Joint Subcarrier and Power Allocation for Cooperative Communications in LTE-Advanced Networks

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***Abstract*—This paper considers an LTE-Advanced cooperative cellular network where a Type II relay station (RS) is deployed to enhance the cell-edge throughput and to extend the coverage area. To better exploit the existing resources, the RS and the eNodeB (eNB) transmit in the same channel (In-Band) with decode-and-forward relaying strategy. For such a network, this paper proposes joint Orthogonal Frequency Division Multiplexing (OFDM) subcarrier and power allocation schemes to optimize the downlink multi-user transmission efficiency. Firstly, an optimal power dividing method between eNB and RS is proposed to maximize the achievable rate on each subcarrier. Based on this result, we show that the optimal joint resource allocation scheme for maximizing the overall throughput is to allocate each subcarrier to the user with the best channel quality and to distribute power in a water-filling manner. Since QoS provision is one of the major design objectives in cellular networks, we further formulate a lexicographical optimization problem to maximize the minimum rate of all users while improving the overall throughput. A sufficient condition for optimality is derived. Due to the complexity of searching for the optimal solution, we propose an efficient, low-complexity suboptimal joint resource allocation algorithm, which outperforms the existing suboptimal algorithms that simplify the joint design into separate allocation. Both theoretical and numerical analyses demonstrate that our proposed scheme can drastically improve the fairness as well as the overall throughput.**

***Index Terms*—LTE-Advanced, cooperative communication, spectrum efficiency, relay channel, resource allocation, user fairness.**

I. Introduction

THIRD-GENERATION (3G) wireless systems have been deployed on a broad scale around the world to provide enhanced downlink (DL) and uplink (UL) transmissions. However, due to the emerging technologies and evolving Quality of Service (QoS) requirement, future-generation wireless communication systems are expected to meet even more challenging demands of high data rate and reliable multimedia communications. As a consequence, the Third Generation Partnership Project (3GPP) has launched the long-term evolution (LTE) standard of 3G for wireless communications. The target is to enable high-speed data transmission for mobile phones and data terminals at substantially reduced cost compared to current radio access technologies [1], [2]. In order to improve the spectrum efficiency, the physical layer technologies specified in LTE Release 8 incorporate new techniques such as Orthogonal Frequency Division Multiplexing (OFDM) as the DL multiple access scheme and Single-Carrier Frequency Division Multiple Access (SC-FDMA) as the UL scheme. Currently, further enhancements are being studied to improve the existing LTE Release 8 standard. These enhancements are included in LTE-Advanced (also known as LTE Release 10) standard, which is targeted to support much higher peak rates, higher throughput and coverage, and lower latencies, resulting in a better user experience [3].

Among the new techniques, the cooperative communication with the help of relay stations is of primary significance due to its ability to enhance the cell-edge throughput and to extend the coverage by the utilization of cooperative communications between the base stations and the relays [4], [5]. There are mainly two types of relays being discussed in the context of 3GPP standards. A Type I relay creates its own physical cell and becomes distinct from the donor macrocell. The RS appears as an eNB or base station (BS) to all UEs within its transmission range. Type I relays are half-duplex, thus are unable to transmit to the user equipments (UEs) and receive from the donor eNB simultaneously. A Type II relay is a full-duplex relay which does not create a new cell. It is transparent to all UEs within its coverage area and the UEs are not aware of its existence. With spatial separation, filtering, or enhanced interference cancellation, the full-duplex relays require no specific resource partitioning [6]. In this paper, we focus on Type II relays since only this type of relays can achieve multipath diversity and transmission gain for the UEs. Two backhaul connections are supported in the LTE-Advanced system: In-Band (IB), where the eNB-RS link shares the same frequency bands with the direct eNB-UE link within a cell, and Out-of-Band (OOB), where the eNB-RS link transmits in other frequency bands.

In terms of cooperative protocols, the most popular ones remain Amplify-and-Forward (AF) and Decode-and-Forward (DF). AF relays simply amplify the received signals and forward them to the destination. To avoid propagating the interference and noise from the source-relay link, extra resources such as frequency bands or time slots are employed at the relay for orthogonal transmission. On the other hand, DF relays can completely eliminate the noise since relays decode the received signal before forwarding it. With DF protocol, the

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source and the relay are able to transmit in the same frequency band (channel) to improve the spectral efficiency.

Considering the unique features of relays in LTE-Advanced systems, extensive research efforts have been dedicated from various aspects including relay architectures [7]–[10], cooperative protocols [11], [12], and cooperative gain [13], [14]. Driven by the user demand of higher data rate communications, improving the network efficiency through resource allocation attracts an upsurge of research interest. Specifically, power allocation schemes to improve the achievable rate for various relay channels are investigated in [15]–[18]. [19]–[22] study the joint OFDM subchannel and power allocation problem assuming the source and the relay transmit in two orthogonal channels. However, in terms of the In-Band DF relay networks where the source and the relay occupy the same channel, few works address the resource allocation issue.

In this paper, we investigate the adaptive joint subcarrier and power allocation to improve the downlink transmission efficiency in LTE-Advanced relay systems. We focus on the In-Band Type II full-duplex relay stations with decode-and-forward strategy, since this type of relays can better exploit the broadcast nature of wireless signals while improving the utilization of existing allocated spectral resources. The loop interference associated with full-duplex relays can be avoided by spatial isolation of the transmitting and receiving transceivers, and utilizing a loop interference suppression technique [11].

As OFDM divides the frequency band into orthogonal narrowband subchannels (subcarriers), each subchannel can be viewed as a conventional relay channel where the eNB and the RS cooperate to transmit to the UE (destination). We first investigate the power allocation on each subcarrier and propose optimal power dividing schemes between the eNB and RS to maximize the relay channel's achievable rate.

With the optimal power dividing schemes on each subcarrier, we then jointly allocate subcarrier and power in the multi-user OFDM network. In the traditional multi-user OFDM, a few resource allocation algorithms have been proposed. For example, [23] provides a sum capacity maximization scheme while [24], [25] proposes algorithms to consider fairness of the system. [26]–[28] investigate resource allocation to balance both throughput and fairness by maximizing a utility function. These algorithms either iteratively search for the joint resource allocation or decompose the joint allocation into separate processes where a uniform power distribution is assumed when allocating subcarriers. Ideally, the subchannel and power allocation should be designed jointly since they are mutually dependent, especially when fairness needs to be considered.

In LTE-Advanced cellular networks, as the eNB needs to perform resource allocation in a rapidly changing environment, efficient joint resource allocation schemes with low computational cost are preferred, especially for cost-effective and delay-sensitive implementations. In our work, we first come up with an optimal joint subchannel and power allocation scheme to maximize the overall throughput. Then, user fairness is taken into consideration. A lexicographical optimization problem is formulated to guarantee the max-min fairness while improving the transmission efficiency and a sufficient condition of the optimal solution is provided. Due to the complexity of finding the optimal solution, we propose an efficient two-step joint resource allocation suboptimal algorithm with low computational complexity.

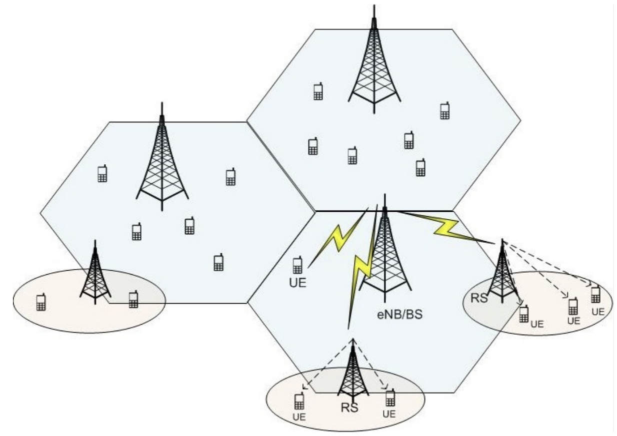

Fig. 1. LTE-Advanced cellular network structure with the deployment of RSs.

The main contributions of the paper are three-fold:

- The optimal power dividing schemes between the eNB and the RS can increase the transmission rate on each subcarrier. With this scheme, the cooperation between the eNB and the RS can be maximized to improve the transmission efficiency.
- In the multi-user OFDM networks, improving the fairness among users is usually at the expense of the reduced overall throughput. To tackle this issue, we formulate a novel lexicographical optimization problem where throughput will be optimized when maximum fairness is guaranteed.
- Due to the complexity of the optimal resource allocation, an efficient suboptimal joint allocation algorithm with low computational complexity is proposed. Compared with most of the two-step procedures, our suboptimal algorithm can effectively improve the minimum rate of all users as well as the average user rate.

The remainder of this paper is organized as follows. Section II introduces the system model, where the resource allocation problem is formulated as an optimization problem. The maximization problem of the relay channel's achievable rate on a single subcarrier is proposed in Section III and an optimal power dividing scheme between eNB and RS is provided. Section IV aims at maximizing the overall throughput, and Section V studies the fairness issue. Section VI contains the numerical results and in Section VII, we provide some concluding remarks and possible future work.

II. System Model

The cellular network structure considered in this paper is shown in Fig. 1. In each LTE-Advanced cell, an eNB is installed in the center to serve several UEs. RSs that usually have smaller coverage area are deployed near the cell edge to improve the cell-edge users' throughput or to extend the cellular radio coverage. To simplify the problem, suppose no cooperation exists among adjacent RSs, so any user can only be served by its affiliated eNB and RS if possible.

In the downlink transmission, users receive signals from their serving eNB. If a user is also located within the RS's coverage range, it can be reached by the eNB via two paths, the direct transmission link and the relay transmission link, which is modeled as the three-node cooperative relay channel. The desired downlink transmission is from the eNB to the user, while the affiliated RS aids the communication by capturing the signals sent from the eNB and forwarding them to the user. Upon receiving the signals, the relay decodes the original message and retransmits it in its own codes during the subsequent transmission block. Two-hop transmissions are discussed in this paper from a practical perspective. As the number of hops grows in the relay networks, the decoding complexity would be drastically increased [29].

In LTE-Advanced cellular systems, the downlink frequency domain transmission technique is OFDM where the available bandwidth B is divided into n orthogonal subchannels operating at different subcarriers (tones). Each subchannel has a bandwidth of B/n. The tones can be dynamically allocated to users to exploit both multi-user diversity and frequency diversity at a finer granularity.

Let $\mathcal{M} = \{1, \ldots, m\}$ and $\mathcal{N} = \{1, \ldots, n\}$ denote the user set and the frequency-domain subchannel set, respectively. At the beginning of each transmission block, the subchannels are allocated by the eNB and the transmission power can be dynamically adjusted at both eNB and RS to improve the transmission efficiency.

Suppose all wireless channels are independent additive white Gaussian noise (AWGN) channels where the noise variances are normalized to 1. The channel gain coefficients for user k, $k \in \mathcal{M}$ on subchannel l, $l \in \mathcal{N}$ are denoted by $h_{sr}^{(k,l)}$, $h_{rd}^{(k,l)}$ and $h_{sd}^{(k,l)}$ representing the channel conditions of the eNB-RS, RS-UE and eNB-UE links, respectively. If a user is not within the RS coverage, there is only one direct transmission channel gain coefficient, $h_{sd}^{(k,l)}$. The channel state information is assumed to be known at both the transmitter and the receiver so that eNB can adaptively allocate the transmission power and subchannels according to the instantaneous channel state information.

The average transmission power at the eNB and at the relay are bounded by

$$\begin{aligned} \frac{1}{T}\sum_{t=1}^{T} x_s^2(t) &\leq P_s \\ \frac{1}{T}\sum_{t=1}^{T} x_r^2(t) &\leq P_r, \end{aligned} \tag{1}$$

where x_s and x_r are signals transmitted from the eNB and from the RS, respectively, and t is the time index ranging from 1 to T. Suppose the power allocated to user k for transmission in the l^{th} subchannel at the eNB is $P_s^{(k,l)}$ and that at the RS is represented by $P_r^{(k,l)}$. They must satisfy

$$\begin{aligned} \sum_{k,l} P_s^{(k,l)} &\leq P_s, \\ \sum_{k,l} P_r^{(k,l)} &\leq P_r. \end{aligned} \tag{2}$$

To indicate whether the l^{th} subchannel is allocated to user k, we introduce a binary variable $\rho^{(k,l)}$:

$$\rho^{(k,l)} = \begin{cases} 1, & \text{the } l^{th} \text{ subchannel is allocated to user } k; \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

To avoid the interference, suppose each subchannel can only be allocated to one user during each transmission block.

Then the maximum achievable rate of cooperative communication for user k in subchannel l in the AWGN environment is given by:

$$\begin{aligned} R^{(k,l)} = \max_{0 \leq \beta^{(k,l)} \leq 1} \min \Bigg\{ & \frac{\rho^{(k,l)}}{n} \log\left(1 + \frac{|h_{sr}^{(k,l)}|^2 \beta^{(k,l)} P_s^{(k,l)}}{B/n}\right), \\ & \frac{\rho^{(k,l)}}{n} \log\Bigg(1 + \frac{|h_{sd}^{(k,l)}|^2 \beta^{(k,l)} P_s^{(k,l)}}{B/n} + \\ & \frac{\left(\sqrt{|h_{sd}^{(k,l)}|^2 \bar{\beta}^{(k,l)} P_s^{(k,l)}} + \sqrt{|h_{rd}^{(k,l)}|^2 P_r^{(k,l)}}\right)^2}{B/n}\Bigg)\Bigg\}, \end{aligned} \tag{4}$$

where $\beta^{(k,l)} \in \{0,1\}$ is a coefficient determined by the source to adjust the portion of its transmission power used for cooperation with the relay in order to achieve the maximum rate. $\bar{\beta}^{(k,l)} = 1 - \beta^{(k,l)}$. The derivation of this achievable rate in the AWGN environment can be found in Appendix A.

For non-cooperative transmission, $P_r^{(k,l)} = 0$. The maximum transmission rate is the capacity of the direct eNB-UE link:

$$R^{(k,l)} = \frac{\rho^{(k,l)}}{n} \log\left(1 + \frac{|h_{sd}^{(k,l)}|^2 P_s^{(k,l)}}{B/n}\right). \tag{5}$$

Different from the power assignment problem that assumes each node has a unique power constraint, we focus on the fundamental performance limitation of the system with the existing allocated resources, e.g., bandwidth and total power. Our objective is to maximize the system's transmission efficiency; in other words, to find the minimum power (energy) needed to transmit a given amount of information at a certain rate under a bandwidth constraint. This is equivalent to finding the rate under a total power and bandwidth constraint [16]. Towards this end, we first investigate the joint OFDM subchannel and power allocation to maximize the sum rate of all users on all subchannels. Then we further consider fairness issue in the subsequent section. Specifically, the throughput maximization problem can be formulated mathematically as:

$$\begin{aligned} \max_{P_s^{(k,l)}, P_r^{(k,l)}, \rho^{(k,l)}} & \sum_{k \in \mathcal{M}, l \in \mathcal{N}} R^{(k,l)} \\ \text{subject to } & \sum_{k,l} P_s^{(k,l)} + \sum_{k,l} P_r^{(k,l)} \leq P_{total} \\ & P_s^{(k,l)} \geq 0, \text{for all } k,\ l \\ & P_r^{(k,l)} \geq 0, \text{for all } k,\ l \\ & \rho^{(k,l)} \in \{0,1\}, \text{for all } k,\ l \\ & \sum_{k \in \mathcal{M}} \rho^{(k,l)} \leq 1, \text{for all } l \end{aligned} \tag{6}$$

where P_{total} is the total available power at the eNB and the RS.

Note that the optimal power and subchannel allocation are functions of the instantaneous channel gain coefficients. Since we assume the channel gain coefficients remain unchanged during each transmission block, the allocation $P_s^{(k,l)}(h_{sr}^{(k,l)}, h_{rd}^{(k,l)}, h_{sd}^{(k,l)})$, $P_r^{(k,l)}(h_{sr}^{(k,l)}, h_{rd}^{(k,l)}, h_{sd}^{(k,l)})$, $\rho^{(k,l)}(h_{sr}^{(k,l)}, h_{rd}^{(k,l)}, h_{sd}^{(k,l)})$ are simplified as $P_s^{(k,l)}$,$P_r^{(k,l)}$ and $\rho^{(k,l)}$ in the formulation.

Problem (6) can be categorized as a mixed integer nonlinear problem (MINLP) which is generally difficult to solve. However, considering the unique feature of the system that orthogonality exists among subcarriers and users, we propose a two-layer resource allocation scheme to solve (6). We first focus on the cooperative communication on a single subcarrier and derive the power dividing scheme between $P_s^{(k,l)}$ and $P_r^{(k,l)}$ to achieve the maximum rate. With the optimal dividing scheme on each single subcarrier, the problem can be transformed to the traditional multi-user OFDM resource allocation problem. Then, we further consider the throughput maximization problem and minimum user rate maximization problem by proposing joint power and subchannel allocation schemes.

III. Power Division for a Single-User Relay Channel

In this section, the first layer of the optimization problem is considered. Since the network can be viewed as the single-user relay channel on each subcarrier, our objective is to maximize the cooperation and the achievable rate for each subcarrier through power dividing between the eNB and the RS. Without loss of generality, suppose the l^{th} subcarrier has been assigned to user k, $\rho^{(k,l)} = 1$. The problem can be written as:

$$\begin{aligned} \max_{P_s^{(k,l)}, P_r^{(k,l)}} \quad & R^{(k,l)} \\ \text{subject to } \quad & P_s^{(k,l)} + P_r^{(k,l)} \le P^{(k,l)} \\ & P_s^{(k,l)} \ge 0, \\ & P_r^{(k,l)} \ge 0. \end{aligned} \tag{7}$$

where $P^{(k,l)}$ is the total transmission power assigned to this subcarrier.

Taking a closer look at (4) where $R^{(k,l)}$ is the minimum of two rates, the first rate is the eNB-RS channel capacity (relay decoding rate) and the second rate is the capacity of the multiple access channel (MAC) from eNB and RS to the UE (destination decoding rate).

Since the highest achievable rate is obtained when the relay decoding rate equals the destination decoding rate, the optimal power division scheme tries to balance these two rates by jointly designing $P_s^{(k,l)}$, $P_r^{(k,l)}$ and $\beta^{(k,l)}$. Depending on which rate is the bottleneck, there are two power dividing strategies that the source node can select [30]:

- If the destination decoding rate is the bottleneck, the source node can reduce $\beta^{(k,l)}$ until the relay decoding rate equals the destination decoding rate.
- If the relay decoding rate is the bottleneck, the source node will set $\beta^{(k,l)} = 1$.

Note that when $\beta^{(k,l)} = 1$, the source node and the relay node will transmit independent codes. Therefore, the second case is also known as the "asynchronous case" while the first case is called the "synchronous case". The asynchronous case is more empirical to implement due to the reduction of coding complexity.

In the rest of the section, we will allocate $P_s^{(k,l)}$, $P_r^{(k,l)}$ and $\beta^{(k,l)}$ for both cases. However, with the optimal power allocation, whether synchronous case or asynchronous case achieves higher rate depends on the specific channel state information.

A. Synchronous Case

In this case, the destination decoding rate is the bottleneck, and $\beta^{(k,l)} \neq 1$. Following the same argument in [30], we first divide the power between $\bar{\beta}^{(k,l)} P_s^{(k,l)}$ and $P_r^{(k,l)}$ when the total power for the MAC channel is fixed. Then the relay decoding rate and the destination decoding rate are balanced when the total power is fixed. The details of the derivation can be found in Appendix B and the result is shown as follows. When $|h_{sr}^{(k,l)}| \ge |h_{sd}^{(k,l)}|$, the optimal power dividing scheme is given by

$$\begin{cases} \beta^{(k,l)} P_s^{(k,l)} = \frac{|h_{sd}^{(k,l)}|^2 + |h_{rd}^{(k,l)}|^2}{|h_{sr}^{(k,l)}|^2 + |h_{rd}^{(k,l)}|^2} P^{(k,l)}, \\ \bar{\beta}^{(k,l)} P_s^{(k,l)} = \frac{|h_{sd}^{(k,l)}|^2 (|h_{sr}^{(k,l)}|^2 - |h_{sd}^{(k,l)}|^2)}{(|h_{sd}^{(k,l)}|^2 + |h_{rd}^{(k,l)}|^2)(|h_{sr}^{(k,l)}|^2 + |h_{rd}^{(k,l)}|^2)} P^{(k,l)}, \\ P_r^{(k,l)} = \frac{|h_{rd}^{(k,l)}|^2 (|h_{sr}^{(k,l)}|^2 - |h_{sd}^{(k,l)}|^2)}{(|h_{sd}^{(k,l)}|^2 + |h_{rd}^{(k,l)}|^2)(|h_{sr}^{(k,l)}|^2 + |h_{rd}^{(k,l)}|^2)} P^{(k,l)}. \end{cases} \tag{8}$$

With the optimal power dividing scheme, the largest achievable rate obtained in the synchronous case is given by

$$R^{(k,l)} = \frac{1}{n} \log \left(1 + \frac{|h_{sr}^{(k,l)}|^2 (|h_{sd}^{(k,l)}|^2 + |h_{rd}^{(k,l)}|^2)}{|h_{sr}^{(k,l)}|^2 + |h_{rd}^{(k,l)}|^2} \cdot \frac{P^{(k,l)}}{B/n} \right). \tag{9}$$

Note that if $|h_{sr}^{(k,l)}| < |h_{sd}^{(k,l)}|$, the eNB-UE link has a better channel condition than the eNB-RS link. In this case, any direct eNB-UE transmission is more reliable than the cooperative transmission. When the relay obtains the channel state information, it first compares $|h_{sr}^{(k,l)}|$ with the channel condition of the direct transmission link to decide whether decode-and-forward needs to be performed to avoid any waste of the resources. In this scenario, the highest end user achievable rate is also the channel capacity of the direct eNB-UE transmission link:

$$R^{(k,l)} = \frac{1}{n} \log \left(1 + \frac{|h_{sd}^{(k,l)}|^2 P_s^{(k,l)}}{B/n} \right). \tag{10}$$

B. Asynchronous Case

In this case, the source and the relay employ independent codes, so $\beta^{(k,l)} = 1$. By the same argument, the maximum achievable rate is obtained when the relay decoding rate equals the destination decoding rate.

When $|h_{sr}^{(k,l)}| \ge |h_{sd}^{(k,l)}|$, the following power dividing scheme is optimal:

$$\begin{cases} P_s^{(k,l)} = \frac{|h_{rd}^{(k,l)}|^2}{|h_{sr}^{(k,l)}|^2 - |h_{sd}^{(k,l)}|^2 + |h_{rd}^{(k,l)}|^2} P^{(k,l)}, \\ P_r^{(k,l)} = \frac{|h_{sr}^{(k,l)}|^2 - |h_{sd}^{(k,l)}|^2}{|h_{sr}^{(k,l)}|^2 - |h_{sd}^{(k,l)}|^2 + |h_{rd}^{(k,l)}|^2} P^{(k,l)}. \end{cases} \tag{11}$$

The highest achievable rate in the asynchronous case is given by

$$R^{(k,l)} = \frac{1}{n}\log\left(1+\frac{|h_{sr}^{(k,l)}|^2|h_{rd}^{(k,l)}|^2}{|h_{sr}^{(k,l)}|^2-|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2}\cdot\frac{P^{(k,l)}}{B/n}\right). \tag{12}$$

IV. Joint Power and Subcarrier Allocation for Throughput Maximization

With the optimal power dividing scheme between eNB and RS for each subcarrier, the optimization problem (6) can be simplified to the problem of jointly allocating subcarriers and power among all users in the network to attain the maximum overall throughput.

The simplified problem can be written as:

$$\begin{aligned} \max_{P^{(k,l)},\rho^{(k,l)}} & \sum_{k\in\mathcal{M},l\in\mathcal{N}} R^{(k,l)} \\ \text{subject to } & \sum_{k,l} P^{(k,l)} \le P_{total} \\ & P^{(k,l)} \ge 0, \text{for all } k,\ l \\ & \rho^{(k,l)} \in \{0,1\}, \text{for all } k,\ l \\ & \sum_{k\in\mathcal{M}} \rho^{(k,l)} \le 1, \text{for all } l \end{aligned} \tag{13}$$

Define the unit power signal-to-noise ratio (SNR) for the single-user subchannel as:

$$H^{(k,l)} = \begin{cases} \frac{|h_{sr}^{(k,l)}|^2(|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2)}{(|h_{sr}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2)B/n}, & \text{the synchronous case,} \\ \frac{|h_{sr}^{(k,l)}|^2|h_{rd}^{(k,l)}|^2}{(|h_{sr}^{(k,l)}|^2-|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2)B/n}, & \text{the asynchronous case,} \\ \frac{|h_{sd}^{(k,l)}|^2}{B/n}, & \text{the non-cooperative case.} \end{cases} \tag{14}$$

Then the highest achievable rate $R^{(k,l)}$ has a unified expression similar to the direct transmission capacity, which is given by

$$R^{(k,l)} = \frac{\rho^{(k,l)}}{n}\log\left(1+H^{(k,l)}P^{(k,l)}\right). \tag{15}$$

Under the assumption that at most one user can be allocated to each subchannel (actually this assumption has been proved to be optimal in [23]), for any deterministic subchannel allocation, the multi-carrier transmission can be viewed as a Gaussian parallel channel with n independent channels coexisting in the network. For this type of network with a common power consumption constraint, the water-filling power allocation scheme has been proved to be optimal in terms of achieving the maximum overall throughput [31]. The fundamental spirit of water-filling is to allow higher transmission power to be allocated to the channel with a better quality. For the subchannel allocation, it can be derived that assigning each subchannel to the user with the best channel quality is the optimal solution to the problem. The joint subchannel and power allocation scheme in terms of maximizing the overall throughput is concluded in Algorithm 1.

$(\cdot)^+$ is defined as:

$$(x)^+ = \begin{cases} x, & x \ge 0, \\ 0, & x < 0. \end{cases} \tag{16}$$

Algorithm 1 Optimal resource allocation algorithm for maximization of the overall throughput

1: For $l = 1, \ldots, n$, find a $k(l)$ satisfying $H^{(k(l),l)} \ge H^{(k',l)}$ for all $k' \in \mathcal{M}$. Assign subchannel l to user $k(l)$, i.e., set $\rho^{(k(l),l)} = 1$.
2: Allocate $P^{(k(l),l)} = \left(\lambda - \frac{1}{H^{(k(l),l)}}\right)^+$ as the transmitting power for user $k(l)$ in subchannel l. λ is the water-filling level that is chosen to satisfy the total power constraint $\sum_{k(l),l} P^{(k(l),l)} = P_{total}$.

[23] has proposed a similar algorithm where it has proved that assigning exclusively one user with the best channel quality to every subchannel is optimal by induction. Thus the proof of the optimality of Algorithm 1 is omitted here.

V. Joint Power and Subchannel Allocation with Fairness Concern

Although Algorithm 1 can maximize the overall throughput, there is no minimum achievable rate guaranteed for each user. Especially, if there exists one user whose channel quality outperforms all other users on every subchannel, all the available subchannels and power would be assigned to this user. In this case, other users' data rate cannot be guaranteed.

As ensuring the mobile users' QoS requirements, e.g., a minimum transmission rate requirement, is one of the major design objectives in the cellular network, the subchannel and power allocation schemes should be adapted accordingly to meet these requirements. This issue has been addressed in some existing works. For example, [24] investigates the maximization of absolute fairness and [25] takes into account each individual user's QoS requirement. [26], [28] employ the network utility maximization (NUM) framework to trade off throughput and fairness by maximizing a utility function which is a concave and increasing function of data rates.

In this paper, we investigate both fairness and efficiency issue to come up with an improved resource allocation scheme to balance the transmission rates among different users in the network while maximizing the overall throughput. Different from the single-objective optimization in most of the literatures, we propose a novel lexicographic optimization problem where the overall throughput is also optimized after maximum fairness has been achieved. The two objectives have a hierarchical structure: the fairness objective has the highest priority to be optimized and among the feasible solutions, the overall throughput is further maximized.

We adopt the fairness definition in [24] where the maximum fairness is achieved when all users have the same data rate. The maximum fairness can be obtained by solving the *max-min* problem to maximize the worst user's achievable rate. It is worth mentioning that the popular proportional fairness [25] can also be included in the *max-min* problem if a set of factors representing the weight of each user's rate is employed.

The problem can be formulated as a lexicographic optimization problem:

$$\begin{aligned} \text{lex} \max_{P^{(k,l)},\rho^{(k,l)}} & \left(\min_k R_k, \sum_{k\in\mathcal{M}} R_k \right) \\ \text{subject to } & \sum_{k,l} P^{(k,l)} \le P_{total} \\ & P^{(k,l)} \ge 0, \text{for all } k,\ l \\ & \rho^{(k,l)} \in \{0,1\}, \text{for all } k,\ l \\ & \sum_{k\in\mathcal{M}} \rho^{(k,l)} \le 1, \text{for all } l \end{aligned} \tag{17}$$

where R_k is the achievable rate for user k and

$$R_k = \sum_{l\in\mathcal{N}} \frac{\rho^{(k,l)}}{n} \log\left(1 + H^{(k,l)} P^{(k,l)}\right). \tag{18}$$

To solve this problem, the subchannel and power allocation should be jointly designed. Whether a subchannel is assigned to a user or not depends on the user's rate, which is controlled by the power allocated. On the other hand, allocating power among subchannels depends on the channel condition of each subchannel, which is decided by the subchannel allocation. In this paper, we first try to find the optimal solution. Then as the eNB needs to allocate resources for the rapidly changing wireless channels, low-complexity suboptimal algorithms are preferred, especially for cost-effective and delay-sensitive implementations. Therefore, an efficient suboptimal algorithm is proposed as well.

A. Optimal Power and Subchannel Allocation

To solve this lexicographic maximization problem, a sufficient condition that the optimal joint power and subchannel allocation satisfies is described in the following theorem.

Theorem 1. *Let S_k denote the set of subchannels allocated to user k. $S_k^+ \subseteq S_k$ is the set containing only the subchannels associated with nonzero transmission powers, i.e., $S_k^+ = \{l : \rho^{(k,l)} = 1$ and $P^{(k,l)} > 0\}$. The cardinality of S_k^+ is $|S_k^+|$. λ is the water-filling level. A subchannel and power allocation is optimal for the lexicographic optimization problem (17) if it satisfies the following condition.*

$$H^{(S_1^+)} \lambda^{|S_1^+|} = H^{(S_2^+)} \lambda^{|S_2^+|} = \cdots = H^{(S_m^+)} \lambda^{|S_m^+|} \tag{19}$$

where

$$H^{(S_k^+)} = \prod_{l\in S_k^+} H^{(k,l)} \tag{20}$$

Proof: Since water-filling is always optimal for power allocation in spite of any subchannel assignment, in order to maximize the overall throughput, the power allocation should still follow a water-filling manner. Suppose that the water-filling level is λ which is a function of the channel gains on all subchannels. The achievable rate for user k can be derived as follows.

$$\begin{aligned} R_k &= \sum_{l\in S_k^+} \frac{1}{n} \log\left(1 + H^{(k,l)} P^{(k,l)}\right) \\ &= \sum_{l\in S_k^+} \frac{1}{n} \log\left(1 + H^{(k,l)} \left(\lambda - \frac{1}{H^{(k,l)}}\right)\right) \\ &= \sum_{l\in S_k^+} \frac{1}{n} \log\left(\lambda H^{(k,l)}\right) \\ &= \frac{1}{n} \log\left(H^{(S_k^+)} \lambda^{|S_k^+|}\right). \end{aligned} \tag{21}$$

Therefore, the ideal case is that $H^{(S_k^+)} \lambda^{|S_k^+|}$ is balanced to attain the same value for all users $k \in \mathcal{M}$. ■

However, since λ is a function of all channel gains which will be determined by the subchannel assignment, it is very difficult to jointly allocate subchannels and power due to the computational complexity. Instead, most suboptimal solutions to the similar problem [24], [25] separate the joint allocation into two steps. Firstly, subchannels are allocated while assuming the transmission power is evenly distributed on all subchannels. Then, based on the subchannel allocation, power distribution is optimized according to water-filling. In our work, we propose a joint suboptimal subchannel allocation algorithm where the subchannels are allocated based on the water-filling power distribution.

B. Suboptimal Power and Subchannel Allocation

The suboptimal resource allocation scheme contains two steps as well. We first allocate subchannels among all users while assuming powers are distributed in a water-filling manner with water-filling level λ. Then in the second step, λ can be decided based the subchannel allocation in the first step.

1) Suboptimal Subchannel Allocation Algorithm: The first step only deals with the subchannel allocation scheme. Before proposing the subchannel allocation algorithm, we first introduce two new sets. Suppose $\mathcal{N}_s$ is the remaining subchannel set after allocation. Initially, $\mathcal{N}_s = \mathcal{N}$. Define $\mathcal{M}_s$ as an ordered set containing all users $1, 2, \ldots, m$ where for any $i, j \in \mathcal{M}_s$, $i < j$ if and only if $H^{(S_i)} \le H^{(S_j)}$. The suboptimal subchannel allocation process is summarized in Algorithm 2.

Algorithm 2 Suboptimal subchannel allocation algorithm

1: Initialize: $S_k = \emptyset$, $H^{(S_k)} = 1$ for all users $k = 1, 2, \ldots, m$. Let $\mathcal{N}_s = \mathcal{N}$ and $\mathcal{M}_s = \mathcal{M}$.
2: While $\mathcal{N}_s \ne \emptyset$, for $k = \mathcal{M}_s(1), \ldots, \mathcal{M}_s(m)$, find a subchannel $l(k) \in \mathcal{N}_s$ satisfying $H^{(k,l(k))} \ge H^{(k,l')}$ for all $l' \in \mathcal{N}_s$. Assign $l(k)$ to user k. Update $\mathcal{N}_s = \mathcal{N}_s - \{l(k)\}$ and $H^{(S_k)} = H^{(S_k)} \cdot H^{(k,l(k))}$.
3: Reorder the user set $\mathcal{M}_s$. Then go back to 2.

Compared with most suboptimal subchannel allocation algorithms assuming equal power distribution, Algorithm 2 reduces the complexity from $\mathcal{O}(mn)$ to $\mathcal{O}(n)$.

2) Optimal Power Allocation: After allocating the subchannels according to Algorithm 2, suppose the channel gain coefficient on each subchannel is $H^{(k,l)}$. According to the

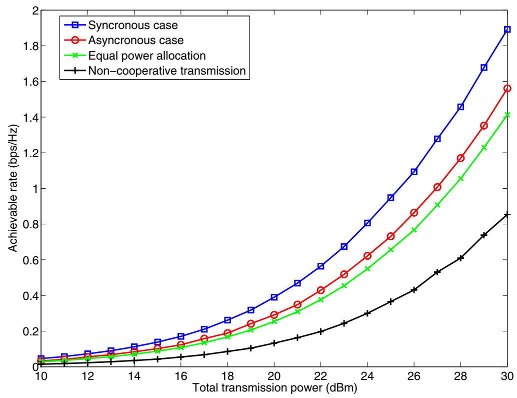

Fig. 2. Comparison of single-user achievable rates in Rayleigh fading environments.

optimal water-filling power allocation, the power allocated to each subchannel is

$$P^{(k,l)} = \left(\lambda - \frac{1}{H^{(k,l)}}\right)^{+} \tag{22}$$

and λ is chosen to satisfy the total power constraint $\sum P^{(k,l)} = P_{total}$.

Remark 1. *It is worth mentioning that although our proposed subchannel and power allocation schemes in this paper are based on a total power constraint, they are also applicable in practical networks where nodes have their individual power constraints. The only difference is that we need to allocate power for both eNB and RS and their water-filling levels are determined according to their respective power constraints.*

VI. Numerical Results

To evaluate the performance of our subchannel and power allocation schemes derived in the previous sections, we present our simulation results in three parts. The first part depicts the achievable rates obtained by the power dividing schemes in the single-user relay channel. The second part demonstrates the superiority of the optimal joint subchannel and power allocation scheme for maximizing the overall throughput in a cellular network. The suboptimal allocation scheme to maximize the minimum rate of all users is evaluated in the third part. We assume that all wireless channels suffer from independent Rayleigh fading.

A. Performance of Power Dividing Scheme for Single-User Relay Channel

The single-user achievable rates are attained through the optimal power dividing schemes (8) and (11) for the synchronous and asynchronous case, respectively. The performance is compared with an equal power dividing scheme where the eNB and the RS transmit with the same power. Besides, in order to illustrate the performance enhancement of introducing the relay station, we always set the capacity of non-cooperative transmission as a baseline for comparison as shown in Fig. 2.

Fig. 2 shows that cooperative transmission with the help of a relay outperforms non-cooperative transmission in terms of achieving higher data rate, which demonstrates the benefits of deploying relay stations in the cellular network. In terms of the comparison of the two cases, higher achievable rate can be obtained in synchronous case than in asynchronous case in most of the network environment since the former one fully exploits the cooperation between the eNB and RS in the coding process. However, synchronous case leads to high coding complexity that will increase the implementation cost significantly. Compared with the equal power dividing scheme, our optimal power dividing schemes achieve higher rate in both asynchronous case and synchronous case.

TABLE I
SIMULATION PARAMETERS.

Parameter	Value
Cellular layout	Hexagonal grid, 6 sectors per cell
Relay station layout	1 relay station per sector
Relay protocol	Decode-and-Forward
Transmission bandwidth	20MHz
Subcarrier separation	15kHz
# of subcarriers in a resource block	12
Carrier frequency	2.7 GHz for downlink
Channel model	Frequency selective Rayleigh fading
UE distribution	Uniformly random
Number of UEs per sector	10
Total transmission power	80W

B. Performance of Optimal Resource Allocation for Throughput Maximization

To illustrate the superiority of our resource allocation scheme in terms of the cell overall throughput improvement, we compare the throughput achieved by the optimal resource allocation in Algorithm 1 with the following two resource allocation schemes:

1) The user with the best channel quality is picked in each subchannel and power is distributed on the subchannels evenly;
2) Both the subchannels and power are allocated equally among all users without any consideration of the channel conditions of each single user.

Some simulation parameters are summarized in Table I:

Fig. 3 illustrates the overall throughput comparison of these three schemes for synchronous relay station case and Fig. 4 depicts the same comparison when the relay station operates in the asynchronous mode. From these figures, it can be seen that the optimal resource allocation scheme achieves the highest throughput in both the synchronous case and the asynchronous case. Furthermore, scheme 1 outperforms scheme 2 since scheme 1 takes the variance of channel conditions into consideration as well.

Notice that in both figures, the throughput achieved by the optimal resource allocation scheme and that achieved by scheme 1 increase with the number of users. However, the throughput achieved by scheme 2 remains almost unchanged when the number of users grows. This indicates that our allocation scheme and scheme 1 can better exploit the user diversity. As the number of users goes up, there is a higher chance that a user with better channel condition can be picked.

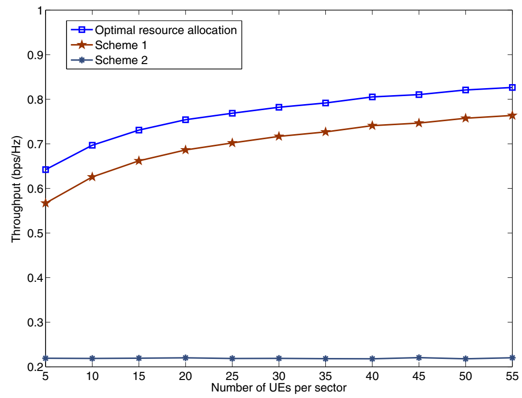

Fig. 3. Comparison of overall throughput when RS operates in synchronous mode.

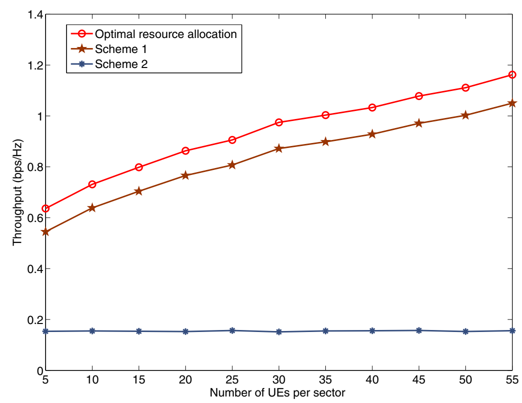

Fig. 4. Comparison of overall throughput when RS operates in asynchronous mode.

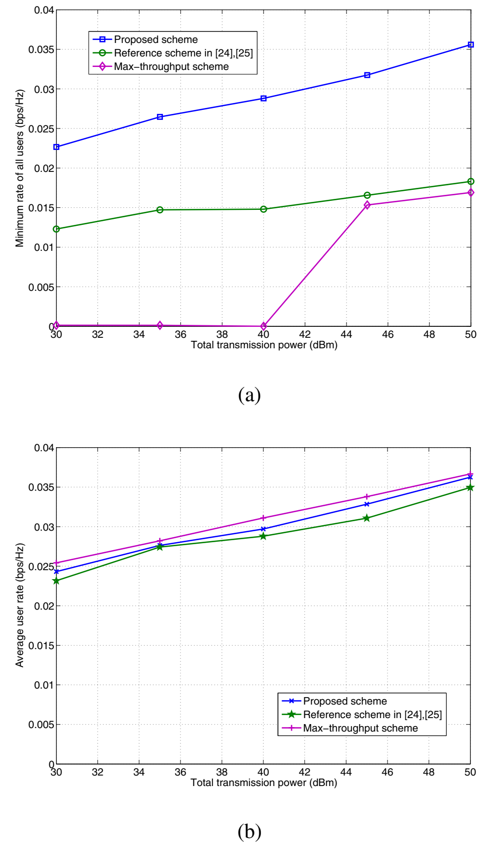

Fig. 5. Comparison of minimum rate of all users and average user rate.

C. Performance of Suboptimal Resource Allocation with Fairness Concern

In order to evaluate the performance of our proposed suboptimal subchannel and power allocation algorithm in improving fairness as well as the overall throughput, we compare both the minimum rate of all users and the average user rate achieved by our suboptimal algorithm with other suboptimal schemes. The total transmission power ranges from 30 to 50 dBm and assume that there are 30 users randomly distributed in each sector. The rest of the network parameters are unchanged. We only take the synchronous relay case as an example and similar results can be obtained in the asynchronous case.

We compare our proposed resource allocation scheme with the scheme which assumes equal power distribution while allocating subchannels [24], [25] in Fig. 5. Note that the optimization problem in [25] aims at maximizing the overall throughput while each user's rate satisfies a proportional constraint. If maximum fairness is desired which can be attained by setting all the proportion coefficients equal to 1, the optimization in [25] is identical to (17).

In the subchannel allocation step, the reference scheme assigns subchannels by assuming that the transmission power is distributed equally; however, our proposed scheme is based on the optimal water-filling power allocation. In the power allocation step, all schemes employ the water-filling method. As a baseline for comparison, the optimal joint power and subchannel allocation scheme for throughput maximization is also depicted.

Fig. 5(a) shows the minimum rate of all users obtained by the three schemes. It can be seen that our proposed scheme can increase the minimum rate of all users drastically. Employing the optimal throughput maximization scheme, it is possible that some users could not have any chance to transmit at all.

In terms of the average user rate which is shown in Fig. 5(b), the optimal throughput maximization scheme can always outperform other schemes. All the three schemes achieve similar rates since in each scheme, the subchannel allocation tends to pick a user with good channel quality.

VII. Conclusion

In this paper, we investigate the downlink resource allocation to improve the transmission efficiency in an LTE-Advanced cellular system where relays are installed. Instead of simply amplifying and forwarding the received signal, relays are expected to have more coding capability to achieve higher data rates. Thus we focus on the In-Band Type II decode-and-forward relays where the source and the relay transmit in the same frequency band (channel). An optimal subcarrier and power allocation scheme is proposed to maximize the overall throughput. Then we consider the fairness issue and come up with a two-step suboptimal algorithm that jointly allocate subcarrier and power to maximize the minimum rate of all users

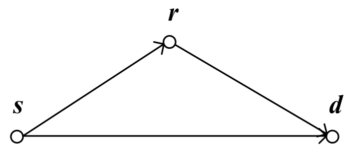

Fig. 6. The relay channel.

while improving the system throughput. Numerical results show that our proposed algorithm can improve the minimum rate of all users by about 80% compared with the reference scheme. Furthermore, in terms of the overall throughput, our scheme can achieve about 90% of the maximum throughput while the reference obtains around 84%.

For the possible future work, we will consider the joint uplink and downlink resource allocation where the relay station is capable of two-way communications. Also, investigation of the cooperation among adjacent relays for DL coordinated multipoint transmission (CoMP) might also be a future topic.

APPENDIX A
DERIVATION OF ACHIEVABLE RATE FOR A SINGLE-USER RELAY CHANNEL IN AWGN ENVIRONMENT

We consider a three node relay network where the source node ***s*** intends to send information to the destination ***d*** via the relay node ***r*** as shown in Fig. 6.

The information theoretic achievable rate for the decode-and-forward relay channel is

$$R = \max_{p(x_s,x_r)} \min\{I(X_s;Y_r|X_r), I(X_s,X_r;Y_d)\}, \qquad (23)$$

where x_s, y_d, y_r and x_r denote the input to the channel, the output of the channel, the observation by the relay and the input symbol chosen by the relay, respectively. This rate is achieved by the joint encoding at the source and relay. $I(X_s;Y_r|X_r)$ is the rate that the relay is able to decode (relay decoding rate), and $I(X_s,X_r;Y_d)$ is the rate that the destination can successfully decode the message (destination decoding rate).

In the AWGN environment, assume that each transmission link is corrupted by a multiplicative fading gain coefficient in addition to an additive white Gaussian noise. The channel gain coefficients of the source-relay channel, the relay-destination channel and the source-destination channel are denoted by h_{sr}, h_{rd} and h_{sd} respectively. h_{sr}, h_{rd} and h_{sd} are independent complex random variables and the fading processes $h_{sr}(t)$, $h_{rd}(t)$ and $h_{sd}(t)$ are stationary and ergodic over time, where t is the time index. Assume the channel gain coefficients remain unchanged during each transmission block. Then the received signals at the relay and at the destination are given by,

$$\begin{aligned} y_r(t) =& h_{sr}x_s(t) + z_r(t), \text{ and} \\ y_d(t) =& h_{sd}x_s(t) + h_{rd}x_r(t) + z_d(t), \end{aligned} \qquad (24)$$

respectively. $z_r(t)$ and $z_d(t)$ are independent zero-mean Gaussian noises received at the relay node ***r*** and at the destination node ***d*** with variances both normalized to 1.

The maximum rate is achieved by the joint superposition coding which includes a consecutive transmission of B blocks. At each transmission block, a message w_b, $b = 1,\ldots,B$ is to be sent into the relay channel. The joint superposition encoding process for the relay channel at transmission block b consists of the generation of two codes: one code $\underline{u}$ containing the current block's message w_b and the other code $\underline{x_r}$ representing the previous block's message w_{b-1}. During the transmission block b, the relay node ***r*** only sends $\underline{x_r}$ containing message w_{b-1} with its maximum transmission power P_r. For the source node ***s***, it divides the total transmission power P_s into two parts, βP_s and $\bar{\beta}P_s$ with different purposes, where $\bar{\beta} = 1-\beta$. βP_s is used for transmitting $\underline{u}$ and $\bar{\beta}P_s$ is devoted to corporate with the relay to transmit $\underline{x_r}$ to the destination. The code $\underline{x_s}$ sent by the source node is the superposition of two codes: $\underline{u}$ and $\underline{x_r}$. Employing the joint superposition encoding and decoding, the following rate can be achieved for the single-user relay channel if all codes are generated according to Gaussian distribution:

$$\begin{aligned} R =& \max_{p(x_s,x_r)} \min\left\{I\left(X_s;Y_r|X_r\right), I\left(X_s,X_r;Y_d\right)\right\} \\ =& \max_{0\le\beta\le1} \min\left\{\frac{1}{2}\log\left(1+|h_{sr}|^2\beta P_s\right),\right. \\ & \left.\frac{1}{2}\log\left(1+|h_{sd}|^2\beta P_s + (\sqrt{|h_{sd}|^2\bar{\beta}P_s} + \sqrt{|h_{rd}|^2P_r})^2\right)\right\}. \end{aligned}$$

APPENDIX B
OPTIMALITY PROOF OF PROPOSED POWER DIVIDING SCHEME FOR SINGLE-USER RELAY CHANNEL

In the synchronous relay case, the destination decoding rate is the bottleneck, and $\beta^{(k,l)} \neq 1$. Denote $P_{s_1}^{(k,l)} = \beta^{(k,l)}P_s^{(k,l)}$ and $P_{s_2}^{(k,l)} = \bar{\beta}^{(k,l)}P_s^{(k,l)}$ as the two components of $P_s^{(k,l)}$. Since the signal received at the destination contains a combined strength, we first maximize the destination decoding rate with fixed $P_0^{(k,l)}$, $P_0^{(k,l)} = P_{s_2}^{(k,l)} + P_r^{(k,l)}$. Then, $P_{s_1}^{(k,l)}$ and $P_0^{(k,l)}$ are allocated under the total power constraint to achieve maximum rate. The destination decoding rate is given by

$$\frac{1}{n}\log\left(1+\frac{|h_{sd}^{(k,l)}|^2\beta^{(k,l)}P_s^{(k,l)}}{B/n}+\frac{(\sqrt{|h_{sd}^{(k,l)}|^2\bar{\beta}^{(k,l)}P_s^{(k,l)}}+\sqrt{|h_{rd}^{(k,l)}|^2P_r^{(k,l)}})^2}{B/n}\right), \qquad (25)$$

which is equivalent to

$$\frac{1}{n}\log\left(1+\frac{|h_{sd}^{(k,l)}|^2P_{s_1}^{(k,l)}}{B/n}+\frac{(\sqrt{|h_{sd}^{(k,l)}|^2P_{s_2}^{(k,l)}}+\sqrt{|h_{rd}^{(k,l)}|^2P_r^{(k,l)}})^2}{B/n}\right). \qquad (26)$$

With fixed $P_0^{(k,l)}$, it can be derived that the optimum power allocation between $P_{s_2}^{(k,l)}$ and $P_r^{(k,l)}$ is given by

$$\begin{aligned} P_{s_2}^{(k,l)} =& \frac{|h_{sd}^{(k,l)}|^2}{|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2}P_0^{(k,l)}, \\ P_r^{(k,l)} =& \frac{|h_{rd}^{(k,l)}|^2}{|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2}P_0^{(k,l)}, \end{aligned} \qquad (27)$$

and the destination decoding rate becomes:

$$\frac{1}{n}\log\left(1+\frac{|h_{sd}^{(k,l)}|^2 P_{s_1}^{(k,l)}+\left(|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2\right)P_0^{(k,l)}}{B/n}\right). \tag{28}$$

For the optimization problem (7), since the optimum of the achievable rate $R^{(k,l)}$ is attained when $P_{s_1}^{(k,l)}+P_0^{(k,l)}=P^{(k,l)}$ and when the relay decoding rate equals the destination decoding rate, i.e.,

$$\frac{1}{n}\log\left(1+\frac{|h_{sr}^{(k,l)}|^2 P_{s_1}^{(k,l)}}{B/n}\right) \tag{29}$$

$$=\frac{1}{n}\log\left(1+\frac{|h_{sd}^{(k,l)}|^2 P_{s_1}^{(k,l)}+(|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2)P_0^{(k,l)}}{B/n}\right), \tag{30}$$

the optimization problem can be rewritten as:

$$\max_{P_{s_1}^{(k,l)},P_0^{(k,l)}} \frac{1}{n}\log\left(1+\frac{|h_{sr}^{(k,l)}|^2 P_{s_1}^{(k,l)}}{B/n}\right)$$

$$\text{subject to } P_{s_1}^{(k,l)}+P_0^{(k,l)}=P^{(k,l)}$$

$$|h_{sr}^{(k,l)}|^2 P_{s_1}^{(k,l)} = |h_{sd}^{(k,l)}|^2 P_{s_1}^{(k,l)}+(|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2)P_0^{(k,l)} \tag{31}$$

$$P_{s_1}^{(k,l)}\geq 0$$

$$P_0^{(k,l)}\geq 0. \tag{32}$$

Actually, the two constraints lead to only one feasible solution when $|h_{sr}^{(k,l)}|\geq|h_{sd}^{(k,l)}|$:

$$P_{s_1}^{(k,l)}=\frac{|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2}{|h_{sr}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2}P^{(k,l)},$$

$$P_0^{(k,l)}=\frac{|h_{sr}^{(k,l)}|^2-|h_{sd}^{(k,l)}|^2}{|h_{sr}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2}P^{(k,l)}, \tag{33}$$

and the highest achievable rate is given by

$$R^{(k,l)}=\frac{1}{n}\log\left(1+\frac{|h_{sr}^{(k,l)}|^2\left(|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2\right)}{|h_{sr}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2}\cdot\frac{P^{(k,l)}}{B/n}\right). \tag{34}$$

In the asynchronous case, the source and the relay employ independent codes, so that $\beta = 1$. By the same argument, the maximum achievable rate can be obtained when the relay decoding rate equals the destination decoding rate. The optimization problem in this scenario can be formulated as:

$$\max_{P_s^{(k,l)},P_r^{(k,l)}} \frac{1}{n}\log\left(1+\frac{|h_{sr}^{(k,l)}|^2 P_s^{(k,l)}}{B/n}\right)$$

$$\text{subject to } P_s^{(k,l)}+P_r^{(k,l)}=P^{(k,l)}$$

$$|h_{sr}^{(k,l)}|^2 P_s^{(k,l)}=|h_{sd}^{(k,l)}|^2 P_s^{(k,l)}+|h_{rd}^{(k,l)}|^2 P_r^{(k,l)}$$

$$P_s^{(k,l)}\geq 0$$

$$P_r^{(k,l)}\geq 0. \tag{35}$$

When $|h_{sr}^{(k,l)}|\geq|h_{sd}^{(k,l)}|$, the feasible region gives rise to the following optimal power dividing scheme:

$$P_s^{(k,l)}=\frac{|h_{rd}^{(k,l)}|^2}{|h_{sr}^{(k,l)}|^2-|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2}P^{(k,l)},$$

$$P_r^{(k,l)}=\frac{|h_{sr}^{(k,l)}|^2-|h_{sd}^{(k,l)}|^2}{|h_{sr}^{(k,l)}|^2-|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2}P^{(k,l)}, \tag{36}$$

and the highest achievable rate is given by

$$R^{(k,l)}=\frac{1}{n}\log\left(1+\frac{|h_{sr}^{(k,l)}|^2|h_{rd}^{(k,l)}|^2}{|h_{sr}^{(k,l)}|^2-|h_{sd}^{(k,l)}|^2+|h_{rd}^{(k,l)}|^2}\cdot\frac{P^{(k,l)}}{B/n}\right). \tag{37}$$

If $|h_{sr}^{(k,l)}|<|h_{sd}^{(k,l)}|$, the direct eNB-UE transmission link has a better channel condition than the cooperative link. In this case, direct transmission can obtain higher transmission rate.

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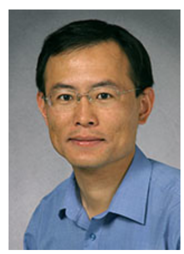

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