdot \rho(w_{i-1}).$$

Since  $\frac{G(w)}{\rho(w)} > \beta$  and  $G(w_0) = n\delta$ , for the  $i$ th iteration

$$G(w_i) \leq G(w_{i-1})(1 + \epsilon/\beta) \leq G(w_{i-1})e^{\epsilon/\beta} \leq n\delta e^{i\epsilon/\beta}.$$

Suppose that the procedure stops at the  $t$ th iteration for which  $G(w_t) \geq 1$ ; we have

$$1 \leq G(w_t) \leq n\delta e^{t\epsilon/\beta} \Rightarrow \frac{\beta}{t} \leq \frac{\epsilon}{\ln(n\delta)^{-1}}. \quad (33)$$

Note that  $t$  is not necessarily the lifetime we achieve because it is possible that the flow already violates the constraints, namely (18), of the primal problem.<sup>11</sup> Let us consider a certain node  $l$ . For every  $E_l/(e_l^{\text{tx}} + e^{\text{rx}})$  units of flow routed through  $l$ , the weight  $w(l)$  is increased by at least  $1 + \epsilon$ . In addition, the weight  $w(l)$  is increased by at most  $1 + \epsilon$  each iteration due to the assumption that  $\sum_i \lambda_i \leq E_l/(e_l^{\text{tx}} + e^{\text{rx}})$ . Since  $w_0(l) = \delta/E_l$  and  $w_{t-1}(l) < 1/E_l$  (due to the fact that  $G(w_{t-1}) < 1$ ), the total amount of flow through  $l$  during the first  $t$  iteration is strictly less than  $\frac{E_l}{e_l^{\text{tx}} + e^{\text{rx}}} \log_{1+\epsilon} \frac{1+\epsilon}{\delta}$ . Therefore, (18) can be violated by at most a multiple of  $\log_{1+\epsilon} \frac{1+\epsilon}{\delta}$ , and thus  $t \cdot \log_{1+\epsilon}^{-1} \frac{1+\epsilon}{\delta}$  gives a feasible primal solution. If we denote the ratio of the dual and the primal solutions by  $\gamma$  and apply the bound on  $\beta/t$  given in (33), we have

$$\gamma = \frac{\beta}{t} \log_{1+\epsilon} \frac{1+\epsilon}{\delta} \leq \frac{\epsilon \log_{1+\epsilon} \frac{1+\epsilon}{\delta}}{\ln(n\delta)^{-1}} = \frac{\epsilon}{\ln(1+\epsilon)} \cdot \frac{\ln \frac{1+\epsilon}{\delta}}{\ln(n\delta)^{-1}}.$$

For  $\delta = (1 + \epsilon)[(1 + \epsilon)n]^{-1/\epsilon}$ , we have

$$\gamma \leq \frac{\epsilon}{(1 - \epsilon) \ln(1 + \epsilon)} \leq \frac{\epsilon}{(1 - \epsilon)(\epsilon - \epsilon^2/2)} < (1 - \epsilon)^{-2}$$

which means that if the maximal lifetime is  $T$  ( $= \beta$  according to strong duality), the algorithm achieves a lifetime  $\hat{T} = t \cdot \log_{1+\epsilon}^{-1} \frac{1+\epsilon}{\delta} > (1 - \epsilon)^2 T \geq (1 - 2\epsilon)T$ .

The time complexity can be obtained by weak duality

$$1 \leq \gamma = \frac{\beta}{t} \log_{1+\epsilon} \frac{1+\epsilon}{\delta}$$

and thus the number of iterations in our algorithm  $t$  is less than  $\beta \log_{1+\epsilon} \frac{1+\epsilon}{\delta}$ . Given  $\delta = (1 + \epsilon)[(1 + \epsilon)n]^{-1/\epsilon}$ , we have  $t \leq \lceil \frac{\beta}{\epsilon} \log_{1+\epsilon}(1 + \epsilon)n \rceil \cdot T_{\text{oracle}}$ .

Notice that the run time depends on  $\beta$ , which can be very large if  $E_l \gg e_l^{\text{tx}} + e^{\text{rx}}$  for each node  $l$  (since  $\beta \leq \min_l [E_l/(e_l^{\text{tx}} + e^{\text{rx}})\lambda_l]$ ). Fortunately, we do not need to solve a full-scale problem. If we scale down every  $E_l$  by a factor of  $\phi$ , we actually scale down the dual objective  $\sum_l E_l w_l$  (and thus the lifetime  $\sum_k t_k$ ) by  $\phi$ . Therefore, we choose the largest  $\phi$  without violating  $\sum_i \lambda_i \leq E_l/(e_l^{\text{tx}} + e^{\text{rx}})$ , so  $\phi = E_l/[(e_l^{\text{tx}} + e^{\text{rx}}) \sum_i \lambda_i]$ , where  $\hat{l} = \arg \min_l E_l/[(e_l^{\text{tx}} + e^{\text{rx}})]$ . Now,  $\beta = \Theta(n)$ , and thus the time complexity becomes  $\Theta(n \log n) \cdot T_{\text{oracle}}$ . Note that the solution of this reduced-scale problem should be scaled up by  $\phi$  to obtain the real solution. Q.E.D.

## APPENDIX C

### PROOF OF PROPOSITION 4

We have already shown in Appendix A that a  $p$ -median oracle is indeed an oracle that computes

$$\begin{aligned} \rho(w) &\equiv \min_k \rho_k(w) \\ &= \min_k \left( \sum_i \lambda_i \sum_{j \in \min\{p | p \in P_{ik}\}} w(j)(e_j^{\text{tx}} + e^{\text{rx}}) \right). \end{aligned}$$

<sup>11</sup>Constraints (17) are always satisfied because the iteration procedure increases  $t_k$  by one only if  $\sum_i \lambda_i$  units of commodities from all nodes are admitted.

Now, suppose we have an  $\alpha$ -approximation for the oracle. It means that the oracle always returns  $\tilde{\rho}(\omega) \leq \alpha\rho(\omega)$  for  $\alpha > 1$ . Again, we have the following relation from (24):

$$G(w_i) = G(w_{i-1}) + \epsilon \cdot \tilde{\rho}(w_{i-1}).$$

Since  $\frac{G(w)}{\rho(w)} > \beta$  (hence  $\frac{G(w)}{\tilde{\rho}(w)} > \tilde{\beta}$ , where  $\tilde{\beta} = \frac{\beta}{\alpha}$ ) and  $G(w_0) = n\delta$ , for the  $i$ th iteration

$$G(w_i) \leq G(w_{i-1}) \left(1 + \epsilon/\tilde{\beta}\right) \leq G(w_{i-1})e^{\epsilon/\tilde{\beta}} \leq n\delta e^{i\epsilon/\tilde{\beta}}.$$

Suppose that the procedure stops at the  $t$ th iteration for which  $G(w_t) \geq 1$ ; we have

$$1 \leq G(w_t) \leq n\delta e^{t\epsilon/\tilde{\beta}} \Rightarrow \frac{\tilde{\beta}}{t} \leq \frac{\epsilon}{\ln(n\delta)^{-1}}.$$

The derivation of the lower bound for the primal solution is not affected by using the  $\alpha$ -approximation oracle. It follows the same line as the proof in Appendix B, and the lower bound is again  $t \cdot \log_{1+\epsilon}^{-1} \frac{1+\epsilon}{\delta}$ . Finally, we have

$$\gamma = \frac{\beta}{t} \log_{1+\epsilon} \frac{1+\epsilon}{\delta} = \frac{\alpha\tilde{\beta}}{t} \log_{1+\epsilon} \frac{1+\epsilon}{\delta} \leq \frac{\alpha}{(1-\epsilon)^2}.$$

This gives an upper bound for the gap between primal and dual solutions, which is the required result: If the maximal lifetime is  $T$  ( $= \beta$  according to strong duality), the algorithm achieves a lifetime  $\hat{T} = t \cdot \log_{1+\epsilon}^{-1} \frac{1+\epsilon}{\delta} > \frac{(1-\epsilon)^2}{\alpha} T$ . Q.E.D.

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