Joint Subcarrier-Relay Assignment and Power Allocation for Decode-and-Forward Multi-Relay OFDM Systems

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***Abstract*—Joint power allocation, relay selection, and subcarrier assignment are critical and challenging for achieving full benefits of OFDM based cooperative relay networks. In this paper, we study such a problem in a dual-hop multi-relay OFDM system with an objective of maximizing the spectral efficiency under a total power constraint. The system consists of a pair of source and destination and multiple decode-and-forward relays. We formulate the joint optimization of the three types of resources: power, subcarrier and relay nodes, as a problem of subcarrier-relay assignment and power allocation. We show that it can be decomposed into $2N+1$ sub-problems through dual relaxation, with N being the total number of subcarriers. An optimal algorithm with polynomial complexity is presented. A suboptimal algorithm that decouples the subcarrier-relay assignment and power allocation is also proposed to tradeoff between performance and computational complexity.**

I. Introduction

The technology combination of relay-based network architecture and orthogonal frequency division multiplexing (OFDM) has been considered as a promising architecture choice for future wireless broadband networks [1]–[3]. One of the key problems in relay-based OFDM communication systems is dynamic resource allocation. Compared with traditional single-hop OFDM or OFDMA systems, the resource allocation in relay-based multi-hop OFDM systems is much more challenging. It involves a coordination of power and subcarrier adaptation between different hops [4]–[7]. In addition, when multiple relay candidates are available, relay selection needs to be done. In this paper we consider a dual-hop multi-relay OFDM system, which consists of a single pair of source-destination and multiple relay nodes. The aim is to seek the joint optimization of relay selection, subcarrier assignment and power control. Such joint optimization regarding the three types of resources: power, subcarrier and relay nodes, is crucial in terms of achieving the maximum system performance.

Relay selection has been well investigated in the literature for narrow band wireless multi-relay networks. A common relay selection strategy is to choose the relay with the best equivalent end-to-end channel gain [8]. Extending the similar idea to broadband multi-relay OFDM networks is simple but may not be efficient. This is because selecting one relay based on the equivalent channel gain over the whole OFDM band, i.e. the so-called *symbol-based* relay selection, cannot exploit the frequency diversity among different subcarriers. *Subcarrier-based* relay selection, which selects one best relay for each subcarrier, was then proposed in [9] to exploit both node diversity and frequency diversity. Note, however, the subcarrier-based relay selection in [9] is based on the assumption that the signals received on different subcarriers of the same relay are processed individually, rather than jointly. Such approach is suboptimal in the decode-and-forward (DF) relay protocol, wherein the OFDM symbol in the first hop is decoded as a whole at the relay node and then transmitted over the second hop. This motivates us to consider *subcarrier-set based* relay selection in this paper, which is also referred to as *subcarrier-relay assignment*.

The contribution of this paper is to formulate the joint optimization of subcarrier-relay assignment and power allocation under a total power constraint in the DF multi-relay OFDM system. The total power constraint is motivated by the fact that in some networks, such as sensor networks, where long-term total power consumption is a major concern, restricting the total transmit power is usually a convenient and effective approach to satisfy the long-term power constraint. In the context of wireless spectrum optimization problem for multicarrier systems, dual decomposition is widely adopted [10], [11]. A key result shown in [12] is that as the number of subcarriers increases, the duality gap for the capacity maximization problem vanishes, thus dual decomposition leads to the global optimal resource allocation even if the primal problem is not convex. In this paper, we show that the joint subcarrier-relay assignment and power allocation optimization problem can be decomposed into $2N+1$ sub-problems through dual relaxation, where N is the number of subcarriers. Each sub-problem has a closed-form solution, and the global optimal solution can be achieved by jointly optimizing the subproblems through proper adjustment of dual variables.

The rest of the paper is organized as follows. In Section II we introduce the system model and formulate the joint optimization problem as a mixed integer programming problem. This problem is solved by introducing the time-sharing concept and using dual decomposition in Section III. Then in Section IV, a suboptimal solution is presented to tradeoff complexity with performance. Simulation results are shown in

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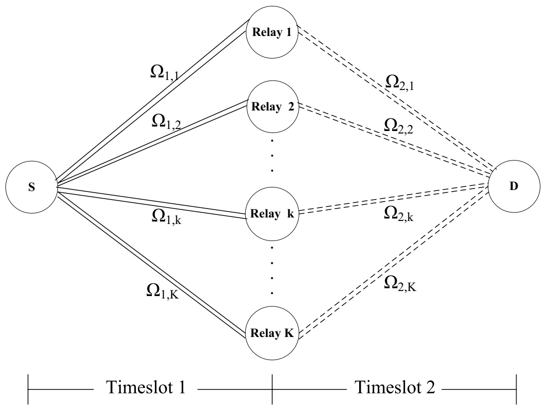

Fig. 1. System model

Section V. Finally we conclude this paper in Section VI.

II. System Model and Problem Formulation

Consider a dual-hop OFDM relay system in Fig. 1, which consists of one source node, one destination node, and a set $\mathcal{K} = \{1, \dots, K\}$ of relay nodes. Assume that perfect channel state information can be obtained in this system. Let $\mathcal{N} = \{1, 2, \cdots N\}$ denote the set of orthogonal subcarriers for each hop. A time-division based half duplex DF protocol is utilized. In the first timeslot, the source transmits signals over all the subcarriers while all relay nodes listen. In the second timeslot, the relay nodes decode the received signal, re-encode it and then transmit it to the destination. Let $\Omega_{i,k}$ denote the subcarrier set assigned to relay k over the i-th hop. To avoid interference cancellation, in each hop a subcarrier is only assigned to one relay. In other words, subcarrier sets assigned to different relays over each hop must be mutually exclusive.

Let $\tilde{\alpha}_{n,k,i}$ denote the channel gain of relay k on subcarrier n over hop i, which is assumed to remain invariant during a frame transmission and be independent for different k and i. The transmission rate $r_{n,k,i}$ (nat/sec/Hz) achieved by relay k on subcarrier n over hop i can be expressed as

$$r_{n,k,i} = \frac{1}{N} \ln \left(1 + \frac{\tilde{\alpha}_{n,k,i} p_{n,k,i}}{N_0 \Gamma B / N} \right),$$

where N_0 denotes the noise spectrum density, Γ denotes the SNR gap, B denotes the system bandwidth, and $p_{n,k,i}$ is the power allocated on subcarrier n when it is assigned to relay k over hop i. For notation brevity, we redefine $\alpha_{n,k,i} := \tilde{\alpha}_{n,k,i} N / N_0 \Gamma B$ as the normalized channel gain in the remainder of this paper. The end-to-end transmission rate achieved by relay k is the minimum of the rates achieved over the two hops,

$$R_k = \frac{1}{2} \min \left\{ \sum_{n \in \Omega_{1,k}} r_{n,k,1}, \sum_{n \in \Omega_{2,k}} r_{n,k,2} \right\} .$$

Our objective is to maximize the end-to-end transmission rate under a total network power constraint, given by P_T. This optimization problem can be formulated as follow

$$\text{P1}: \quad \max_{\{\boldsymbol{p}, \boldsymbol{\Omega}\}} \sum_{k=1}^{K} \frac{1}{2} \min \left\{ \sum_{n \in \Omega_{1,k}} r_{n,k,1}, \sum_{n \in \Omega_{2,k}} r_{n,k,2} \right\} \quad (1)$$

$$\text{s.t.} \quad \sum_{n=1}^{N} \sum_{i=1}^{2} \sum_{k \in \Omega_{i,k}} p_{n,k,i} \le P_T, \quad (2)$$

$$\Omega_{i,k} \cap \Omega_{i,k'} = \emptyset, \qquad \forall i, \forall k \neq k', \quad (3)$$

$$\cup_{k=1}^{K} \Omega_{i,k} = \mathcal{N}, \qquad \forall i, \quad (4)$$

$$p_{n,k,i} \ge 0, \qquad \forall k, i \ , \quad (5)$$

where $\boldsymbol{p} = \{p_{n,k,i}\}$ and $\boldsymbol{\Omega} = \{\Omega_{i,k}\}$ denote the sets of optimization variables. This is a mixed integer nonlinear programming problem. Since each subcarrier can be assigned to any of the K relays for both hops, there are K^{2N} possible subcarrier-relay assignments. The complexity is thus prohibitive for large N and K. In the next section, we transform it into a convex optimization problem and solve it with polynomial complexity.

III. Optimal Resource Allocation

To solve P1, we first relax the constraint of exclusive subcarrier assignment and introduce time-sharing parameters $\{\rho_{n,k,i}\}$, for $n \in \mathcal{N}, k \in \mathcal{K}$, and $i = 1, 2$. Each $\rho_{n,k,i}$ denotes the portion of time that subcarrier n is assigned to relay k in the i-th hop and satisfies $\sum_{k=1}^{K} \rho_{n,k,i} = 1, \forall n, i$. Besides, we introduce a new variable $s_{n,k,i} = p_{n,k,i} \rho_{n,k,i}$ to denote the actual power consumed on subcarrier n when it is assigned to relay k over hop i. Variables r_k, for $k \in \mathcal{K}$ are also introduced to transform the max-min problem into a convex one, whose optimality is guaranteed by allowing time-sharing. Then the relaxed optimization problem can be written as

$$\text{P2}: \qquad \max_{\{\boldsymbol{r}, \boldsymbol{\rho}, \boldsymbol{s}\}} \qquad \sum_{k=1}^{K} r_k \quad (6)$$

$$\text{s.t.} \quad \sum_{n=1}^{N} \rho_{n,k,i} \ln \left(1 + \alpha_{n,k,i} \frac{s_{n,k,i}}{\rho_{n,k,i}} \right) \ge r_k, \quad \forall k, i, \quad (7)$$

$$\sum_{n=1}^{N} \sum_{k=1}^{K} \sum_{i=1}^{2} s_{n,k,i} \le P_T, \quad (8)$$

$$\sum_{k=1}^{K} \rho_{n,k,i} = 1, \quad \forall n, i, \quad (9)$$

$$s_{n,k,i} \ge 0, \quad \rho_{n,k,i} \ge 0, \quad \forall n, k, i \ . \quad (10)$$

As shown in [12] this optimal solution achieved by time sharing will be very close to the value that can be achieved subject to integer channel allocations. In fact, this gap has been empirically shown to be close to zero when there are only 8 tones in practice (e.g., [13]). Thus, we will concentrate on solving the relaxed problem P2. Our simulation in Section V also justifies this approach.

It can be shown that all the inequality constraints in P2 are convex. The equality constraints and the objective function are all linear. Therefore, P2 is convex and the strong duality holds. The Lagrangian of P2 can be expressed as

$$\begin{aligned}
&J(\boldsymbol{r},\boldsymbol{s},\boldsymbol{\rho},\boldsymbol{\mu},\beta,\boldsymbol{\nu})\\
&=\sum_{i=1}^{2}\sum_{k=1}^{K}\mu_{k,i}\left[\sum_{n=1}^{N}\rho_{n,k,i}\ln\left(1+\alpha_{n,k,i}\frac{s_{n,k,i}}{\rho_{n,k,i}}\right)-r_k\right]\\
&+\sum_{i=1}^{2}\sum_{n=1}^{N}\nu_{n,i}\left(1-\sum_{k=1}^{K}\rho_{n,k,i}\right)\\
&+\beta\left(P_T-\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{i=1}^{2}s_{n,k,i}\right)+\sum_{k=1}^{K}r_k\ ,
\end{aligned}\tag{11}$$

where $\boldsymbol{\mu}=\{\mu_{k,1},\mu_{k,2}\}\succeq 0$, $\beta\geq 0$, and $\boldsymbol{\nu}=\{\nu_{n,1},\nu_{n,2}\}$ are the Lagrange multipliers for the constraints (7)-(9). Define $\mathcal{D}$ as the set of all primal variables that satisfy $\rho_{n,k,i}\geq 0$, $s_{n,k,i}\geq 0$ and $r_k\geq 0$. Then the dual objective function can be expressed as:

$$g(\boldsymbol{\mu},\beta,\boldsymbol{\nu})\triangleq\max_{\{\boldsymbol{r},\boldsymbol{s},\boldsymbol{\rho}\}\in\mathcal{D}}J(\boldsymbol{r},\boldsymbol{s},\boldsymbol{\rho},\boldsymbol{\mu},\beta,\boldsymbol{\nu})\ ,\tag{12}$$

and the dual optimization problem is

$$\begin{aligned}
&\min_{\{\boldsymbol{\mu},\beta,\boldsymbol{\nu}\}}\qquad g(\boldsymbol{\mu},\beta,\boldsymbol{\nu})\\
&\text{s.t.}\qquad \boldsymbol{\mu}\succeq 0,\qquad \beta\geq 0\ .
\end{aligned}\tag{13}$$

In the following subsections we solve this dual problem.

A. Computing dual function

Observing (11), we find that the dual function defined in (12) can be decomposed into $2N+1$ independent functions, as follows

$$\begin{aligned}
g(\boldsymbol{\mu},\beta,\boldsymbol{\nu})&=g_0(\boldsymbol{\mu})+\sum_{n=1}^{N}g_{n,1}(\boldsymbol{\mu},\beta,\boldsymbol{\nu})+\sum_{n=1}^{N}g_{n,2}(\boldsymbol{\mu},\beta,\boldsymbol{\nu})\\
&+\beta P_T+\sum_{i=1}^{2}\sum_{n=1}^{N}\nu_{n,i}\ .
\end{aligned}\tag{14}$$

Here

$$g_0(\boldsymbol{\mu})\triangleq\max_{\boldsymbol{r}\in\mathcal{D}}J_0(\boldsymbol{r},\boldsymbol{\mu})=\max_{\boldsymbol{r}\in\mathcal{D}}\sum_{k=1}^{K}(1-\mu_{k,1}-\mu_{k,2})r_k\ ,\tag{15}$$

and

$$\begin{aligned}
g_{n,i}(\boldsymbol{\mu},\beta,\boldsymbol{\nu})&\triangleq\max_{\{\boldsymbol{s},\boldsymbol{\rho}\}\in\mathcal{D}}J_{n,i}(\boldsymbol{s},\boldsymbol{\rho},\boldsymbol{\mu},\beta,\boldsymbol{\nu})\\
&=\max_{\{\boldsymbol{s},\boldsymbol{\rho}\}\in\mathcal{D}}\sum_{k=1}^{K}\left[\mu_{k,i}\rho_{n,k,i}\ln\left(1+\alpha_{n,k,i}\frac{s_{n,k,i}}{\rho_{n,k,i}}\right)\right.\\
&\left.-\nu_{n,i}\rho_{n,k,i}-\beta s_{n,k,i}\right],\forall n,\ i=1,2.
\end{aligned}\tag{16}$$

With the above decomposition, we show that for given dual variables $\boldsymbol{\mu}$ and β, we can obtain a closed-form expression of $p_{n,k,i}$. Moreover, finding the optimal value of time-sharing factors $\rho_{n,k,i}$ as well as determining the optimal value of $\nu_{n,i}$ has a complexity $O(NK)$.

We solve $g_0(\boldsymbol{\mu})$ first. $J_0(\boldsymbol{r},\boldsymbol{\mu})$ in (15) is a linear function of r_k. The optimal r_k^* that maximizes $J_0(\boldsymbol{r},\boldsymbol{\mu})$ should satisfy

$$r_k^*=\begin{cases}0, & \text{when } \mu_{k,1}+\mu_{k,2}>1\\ \text{any}, & \text{when } \mu_{k,1}+\mu_{k,2}=1,\quad \forall k\in\mathcal{K}.\\ \infty, & \text{when } \mu_{k,1}+\mu_{k,2}<1\end{cases}\tag{17}$$

When $\mu_{k,1}+\mu_{k,2}<1$, then $g(\boldsymbol{\mu},\beta,\boldsymbol{\nu})=\infty$. This means that the dual function cannot be minimized. Hence, the optimal dual variables cannot lie in the region $\{\mu_{k,1},\mu_{k,2}\mid\mu_{k,1}+\mu_{k,2}<1\}$. When $\mu_{k,1}+\mu_{k,2}\geq 1$, we have $g_0(\boldsymbol{\mu})\equiv 0$.

Now let us solve for $g_{n,i}(\boldsymbol{\mu},\beta,\boldsymbol{\nu})$. Assume that subcarrier n is assigned to relay k in hop i for a time period $\rho_{n,k,i}$. Since $J_{n,i}(\boldsymbol{s},\boldsymbol{\rho},\boldsymbol{\mu},\beta,\boldsymbol{\nu})$ is a concave function in $s_{n,k,i}$, the KKT condition can be applied. Taking the derivative of $J_{n,i}(\boldsymbol{s},\boldsymbol{\rho},\boldsymbol{\mu},\beta,\boldsymbol{\nu})$ in (16) with respect to $s_{n,k,i}$, equating it to zero and considering the boundary constraint $s_{n,k,i}^*\geq 0$, we obtain $p_{n,k,i}^*$ as

$$p_{n,k,i}^*=\frac{s_{n,k,i}^*}{\rho_{n,k,i}}=\left(\frac{\mu_{k,i}}{\beta}-\frac{1}{\alpha_{n,k,i}}\right)^+\ ,\tag{18}$$

where $(.)^+=\max(0,.)$. Substituting (18) into (16), we obtain that

$$J_{n,i}(\boldsymbol{\rho},\boldsymbol{\mu},\beta,\boldsymbol{\nu})=\sum_{k=1}^{K}\rho_{n,k,i}(H_{n,k,i}-\nu_{n,i})\ ,\tag{19}$$

where

$$H_{n,k,i}=\mu_{k,i}\left[\ln\left(\frac{\mu_{k,i}\alpha_{n,k,i}}{\beta}\right)\right]^+-\beta\left(\frac{\mu_{k,i}}{\beta}-\frac{1}{\alpha_{n,k,i}}\right)^+\ .\tag{20}$$

It is clear that $J_{n,i}(\boldsymbol{\rho},\boldsymbol{\mu},\beta,\boldsymbol{\nu})$ is a linear function of $\rho_{n,k,i}$. In order to maximize $J_{n,i}(\boldsymbol{\rho},\boldsymbol{\mu},\beta,\boldsymbol{\nu})$, the optimal value $\rho_{n,k,i}^*$ should satisfy

$$\rho_{n,k,i}^*\in\begin{cases}\{1\}, & \text{when}\quad H_{n,k,i}>\nu_{n,i}\\ [0,1], & \text{when}\quad H_{n,k,i}=\nu_{n,i}\\ \{0\}, & \text{when}\quad H_{n,k,i}<\nu_{n,i}\ .\end{cases}\tag{21}$$

Therefore we have

$$g_{n,i}(\boldsymbol{\mu},\beta,\boldsymbol{\nu})=\sum_{k=1}^{K}(H_{n,k,i}-\nu_{n,i})^+\ .\tag{22}$$

Substituting (22) into (14) we obtain the closed-form expression of the dual function

$$g(\boldsymbol{\mu},\beta,\boldsymbol{\nu})=\beta P_T+\sum_{n=1}^{2}\sum_{n=1}^{N}\left[\sum_{k=1}^{K}(H_{n,k,i}-\nu_{n,i})^++\nu_{n,i}\right].$$

To minimize $g(\boldsymbol{\mu},\beta,\boldsymbol{\nu})$ over all dual variables $\{\boldsymbol{\mu},\beta,\boldsymbol{\nu}\}$, we first solve $g(\boldsymbol{\mu},\beta)=\min_{\boldsymbol{\nu}}g_(\boldsymbol{\mu},\beta,\boldsymbol{\nu})$. This process easily leads to

$$\nu_{n,i}^*=\max_k\{H_{n,k,i}\}\ ,\tag{23}$$

which follows a similar result in [11]. The corresponding dual function is

$$g(\boldsymbol{\mu},\beta)=\sum_{i=1}^{2}\sum_{n=1}^{N}\max_k\{H_{n,k,i}\}+\beta P_T.\tag{24}$$

From above, we conclude that for subcarrier n over hop i, if there exists a unique relay $k^*_{n,i}$ which has the largest $H_{n,k,i}$, then the optimal subcarrier-relay assignment is to assign this subcarrier to the relay $k^*_{n,i}$ with $\rho^*_{n,k^*_{n,i},i} = 1$, and $\rho^*_{n,k,i} = 0$, $\forall k \neq k^*_{n,k}$. If there exists more than one relay which has the maximal $H_{n,k,i}$ simultaneously, then this subcarrier n should be time-shared by these relays. However, the values of time-sharing factors $\rho^*_{n,k,i}$ do not affect the value of dual function $g(\boldsymbol{\mu}, \beta, \boldsymbol{\nu})$ as long as $\sum_{k=1}^{K} \rho_{n,k,i} = 1$ is satisfied. Thus we can obtain the optimal end-to-end transmission rate achieved by time-sharing by evaluating the optimal value of dual function $g^*(\boldsymbol{\mu}, \beta)$.

B. Finding dual variables

After obtaining the closed-form expression of $g(\boldsymbol{\mu}, \beta)$ in (24), we solve the dual problem:

$$\begin{aligned} \min \quad & g(\boldsymbol{\mu}, \beta) \\ \text{s.t.} \quad & \boldsymbol{\mu} \succeq 0, \qquad \beta \geq 0. \end{aligned} \tag{25}$$

Since the dual problem is always convex, it can be readily solved by a gradient-based method. In the following we present a subgradient of $g(\boldsymbol{\mu}, \beta)$, based on which subgradient method or ellipsoid method can be employed to find the optimal dual variables β^* and $\boldsymbol{\mu}^*$.

We firstly introduce a lemma to reduce the number of variables to solve, the proof of which is given in the Appendix.

Lemma 1: There always exists the optimal dual variables $(\beta^*, \boldsymbol{\mu}^*)$ that satisfy $\mu_{k,1} + \mu_{k,2} = 1,$ for all $k \in \mathcal{K}$.

Define $r_{k,i} = \sum_{n=1}^{N} \rho_{n,k,i} \ln\left(1 + \frac{s_{n,k,i}\alpha_{n,k,i}}{\rho_{n,k,i}}\right)$, which is the transmission rate achieved by relay k over hop i. Let $\{r^*_{k,i}(\boldsymbol{\mu}, \beta),\ s^*_{n,k,i}(\boldsymbol{\mu}, \beta)\}$ denote the rates and powers that maximize $J(\boldsymbol{\mu}, \beta, \boldsymbol{r}, \boldsymbol{s})$ over $\mathcal{D}$ for fixed values of $\{\boldsymbol{\mu}, \beta\}$. Using Lemma 1, we obtain a subgradient of $g(\boldsymbol{\mu}, \beta)$ similar as Proposition 1 in [10] and Proposition 1 in [11]

$$\begin{aligned} \nabla \beta &= P_T - \sum_{k=1}^{K} \sum_{n=1}^{N} \left[s^*_{n,k,1}(\boldsymbol{\mu}, \beta) + s^*_{n,k,2}(\boldsymbol{\mu}, \beta) \right], \\ \nabla \mu_{k,1} &= r^*_{k,1}(\boldsymbol{\mu}, \beta) - r^*_{k,2}(\boldsymbol{\mu}, \beta), \qquad \forall k. \end{aligned}$$

C. Obtaining primal variables

After finding the optimal dual variables β^* and $\boldsymbol{\mu}^*$ of problem P2, we now turn to recover the optimal primal variables. Since the optimal objective value achieved by time sharing is very close to the one achieved by integer channel assignment as we mentioned earlier, here we focus on the integer constrained case, i.e., the power allocation and subcarrier-relay assignment satisfying (2)-(5) in problem P1 using the optimal dual variables of problem P2. Substituting β^* and $\boldsymbol{\mu}^*$ into (22) and then as discussed in Section III-A, without loss of the optimality for the dual problem, we can simply let

$$\rho^*_{n,k} = \begin{cases} 1, & \text{if } k = \arg\max_k \{H_{n,k,i}\} \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

If there are more than one relay that achieves the maximum value in (26), we will just randomly assign the channel to one of them.

Then the optimal subcarrier sets $\Omega^*_{k,1}$ and $\Omega^*_{k,2}$ are obtained and the optimal power allocation can be performed based on this subcarrier-relay assignment. This problem can be expressed as

$$\begin{aligned} \text{P3}: \quad & \max_{\{\boldsymbol{p}, \boldsymbol{r}\}} \quad \sum_{k=1}^{K} r_k \\ \text{s.t.} \quad & \sum_{n \in \Omega^*_{k,i}} \ln(1 + \alpha_{n,k,i} p_{n,k,i}) \geq r_k, \quad \forall k, i \\ & \sum_{k=1}^{K} \sum_{i=1}^{2} \sum_{n \in \Omega^*_{k,i}} p_{n,k,i} = P_T, \\ & p_{n,k,i} \geq 0, \quad \forall n, k, i\ , \end{aligned}$$

which is a convex optimization problem. This problem can be solved by utilizing a similar two-nested binary search as in [14]. Note that in this paper, no direct link between source and destination exists and there is only one destination (user).

The computational complexity of this optimal algorithm is mainly determined by the complexity of solving the dual problem, since it is much higher than the complexity of recovering the optimal primal variables after the optimal dual variables are found. As we know, the complexity of ellipsoid method is polynomial in its dimension, which is $K+1$ in our algorithm. Besides, in each iteration of the ellipsoid method, solving the dual function has a complexity $O(NK)$. Therefore, the computational complexity of the optimal algorithm is polynomial in the number of relay nodes and subcarriers.

IV. Suboptimal Solution

Although the algorithm in Section III provides an optimal resource allocation solution, its computational complexity may still be too high for practical implementation. In this section, we propose a suboptimal solution, which performs subcarrier-relay assignment first, and then allocates the power optimally with the given assignment. This suboptimal solution employs subcarrier-based relay selection as well as subcarrier pairing.

The algorithm is sketched as follows.

(1) For each relay k, compute the equivalent channel gain for all the N^2 possible subcarrier pairs (n, n') under a total power constraint, which is given by [4]

$$\alpha^{eq}_{n,k,n'} = \frac{\alpha_{n,k,1}\alpha_{n',k,2}}{\alpha_{n,k,1} + \alpha_{n',k,2}}\ .$$

(2) For each pair (n, n'), select the relay $k^*(n, n') = \arg\max_k \{\alpha^{eq}_{n,k,n'}\}$ to occupy this pair and let $\alpha^*_{n,n'} = \max_k \{\alpha^{eq}_{n,k,n'}\}$.

(3) Form an $N \times N$ matrix $A = [\alpha^*_{i,j}]_{N \times N}$. Since in each hop a subcarrier can be assigned to only one relay, from each row and each column only one element can be selected. The Hungarian method can be used to select the subcarrier pairs from this matrix. Alternatively, the greedy algorithm that always selects the maximum element at each time can also be used.

(4) Using the subcarrier pairs obtained from Step (3), we now have accomplished the subcarrier-relay assignment. Then

power allocation is performed for this given assignment, as described in Subsection III-C.

V. Numerical Results

The DF based dual-hop OFDM system under consideration includes one source, multiple relays and one destination. The distance between the source and the destination is 2Km. All relays are allocated on a circle centered at the middle of the line connecting the source and the destination. The radius of the circle is 200m. Stanford University Interim (SUI) channel model with a central frequency at around 1.9 GHz is used to approximate the fixed broadband wireless channel with 1MHz bandwidth. The broadband channel consists of 6 taps. The signal fading on the first tap is characterized by a Ricean distribution with K-factor equal to 1, while fading on the other five taps follows a Rayleigh distribution. No shadowing is considered here. The noise spectrum density is set to 4.14×10^{-21}W/Hz and the pathloss exponent is set to be 3.5. The number of subcarriers is fixed at $N = 64$.

As a benchmark scheme, the OFDM symbol based relay selection is also presented. In this scheme, we first assume uniform power allocation and then select the relay k^* that maximizes the end-to-end transmission rate:

$$k^* = \arg\max_k \left\{ \frac{1}{2} \min \left\{ \sum_{n=1}^{N} \ln\left(1 + \alpha_{n,k,1} P_T / 2N\right), \right.\right.$$
$$\left.\left. \sum_{n=1}^{N} \ln\left(1 + \alpha_{n,k,2} P_T / 2N\right) \right\} \right\}.$$

Once k^* is found, we then perform the optimal power allocation, which can be carried out in a similar way to the two-nested binary search algorithm in Subsection III-C. Note that only one relay node is selected for use in each transmission in the benchmark scheme.

Fig. 2 shows the end-to-end average transmission rate of different schemes using 8 relay nodes. We can observe that the proposed optimal subcarrier-relay assignments with and without time-sharing (denoted as "Opt-TS" and "Opt-no TS" in the figure respectively) have almost the same performance. The subcarrier-based suboptimal solution proposed in Section IV also performs well although it has a much lower complexity, compared with the optimal algorithm. From this figure it is also observed the proposed optimal algorithm achieves a gain of 0.5 bit/s/Hz in spectral efficiency over the symbol-based benchmark scheme.

Fig. 3 illustrates the end-to-end average transmission rate achieved by using different number of relay nodes. The average rate increases as the relay number K increases, however, the improvement in transmission rate is diminishing when K is large enough.

Fig. 4 shows the information outage probability of the proposed schemes when the target transmission rate is $R_0 = 4$ bit/s/Hz and the relay number is $K = 8$. We can observe that the optimal algorithm (Opt-no TS) outperforms both subcarrier-based and symbol-based relay selections considerably. Specifically, when outage probability is set to 10^{-3}, the optimal algorithm can save 0.5 dB total transmit power when compared with subcarrier-based solution and save more than 4.5 dB when compared with the symbol-based solution.

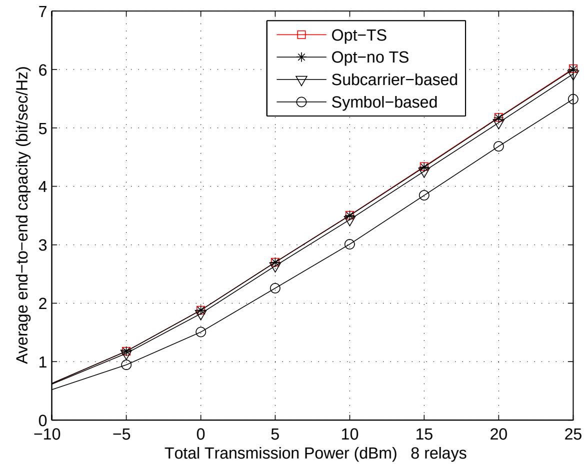

Fig. 2. Average end-to-end transmission rate with 8 relay nodes using different resource allocation algorithms

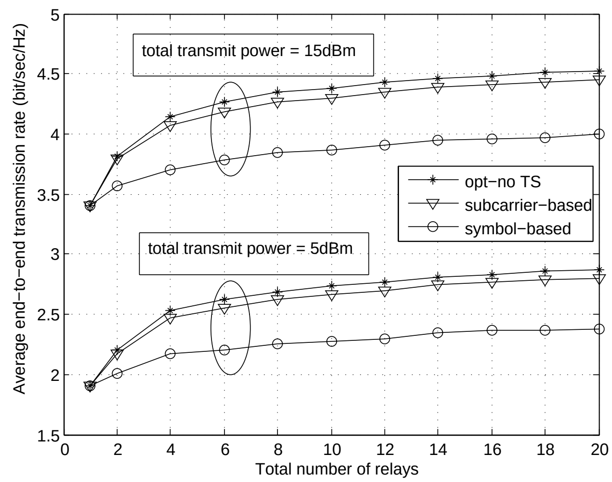

Fig. 3. Average end-to-end transmission rate achieved by different algorithms with different number of relay nodes

VI. Conclusion

We study the optimal resource allocation problem in a relay-based OFDM system. The source transmits data over orthogonal subcarriers to multiple relays, while each relay fully decodes the information bits and forwards them to the destination. A joint subcarrier-relay assignment and power allocation problem is presented for maximizing the system end-to-end transmission rate under a total power constraint. We transform the mixed integer nonlinear programming problem into a convex optimization problem, and solve it using dual decomposition. A suboptimal algorithm that first performs subcarrier-based relay selection and then carries out power allocation is also proposed. Both average transmission rate and outage probability are studied in simulation to evaluate

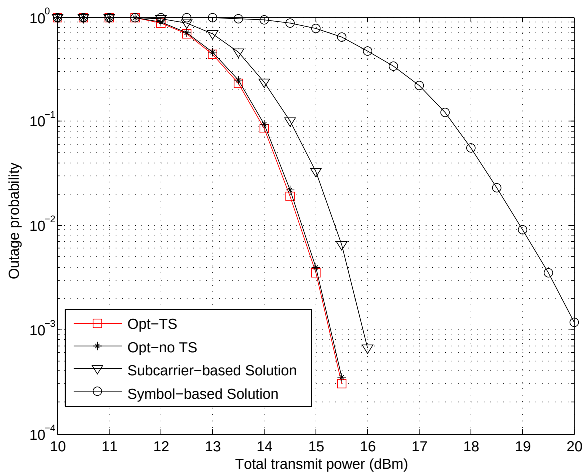

Fig. 4. Outage probability with 8 relay nodes using different resource allocation algorithms when $R_0 = 4$ bit/sec/Hz

the performance of the proposed schemes. It is shown that the suboptimal algorithm performs closely to the optimal algorithm with much less complexity. The numerical results also show that they both outperform significantly the OFDM symbol based relay selection with power control. Although it is a centralized algorithm, the optimal resource allocation algorithm presented in this paper defines the performance bounds for any distributed algorithm.

APPENDIX
PROOF OF LEMMA 1

From (20), we can observe that $H_{n,k,i}$ is a continuous function of $\mu_{k,i}$. Evaluating its derivative with respect to $\mu_{k,i}$, we have

$$\frac{\partial H_{n,k,i}}{\partial \mu_{k,i}} = \begin{cases} 0, & \text{when } \frac{\mu_{k,i}}{\beta} < \frac{1}{\alpha_{n,k,i}} \\ \ln\left(\frac{\mu_{k,i}\alpha_{n,k,i}}{\beta}\right) > 0, & \text{when } \frac{\mu_{k,i}}{\beta} > \frac{1}{\alpha_{n,k,i}} \end{cases}. \quad (27)$$

Thus $H_{n,k,i}$ is a non-decreasing function of $\mu_{k,i}$ and so is $g(\boldsymbol{\mu}, \boldsymbol{\beta})$ in (24). From Subsection III-A, we have concluded that the optimal dual variables $\{\mu_{k,1}, \mu_{k,2}\}$ always lie in the region $\{\mu_{k,1}, \mu_{k,2} \mid \mu_{k,1} + \mu_{k,2} \geq 1\}$. Suppose we update dual variables $\{\mu_{k,i}\}$ within the region $\{\mu_{k,1}, \mu_{k,2} | \mu_{k,1} + \mu_{k,2} \geq 1\}$ and reach an optimal point $\{\boldsymbol{\mu}^*, \boldsymbol{\beta}^*\}$ with $g^*(\boldsymbol{\mu}^*, \boldsymbol{\beta}^*) = \min_{\boldsymbol{\mu},\boldsymbol{\beta}} g(\boldsymbol{\mu}, \boldsymbol{\beta})$. Assume for relay k_0, we have $\mu^*_{k_0,1} + \mu^*_{k_0,2} > 1$. Then we can find $\mu'_{k_0,1}$ and $\mu'_{k_0,2}$ such that $\mu'_{k_0,1} \leq \mu^*_{k_0,1}$ and $\mu'_{k_0,2} \leq \mu^*_{k_0,2}$, and $\mu'_{k_0,1} + \mu'_{k_0,2} = 1$. It is clear that $g(\{\mu'_{k_0,i}, \mu^*_{-k_0,i}\}_{i=1,2}, \beta) \leq g^*$, where $\mu^*_{-k_0,i}$ denotes that all relays except relay k_0 will use dual variable in $\{\boldsymbol{\mu}^*\}$. Since $g^* = \min_{\{\boldsymbol{\mu},\boldsymbol{\beta}\}} g(\boldsymbol{\mu}, \beta)$, we have $g(\{\mu'_{k_0,i}, \mu^*_{-k_0,i}\}_{i=1,2}, \boldsymbol{\beta}) \geq g^*$. Therefore, $g(\{\mu'_{k_0,i}, \mu^*_{-k_0,i}\}_{i=1,2}, \boldsymbol{\beta}) = g^*$, which means $\{\mu'_{k_0,i}, \mu^*_{-k_0,i}\}_{i=1,2}$ is also an optimal dual variable which can minimize dual function $g(\boldsymbol{\mu}, \boldsymbol{\beta})$. This proves the lemma.

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