Joint Spectrum and Power Allocation for D2D Communications Underlaying Cellular Networks

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Abstract

This paper addresses the joint spectrum sharing and power allocation problem for *device-to-device* (D2D) communications underlaying a *cellular network* (CN). In the context of *orthogonal frequency-division multiple-access* (OFDMA) systems, with the uplink resources shared with D2D links, both centralized and decentralized methods are proposed. Assuming global *channel state information* (CSI), the resource allocation problem is first formulated as a non-convex optimization problem, which is solved using convex approximation techniques. We prove that the approximation method converges to a sub-optimal solution, and is often very close to the global optimal solution. On the other hand, by exploiting the decentralized network structure with only local CSI at each node, the Stackelberg game model is then adopted to devise a distributed resource allocation scheme. In this game-theoretic model, the base station (BS), modeled as the leader, coordinates the interference from the D2D transmission to the *cellular users* (CUs) by pricing the interference. Subsequently, the D2D pairs, regarded as followers, compete for the spectrum in a non-cooperative fashion. Sufficient conditions for the existence of the *Nash equilibrium* (NE) and the uniqueness of the solution are presented, and an iterative algorithm is proposed to solve the problem. In addition, the signaling overhead is compared between the centralized and decentralized schemes. Finally, numerical results are presented to verify the proposed schemes. It is shown that the distributed scheme is effective for the resource allocation and could protect the CUs with limited signaling overhead.

Index Terms

Cellular networks, device-to-device communications, resource allocation, Stackelberg game.

I. INTRODUCTION

Device-to-device (D2D) communications underlaying *cellular networks* (CNs) has in recent years gained significant interest both from the academia and industry due to its potential to improve spectrum efficiency, offload the cellular system, enhance the cell throughput and save the energy consumption of *user equipments* (UE)s [1]–[3]. Different from the traditional CNs where UEs receive the services from the base station (BS) directly, for D2D communications, UEs may communicate directly via the D2D links under the control of the BS. In general, there are two different types of access policy for the D2D links, namely, orthogonal access where the D2D pairs (including the D2D transmitters and D2D receivers) and the *cellular users* (CU)s are allocated with orthogonal frequency bands [4], and non-orthogonal access where the D2D pairs share the same spectrum with the CUs [5]–[7].

Due to the significant enhancement of the spectral efficiency with non-orthogonal access policy, enormous efforts have been devoted to the analysis and design of efficient spectrum sharing D2D systems. Assuming global *channel state information* (CSI), centralized power and channel allocation schemes were proposed in [6], [7] to coordinate the interference caused by D2D transmission to the CUs. For D2D communications underlaying LTE-A systems, resource allocation schemes combined with the mode selection were proposed in [8], [9], which demonstrated substantial capacity improvement. A distributed suboptimal joint resource allocation and mode selection scheme was proposed and analyzed in [10].

A common feature of the aforementioned works is that they only concerned the interference from the D2D users to the CUs, while the interference from the CUs to the D2D users was ignored. Recently, a number of works have appeared which investigated more practical scenarios with mutual interference between the D2D users and the CUs. In [11], with an emphasis on a local interference situation, the authors designed a novel scheme to remove the near-far interference to D2D users from CUs. Also, an adaptive receive mode selection scheme was proposed in [12] to improve the reliability of D2D communications with the assumption that only one CU and one D2D pair share the same radio resource. To avoid causing interference at the D2D users from the CUs, a novel interference limited area control scheme was designed in [13].

In summary, for non-orthogonal D2D underlaying systems, controlling the mutual interference between the D2D users and the CUs is the most critical problem. Without proper interference coordination, the spectrum efficiency of the D2D underlaying systems may be deteriorated rather than improving.

The major efforts, so far, have mainly concentrated on the design of centralized interference coordination schemes. In such schemes, the BS which acts as a central controller, has to obtain global CSI which incurs a huge system signalling overhead. Hence, the benefit of improved spectrum efficiency brought by the D2D communications may be overshadowed because of the expensive overhead. Also, there are practical scenarios where certain CSI is difficult to obtain. In addition, most of prior works [11] assume that only one CU and one D2D pair share the same frequency spectrum, and that the general case where one D2D pair is allowed to share the frequency spectrum with more than one CUs has not been investigated. In [14], a joint scheduling and resource allocation scheme has been proposed to improve the performance of D2D communications. Stackelberg game has been utilized to model the problem where the CUs are viewed as leaders while the D2D pairs are modelled as followers. It was also assumed that a channel occupied by a CU is only allowed to be reused by one D2D pair.

In this paper, we consider the D2D communications underlaying cellular networks using OFDMA technology, and investigate the problem of designing joint power and channel allocation scheme to maximize the sum data rates of D2D users while guaranteeing each CU's data rate requirement. First, we present a centralized resource allocation scheme via the convex approximation method, which serves as a benchmark for the system performance. Then a decentralized scheme is proposed by modeling the system as a Stackelberg game. In the game, the BS, regarded as a leader, decides the price of the interference on each subchannel brought by D2D communications in the uplink to maximize its own profit, while the uplink transmission from the CUs to the BS are protected through the pricing. On the other hand, the D2D pairs, as followers, compete selfishly in a non-cooperative Nash game to maximize their individual data rates based on the prices set by BS. Capitalizing on the variational inequality approach, we derive the sufficient condition for the uniqueness of *Nash equilibrium* (NE) in the non-cooperative game among the D2D pairs. Then we propose a distributed iterative algorithm which is proved to converge to the unique NE. Finally, combining this distributed iterative scheme and the pricing mechanism at the BS side, the distributed resource allocation scheme is concluded.

Simply put, we apply the Stackelberg game-theoretic method into the D2D communications underlaying cellular systems, since it is naturally compatible with the semi-centralized network structure. Consequently, a practical decentralized joint spectrum and power allocation scheme is proposed with limited overhead incurred to the system.

Part of this works has been published in [15]. In this journal version, we include proofs, derivations, centralized scheme design and signaling overhead analysis that are omitted in the conference version.

The remainder is organized as follows. Section II introduces the system model for D2D communications. Section III presents the resource allocation problem formulation and a centralized optimal scheme based on the successive convex approximation method. In Section IV, we design a decentralized scheme by modeling the resource allocation problem using the Stackelberg game. The analysis of signaling overhead for both centralized and distributed schemes is presented in Section V. In Section VI, the numerical simulations are presented to verify the proposed schemes. Finally, Section VII concludes the paper.

II. System Model

In this paper, we consider the D2D communications underlaying the cellular networks where the uplink radio resource is shared by the D2D pairs as depicted in Fig. 1. For uplink channel reuse, the victim of interference at the cellular side is the BS, which likely has the processing capability of sensing and dealing with co-channel interference. It is assumed that OFDM is used so that the frequency band is divided into N narrowband subchannels. In our model, we consider that there are K D2D pairs coexisting with the CUs in the system. Furthermore, we also assume that

1) The subchannels in the system are either pre-allocated to the CUs by the BS or unoccupied. Each subchannel is dedicated to one CU at most;
2) The transmission powers of the CUs on those occupied subchannels are fixed.

Considering that D2D transmission is a complementary transmission mode and the resource optimization for cellular uplink transmission has been studied extensively, we only study the joint channel and power allocation for D2D transmission in the paper.

If subchannel n is allocated to the ith CU (CU_i) and if it is reused by the kth D2D pair (D2D_k), then for D2D_k, the received signal at the receiver (denoted as $D2DR_k$) on subchannel n is expressed as

$$y_k^n = \sqrt{p_k^n} h_k^n x_k^n + \sum_{\substack{j=1 \\ j \neq k}}^{K} \sqrt{p_j^n} g_{j,k}^n x_j^n + \sqrt{\tilde{p}_i^n} f_{i,k}^n s_i^n + \mathcal{N}_k^n, \tag{1}$$

where p_k^n is the transmission power of the transmitter for D2D pair k (denoted as D2DT_k) on subchannel n, h_k^n denotes the channel gain from D2DT_k to D2DR_k, $g_{j,k}^n$ is the channel gain from D2DT_j of the jth D2D pair to D2DR_k, $\tilde{p}_i^n$ is the transmission power of CU_i on subchannel n, $f_{i,k}^n$ denotes the interference

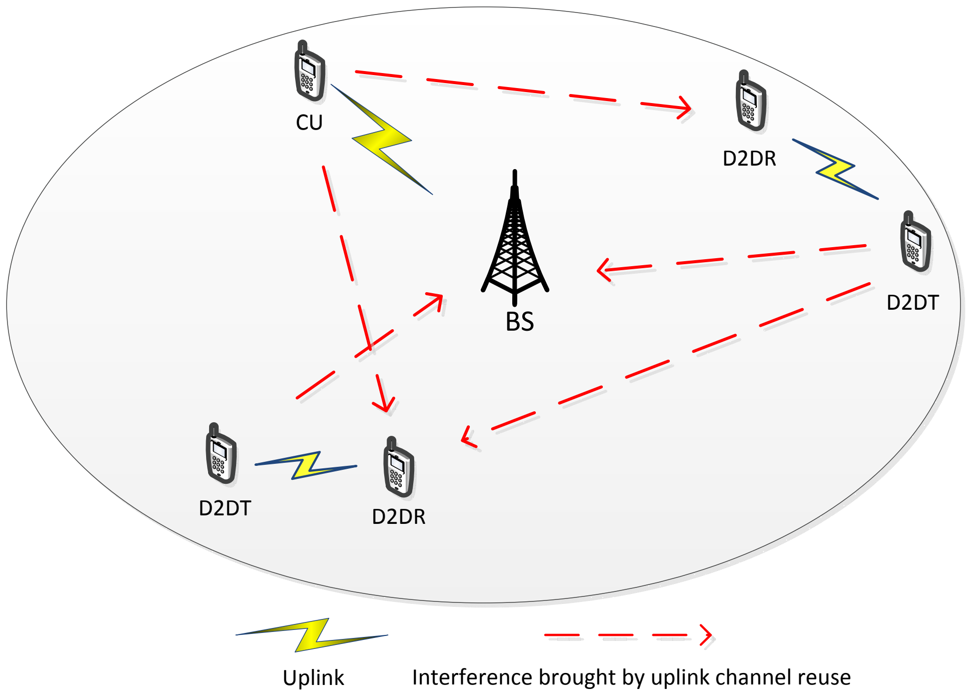

Fig. 1. The model of D2D communications underlaying a cellular network in the uplink.

channel from CU_i to D2DR_k, $\mathcal{N}_k^n$ represents the Gaussian noise with zero mean and variance of N_k^n at D2DR_k on subchannel n, and x_j^n and s_i^n are the transmitted symbols by D2DT_j and CU_i, respectively. Therefore, the signal-to-interference plus noise ratio (SINR) achieved at D2DR_k on subchannel n is given by

$$\text{SINR}_k^n = \frac{p_k^n H_k^n}{\sum\limits_{\substack{j=1\\ j\neq k}}^{K} p_j^n G_{j,k}^n + \tilde{p}_i^n F_{i,k}^n + N_k^n} = \frac{p_k^n}{\sum\limits_{\substack{j=1\\ j\neq k}}^{K} p_j^n \alpha_{j,k}^n + \sigma_k^n}, \tag{2}$$

where $H_k^n \triangleq |h_k^n|^2$, $G_{j,k}^n \triangleq |g_{j,k}^n|^2$, $F_{i,k}^n \triangleq |f_{i,k}^n|^2$, $\alpha_{j,k}^n \triangleq \frac{G_{j,k}^n}{H_k^n}$ is the normalized interference channel gain from D2DT_j to D2DR_k and $\sigma_k^n \triangleq \frac{\tilde{p}_i^n F_{i,k}^n + N_k^n}{H_k^n}$ is the normalized noise power. The interference from the CU is treated as part of the noise. As a result, the achievable data rate of the kth D2D pair on subchannel n can be expressed as

$$R_k^n(p_k^n) = \log_2\left(1 + \frac{p_k^n}{\Gamma_k\left(\sum\limits_{\substack{j=1\\ j\neq k}}^{K} p_j^n \alpha_{j,k}^n + \sigma_k^n\right)}\right), \tag{3}$$

where Γ_k is a constant SINR gap, which links the achievable rate expression with the bit-error-rate (BER) in practical systems. According to [16], with the required BER, BER_k, we have

$$\Gamma_k = -\frac{\ln(5\mathrm{BER}_k)}{1.5}. \tag{4}$$

On the other hand, the received signal at the BS from CU_i on subchannel n is expressed as

$$\tilde{y}_i^n = \sqrt{\tilde{p}_i^n}\tilde{h}_i^n s_i^n + \sum_{k=1}^{K}\sqrt{p_k^n}\tilde{f}_k^n x_k^n + \tilde{\mathcal{N}}_i^n, \tag{5}$$

where $\tilde{h}_i^n$ is the channel gain between CU_i and the BS, $\tilde{f}_k^n$ is the interference channel gain on subchannel n between $\mathrm{D2DT}_k$ and the BS, $\tilde{\mathcal{N}}_i^n$ is the Gaussian noise with zero mean and variance of $\tilde{N}_i^n$ received at the BS on channel n. Therefore, the SINR achieved at the BS from CU_i on subchannel n in the uplink is given by

$$\mathrm{SINR}_{b,i}^n = \frac{\tilde{p}_i^n\tilde{H}_i^n}{\sum\limits_{k=1}^{K} p_k^n\tilde{F}_k^n + \tilde{N}_i^n} = \frac{\tilde{p}_i^n}{\sum\limits_{k=1}^{K} p_k^n\tilde{\alpha}_k^n + \tilde{\sigma}_i^n}, \tag{6}$$

where $\tilde{H}_i^n \triangleq |\tilde{h}_i^n|^2$, $\tilde{F}_k^n \triangleq |\tilde{f}_k^n|^2$, $\tilde{\alpha}_k^n \triangleq \frac{\tilde{F}_k^n}{\tilde{H}_i^n}$, and $\tilde{\sigma}_i^n \triangleq \frac{\tilde{N}_i^n}{\tilde{H}_i^n}$. Then similar to (3), the data rate of CU_i on subchannel n can be expressed as

$$R_{b,i}^n(\tilde{p}_i^n) = \log_2\left(1 + \frac{\tilde{p}_i^n}{\tilde{\Gamma}_i\left(\sum\limits_{k=1}^{K} p_k^n\tilde{\alpha}_k^n + \tilde{\sigma}_i^n\right)}\right), \tag{7}$$

where $\tilde{\Gamma}_i$ can be derived according to (4).

In the paper, although we aim at designing the resource allocation for D2D transmission in the single MBS scenario, the proposed schemes can be extended to the multi-MBS scenario where the *inter-cell interference* (ICI) is present. The *fractional frequency reuse* (FFR) can be applied to avoid the ICI or it can be treated as noise.

III. A BS-Controlled Centralized Scheme

If the BS has global CSI, power and channel allocation can be optimized jointly in a centralized manner, to maximize the D2D sum-rate under the constraint of each CU's data rate.

Define the power allocation matrix $\mathbf{P} \triangleq [\mathbf{P}_1, \ldots, \mathbf{P}_k, \ldots, \mathbf{P}_K]^T$, where $\mathbf{P}_k \triangleq [p_k^1, \ldots, p_k^n, \ldots, p_k^N]$,

$\forall k \in \{1,\ldots,K\}$. Then the problem can be formulated as a non-convex optimization problem:

$$\max_{\mathbf{P}} \sum_{k=1}^{K}\sum_{n=1}^{N} R_k^n(p_k^n) \quad \text{s.t.} \quad \begin{cases} 0 \le p_k^n \le p_k^{\max}, \forall k \in \{1,\ldots,K\}, \forall n \in \{1,\ldots,N\}; \text{(C1)} \\ \sum\limits_{n=1}^{N} p_k^n \le P_k, \forall k \in \{1,\ldots,K\}; \text{(C2)} \\ \sum\limits_{n=1}^{N} R_{b,i}^n(\tilde{p}_i^n) \ge r_i, \forall CU_i; \text{(C3)} \end{cases} \tag{8}$$

where C1 is the local spectral mask constraint and $p_k^{\max}$ is the maximum transmission power D2D pair k can use on each subchannel, C2 represents that the total transmit power of each D2D pair k is limited by P_k, and C3 guarantees the data rates of the CUs. Since the objective function and the functions $R_{b,i}^n$ in constraints C3 are non-concave, the problem is not convex. However, some approximations could be applied. Based on [17], the following inequality is used to approximate the $\ln$ function:

$$a \ln x + b \le \ln(1+x). \tag{9}$$

When $a = \frac{x}{1+x}$ and $b = \ln(1+x) - \frac{x}{1+x}\ln x$, the above approximation is exact. With the approximation (9), the non-concave function in (8) is converted into:

$$\begin{aligned} R_k^n &= \ln\left(1 + \frac{p_k^n}{\Gamma_k\left(\sum\limits_{\substack{j=1\\j\neq k}}^{K} p_j^n \alpha_{j,k}^n + \sigma_k^n\right)}\right) \cdot \log_2 e \\ &= \left(a_k^n \ln\left(\frac{p_k^n}{\Gamma_k\left(\sum_{\substack{j=1\\j\neq k}}^{K} p_j^n \alpha_{j,k}^n + \sigma_k^n\right)}\right) + b_k^n\right) \cdot \log_2 e \\ &= \left(a_k^n \ln p_k^n - a_k^n \ln\left(\Gamma_k\left(\sum\limits_{\substack{j=1\\j\neq k}}^{K} p_j^n \alpha_{j,k}^n + \sigma_k^n\right)\right) + b_k^n\right) \cdot \log_2 e. \end{aligned} \tag{10}$$

Then defining $p_k^n \triangleq e^{\hat{p}_k^n}$, (8) becomes a convex optimization problem in which parameters a_k^n and b_k^n can be estimated by $a_k^n = \frac{\text{SINR}_k^n}{1+\text{SINR}_k^n}$ and $b_k^n = \ln(1+\text{SINR}_k^n) - \frac{\text{SINR}_k^n}{1+\text{SINR}_k^n}$. Hence, standard convex optimization methods can be used to solve it. However, since (9) is employed here, the choices of a_k^b and b_k^n might not lead to the best result. Thus, iterative procedures should be applied to tighten the approximation. The centralized algorithm is formally described as Algorithm 1.

Algorithm 1 Centralized Scheme

1: Set a counter $c = 0$ and initialize the power allocation vector as $\mathbf{P}_k(c+1), \forall k \in \{1, \ldots, K\}$;
2: **repeat**
3: Update $c := c + 1$;
4: Calculate $a_k^n(c), b_k^n(c), \forall k \in \{1, \ldots, K\}, \forall n \in \{1, \ldots, N\}$ with $\mathbf{P}_k(c-1), \forall k \in \{1, \ldots, K\}$;
5: With (9) and $a_k^n(c),\ b_k^n(c),\ p_k^n \triangleq e^{\hat{p}_k^n}, \forall k, n$, transform (8) into a convex optimization problem;
6: Using standard convex optimization techniques, such as the Lagrangian dual method, to solve the problem obtained in **Line 5** and obtain the optimal solution which is assigned as $\mathbf{P}(c+1)$;
7: **until** $\|\mathbf{P}(c+1) - \mathbf{P}(c)\| \leq \kappa$, for some prescribed κ
8: **return** $\mathbf{P}(c+1)$;

Note the successive convex approximation method employed here would converge to the point which satisfies the Karush-Kuhn-Tucker (KKT) conditions of the original problem based on the analysis in Appendix A. Although it is a heuristic algorithm and can only converge to a suboptimal solution, as observed in [17], this approximation method often computes the solution close to the global optimum.

IV. A BS-Supervised Distributed Algorithm

This section presents a decentralized joint power and subchannel allocation scheme using the Stackelberg game-theoretic model. Stackelberg game is a strategic game which includes a leader and some followers competing with each other on certain resources. The leader sets the price of the resource first and then the followers compete with each other according to the price.

Although the Stackelberg game model has been applied to the cognitive radio (CR) system [19], there are two main differences between CR and D2D system. Firstly, the D2D users are also authorized users in D2D system. Secondly, in CR system, the secondary users are not controlled by any central controller. But the D2D communication is dominated by the BS in D2D systems. Note, although the BS dominates the system, it is unwise to let BS decide the resource allocation scheme in the system completely unless it is easy for BS to achieve the full CSI. Therefore, compared to its application in the areas of CR for designing distributed resource allocation methods, the Stackelberg game is more effective to model the semi-centralized structure of D2D systems, and the powerful variational inequality (VI) method is applied to the analyze the model.

Stackelberg game has also been used for resource allocation in two-tier femtocell networks [20], where the macrocell was viewed as a leader and the femtocell users as followers. The received interference from the femtocell users was controlled through pricing the interference. In this case, the Stackelberg game can be converted into a bargaining game due to the assumption of ignoring or fixed cross-femtocell interference.

A. Stackelberg Game Formulation for the D2D System

In our model, the BS plays the role as the leader to establish a set of "prices" for the received interference power from the D2D transmission on each subchannel. The purpose of setting the price is to maximize its own profit, meanwhile to protect the CUs by limiting the interference caused by the D2D transmission on each subchannel. Then according to the prices, the D2D pairs as followers compete selfishly for the available bandwidth in a non-cooperative game to maximize their individual data rates.

The objective of the BS here is to maximize its "profit" by selling the spectrum to the D2D pairs for accepting "interference" on the subchannels. Mathematically, it can be formulated as

$$U_{\mathrm{BS}}(\boldsymbol{\theta}, \mathbf{P}) = \sum_{k=1}^{K} \sum_{n=1}^{N} \theta^n p_k^n \tilde{\alpha}_k^n, \tag{11}$$

where $\boldsymbol{\theta} \triangleq [\theta^1, \ldots, \theta^N]^T$ denotes the interference price vector on the N subchannels with θ^n being the interference price on subchannel n. The BS requires that the interference brought by the D2D transmission will not violate the CUs' target rates. Therefore, the BS needs to derive the optimal price $\boldsymbol{\theta}$ to maximize its revenue, while satisfying the CUs' rate requirements. The price $\boldsymbol{\theta}$ can be in the form of real money which can not only assist the decentralized implementation of the algorithm but also compensate the D2D pairs for biasing the CUs. The game for the BS aims to solve

$$\text{Problem 1}: \quad \max_{\boldsymbol{\theta}} U_{\mathrm{BS}}(\boldsymbol{\theta}, \mathbf{P}) \quad \text{s.t.} \quad \sum_{n=1}^{N} R_{b,i}^n \geq r_i, \forall CU_i. \tag{12}$$

As a follower, with the price θ^n, the utility function of the kth D2D pair on subchannel n is defined as

$$u_k^n \triangleq R_k^n(p_k^n, \overline{\boldsymbol{p}}_k^n) - \theta^n p_k^n \tilde{\alpha}_k^n, \tag{13}$$

where $R_k^n(p_k^n, \overline{\boldsymbol{p}}_k^n)$ is the data rate achieved by D2D pair k on the nth subchannel defined in (3), and $\overline{\boldsymbol{p}}_k^n$ denotes the power allocation vector of all D2D transmitters except $\mathrm{D2DT}_k$ on the nth subchannel. In addition, $p_k^n \tilde{\alpha}_k^n$ represents the normalized interference caused by $\mathrm{D2DT}_k$ to the BS.

The utility function (13) for the D2D pairs includes two parts: the achievable data rate and the cost. On one hand, with more transmit power utilized on subchannel n, a higher data rate can be achieved by D2D pair k. On the other hand, however, more interference would be experienced at the BS so more money should be paid by D2D pair k. Thus, there exists a tradeoff between the data rates and the "cost" for D2D pair k. Hence, to maximize its utility, the optimization problem at each D2D pair k is formulated as

$$\text{Problem 2}: \max \sum_{n=1}^{N} u_k^n \quad \text{s.t.} \quad \begin{cases} 0 \le p_k^n \le p_k^{\max}; \text{ (C1)} \\ \sum_{n=1}^{N} p_k^n \le P_k. \text{ (C2)} \end{cases} \tag{14}$$

The constraints C1 and C2 are similar to those in (8).

B. Stackelberg Equilibrium

Under the Stackelberg game model above, the Stackelberg equilibrium is defined as follows.

Definition 1: Let $\boldsymbol{\theta}$ be a solution for Problem 1 and $\mathbf{P}_k$ denote a solution for Problem 2 for the kth D2D pair with $\mathbf{P} = \{\mathbf{P}_k\}_{k=1}^K$. Then the point $(\boldsymbol{\theta}^*, \mathbf{P}^*)$ (with the superscript $*$ specifying the corresponding parameters at the equilibrium) is a Stackelberg equilibrium for the game if for any $(\boldsymbol{\theta}, \mathbf{P})$ with $\boldsymbol{\theta}, \mathbf{P} \succeq \mathbf{0}$, the following conditions are satisfied:

$$U_{\text{BS}}(\boldsymbol{\theta}^*, \mathbf{P}^*) \ge U_{\text{BS}}(\boldsymbol{\theta}, \mathbf{P}), \tag{15a}$$

$$U_k(\boldsymbol{\theta}^*, \mathbf{P}^*) \ge U_k(\boldsymbol{\theta}, \mathbf{P}), \forall k \in \{1, \ldots, K\}, \tag{15b}$$

where $U_k(\boldsymbol{\theta}, \mathbf{P}) \triangleq \sum_{n=1}^N u_k^n$.

From Definition 1, we see that in order to reach a Stackelberg equilibrium, a two-stage iterative algorithm is required. In the first stage, the leader sets a price and broadcasts it in the system. Then the followers compete in a non-cooperative fashion in the following stage. After the NE (Nash Equilibrium) is reached, leader will reset the price based on the strategies adopted by the followers and the interference on each subchannel. This two-stage update will continue until the two conditions in Definition 1 are satisfied.

Therefore, for the proposed game defined in Section IV-A, the BS sets the "price" θ^n for each subchannel n, and then the D2D pairs compete for the subchannel in a non-cooperative fashion. After the NE is reached, the BS updates the price θ^n according to the aggregate interference received at subchannel n. These two processes will be repeated until convergence. In the following sections, the non-cooperative game for the D2D pairs and the price updating strategy of the BS will be studied, respectively.

C. Non-Cooperative Game for D2D Pairs

After receiving the price vector $\boldsymbol{\theta}$ broadcasted by the BS, the non-cooperative game for D2D pairs is defined as $\mathcal{G} = \{\Omega, (\mathbb{P}_k)_{k\in\Omega}, (U_k(\boldsymbol{\theta}, \mathbf{P}))_{k\in\Omega}\}$, where $\Omega = \{1, \ldots, K\}$ is the player set, $\mathbb{P}_k$ is the D2D pair k's admissible strategy set, and $U_k(\boldsymbol{\theta}, \mathbf{P})$ is the payoff function of D2D pair k.

According to Problem 2, the game for each D2D pair k aims to

$$\max_{\mathbf{P}_k} U_k \quad \text{s.t.} \quad \mathbf{P}_k \in \mathbb{P}_k, \tag{16}$$

where

$$\mathbb{P}_k \triangleq \left\{ \mathbf{P}_k \in \Re^N : \sum_{n=1}^{N} p_k^n \le P_k, 0 \le p_k^n \le p_k^{\max} \right\}. \tag{17}$$

Since it is a non-cooperative game among the D2D pairs, for each D2D pair k, the interference from other D2D transmission is treated as noise. Therefore, the game (16) for D2D pair k is a convex optimization problem. To solve this, we define

$$\mathbf{F}_k(\mathbf{P}) \triangleq -\nabla U_k(\mathbf{P}) = \left\{ -\frac{1}{p_k^n + \Gamma_k \left(\sum_{\substack{j=1 \\ j\neq k}}^{K} p_j^n \alpha_{j,k}^n + \sigma_k^n \right)} + \theta^n \tilde{\alpha}_k^n \right\}_{n=1}^{N}. \tag{18}$$

Then according to the first order (necessary and sufficient) optimality conditions of the convex optimization problem, for each $k \in \{1, \ldots, K\}$, $\mathbf{P}_k^* \in \mathbb{P}_k$ is an optimal solution if and only if

$$(\mathbf{p}_k - \mathbf{p}_k^*)^T \mathbf{F}_k(\mathbf{P}) \ge 0, \forall \mathbf{P}_k \in \mathbb{P}_k. \tag{19}$$

Then with the Lagrangian method, the solution of (16) for D2D user k has a water-filling interpretation

$$p_k^n = \mathrm{WF}(\overline{\boldsymbol{p}}_k; \boldsymbol{\theta})_n \triangleq \left[\frac{1}{\theta^n \tilde{\alpha}_k^n + \lambda_k} - \Gamma_k \left(\sum_{\substack{j=1 \\ j\neq k}}^{K} p_j^n \alpha_{j,k}^n + \sigma_k^n \right) \right]_0^{p_k^{\max}}, \forall n \in \{1, \ldots, N\}, \tag{20}$$

where $[a]_x^y = \min(y, \max(a, x))$, and λ_k is regarded as the Lagrange multiplier which is chosen to ensure that the total power constraint $\sum_{n=1}^{N} p_k^n \le P_k$ is satisfied.

Now, we study the sufficient condition for the existence and uniqueness of NE in $\mathcal{G}$. First, according to [21], the game $\mathcal{G}$ can be converted into the variation inequality problem. To do so, define the joint strategy as the Cartesian product set of D2D pairs' strategies:

$$\mathbb{P} \triangleq \mathbb{P}_1 \times \cdots \times \mathbb{P}_K \tag{21}$$

and the vector function

$$\mathbf{F}(\mathbf{P}) \triangleq (\mathbf{F}_1(\mathbf{P}), \ldots \mathbf{F}_K(\mathbf{P}))^T. \tag{22}$$

Since for each D2D pair k, the game is a convex optimization problem, the condition (19) will be satisfied. Therefore, if $\mathbf{P}^*$ is an NE of the game $\mathcal{G}$, then for each $k \in \{1, \ldots, K\}$, the inequality (19) must be satisfied. The set of inequalities (19) can be treated as the variational inequality problem $VI(\mathbb{P}, \mathbf{F}(\mathbf{P}))$ according to [21], [22]. Therefore, to prove the existence and uniqueness of NE in $\mathcal{G}$, it is equivalent to prove that there is one unique solution in $VI(\mathbb{P}, \mathbf{F}(\mathbf{P}))$. Therefore, we have the following theorem.

Theorem 1: Given $\boldsymbol{\theta} \succeq \mathbf{0}$, the game $\mathcal{G}$ always admits NEs for any channel matrices and power constraints of the D2D pairs. In addition, Every NE solution satisfies the following water-filling like fixed-point equation:

$$\mathbf{p}_k^* = \mathrm{WF}(\overline{\boldsymbol{p}}_k^*; \boldsymbol{\theta})_{n=1}^N. \tag{23}$$

Proof: As we analyzed above, $\mathcal{G}$ is equivalent to $VI(\mathbb{P}, \mathbf{F}(\mathbf{P}))$. If there exists solutions in $VI(\mathbb{P}, \mathbf{F}(\mathbf{P}))$, then $\mathcal{G}$ has NEs. According to [21], if $\mathbb{P}$ is compact and convex and $\mathbf{F}$ is continuous, then there have solutions for $VI(\mathbb{P}, \mathbf{F}(\mathbf{P}))$. According to the definitions of U_k, (17) and (19), (22), it is easy to show that $\mathbb{P}$ is compact and convex and $\mathbf{F}$ is continuous. Hence, we can conclude that there always have NEs in $\mathcal{G}$.

Note the water-filling like solution for each D2D pair k directly follows from the fact that for fixed $\overline{\boldsymbol{p}}_k$, there is only one unique solution to the optimization problem (16) as presented in (20). ■

In the following theorem, the sufficient condition for the uniqueness of the NE in $\mathcal{G}$ is derived.

Theorem 2: Given $\boldsymbol{\theta} \succeq \mathbf{0}$, the game $\mathcal{G}$ has a unique NE, if the $K \times K$ matrix $\mathbf{R}$ is positive definite, where it is defined that

$$[\mathbf{R}]_{k,j} \triangleq \begin{cases} 1, & \text{if } j = k, \\ -\max_{1 \le n \le N}\{\Gamma_k \alpha_{j,k}^n \phi_{j,k}^n\}, & \text{if } j \neq k, \end{cases} \tag{24}$$

where

$$\phi_{j,k}^n \triangleq \frac{p_j^{\max} + \Gamma_k \left(\sum_{\substack{j'=1 \\ j' \neq j}}^K p_{j'}^{\max} \alpha_{j',k}^n + \sigma_j^n \right)}{\sigma_k^n}. \tag{25}$$

In this case, the mapping $\mathbf{F}(\mathbf{P})$ is also a strongly monotonic function on $\mathbb{P}$.

Proof: See Appendix B. ■

A distributed algorithm to reach the NE is possible, where each player in $\mathcal{G}$ updates its strategy according to the best-response solution (23). The distributed scheme is formalized as Algorithm 2 below.

Algorithm 2 Distributed Asynchronous Iterative Water-Filling Algorithm (AIWA)

1: Given the price vector $\boldsymbol{\theta}$, initialize the power allocation strategy $\mathbf{P}_k(\in \mathbb{P}_k)$;
2: Set a counter $m = 0$ and $\mathbf{P}(m) = [\mathbf{P}_1, \ldots, \mathbf{P}_K]$;
3: **repeat**
4: Update $m := m + 1$;
5: For each $k \in \{1, \ldots, K\}$, according to (23), update $\mathbf{P}_k$ and form a new $\mathbf{P}(m)$;
6: **until** $\|\mathbf{P}(m) - \mathbf{P}(m-1)\| \le \varsigma$, for some prescribed ς
7: **return** $\mathbf{P}(m)$

Theorem 3: The AIWA algorithm will converge to the unique NE of $\mathcal{G}$ if

$$\delta(\mathbf{S}^{\max}) < 1, \tag{26}$$

where $\mathbf{S}^{\max}$ is a $K \times K$ matrix with

$$[\mathbf{S}^{\max}]_{k,j} = \begin{cases} \Gamma_k \max_{n\in\{1,\ldots,N\}} \alpha_{j,k}^n \frac{p_j^{\max}}{p_k^{\max}}, & \text{if } j \neq k, \\ 0, & \text{otherwise,} \end{cases} \tag{27}$$

and $\delta(\mathbf{S}^{\max})$ denotes the spectral radius of $\mathbf{S}^{\max}$.

Proof: The basic idea of the proof is to treat (20) as a projector, and then based on the contraction property of the projector, we can show that the nesting condition, synchronous convergence condition and the box condition of the asynchronous convergence theorem [24] in AIWA are all satisfied under the condition (26). For more details regarding the proof, the readers are referred to [25]. ■

Then comparing Theorem 2 with Theorem 3, we have the following corollary.

Corollary 1: The condition (26) for the convergence of AIWA is less stringent than the sufficient condition (24) for the uniqueness of NE in Theorem 2.

Proof: According to Theorem 2, if $\mathbf{R}$ is positive definite, the NE in $\mathcal{G}$ is unique. Since the positive definite matrix is also a $\mathbf{P}$-matrix, $\delta(\mathbf{I}-\mathbf{R}) < 1$ [26], where $\mathbf{I}$ is the identity matrix. Then $\lim_{l\to\infty}(\mathbf{I}-\mathbf{R})^l = \mathbf{0}$ [27]. Since $\phi_{j,k}^n > \frac{p_j^{\max}}{p_k^{\max}}$, then $\lim_{l\to\infty}(\mathbf{S}^{\max})^l = \mathbf{0}$. Because $\delta(\mathbf{S}^{\max}) < 1$ if only if $\lim_{l\to\infty}(\mathbf{S}^{\max})^l = \mathbf{0}$ [27], we conclude that the condition in Theorem 2 is more stringent than that in Theorem 3. As such, if the condition in Theorem 2 is satisfied, then both the uniqueness of NE and the convergence to NE by

AIWA will be guaranteed, which completes the proof. ∎

The sufficient conditions for Theorems 2 and 3 can be derived based on the inequality [26], [27]:

$$\delta(\mathbf{I}-\mathbf{R}) = \delta((\mathbf{I}-\mathbf{R})^T) \leq \|\mathbf{I}-\mathbf{R}\|, \tag{28}$$

where $\|\cdot\|$ can be any matrix norm. Therefore, a sufficient condition for (26) is $\|\mathbf{I}-\mathbf{R}\|_\infty^{\mathbf{w}} \leq 1$, with $\|\cdot\|_\infty^{\mathbf{w}}$ denoting the weighted block maximum norm, defined as

$$\|\mathbf{I}-\mathbf{R}\|_\infty^{\mathbf{w}} \triangleq \max_i \frac{1}{w_i} \sum_{j\neq i} w_j [\mathbf{I}-\mathbf{R}]_{i,j}, \tag{29}$$

where $\mathbf{w} \triangleq [w_1, \ldots, w_K]$ is any positive vector.

With (28) and (29), the sufficient conditions for Theorem 2 are

$$\max_k \frac{1}{w_k} \sum_{j\neq k} w_j \max_{n\in\{1,\ldots,N\}} \{\Gamma_k \alpha_{j,k}^n \phi_{j,k}^n\} < 1, \tag{30}$$

and

$$\max_j \frac{1}{w_j} \sum_{k\neq j} w_k \max_{n\in\{1,\ldots,N\}} \{\Gamma_k \alpha_{j,k}^n \phi_{j,k}^n\} < 1. \tag{31}$$

Similarly, the sufficient conditions for Theorem 3 can be obtained as

$$\max_k \frac{1}{w_k} \sum_{j\neq k} w_j \max_{n\in\{1,\ldots,N\}} \left\{\Gamma_k \alpha_{j,k}^n \frac{p_j^{\max}}{p_k^{\max}}\right\} < 1, \tag{32}$$

and

$$\max_j \frac{1}{w_j} \sum_{k\neq j} w_k \max_{n\in\{1,\ldots,N\}} \left\{\Gamma_k \alpha_{j,k}^n \frac{p_j^{\max}}{p_k^{\max}}\right\} < 1. \tag{33}$$

The set of conditions (30)–(33) have the same physical explanation that the uniqueness of NE in $\mathcal{G}$ and the convergence of AIWA are ensured if the interference among the D2D pairs is sufficiently small. The price set by the BS and the interference brought by the CUs to the D2D transmission do not affect the sufficient conditions. These conditions presented above can be treated as the admission conditions to allow users to communicate with each other on D2D mode when the spectrums are to be reused.

D. Pricing Mechanism at the BS

In this subsection, the pricing strategy of the BS is studied. The BS sets the price primarily to maximize its own profit. Another function of the pricing at the BS is to differentiate the D2D pairs from the CUs.

In our model, as the CUs have priority over the D2D users, the price charged for the communications among the D2D pairs should be less when the channels are not dedicated to the CUs. Substituting (20) into Problem 1, the optimization problem for the BS side can be formulated as

$$\max_{\boldsymbol{\theta}} \sum_{k=1}^{K} \sum_{n=1}^{N} \theta^n p_k^n \tilde{\alpha}_k^n \quad \text{s.t.} \quad \begin{cases} p_k^n = \left[\dfrac{1}{\theta^n \tilde{\alpha}_k^n + \lambda_k} - \Gamma_k \left(\sum\limits_{\substack{j=1 \\ j\neq k}}^{K} p_j^n \alpha_{j,k}^n + \sigma_k^n \right) \right]_0^{p_k^{\max}} ; \text{ (C1)} \\ \lambda_k \left(P_k - \sum\limits_{n=1}^{N} p_k^n \right) = 0, \forall k \in \{1, \ldots, K\}, \forall n \in \{1, \ldots, N\}; \text{ (C2)} \\ \sum\limits_{n=1}^{N} R_{b,i}^n \geq r_i, \forall CU_i; \text{ (C3)} \\ \lambda_k \geq 0, \forall k \in \{1, \ldots, K\}. \text{ (C4)} \end{cases} \tag{34}$$

Note that the constraints in (34) are coupled. In C2, the power allocation strategy of each D2D pair is coupled across the subchannels by its total power constraint. In C3, the power allocation strategies of all the D2D pairs are coupled across the subchannels under the rate constraints of the CUs.

The data rate requirement r_i of CU_i can be decomposed into the data rate requirements on each of its occupied subchannels as:

$$\begin{aligned} \log \left\{ 1 + \frac{\tilde{p}_i^n}{\tilde{\Gamma}_i \left(\sum_{k=1}^{K} p_k^n \tilde{\alpha}_k^n + \tilde{\sigma}_i^n \right)} \right\} \geq r_i^n, \\ \sum_{i=1}^{N} r_i^n \geq r_i. \end{aligned} \tag{35}$$

Since the subchannels allocated to each CU are preset and the transmission powers on these subchannels are fixed, the data rates $r_i^n, \forall n \in \{1, \ldots, N\}$ can be set according to the waterfilling strategy. Therefore, the sum interference caused by the D2D transmission on subchannel n should be lower bounded by

$$\log \left\{ 1 + \frac{\tilde{p}_i^n}{\tilde{\Gamma}_i \left(\sum\limits_{k=1}^{K} p_k^n \tilde{\alpha}_k^n + \tilde{\sigma}_i^n \right)} \right\} \geq r_i^n \Rightarrow \sum_{k=1}^{K} p_k^n \tilde{\alpha}_k^n \leq \frac{\tilde{p}_i^n}{\tilde{\Gamma}_i (2^{r_i^n} - 1)} - \tilde{\sigma}_i^n. \tag{36}$$

Since $\frac{\tilde{p}_i^n}{\tilde{\Gamma}_i (2^{r_i^n} - 1)} - \tilde{\sigma}_i^n$ is fixed, we find it convenient to define $T_i^n \triangleq \frac{\tilde{p}_i^n}{\tilde{\Gamma}_i (2^{r_i^n} - 1)} - \tilde{\sigma}_i^n$.

According to the constraint C2 in(34), for each k, either $\lambda_k = 0$ or $P_k = \sum\limits_{n=1}^{N} p_k^n$. If $\lambda_k = 0, \forall k \in$

$\{1,\ldots,K\}$, (34) can be decomposed across the subchannels. Hence, for $\forall n \in \{1,\ldots,N\}$, we have

$$\max_{\theta^n} \theta^n \sum_{k=1}^{K} p_k^n \tilde{\alpha}_k^n \quad \text{s.t.} \quad \begin{cases} \sum_{k=1}^{K} p_k^n \tilde{\alpha}_k^n \leq T_i^n; \text{ (C1)} \\ p_k^n = \left[\dfrac{1}{\theta^n \tilde{\alpha}_k^n} - \Gamma_k \left(\sum_{\substack{j=1 \\ j\neq k}}^{K} p_j^n \alpha_{j,k}^n + \sigma_k^n \right) \right]_0^{p_k^{\max}}; \text{ (C2)} \end{cases} \tag{37}$$

However, (37) is a non-convex optimization problem. When the unique NE is achieved in $\mathcal{G}$, the group of D2D pairs which occupy subchannel $n, \forall n \in \{1,\ldots,N\}$ would be determined. Define the set of those D2D pairs using subchannel $n, \forall n \in \{1,\ldots,N\}$ as $\mathcal{U}^n$. If $k \in \mathcal{U}^n$, then

$$p_k^n = \frac{1}{\theta^n \tilde{\alpha}_k^n} - \Gamma_k \left(\sum_{\substack{j\in\mathcal{U}^n \\ j\neq k}} p_j^n \alpha_{j,k}^n + \sigma_k^n \right). \tag{38}$$

If $k \notin \mathcal{U}^n$, then $p_k^n = 0$. As a result, the objective function of (37) is rewritten as

$$\sum_{k\in\mathcal{U}^n} \frac{1}{\tilde{\alpha}_k^n} - \theta^n \sum_{k\in\mathcal{U}^n} \Gamma_k \left(\sum_{\substack{j\in\mathcal{U}^n \\ j\neq k}} p_j^n \alpha_{j,k}^n + \sigma_k^n \right). \tag{39}$$

Since $\sum_{k\in\mathcal{U}^n} \Gamma_k \left(\sum_{\substack{j\in\mathcal{U}^n \\ j\neq k}} p_j^n \alpha_{j,k}^n + \sigma_k^n \right) \geq 0, \forall \mathcal{U}^n$, then (39) is a monotonic decreasing function with respect to the price θ^n. According to the constraint C1 in (37), the following inequality is derived:

$$\begin{aligned} &\sum_{k\in\mathcal{U}^n} \left(\frac{1}{\theta^n} - \Gamma_k \tilde{\alpha}_k^n \left(\sum_{\substack{j\neq k \\ j\in\mathcal{U}^n}} p_j^n \alpha_{j,k}^n + \sigma_k^n \right) \right) \leq T_i^n \\ &\Rightarrow \theta^n \geq \frac{|\mathcal{U}^n|}{T_i^n + \sum_{k\in\mathcal{U}^n} \Gamma_k \tilde{\alpha}_k^n \left(\sum_{\substack{j\neq k \\ j\in\mathcal{U}^n}} p_j^n \alpha_{j,k}^n + \sigma_k^n \right)}, \end{aligned} \tag{40}$$

where $|\mathcal{U}^n|$ denotes the cardinality of the set $\mathcal{U}^n$.

Corollary 2: If $\lambda_k = 0, \forall k \in \{1,\ldots,K\}$ and the NE achieved in $\mathcal{G}$ is unique, then the optimal solution of (37) is reached when the interference from D2D to CU_i on subchannel n equals the constraint T_i^n.

Note that if the NE in $\mathcal{G}$ is not unique, Corollary 2 will not apply.

The direction to update the price θ^n can be decided according to Corollary 2. Since θ^n is a scalar for each subchannel n, the bisection method can be employed to find its optimal value.

Note that the condition $\lambda_k = 0, \forall k \in \{1,\ldots,K\}$ means that the interference constraint to the CUs on each subchannel (i.e., C2 in (34)) is so stringent that each D2DT$_k$ cannot use all its transmission power. If for some $k \in \{1, \dots, K\}$, $\lambda_k \neq 0$ and $P_k = \sum_{n=1}^{N} p_k^n$, then (34) cannot be decoupled. One suboptimal method is to first assume that $\lambda_k = 0$. With the same approach, the BS updates its price. However, this time, at the D2D pairs side, the sum of the solutions p_k^n's on the N subchannels may be greater than P_k. New solutions, $(p_k^n)_{\text{new}} = \frac{p_k^n}{\sum_{n=1}^{N} p_k^n} \times P_k$, can be used to replace $p_k^n, \forall n \in \{1, \dots, N\}$. However, the optimality of the solution for problem (34) cannot be guaranteed using this manipulation.

E. Distributed Scheme

Based on the above analysis in Sections IV-C and IV-D, a distributed algorithm namely Algorithm 3, is developed. In this scheme, there are two loops. In the inner loop, the D2D pairs compete for the subchannels via a non-cooperative game. For the outer loop, the BS updates the price θ^n for each subchannel to maximize its profit based on the interference constraint on the corresponding subchannel. Since Algorithm 2 is implemented in the inner loop, its convergence is not affected by the price according to Theorem 2 and Theorem 3. In other words, the inner loop would converge with any price as long as the sufficient conditions in Theorem 2 are satisfied. On the other hand, based on (19), the power allocation function for each D2D pair is a monotonic decreasing function with the price. Therefore, in the outer loop, the BS can coordinate the interference from the CUs to the D2D pairs through updating the price. In the paper, the bisection method has been applied to find the optimal price. Since the inner loop will converge with any price, the convergence of Algorithm 3 is guaranteed.

It is noteworthy that the BS needs to sense the aggregate interference in Algorithm 3 to update the price since the bisection method is applied to find the optimal price value. As a result, the BS does not need to have available each individual CSI, such as h_k^n, $g_{j,k}^n$, $\tilde{f}_k^n$, or $f_{i,k}^n$.

V. Implementation Overhead

In this section, we demonstrate the difference in terms of the implementation overhead between Algorithm 1 (the centralized scheme) and Algorithm 3 (the decentralized scheme). As presented in Section III, the BS, as the central controller, needs to acquire full CSI before the resource allocation. This acquisition includes the CSI between D2DTs and BS, between all D2DTs and D2DRs, the interference CSIs among the D2D pairs, and the interference CSI from the CUs to the D2DRs, and from D2DTs to the BS.

Algorithm 3 Decentralized Scheme

Initialize $\theta^n_{\min} = 0$ and $\theta^n_{\max}$ for some sufficiently large value $\forall n \in \{1, \dots, N\}$;
repeat
  Compute $\theta^n = \frac{(\theta^n_{\max} + \theta^n_{\min})}{2}, \forall n \in \{1, \dots, N\}$;
  Run AIWA with the price vector $\boldsymbol{\theta}$;
  for $n \in \{1, \dots, N\}$ **do**
    if $\sum_{k=1}^{K} p_k^n \tilde{\alpha}_k^n < T_i^n$ **then**
      $\theta^n_{\max} = \theta^n$
    else
      $\theta^n_{\min} = \theta^n$
    end if
  end for
until Convergence
return $(\boldsymbol{\theta}, \mathbf{P})$

In comparison, the overhead for feeding back the CSI in the decentralized scheme is considerably less. After the BS broadcasts the price, each D2D pair optimizes its own resource allocation scheme, while treating its interference as noise. Therefore, the CSI required at each D2DT is only the CSI on its own transmission link. At the BS, since the price is updated according to the aggregate interference caused by the D2D transmission, the individual CSI between each D2DT and BS is not needed. The following table summarizes the difference of the signaling overhead between the centralized and decentralized schemes.

Overhead	Decentralized	Centralized
Interference CSI: $D2DT_k \rightarrow D2DR_j, \forall k \neq j$ and $\forall n$	N	Y
Direct CSI: $D2DT_k \rightarrow D2DR_k, \forall k, n$	Y	Y
Interference CSI: CUs$\rightarrow D2DR_k, \forall k, n$	N	Y
Cross CSI: $D2DT_k \rightarrow$ BS, $\forall k, n$	N	Y
Iterations	Y	Y

TABLE I
OVERHEAD COMPARISON FOR CHANNEL ESTIMATION.

It is worth pointing out that not only the amount of the required CSI in the centralized scheme is much higher than that for the decentralized one, but the difficulty to obtain the CSI is higher. In the centralized scheme, the interference CSI among the D2D pairs and the interference CSI from the CUs to the D2DRs are required but they are too complex to obtain in practice. On the contrary, in the decentralized scheme, the CSI between the D2DT and the D2DR is typical and not difficult to acquire.

Note that the iterations may bring extra signalling overhead in the decentralized scheme. Yet even so, during the iteration, only BS needs to broadcast the updated price vector in the system and the necessary iterations for the convergence is also quite low (examined by the simulations in Section VI). Therefore, both the amount and the difficulty in accomplishing it are lower than those in the centralized scheme. The amount of overhead from iterations is still limited. As a result, the iterations would not affect the effectiveness of the Algorithm 3 too much.

VI. Simulation Results

In this Section the numerical results are demonstrated to verify the proposed algorithms. In the simulations, the subchannels are assumed frequency-flat. We model the small-scale fading by a three-path Rayleigh fading channel with an exponential power delay profile. The D2D pairs are randomly located in an area at least $100\mathrm{m}$ away from the BS. The distances between any two D2D pairs are more than $100\mathrm{m}$, while the distance between a D2DT and its receiver is less than $35\mathrm{m}$. Other simulation parameters are provided in Table II.

Parameters	Value
Number of D2D pairs K	5
Number of subchannels N	16
Path loss exponent	3
Radius of the cell	$500\mathrm{m}$
CU transmit power	$24\mathrm{dBmW}$
Total transmit power at D2DT	$24\mathrm{dBmW}$
Maximum transmit power at D2DT on each subchannel	$10\mathrm{dBmW}$
AWGN noise power	$-174\mathrm{dBm}$
BER requirement on each subchannel	10^{-4}
ς	10^{-6}
κ	10^{-6}

TABLE II
SIMULATION PARAMETERS

In Fig. 2, results for the spectral efficiencies (SE) for Algorithms 1 and 3 are shown. The x-axis corresponds to the data rate requirements of the CUs on each subchannel. Obviously, the higher the data rate requirement, the more stringent the interference constraint in (12). For this reason, the performance of both algorithms is decreasing as the target rate increases. As expected, the performance of Algorithm 1 is always superior than that of Algorithm 3 because it is centralized. Nevertheless, one interesting observation is that as the interference constraint becomes more and more stringent, the performance of the decentralized scheme (Algorithm 3) actually approaches more and more closer to Algorithm 1.

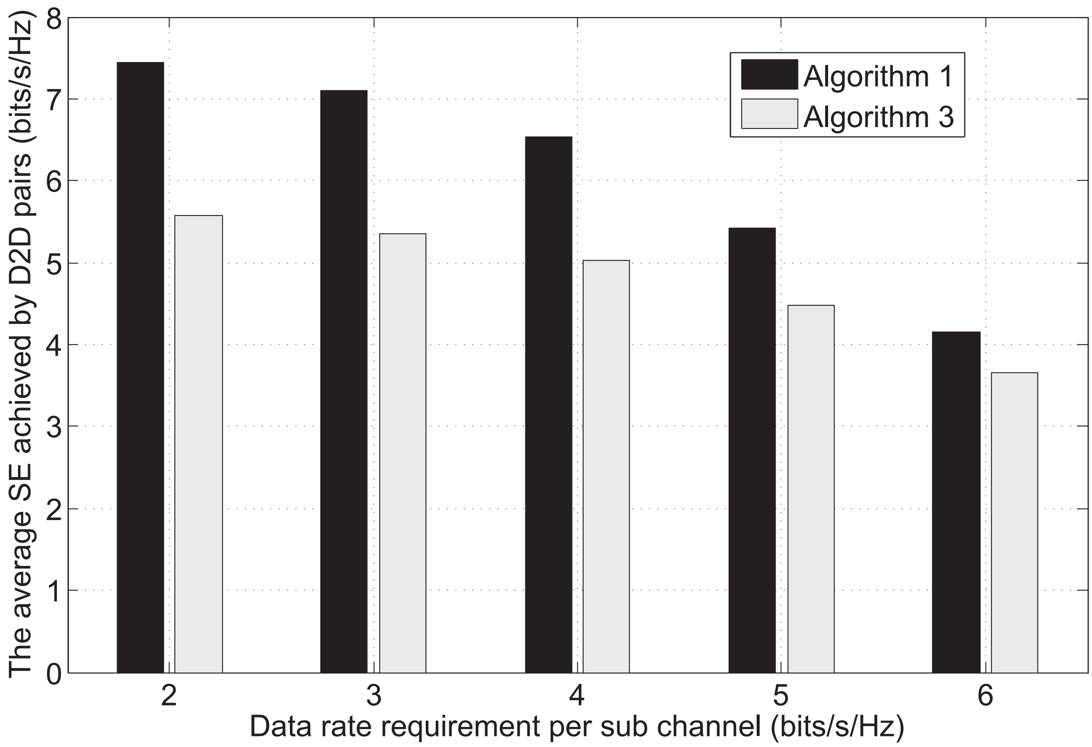

Fig. 2. The spectrum efficiency results for the D2D pairs against the rate requirements of the CUs.

Results in Figs. 3 and 4 demonstrate the average system SE with respect to the number of D2D pairs and subchannels when the data rate requirements of CUs on each subchannel is 3bits/s/Hz. As the number of D2D pairs increases, the system performance is increased in both algorithms due to the inherent multiuser diversity gain. On the contrary, since the total transmission power of each D2D pair is limited, as the number of spectrums increases, the system SE decreases. Therefore, the SE in Fig. 4 when $N = 32$ is lower than that in Fig. 3 when $N = 16$.

Fig. 5 is one snapshot of the simulation for the spectrum reuse when $K = 5$ D2D pairs share $N = 16$ subchannels with CUs. In the figure, the subchannels reused by D2D pairs in Algorithms 1 and 3 are demonstrated, respectively. It is shown that, optimally, one D2D pair would reuse multiple channels of CUs in Algorithm 1. In comparison, one D2D pair also reuse multiple channels in Algorithm 3, but the reuse pattern is different from Algorithm 1. In conclusion, the assumption that each D2D pair is only allowed to share the spectrums with one CU in the traditional works would limit the improvement on the system performance.

Results in Fig. 6 show the convergence speed for the D2D pairs in their rates on a particular subchannel, while Fig. 7 illustrates the change in the interference the BS receives due to the D2D transmission on a subchannel, all for Algorithm 3. Although the interference at the BS is properly controlled, the data rates achieved by the D2D pairs are not balanced and fairness could be an issue for the decentralized scheme.

The updating process of the price θ^n (for some n) is examined by the results in Fig. 8, for various target rates of the CU. As is expected, the higher the target rate, the higher the price the D2D transmission. As we mentioned in Section IV-D, since the CUs have priority in accessing the subchannels, users in the D2D mode should be compensated when the BS sets the price for the interference.

As far as the convergence speed of Algorithm 3 is concerned, the convergence processes of inner loop with different number of D2D pairs are demonstrated in Figs. 9 and 10, respectively. We observe that the convergence speed of inner loop (hence Algorithm 2) will be decreased as the number of D2D pairs increases. For the outer loop, the BS updates the price based on the sensed interference on each subchannel. Therefore, its convergence speed would not affect by the number of D2D pairs, as shown in Fig. 11.

Finally, the convergence of the parameters a_k and b_k (hence Algorithm 1) is demonstrated in Fig. 12. As we can observe, they converge quickly with less than 10 iterations required to reach a steady state.

VII. Conclusion

This paper has studied the resource allocation problem in the D2D communications underlaying cellular networks. We first propose a centralized scheme, referred to as Algorithm 1, which is formulated as a non-convex optimization problem that is solved by using a convex approximation method. The corresponding results serve as a performance benchmark. Then utilizing the Stackelberg model, we proposed a distributed resource allocation strategy, and design an iterative algorithm (Algorithm 3) to solve the proposed game. Moreover, the signaling overheads for both algorithms have been analyzed and compared. Finally, numerical results are provided to verify the convergence rate of the proposed algorithms and their performance. Our results have demonstrated that the distributed Algorithm 3 achieves good performance with significant reduction on the signaling overhead, illustrating its potential for a practical design.

Appendix A
The analysis on convergence and the accuracy of Algorithm 1

The following non-convex optimization problem is studied in [18]:

$$\min_{\mathbf{x}} W_0(\mathbf{x}) \quad \text{s.t.} \quad W_i(\mathbf{x}) \le 1, i = 1, 2, \cdots, m,$$

where the objective function $W_0(\mathbf{x})$ and the constraints $W_i(\mathbf{x}), \forall i = 1, 2, \cdots, m$ are either convex or non-convex. If a series of convex approximations $\tilde{W}_i(\mathbf{x}) \approx W_i(\mathbf{x})$ are applied for any non-convex $W_0(\mathbf{x})$ and $W_i(\mathbf{x})$ if they are non-convex, then the problem could be solved by convex optimization methods. Based on the analysis in [18], if the approximations satisfy the following three conditions, then the solutions of this series of approximations converge to a point satisfying the Karush-Kuhn-Tucker (KKT) conditions of the original problem:

(a) $W_i(\mathbf{x}) \leq \tilde{W}_i(\mathbf{x})$, for any non-convex $W_i(\mathbf{x})$;

(b) $W_i(\mathbf{x}_0) = \tilde{W}_i(\mathbf{x}_0)$, where $\mathbf{x}_0$ is the optimal solution of the approximated problem in the previous iteration, for any non-convex $W_i(\mathbf{x})$;

(c) $\nabla W_i(\mathbf{x}_0) = \nabla \tilde{W}_i(\mathbf{x}_0)$, for any non-convex $W_i(\mathbf{x})$;

Herein, condition (a) guarantees that the approximation $\tilde{W}_i(\mathbf{x}), \forall i = 1, 2, \cdots, m$ tightens the constraints, and any solution of the approximated problem will also be a feasible solution to the original one. Condition (b) guarantees that the solution of each approximated problem will decrease the objective function. Condition (c) guarantees that the KKT conditions of the original problem will be satisfied after the series of approximations converges.

In the problem (8), the objective function is to maximize the sum rates of D2D pairs. Therefore, when examining the effectiveness of approximation (9), the corresponding condition (a) should be changed to

(a*) $W_i(\mathbf{x}) \geq \tilde{W}_i(\mathbf{x})$, for any non-convex $W_i(\mathbf{x})$;

Then, based on the inequality (9), equality (10) and the definition of parameters a_k^n and b_k^n, it is easily verified that the convex approximation employed in Algorithm 1 satisfies conditions a*, b, and c simultaneously. Therefore, the proposed successive approximation method would converge to the solution satisfying the KKT conditions of the original problem. Therefore it at least guarantees a local optimum solution.

Appendix B
Proof of Theorem 2

According to the properties of $VI(\mathbb{P}, \mathbf{F}(\mathbf{P}))$, if $\mathbf{F}(\mathbf{P})$ is strongly monotonic on $\mathbb{P}$, then $VI(\mathbb{P}, \mathbf{F}(\mathbf{P}))$ admits one unique solution. Since $\mathcal{G}$ is equivalent to $VI(\mathbb{P}, \mathbf{F}(\mathbf{P}))$, the sufficient condition under which $\mathcal{G}$ has a unique solution is equivalent to the sufficient condition for the strongly monotonic condition on $\mathbb{P}$ of $VI(\mathbb{P}, \mathbf{F}(\mathbf{P}))$. Here, we use the method in [26], [29], and [30] to derive the sufficient condition.

The mapping is strongly monotonic on $\mathbb{P}$ if there exists a constant $c > 0$ such that for all pairs $\mathbf{P} = \{\mathbf{P}_k\}_{k=1}^K \in \mathbb{P}$ and $\mathbf{P}' = \{\mathbf{P}'_k\}_{k=1}^K \in \mathbb{P}$, the following inequality is satisfied:

$$(\mathbf{P} - \mathbf{P}')^T(\mathbf{F}(\mathbf{P}) - \mathbf{F}(\mathbf{P}')) \geq c\|\mathbf{P} - \mathbf{P}'\|^2. \tag{41}$$

For each $k \in \{1, \dots, K\}$ and $n \in \{1, \dots, N\}$, define

$$\begin{aligned}
\psi_k^n &\triangleq \sqrt{p_k^n + \Gamma_k \left(\sum_{\substack{j=1 \\ j \neq k}}^K p_j^n \alpha_{j,k}^n + \sigma_k^n \right)} \times \\
&\quad \sqrt{(p_k^n)' + \Gamma_k \left(\sum_{\substack{j=1 \\ j \neq k}}^K (p_j^n)' \alpha_{j,k}^n + \sigma_k^n \right)}, && \text{(42a)} \\
\rho_k^n &\triangleq \frac{p_k^n - (p_k^n)'}{\psi_k^n}. && \text{(42b)}
\end{aligned}$$

Then from (41), we can derive that

$$\begin{aligned}
&(\mathbf{P} - \mathbf{P}')^T(\mathbf{F}(\mathbf{P}) - \mathbf{F}(\mathbf{P}')) \\
&= \sum_{k=1}^K \sum_{n=1}^N \frac{p_k^n - (p_k^n)'}{\psi_k^n} \left(\frac{p_k^n - (p_k^n)'}{\psi_k^n} + \frac{\Gamma_k \sum_{j \neq k} \alpha_{j,k}^n (p_k^n - (p_k^n)')}{\psi_k^n} \right) \\
&\quad \geq \sum_{k=1}^K \left(\sum_{n=1}^N (\rho_k^n)^2 - \sum_{j \neq k} \left| \sum_{n=1}^N \rho_k^n \frac{\Gamma_k \alpha_{j,k}^n \psi_j^n}{\psi_k^n} \rho_j^n \right| \right) \\
&\quad \geq \sum_{k=1}^K \left(\sum_{n=1}^N (\rho_k^n)^2 - \sum_{j \neq k} \left(\widetilde{\rho}_k \max_{1 \leq n \leq N} \left(\frac{\Gamma_k \alpha_{j,k}^n \psi_j^n}{\psi_k^n} \right) \widetilde{\rho}_j \right) \right) \\
&\quad \geq \sum_{k=1}^K \left(\widetilde{\rho}_k \sum_{j=1}^K [\mathbf{R}]_{j,k} \widetilde{\rho}_j \right) = \widetilde{\boldsymbol{\rho}}^T \mathbf{R} \widetilde{\boldsymbol{\rho}} \\
&\quad \geq \frac{\eta_{\min}(\mathbf{R})}{\max_{1 \leq k \leq K} \max_{1 \leq n \leq N} (\psi_k^{n,\max})^2} \|\mathbf{P} - \mathbf{P}'\|_2^2, && (43)
\end{aligned}$$

where

$$\begin{cases} \widetilde{\rho}_k = \left(\sum_{n=1}^{N} (\rho_k^n)^2\right)^{\frac{1}{2}}, \\ \widetilde{\boldsymbol{\rho}} = [\widetilde{\rho}_k]_{k=1}^{K}, \\ \psi_k^{n,\max} = p_k^{\max} + \Gamma_k \left(\sum_{\substack{j=1 \\ j\neq k}}^{K} p_j^{\max} \alpha_{j,k}^n + \sigma_k^n\right), \end{cases} \tag{44}$$

and $\eta_{\min}(\mathbf{R})$ denotes the smallest eigenvalue of $\mathbf{R}$. The third inequality of (43) follows from the Cauchy-Shwarz inequality. The last inequality of (43) comes from the fact that if the matrix $\mathbf{R}$ is positive definite, it is also a P-matrix.[1] According to Theorem 3.3.4 in [28] and Lemma 2 in [26], with vector $\mathbf{x}$, $\|\mathbf{x}\|_2 = 1, \mathbf{x}^T\mathbf{R}\mathbf{x} \geq \eta_{\min}(\mathbf{R})$. Then the last inequality can be derived, which completes the proof.

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[1] A matrix $\mathbf{H} \in \Re^{\mathbf{n}\times\mathbf{n}}$ is called a P-matrix if every principal minor of $\mathbf{H}$ is positive [26].

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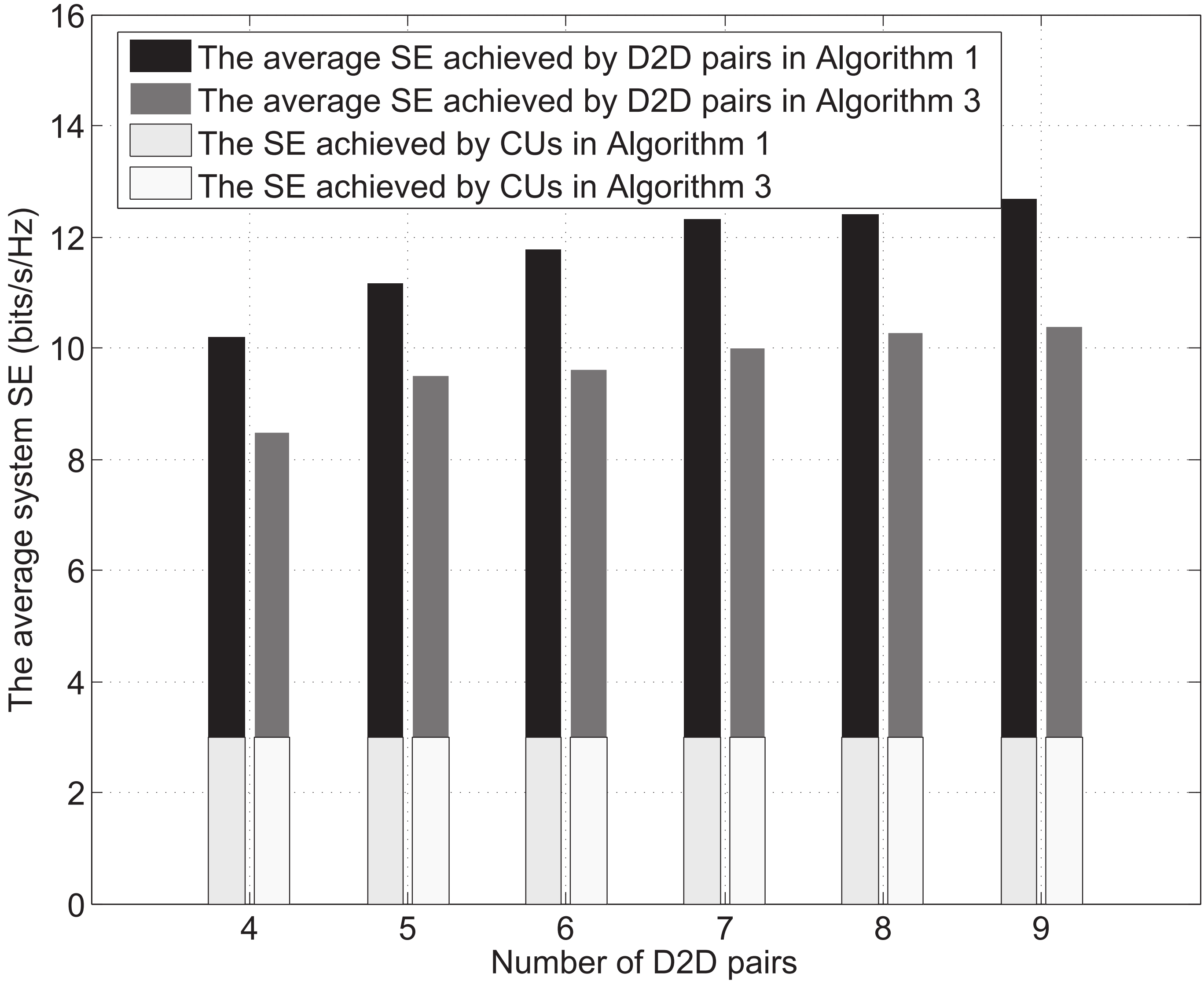

Fig. 3. The average system spectrum efficiency against the number of D2D pairs when $N = 16$.

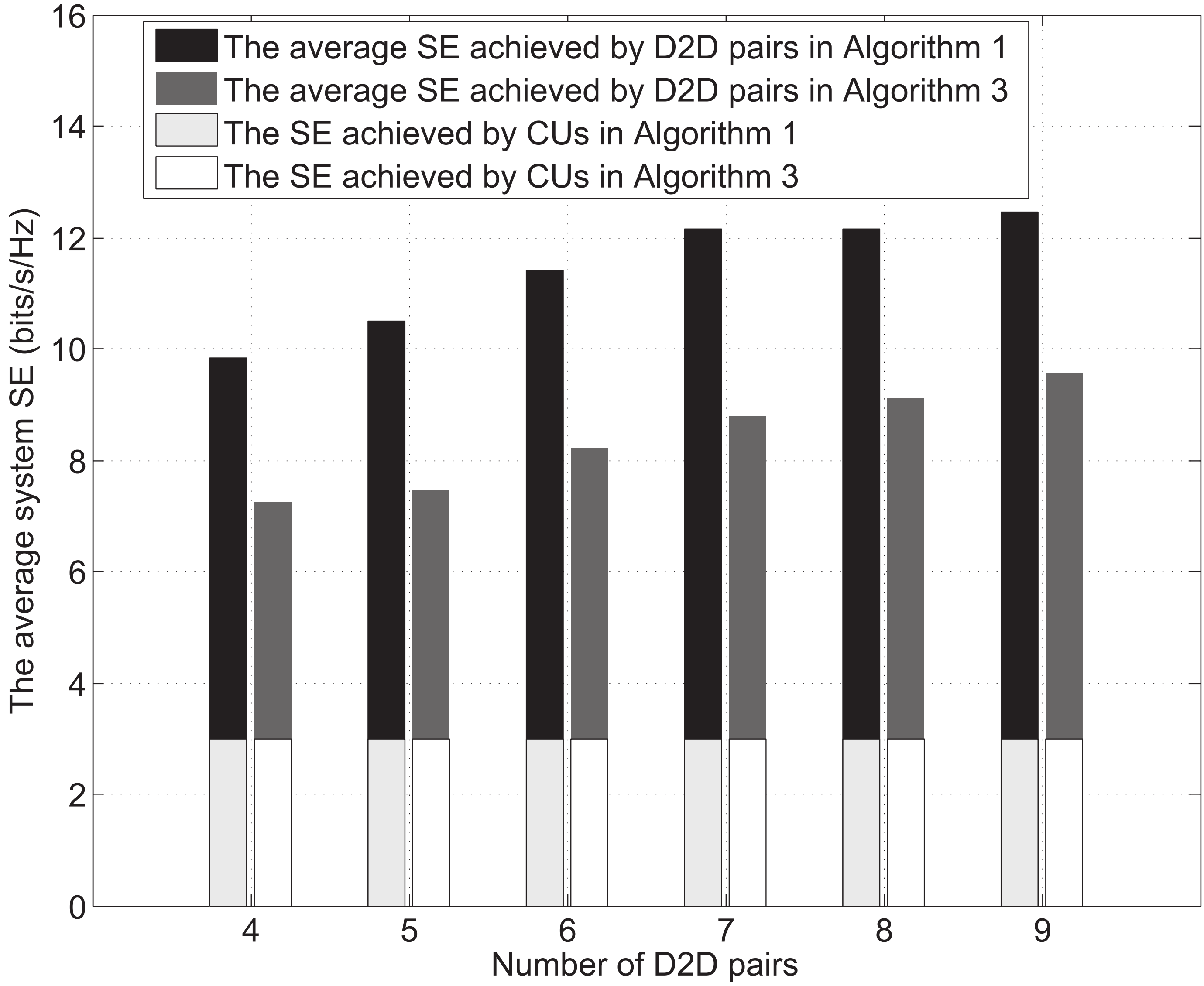

Fig. 4. The average system spectrum efficiency against the number of D2D pairs when $N = 32$.

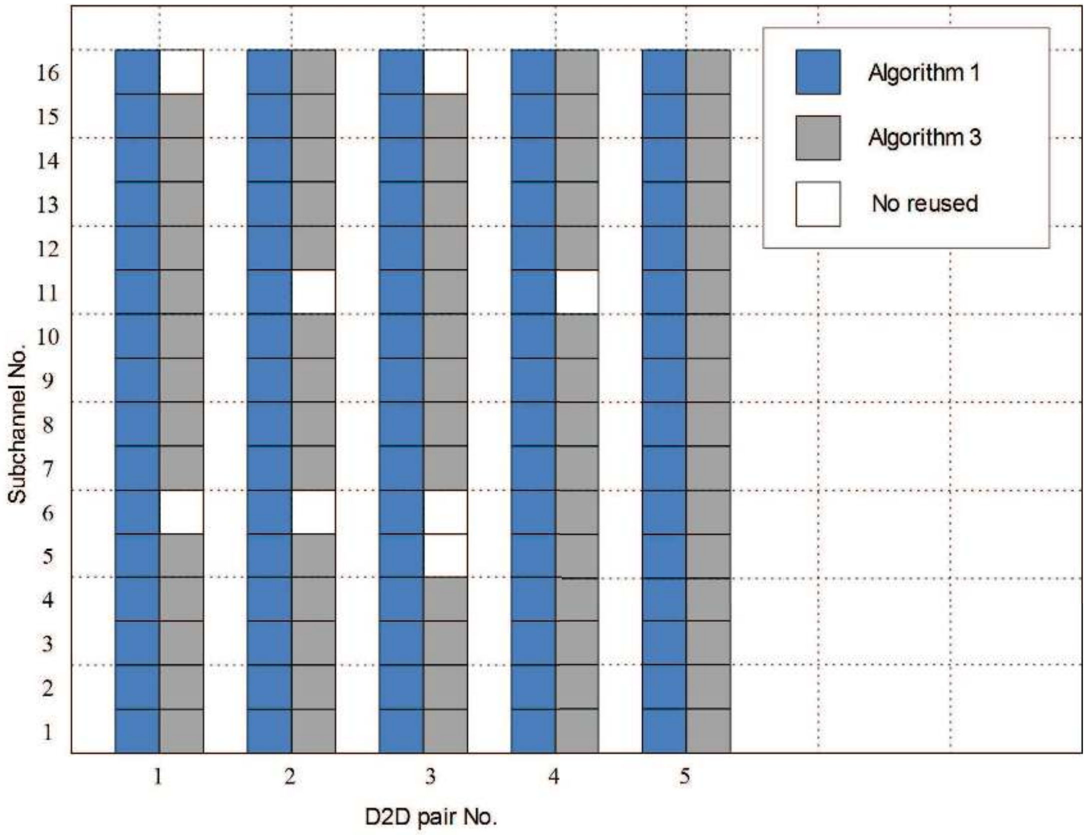

Fig. 5. The indication for the subchannels reused in Algorithm 1 and Algorithm 3.

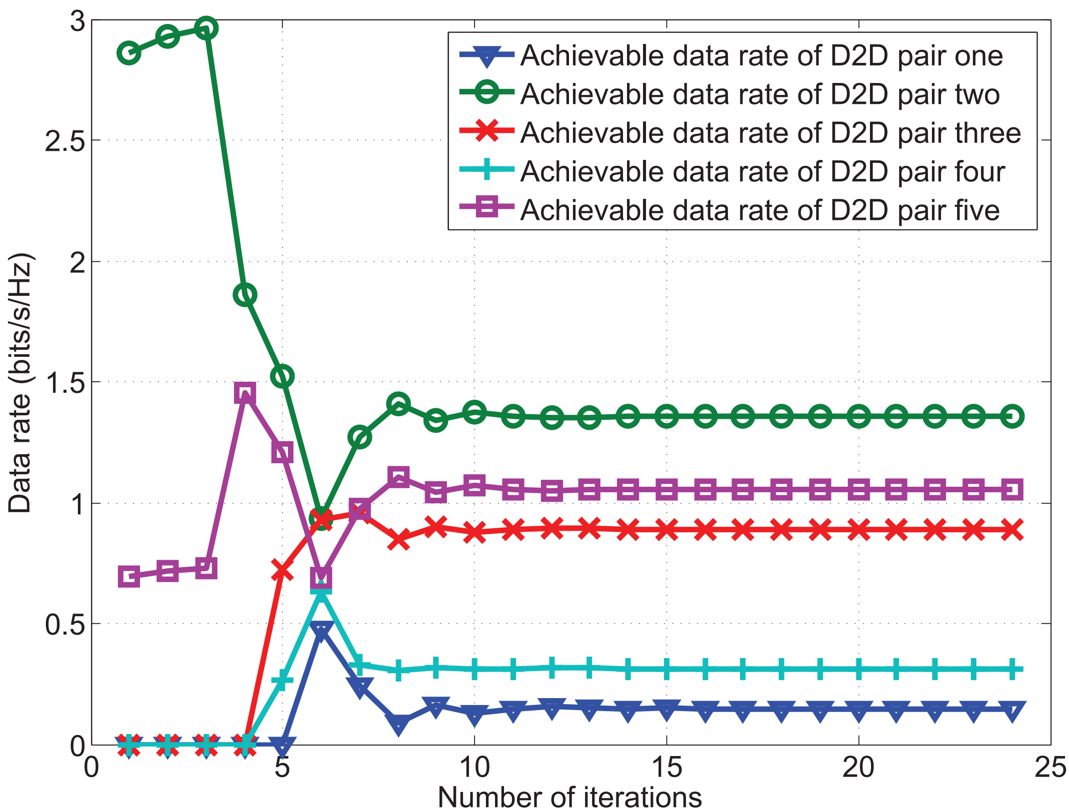

Fig. 6. The convergence behavior for the rates of the D2D pairs using (decentralized) Algorithm 3.

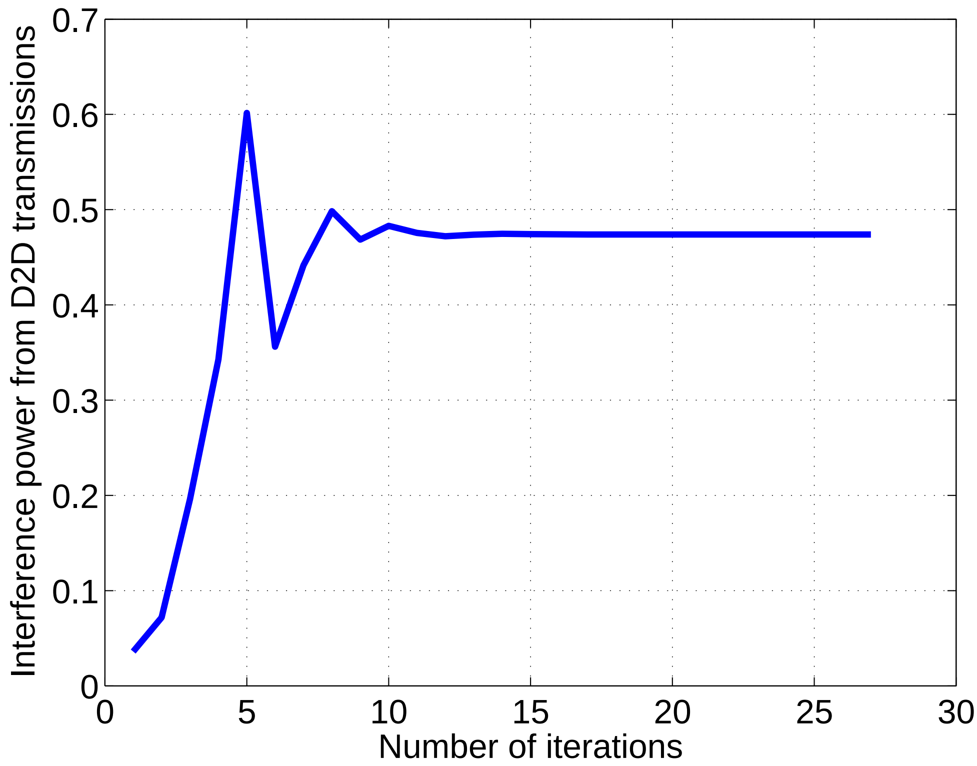

Fig. 7. The convergence behavior for the interference power received at the BS in the case of Algorithm 3.

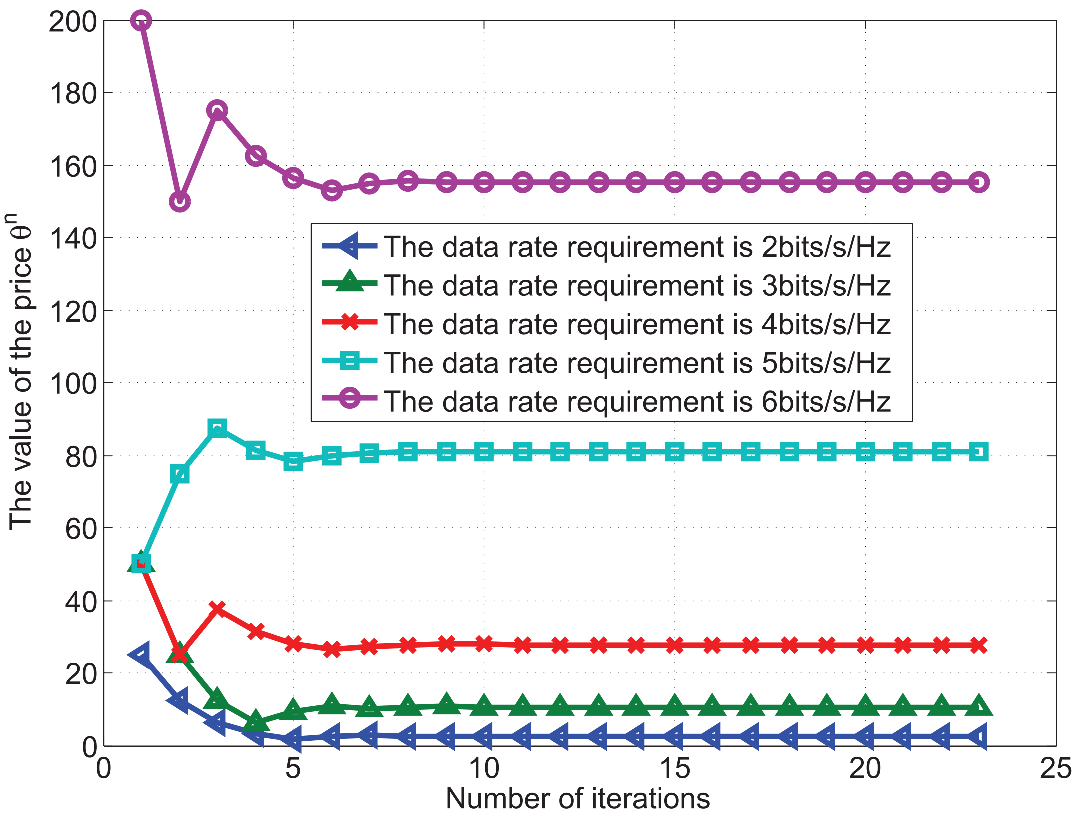

Fig. 8. The price updating process on a subchannel in the case of Algorithm 3.

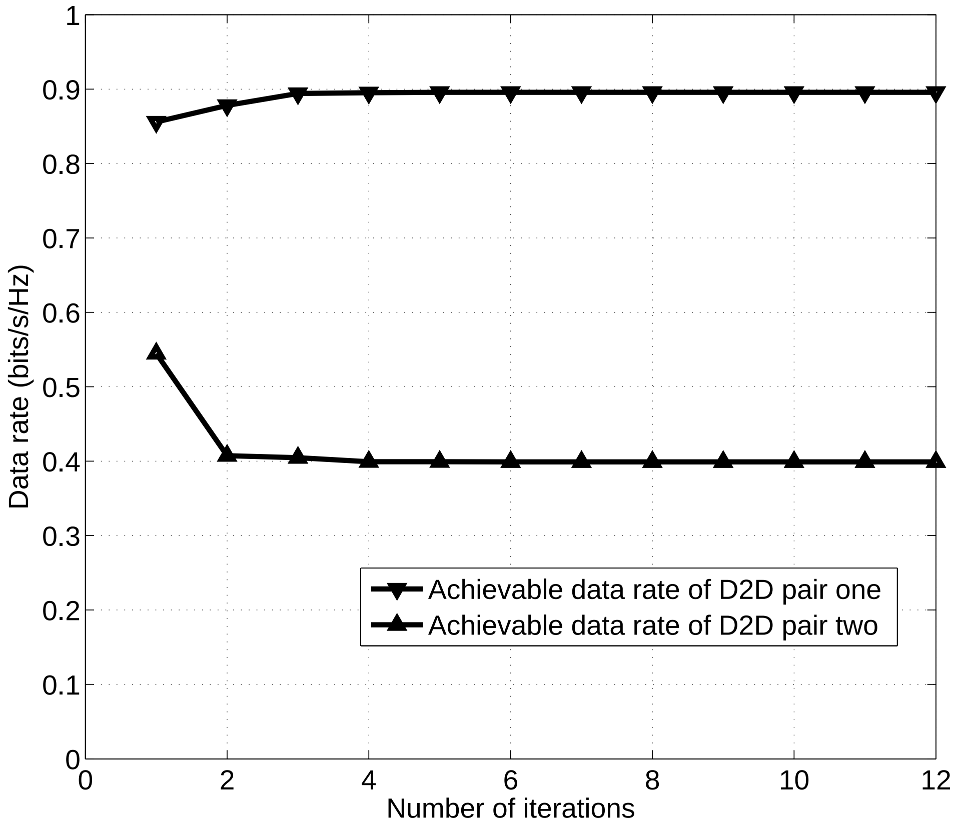

Fig. 9. The convergence process for the rates of D2D pairs when the number of D2D pairs is 2

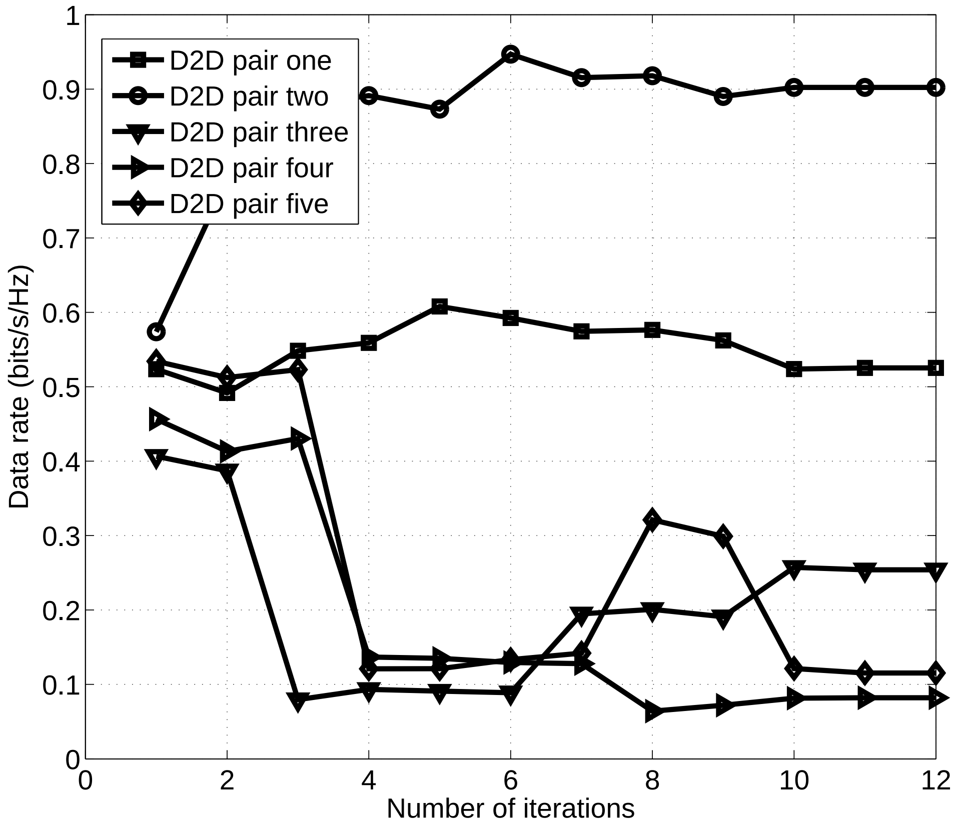

Fig. 10. The convergence process for the rates of D2D pairs when the number of D2D pairs is 5

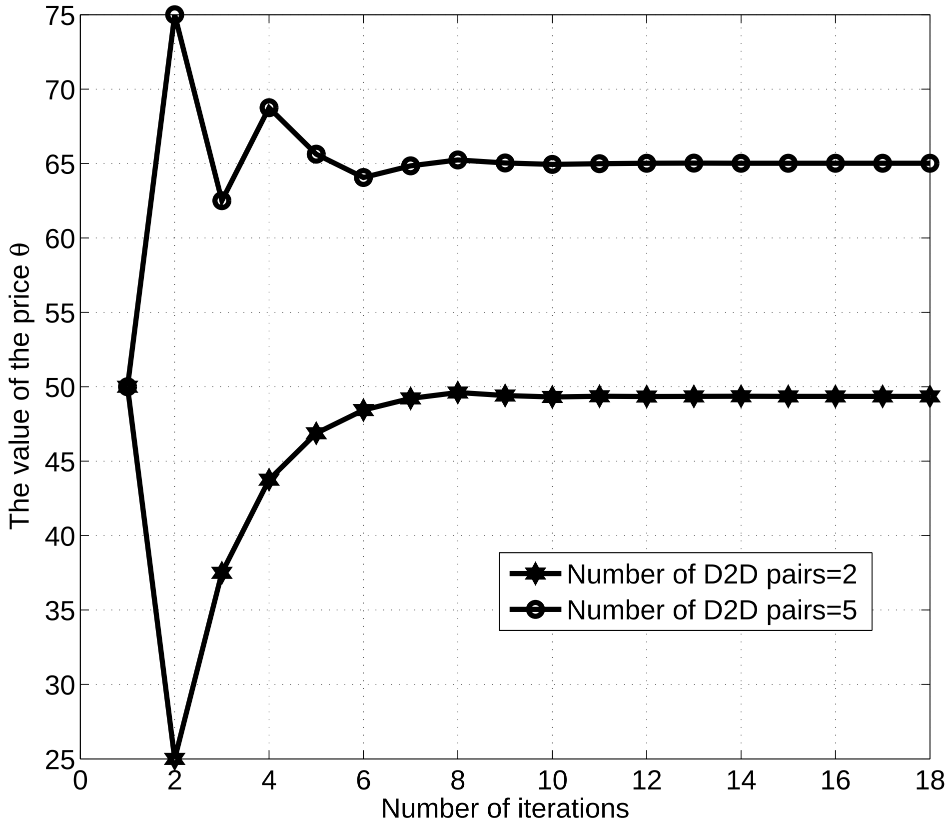

Fig. 11. The convergence process of the price when the number of D2D pairs is 2 and 5, respectively

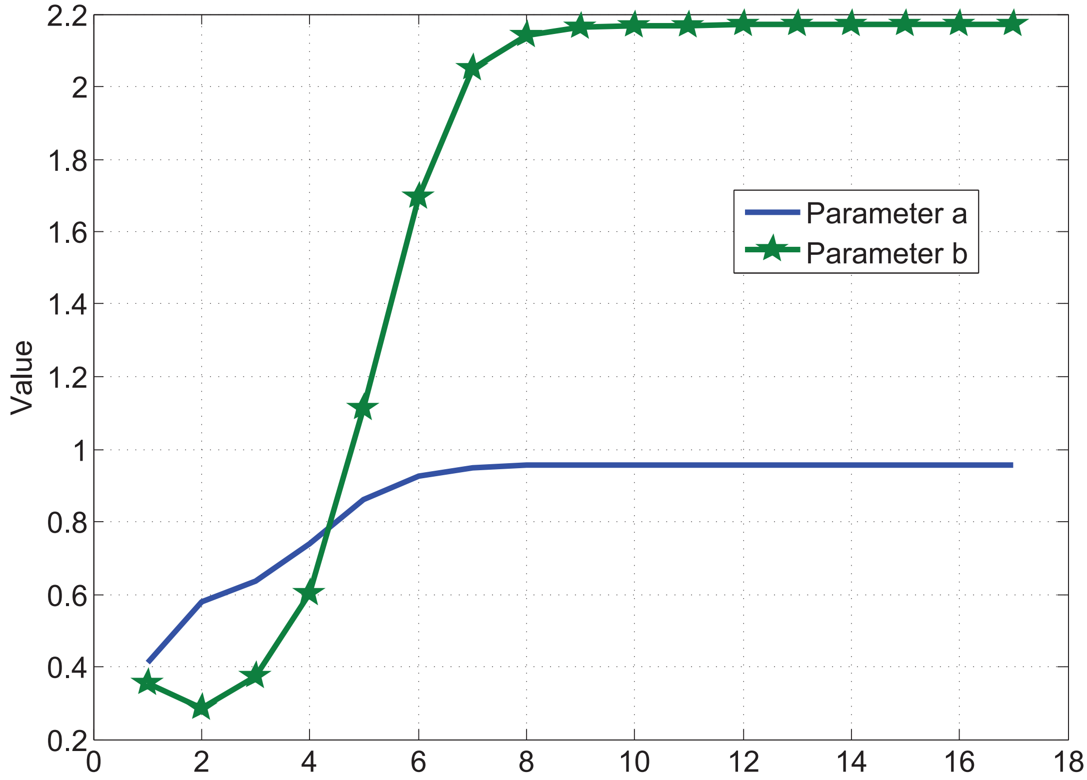

Fig. 12. The convergence behavior for the parameters a_k's and b_k's in the case of (centralized) Algorithm 1.