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Joint User Grouping and Linear Virtual Beamforming: Complexity, Algorithms and Approximation Bounds

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Abstract—In a wireless system with a large number of distributed nodes, the quality of communication can be greatly improved by pooling the nodes to perform joint transmission/reception. In this paper, we consider the problem of optimally selecting a subset of nodes from potentially a large number of candidates to form a virtual multi-antenna system, while at the same time designing their joint linear transmission strategies. We focus on two specific application scenarios: *1)* multiple single antenna transmitters cooperatively transmit to a receiver; *2)* a single transmitter transmits to a receiver with the help of a number of cooperative relays. We formulate the joint node selection and beamforming problems as *cardinality constrained optimization problems* with both discrete variables (used for selecting cooperative nodes) and continuous variables (used for designing beamformers). For each application scenario, we first characterize the computational complexity of the joint optimization problem, and then propose novel semi-definite relaxation (SDR) techniques to obtain approximate solutions. We show that the new SDR algorithms have a guaranteed approximation performance in terms of the gap to global optimality, regardless of channel realizations. The effectiveness of the proposed algorithms is demonstrated via numerical experiments.

Index Terms—Virtual Multi-antenna Systems, Beamforming, User Grouping, Cardinality Constrained Quadratic Program, Semi-definite Relaxation, Approximation Bounds, Computational Complexity

I. Introduction

With the proliferation of rich multimedia services as well as smart mobile devices, the demand for wireless data has been increasing explosively in recent years. To accommodate the growing demand for wireless data, practical techniques that can significantly improve the spectrum efficiency of existing wireless systems must be developed. In this paper, we focus on a combination of two such techniques: partial node cooperation and collaborative beamforming.

In a cellular network, cooperation can be achieved by allowing the neighboring base stations (BSs) to form a virtual multi-antenna system for joint transmission and reception, a scheme known as cooperative multipoint (CoMP) [1]. It can effectively cancel the inter-BS interference, and has been included into the

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next-generation wireless standards such as 3GPP Long Term Evolution-Advanced (LTE-A); see e.g., [1]–[3]. For example, in a downlink network, assuming the users' data signals are known at all BSs, then either the capacity achieving non-linear dirty-paper coding (DPC) (see, e.g., [4], [5]), or simpler linear precoding schemes such as zero-forcing (ZF) (see, e.g., [6]–[8]) can be used for joint transmission. In addition, various cooperation schemes have been proposed to exploit spatial diversity among the mobile users as well [9]–[14]. In these schemes, users assist each other in relaying information to the desired destinations by using various strategies such as amplify-and-forward (AF) and decode-and-forward (DF).

However, the cost of cooperation can outweigh its benefit when the size of the cooperation group grows large. Such costs include the overhead incurred by exchanging control and data signals among the cooperating nodes (either via backhaul networks or air interfaces); it can also include efforts required to maintain system level synchronization [2], [3], [15], [16]. To control the size of cooperation group, various *partial* cooperation schemes have been developed recently. In the setting of a cellular network, partial cooperation among the BSs amounts to judiciously clustering the BSs into (possibly overlapping) small cooperation groups, within which they cooperatively transmit to or receive from the users [7], [17], [18]. In [19]–[21], joint BS clustering and beamforming problems are formulated as certain *sparse* beamformer design problems, in which the sparsity of the virtual beamformer corresponds to the size of the cooperation groups. Partial cooperation in the relay networks has also been studied recently. In [14], [22], the authors propose to select a *single* relay (out of many candidates) so that certain performance metric at the receiver is optimized. Alternatively, references [14], [23]–[25] study the *multiple* relay selection problem. In particular, the authors of [23] propose to increase the number of relays until adding an additional one decreases the received SNR. Reference [25] formulates the relay selection problem as a Knapsack problem [26], and proposes greedy algorithms for this problem. However, these schemes generally assume simplified underlying cooperation schemes after fixing the cooperative set. For example, references [7], [18] use simple zero forcing strategies for intra-cluster transmission, while references [23], [24] assume that the cooperative relays transmit with full power. There has been no performance analysis for these partial cooperation schemes. This is due to the *mixed-integer* nature of the problem when treating the group membership (which is a set of discrete variables) as

optimization variables.

In this paper, we study the problem of optimally partitioning the transmit nodes into cooperation groups, while at the same time designing their cooperation strategies. We focus on two related network settings in which either multiple nodes cooperatively transmit to a receiver, or a single node transmits to the receiver with the help of a set of cooperative relays. In both cases, the cooperative nodes are allowed to form a virtual antenna system, and they can jointly design the virtual transmit beamformers. More specifically, our objective is to find a subset of cooperative nodes (with given cardinality) and their joint linear beamformers so that the system performance measured by the receive signal to noise ratio (SNR) is maximized. We formulate the problem as a cardinality constrained quadratic program and study its computational complexity. Furthermore, we develop novel semi-definite relaxation (SDR) algorithms for this mixed integer quadratic program and prove that they have a guaranteed approximation performance in terms of the gap to global optimality, regardless of channel realization. Compared to the existing SDR algorithms and their analysis [27]–[30] which focus on quadratic problems with continuous variables, our work deals with mixed-integer cardinality constrained quadratic optimization problems and therefore has a significantly broader scope.

The rest of the paper is organized as follows. In Section II, we introduce the virtual beamforming problem without node grouping. Section III–Section V consider the joint node grouping and virtual beamforming problems in various settings. Section VI presents numerical results. The concluding remarks and future works are given in Section VII.

Notations: For a symmetric matrix $\mathbf{X}$, $\mathbf{X} \succeq 0$ signifies that $\mathbf{X}$ is positive semi-definite. We use $\text{Tr}[\mathbf{X}]$, $\mathbf{X}^H$, and $\lambda_i(\mathbf{X})$ to denote the trace, Hermitian transpose, and the i-th largest eigenvalue of $\mathbf{X}$, respectively. The notation $\text{diag}(\mathbf{X})$ denotes a matrix consisting the diagonal value of $\mathbf{X}$. For an index set $\mathcal{S}$, the notations $\mathbf{X}[i,j]$ and $\mathbf{X}[\mathcal{S}]$ denote the (i,j)-th element of $\mathbf{X}$ and the principal submatrix of $\mathbf{X}$ indexed by the set $\mathcal{S}$, respectively. Similarly, we use $\mathbf{x}[i]$ and $\mathbf{x}[\mathcal{S}]$ to denote the i-th element of a vector $\mathbf{x}$ and the subvector of $\mathbf{x}$ with the elements in set $\mathcal{S}$, respectively. For a complex scalar x, the notation $\bar{x}$ denotes its complex conjugate . Let $\delta_{i,j}$ denote the Kronecker function, which takes the value 1 if $i = j$, and 0 otherwise. The notations $\mathbf{I}_n$, $\mathbf{e}$ and $\mathbf{e}_i$ denote, respectively, the $n \times n$ identity matrix, the all one vector in $\mathbb{R}^n$, and the i-th unit vector in $\mathbb{R}^n$. Some other notations are listed in Table I.

II. VIRTUAL BEAMFORMING WITH FULL COOPERATION

A. A Single-hop Network

Let us first describe the virtual beamforming (VB) problem with all transmit nodes fully cooperating in transmission. Suppose there is a set $\mathcal{M} = \{1, \cdots, M\}$ of transmitters each equipped with a single antenna, and there is a single receiver with $N \geq 1$ receive antennas. This setting depicts for instance an uplink cellular network, where the transmit nodes are the users and the receive node is the BS. We are interested in the case $M > N$, where the receiver cannot cancel the interference among the transmit nodes if they transmit

simultaneously and independently. In this case, the benefit of transmit nodes cooperation in improving system performance is more pronounced. Let $h_{i,n} \in \mathbb{C}$ denote the channel between transmitter i and the n-th antenna of the receiver, and define $\mathbf{h}_n \triangleq [h_{1,n}, \cdots, h_{M,n}]^H$. Suppose only second order statistics on the channels are available, that is, both the transmitters and the receiver only know $\mathbb{E}[\mathbf{h}_n\mathbf{h}_n^H] = \mathbf{R}_n \succ 0$, for $n = 1, \cdots, N$. Let $z \sim \mathcal{CN}(0,1)$ denote the (normalized) noise at the receiver.

For tractability, we restrict ourselves to a simple transmit cooperation strategy in which the cooperative transmitters can form a *linear* virtual beamformer for joint transmission. Let $w_i \in \mathbb{C}$ denote the complex antenna gain for transmitter i, which satisfies an individual transmit power constraint: $|w_i|^2 \leq P$. Define $\mathbf{w} \triangleq [w_1, \cdots, w_M]^H$. When all the transmitters participate simultaneously in the beamforming, they transmit *the same* data signal to the receiver by using *distinct* antenna gains. The transmit nodes can share their data signals by the following steps: *1)* identify the node whose data is to be transmitted; *2)* the identified node broadcasts its data to all nearby nodes, who subsequently decode the data.

Assume that the receiver performs spatially matched filtering/maximum ratio combining (which is equivalent to the MMSE receiver in this case), then the total received signal power can be expressed as: $\sum_{n=1}^{N} |\mathbf{h}_n^H\mathbf{w}|^2 = \mathbf{w}^H(\sum_{n=1}^{N} \mathbf{h}_n\mathbf{h}_n^H)\mathbf{w}$. Let $\mathbf{R} \triangleq \sum_{n=1}^{N} \mathbf{R}_n$, and assume that the noise power is normalized to 1, then the averaged signal to noise ratio (SNR) at the receiver is given by: $\text{SNR} = \mathbb{E}\left[\sum_{n=1}^{N} |\mathbf{w}^H\mathbf{h}_n|^2\right] = \mathbf{w}^H\mathbf{R}\mathbf{w}$. To optimize the averaged SNR at the BS, one can solve the following quadratic program (QP)

$$\max_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{R}\mathbf{w} \qquad (1)$$
$$\text{s.t.} \quad |w_i|^2 \leq P, \ i = 1, \cdots, M.$$

At this point, it may appear that solving the above SNR maximization problem with per-antenna power constraints can be done easily, using for example algorithms based on uplink-downlink duality proposed in [5]. Unfortunately, as will be seen later in Section III-A, this seemingly simple problem turns out to be computationally very difficult, for *general* channel covariance matrix $\mathbf{R}$ with rank larger than one. In fact, the uplink-downlink duality theory developed in [5] and related works depends critically on the assumption that $\mathbf{R}$ is of rank one. Later in Claim 1 we will see that indeed in our case when $\text{Rank}(\mathbf{R}) = 1$, solving problem (1) is easy.

It is worth mentioning that the formulation (1) is equally applicable to solving the sum SNR maximization problem when the *instantaneous* channel states $\{\mathbf{h}_n\}$ are available. In this case, $\mathbf{R}$ should be replaced by the instantaneous channel $\widehat{\mathbf{R}} \triangleq \sum_{n=1}^{N} \mathbf{h}_n\mathbf{h}_n^H$.

B. A Two-hop Network

In the previous single hop model, it is assumed that the hop connecting the source and the cooperative nodes is *reliable*, in the sense that all the cooperative nodes can perfectly decode the signals to be jointly transmitted. Alternatively, when the

TABLE I
A LIST OF NOTATIONS

| $\mathcal{M}$	The set of all transmit nodes	N	The number of antennas at the receiver
$\mathbf{R}$	The channel covariance matrix	$\mathbf{w}$	The virtual beamformer
P	The transmit power budget for each node	Q	The size of cooperative group
$\mathbf{x}$	The vector of both discrete and continuous variables	$\mathcal{S}$	The support of a beamformer $\mathbf{w}$
$\widetilde{\mathbf{w}}$	The dimension-reduced beamformer $\mathbf{w}[\mathcal{S}]$	$\mathbf{Y}$	The rank-1 matrix $\widetilde{\mathbf{w}}\widetilde{\mathbf{w}}^H$
$\mathbf{X}$	The rank-1 matrix $\mathbf{x}\mathbf{x}^H$	L	The sample size for randomization
$\mathbf{f}$	The first hop channel	$\mathbf{g}$	The second hop channel
P_0	The transmit power for the transmitter	$\boldsymbol{\nu}$	The noise at the relay
$\mathbf{S}$	The Channel matrix $P_0\mathbb{E}[(\mathbf{g}\odot\mathbf{f})(\mathbf{g}\odot\mathbf{f})^H]$	$\mathbf{F}$	The channel matrix $\mathbb{E}[(\mathbf{g}\odot\boldsymbol{\nu})(\mathbf{g}\odot\boldsymbol{\nu})^H]$
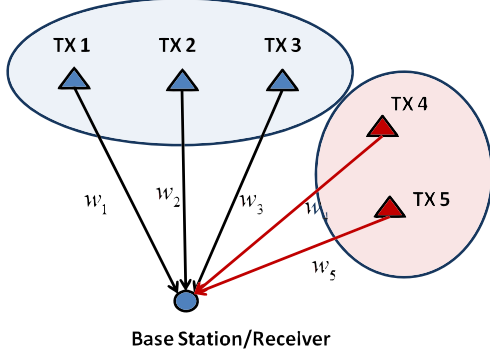
Fig. 1. Illustration of joint node grouping and VB. Nodes 1-3 and 4, 5 are divided into two different groups.
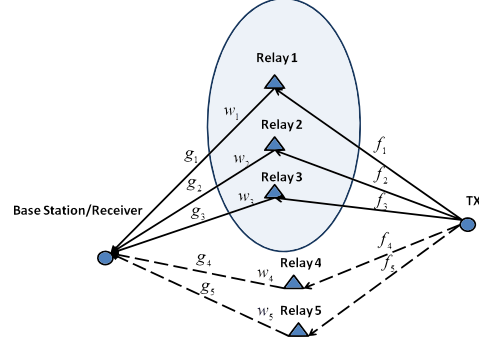

Fig. 2. Illustration of joint relay selection and VB. Relays 1-3 are the serving relays.

quality of the first hop communication also needs to be taken into consideration, the problem can be formulated in the context of the a two-hop relay network, as we explain below.

Consider a network with a pair of transceiver and a set of M relays, each of which has a single antenna (see Fig. 2 for an illustration). Assume that there is no direct link between the transmitter and the receiver. Let $\{f_i\}_{i=1}^M$ and $\{g_i\}_{i=1}^M$ denote the complex channel coefficients between the transmitter and the relays, and between the relays and the receiver, respectively. We focus on a popular AF relay protocol, in which the transmitter broadcasts the desired signal to the relays, who subsequently forward the signals to the receiver. Assume that there is a large number of relays available, and any group of them can form a virtual multi-antenna system for transmission.

Let us use $s \in \mathbb{C}$ to denote the message transmitted by the transmitter; use P_0 to denote the transmit power; use ν_i to denote the noise at the i-th relay with power σ_ν^2. Then the signal x_i received at the i-th relay can be expressed as $x_i = \sqrt{P_0}f_i s + \nu_i$. Again use w_i to denote the complex gain applied by the i-th relay, which satisfies the power constraint $|w_i|^2 \leq P$. It follows that the transmitted signal of i-th relay is given by $y_i = w_i x_i$. Using this expression, the averaged transmit power of relay i can be expressed as

$$\mathbb{E}[|y_i|^2] = |w_i|^2 \mathbb{E}[x_i \bar{x}_i] = |w_i|^2 \left(P_0 \mathbb{E}[|f_i|^2] + \sigma_\nu^2 \right).$$

Let $n \in \mathcal{CN}(0, \sigma_n^2)$ denote the noise at the receiver, then the received signal is given by

$$z = \sum_{i=1}^M g_i y_i + n = \underbrace{\sqrt{P_0}\sum_{i=1}^M w_i f_i g_i s}_{\text{signal}} + \underbrace{\sum_{i=1}^M w_i g_i \nu_i + n}_{\text{noise}}. \quad (2)$$

The averaged signal power at the receiver is then given by

$$\mathbb{E}\left[\left| \sqrt{P_0}\sum_{i=1}^M w_i f_i g_i s \right|^2 \right] = \mathbf{w}^H \mathbf{S} \mathbf{w} \quad (3)$$

where $\mathbf{S} \triangleq P_0 \mathbb{E}[(\mathbf{f}\odot\mathbf{g})(\mathbf{f}\odot\mathbf{g})^H]$, with $\odot$ denoting the componentwise product. When $\{\nu_i\}_{i=1}^M$ and $\{g_i\}_{i=1}^M$ are independent from each other, the averaged noise power is given by [11]

$$\mathbb{E}\left[\left| \sum_{i=1}^M w_i g_i \nu_i + n \right|^2 \right] = \mathbf{w}^H \mathbf{F} \mathbf{w} + \sigma_n^2, \quad (4)$$

where $\mathbf{F} \triangleq \mathbb{E}[(\mathbf{g}\odot\boldsymbol{\nu})(\mathbf{g}\odot\boldsymbol{\nu})^H]$. Additionally, if we further assume that the noises $\{\nu_i\}_{i=1}^M$ are independent, then $\mathbf{F}$ becomes diagonal: $\mathbf{F} \triangleq \sigma_\nu^2 \text{diag}(\mathbb{E}[\mathbf{g}\mathbf{g}^H])$. It follows that the averaged SNR at the receiver is given by [11]: $\text{SNR} = \frac{\mathbf{w}^H \mathbf{S} \mathbf{w}}{\sigma_n^2 + \mathbf{w}^H \mathbf{F} \mathbf{w}}$. To optimize the averaged SNR at the receiver, the following problem needs to be solved

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H \mathbf{S} \mathbf{w}}{\sigma_n^2 + \mathbf{w}^H \mathbf{F} \mathbf{w}} \quad (5)$$
$$\text{s.t.} \quad |w_i|^2 \left(P_0 \mathbb{E}[|f_i|^2] + \sigma_\nu^2 \right) \leq P, \ i = 1, \cdots, M.$$

We remark that when the set of per-relay power constraints is replaced by a single sum-power constraint, the above problem is equivalent to a principal generalized eigenvector problem, which is easily solvable [11]. However, as will be explained in more detail in Section V, when the per-relay power constraints are present, (5) turns out to be computationally difficult.

In practice, when the number of transmit/relay nodes becomes large, allowing all of them to cooperate at the same time induces heavy signaling overhead (related to nodes' exchange of data and control signals) and computational efforts (related

to computing the optimal virtual beamformer for all the nodes) [15]. To address these issues, it is necessary to divide the transmit/relay nodes into different cooperative groups while at the same time optimizing their virtual beamformers. How to do so in either single-hop or two-hop networks will be the focus of the rest of this paper.

III. JOINT ADMISSION CONTROL AND VB

In this section, we consider a basic setting in which the aim is to find a *single* cooperative group with a fixed size. Such admission control problem is important as fixing the size of the group can effectively control the cooperation and computational overhead. Although admission control for wireless networks is a well-studied subject (see [31]–[33]), existing solutions cannot be directly applied in our setting because they are designed for conventional wireless networks *without* node cooperation.

A. Problem Formulation and Complexity Status

Let Q denote the desired size of the cooperation group, and introduce the set of binary variables $a_i \in \{0,1\}, i \in \mathcal{M}$ to indicate the transmit nodes' group membership: when $a_i = 1$, node i is being assigned to the cooperation group. Let $\mathbf{a} \triangleq [a_1, \cdots, a_M]$. Then the joint admission control and VB problem is given as the following cardinality constrained program

$$v_1^{\text{CP}} = \max_{\mathbf{w}, \mathbf{a}} \quad \mathbf{w}^H \mathbf{R} \mathbf{w} \qquad \text{(CP1)}$$
$$\text{s.t.} \quad |w_i|^2 \le a_i P, \ i = 1, \cdots, M$$
$$\sum_{i=1}^M a_i = Q, \quad a_i \in \{0,1\}, \ i = 1, \cdots, M.$$

Note that $a_i = 0$ implies $|w_i|^2 = 0$, that is, node i does not transmit. In the following, we will use $v_1^{\text{CP}}(\mathbf{w})$ to indicate the objective value achieved by a feasible solution $\mathbf{w}$.

In order to express the problem in a simpler form, we introduce a homogenizing variable $\gamma \in \{-1, 1\}$ and change the domain of the discrete variables to $\{-1, 1\}$. By doing so problem (CP1) can be equivalently reformulated in the following quadratic form:

$$\max_{\mathbf{w}, \mathbf{a}, \gamma} \quad \mathbf{w}^H \mathbf{R} \mathbf{w}$$
$$\text{s.t.} \quad \mathbf{w}^H \mathbf{e}_i \mathbf{e}_i^T \mathbf{w} + \frac{P}{4}(a_i - \gamma)^2 \le P, \ i = 1, \cdots, M$$
$$\sum_{i=1}^M (a_i + \gamma)^2 = 4Q,$$
$$a_i \in \{-1, 1\}, \ i = 1, \cdots, M, \ \gamma \in \{-1, 1\}.$$

To see the equivalence, we first perform a change of variable domain by defining: $\hat{a}_i = 2a_i - 1$, for all i, where a_i is the original variable with the domain $\{0, 1\}$. Then we split each $\hat{a}_i$ by $\hat{a}_i = \gamma \tilde{a}_i$ for a new variable $\tilde{a}_i \in \{-1, 1\}$. By doing so the constraints can be shown to be quadratic in both $\tilde{\mathbf{a}}$ and γ. For notational simplicity, below we still use a_i to denote the new variable $\tilde{a}_i \in \{-1, 1\}$.

After such transformation, we see that $\gamma a_i = -1$ implies $w_i = 0$, i.e., node i does not join the cooperative group.

To further express the problem in a standard quadratic form of both the binary and continuous variables, we need the following definitions

$$\mathbf{C}_{i,0} \triangleq \frac{1}{4} \left(\mathbf{e}_i \mathbf{e}_i^T + \mathbf{e}_{M+1} \mathbf{e}_{M+1}^T - \mathbf{e}_i \mathbf{e}_{M+1}^T - \mathbf{e}_{M+1} \mathbf{e}_i^T \right) \in \mathbb{R}^{(M+1) \times (M+1)}, \qquad \text{(6a)}$$

$$\mathbf{C}_{i,1} \triangleq \mathbf{e}_i \mathbf{e}_i^T \frac{1}{P} \in \mathbb{R}^{M \times M} \qquad \text{(6b)}$$

$$\mathbf{D}_i \triangleq \text{blkdg}[\mathbf{C}_{i,0}, \mathbf{C}_{i,1}] \in \mathbb{C}^{(2M+1) \times (2M+1)} \qquad \text{(6c)}$$

$$\widetilde{\mathbf{R}} \triangleq \text{blkdg}[\mathbf{0}, \mathbf{R}] \in \mathbb{C}^{(2M+1) \times (2M+1)} \qquad \text{(6d)}$$

$$\mathbf{B}_0 \triangleq \begin{bmatrix} \mathbf{I} & \mathbf{e} \\ \mathbf{e}^T & M \end{bmatrix} \in \mathbb{R}^{(M+1) \times (M+1)}, \qquad \text{(6e)}$$

$$\mathbf{B} = \text{blkdg}[\mathbf{B}_0, \mathbf{0}] \in \mathbb{R}^{(2M+1) \times (2M+1)} \qquad \text{(6f)}$$

$$\mathbf{x} \triangleq [\mathbf{a}^T, \gamma, \mathbf{w}^T]^T \ \mathbf{x}_0 \triangleq [\mathbf{a}^T, \gamma]^T, \mathbf{x}_1 \triangleq \mathbf{w}. \qquad \text{(6g)}$$

We can now compactly write (CP1) as a quadratic problem of the newly defined vector $\mathbf{x}$, which contains both binary and continuous variables:

$$\max_{\mathbf{x}} \quad \mathbf{x}^H \widetilde{\mathbf{R}} \mathbf{x} \qquad \text{(R1)}$$
$$\text{s.t.} \quad \mathbf{x}^H \mathbf{D}_i \mathbf{x} \le 1, \ i = 1 \cdots, M \qquad \text{(7a)}$$
$$\mathbf{x}^H \mathbf{B} \mathbf{x} = 4Q, \qquad \text{(7b)}$$
$$\mathbf{x}[i] \in \{-1, 1\}, \ i = 1, \cdots, M+1.$$

We emphasize again that in this new notation, $\mathbf{x}[i]\mathbf{x}[M+1] = -1$ implies that node i is not in the cooperative group (i.e., $\mathbf{w}[i] = 0$), or equivalently $\mathbf{x}[M+1+i] = 0$ from definition (6g).

Towards finding a solution for problem (CP1)/(R1), the first task is to analyze their computational complexity. Our analysis, to be presented shortly, shows that these problems are difficult even when fixing the values of the binary variables $\{a_i\}$.

Let $\mathcal{S} \subset \mathcal{M}$ with $|\mathcal{S}| = Q$ denote the *support* of a feasible solution $\mathbf{w}$ to problem (CP1): $\mathcal{S} = \{i : a_i = 1\}$. When $\mathcal{S}$ is fixed, problem (CP1) is equivalent to the following QP

$$\max_{\widetilde{\mathbf{w}} \in \mathbb{C}^Q} \quad \widetilde{\mathbf{w}}^H \mathbf{R}[\mathcal{S}] \widetilde{\mathbf{w}} \qquad \text{(QP1)}$$
$$\text{s.t.} \quad |\widetilde{w}_i|^2 \le P, i = 1, \cdots, Q.$$

In the following we analyze the computational complexity of (QP1).

Proposition 1 *Solving the problem* (QP1) *is NP-hard in the number of transmit nodes.*

Proof: We only prove the claim with real variables. The complex case can be derived similarly. The claim is proved by a polynomial time reduction from a known NP-complete problem called equal partition problem [26], which can be described as follows. Given a vector $\mathbf{c}$ consisting of positive integers $c_1, \cdots, c_Q$, the equal partition problem decides whether there exists a subset $\mathcal{I}$ such that

$$\frac{1}{2} \sum_{i=1}^Q c_i = \sum_{i \in \mathcal{I}} c_i. \qquad \text{(8)}$$

In the following, we will show that a special case of problem (QP1) is equivalent to an instance of equal partition problem.

Suppose $\mathbf{c}^T\mathbf{c} = C > 0$. Let $\mathbf{R}[\mathcal{S}] = (-\mathbf{c}\mathbf{c}^T + 2C\mathbf{I}_Q) \succ 0$. The claim is that the problem (QP1) has the maximum value of $2CQP$ if and only if there exists a set $\mathcal{I}$ satisfying (8). The objective of problem (QP1) can be written as

$$\widetilde{\mathbf{w}}^T\mathbf{R}[\mathcal{S}]\widetilde{\mathbf{w}} = -|\widetilde{\mathbf{w}}^T\mathbf{c}|^2 + 2C\sum_{i=1}^{Q}|\widetilde{w}_i|^2$$
$$\leq 2C\sum_{i=1}^{Q}|\widetilde{w}_i|^2 \leq 2CQP, \text{ whenever } |\widetilde{w}_i|^2 \leq P.$$
$$(9)$$

Consequently, the maximum value for problem (QP1) is $2CQP$ if and only if $-|\widetilde{\mathbf{w}}^T\mathbf{c}|^2 = 0$ and $|\widetilde{w}_i|^2 = P$. This is equivalent to the existence of an index set $\mathcal{I}$ such that (8) is true. ∎
It is important to note that when $\mathcal{S} = \mathcal{M}$, (QP1) is the same as the VB problem (1). Therefore we can readily conclude that solving problem (1) is also NP-hard.

Claim 1 *When* $\mathbf{R}$ *admits certain special structures, both* (QP1) *and* (CP1) *may be easy to solve. One such example is that when* $\text{Rank}(\mathbf{R}) = 1$, *which corresponds to the special case where the receiver has a single antenna, and the instantaneous SNR is considered. Another relevant case is that when* $\mathbf{R}$ *is a diagonal matrix, which happens when all the transmit nodes' channels are independent and zero mean. For both cases, problems* (QP1) *and* (CP1) *are separable among the variables, and their solutions can be easily obtained in closed form.*

B. The Semi-definite Relaxation

Our proposed algorithm is based on the technique called semi-definite relaxation (SDR), which has been widely used to solve problems in communications and signal processing [27]. We emphasize that unlike conventional SDR methods, in which the problems to be relaxed have either all continuous (e.g., [28]) or all discrete (e.g., [34]) variables, our problem (CP1)/(R1) is of *mixed integer* nature. Consequently our algorithm and analysis to be presented differ significantly from those developed in the existing literature.

We first introduce two semi-definite programs (SDPs) which are relaxations of problems (R1) and (QP1). Define a variable $\mathbf{X} \triangleq \mathbf{x}\mathbf{x}^H$. Define two index sets $\mathcal{I} \triangleq \{1, \cdots, M+1\}$ and $\bar{\mathcal{I}} \triangleq \{M+2, \cdots, 2M+1\}$. Then $\mathbf{X}_0 \triangleq \mathbf{X}[\mathcal{I}]$ and $\mathbf{X}_1 \triangleq \mathbf{X}[\bar{\mathcal{I}}]$ denote the leading and trailing principal submatrices of $\mathbf{X}$, respectively. Clearly $\mathbf{X}_0 = \mathbf{x}_0\mathbf{x}_0^T$ and $\mathbf{X}_1 = \mathbf{x}_1\mathbf{x}_1^H$. Moreover, we have $\text{Rank}(\mathbf{X}) = 1$ and $\mathbf{X}_0[i, j] \in \{-1, 1\}$ for all $i, j \in \mathcal{I}$. The following SDP is a relaxation of (R1), by removing the non-convex constraint $\text{Rank}(\mathbf{X}) = 1$ and by replacing $\mathbf{X}_0[i, j] \in \{-1, 1\}$ by $\mathbf{X}_0[i, j] \in [-1, 1]$, for all $i, j \in \mathcal{I}$:

$$v_1^{\text{SDP}} = \max_{\mathbf{X} \succeq 0} \quad \text{Tr}[\widetilde{\mathbf{R}}\mathbf{X}] \tag{SDP1}$$
$$\text{s.t.} \quad \text{Tr}[\mathbf{D}_i\mathbf{X}] \leq 1, \ i = 1 \cdots, M \tag{10a}$$
$$\text{Tr}[\mathbf{B}\mathbf{X}] = 4Q \tag{10b}$$
$$\mathbf{X}[i, i] = 1, i = 1, \cdots, M+1. \tag{10c}$$

Note that we did not explicitly include the conditions $\mathbf{X}_0[i, j] \in [-1, 1]$, for all $i, j \in \mathcal{I}$, as it can be ensured by the set of conditions (10c) and $\mathbf{X} \succeq 0$. As the above problem is a *relaxation* of problem (R1), we must have $v_1^{\text{SDP}} \geq v_1^{\text{CP}}$. Denote the optimal solution for this problem as $\mathbf{X}^*$. Since all the data matrices $\mathbf{B}$, $\mathbf{D}_i$ and $\widehat{\mathbf{R}}$ are block diagonal matrices, removing the off-diagonal blocks of an optimal solution does not change either its optimality or feasibility. Thus, without loss of generality we can assume $\mathbf{X}^* = \text{blkdg}[\mathbf{X}_0^*, \mathbf{X}_1^*]$.

Similarly, let $\mathbf{Y} \triangleq \widetilde{\mathbf{w}}\widetilde{\mathbf{w}}^H \in \mathbb{C}^{Q \times Q}$. The following problem is a relaxation of the problem (QP1), for a given index set $\mathcal{S} \subseteq \mathcal{M}$ with $|\mathcal{S}| = Q$

$$\max_{\mathbf{Y} \succeq 0} \quad \text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}] \tag{11a}$$
$$\text{s.t.} \quad \mathbf{Y}[i, i] \leq P, \ i = 1, \cdots, Q. \tag{11b}$$

Let us denote the optimal solution of this problem by $\mathbf{Y}^*$.

Fig. 3 below shows the relationship among different problem formulations introduced so far. For problems (SDP1) and (11a), the following claims summarize some useful properties of their optimal solutions. These properties will be used later for analyzing the quality of certain approximate solutions for the original problem (CP1)/(R1).
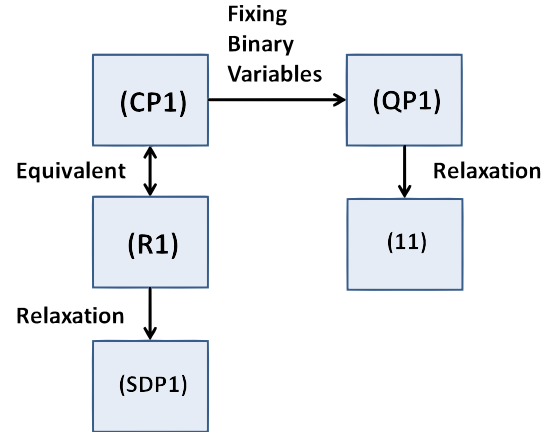

Fig. 3. Relationship among different problem formulations.

Claim 2 *At optimality, the set of constraints* (10a) *and* (11b) *must be all tight. That is*

$$\frac{1}{P}\mathbf{X}_1^*[i, i] = \frac{1}{2} + \frac{1}{2}\mathbf{X}_0^*[i, M+1], \ \forall \ i = 1, \cdots, M, \tag{12}$$
$$\mathbf{Y}^*[i, i] = P, \ \forall \ i = 1, \cdots, Q. \tag{13}$$

Claim 3 *The sum of the last column of* $\mathbf{X}_0^*$ *admits a closed form expression:* $\sum_{i=1}^{M}\mathbf{X}_0^*[i, M+1] = 2Q - M$.

Claim 2 can be shown straightforwardly using a contradiction argument. Claim 3 can be derived using the cardinality constraint (10b). Due to space limitations, we refer the readers to [35] for a formal proof.

C. The Proposed Algorithm

In this section, we propose a randomized algorithm that generates an approximate solution for problem (CP1). To highlight ideas, we list below the main steps of the algorithm:

1) Compute the optimal solution $\mathbf{X}^*$ of the relaxed problem (SDP1);
2) Determine the discrete variables $\mathbf{x}_0$ and the set $\mathcal{S}$ according to $\mathbf{X}_0^*$;
3) Fixing $\mathcal{S}$, compute the optimal solution $\mathbf{Y}^*$ of problem (11a);
4) Randomly generate a sample of feasible $\mathbf{w}$'s using $\mathbf{Y}^*$;
5) Select the solution that achieves the best objective value for problem (CP1).

Intuitively, steps 1)–2) select the set of cooperative nodes, while the rest of the steps determine the virtual beamformer among the selected nodes. To formally describe the proposed algorithm, the following definitions are needed. Let $\mathcal{S} \subseteq \mathcal{M}$ be an index set, and let $\mathbf{Y}^*$ denote the corresponding solution for problem (11a). Let us factorize $\mathbf{Y}^*$ as $\mathbf{Y}^* = \mathbf{\Delta}^H \mathbf{\Delta}$. Then define

$$\mathbf{E}_i \triangleq \mathbf{\Delta} \mathbf{C}_{i,1}[\mathcal{S}] \mathbf{\Delta}^H, \quad \mathbf{E} \triangleq \mathbf{\Delta} \mathbf{R}[\mathcal{S}] \mathbf{\Delta}^H.$$

Let us further decompose $\mathbf{E}$ as $\mathbf{E} = \mathbf{U} \mathbf{\Sigma} \mathbf{U}^H$. Then the diagonal matrix $\mathbf{\Sigma}$ can be expressed as

$$\mathbf{\Sigma} = \mathbf{U}^H \mathbf{E} \mathbf{U} = \mathbf{U}^H \mathbf{\Delta} \mathbf{R}[\mathcal{S}] \mathbf{\Delta}^H \mathbf{U}. \tag{14}$$

Let L denote the sample size of the randomization, and let the superscript (l) denote the index of a random sample. Let $\underline{\mathbf{x}}_{(Q)}$ and $\underline{\mathbf{r}}_{(Q)}$ respectively denote the Q-th largest value in the sets $\{\mathbf{X}_0^*[i, M+1]\}_{i=1}^M$ and $\{\mathbf{R}[i,i]\}_{i=1}^M$. The proposed algorithm is described in Table II.

TABLE II
THE PROPOSED ALGORITHM FOR ADMISSION CONTROL

S1: Compute the solution $\mathbf{X}^*$ of problem (SDP1)
S2: Find a set $\mathcal{T}$ of indices such that $|\mathcal{T}| = Q$ and
$\quad \mathcal{T} = \{j : \mathbf{X}_0^*[j, M+1] \geq \underline{\mathbf{x}}_{(Q)}\}$;
$\quad$ S2a: **If** $\quad \text{Tr}[\mathbf{R}[\mathcal{T}]\mathbf{X}_1^*[\mathcal{T}]] \geq \frac{QP}{M}\text{Tr}[\mathbf{R}]$, let $\mathcal{S} = \mathcal{T}$;
$\quad$ S2b: **Else** $\quad$ Let $\mathcal{S} = \{j : \mathbf{R}[j,j] \geq \underline{\mathbf{r}}_{(Q)}\}$;
$\quad$ Let $\bar{\mathcal{S}} = \mathcal{M} \setminus \mathcal{S}$;
S3: Set $\mathbf{x}_0[M+1] = 1$ and $\mathbf{x}_0[j] = 1$, for all $j \in \mathcal{S}$;
$\quad$ Set $\mathbf{x}_0[i] = -1$ for all $i \in \bar{\mathcal{S}}$;
S4: Compute the solution $\mathbf{Y}^*$ of problem (11a) with index set $\mathcal{S}$;
For $\ell = 1, \cdots, L$
S5: Generate $\boldsymbol{\xi}^{(\ell)} \in \{-1, 1\}^Q$ by randomly and independently generating its components from $\{-1, 1\}$;
S6: Compute $t^{(\ell)} = \sqrt{\max_{i \in \mathcal{S}} (\boldsymbol{\xi}^{(\ell)})^T \mathbf{U}^H \mathbf{E}_i \mathbf{U} \boldsymbol{\xi}^{(\ell)}}$;
S7: Compute $\widetilde{\mathbf{w}}^{(\ell)} = \frac{1}{t^{(\ell)}} \mathbf{\Delta}^H \mathbf{U} \boldsymbol{\xi}^{(\ell)}$;
$\quad$ Let $\mathbf{w}^{(\ell)}[\mathcal{S}] = \widetilde{\mathbf{w}}^{(\ell)}$ and $\mathbf{w}^{(\ell)}[\bar{\mathcal{S}}] = \mathbf{0}$;
End For
S8: Compute $\ell^* = \arg_\ell \max (\mathbf{w}^{(\ell)})^H \mathbf{R} \mathbf{w}^{(\ell)}$; let $\mathbf{w}^* = \mathbf{w}^{(\ell^*)}$;

This algorithm can be viewed as a generalization of the algorithm developed by Nemirovski et al. and Zhang et al. [29], [30] for approximating continuous quadratic programs. The novelty of this algorithm lies in steps S2)–S3), in which discrete variables are determined. Below we motivate Step S2). Without rank relaxation, $\mathbf{X}_0 = \mathbf{x}_0 \mathbf{x}_0^H$ is the block variable representing the discrete variables, so it is reasonable to select cooperative groups using the elements of this matrix. Recall that in problem (R1), a node i joins the cooperative group if $\mathbf{x}_0[i]\mathbf{x}_0[M+1] = 1$, which, combined with the definition $\mathbf{X}_0 = \mathbf{x}_0 \mathbf{x}_0^T$, implies that $\mathbf{X}_0[i, M+1] = 1$. Ideally, we should

form the cooperative group by choosing Q elements in the set $\widetilde{\mathcal{S}} = \{i : \mathbf{X}_0^*[i, M+1] = 1, i \in \mathcal{M}\}$. However it is possible that $|\widetilde{\mathcal{S}}| < Q$, as we have relaxed $\mathbf{X}_0[i,j] \in \{-1, 1\}$ to $\mathbf{X}_0[i,j] \in [-1, 1]$. As a result, we instead choose the largest Q elements in the set $\{\mathbf{X}_0^*[i, M+1]\}_{i=1}^M$. Using Claim 3, it is seen that there is a lower bound on the sum of such Q elements: $\sum_{j \in \mathcal{T}} \mathbf{X}_0^*[j, M+1] \geq \frac{(2Q-M)Q}{M}$. This bound will be instrumental in the following performance analysis. Additionally, steps S2a)–S2b) are some technical refinement of the selection procedure that are needed later for the proof of the approximation bounds.

The reason for using steps S4)–S8) to generate the solution $\mathbf{w}^*$ is twofold: *1)* the optimal objective value $v_1^{\text{CP}}(\mathbf{w}^*)$ can be written down analytically; *2)* $\mathbf{w}^*$ is always feasible. See the following two claims for more details regarding these two properties. Formal arguments for these claims are relegated to Appendix A.

Claim 4 *The objective value of the problem* (CP1) *evaluated at a solution* $\mathbf{w}^{(\ell)}$ *is given by*

$$v_1^{\text{CP}}(\mathbf{w}^{(\ell)}) = \frac{1}{(t^{(\ell)})^2} \text{Tr}[\mathbf{R}[\mathcal{S}] \mathbf{Y}^*]. \tag{15}$$

This result implies that $v_1^{\text{CP}}(\mathbf{w}^*) = \frac{1}{\min_\ell (t^{(\ell)})^2} \text{Tr}[\mathbf{R}[\mathcal{S}] \mathbf{Y}^*]$.

Claim 5 *For all* $l = 1, \cdots, L$, *the solution* $\mathbf{x}^{(\ell)} \triangleq [\mathbf{x}_0^H, (\mathbf{w}^{(\ell)})^H]^H$ *is a feasible solution for the problem* (R1). *Moreover,* $\mathbf{w}^{(\ell)}$ *is a feasible solution to* (CP1).

D. The Analysis of the Quality of the Solution

Clearly the solution $\mathbf{w}^*$ generated by the proposed algorithm is only a feasible solution for (CP1). A natural question then is: how good this solution is in terms of the achieved receive SNR. In the following, we will show that the quality of $\mathbf{w}^*$ can be indeed guaranteed. That is, compared with the globally optimal objective v_1^{CP}, there is a *finite* constant $\alpha_1 > 1$ such that: $v_1^{\text{CP}}(\mathbf{w}^*) \geq \frac{1}{\alpha_1} v_1^{\text{CP}}$. The constant α_1 is referred to as the *approximation ratio* of the solution $\mathbf{w}^*$. The smaller the value of α_1, the better the quality of the solution $\mathbf{w}^*$. The following result provides a finite data independent bound for α_1. The proof is relegated to Appendix B.

Theorem 1 *If* $\mathbf{w}^*$ *is generated using the algorithm in Table II, then with high probability, we have* $v_1^{\text{CP}}(\mathbf{w}^*) \geq \frac{1}{\alpha_1} v_1^{\text{CP}}$, *with* α_1 *bounded above by*

$$\alpha_1 \leq \frac{8M\lambda_1(\mathbf{R})}{\sum_{i=1}^M \lambda_i(\mathbf{R})} \ln(5Q) < 8M \ln(5Q). \tag{16}$$

It is interesting to see that for any channel realization, α_1 is finite. Moreover, when the eigenvalues of $\mathbf{R}$ are roughly uniformly distributed, the derived bound is of the order $\mathcal{O}(\ln(Q))$, which is better than the case where $\mathbf{R}$ has a single dominant eigenvalue. Nevertheless, it is important to note that the theoretical approximation ratio obtained above characterizes the quality of the *worst* solutions. It implies that, compared to the global optimal solution or the cardinality constrained problem (CP1)/(R1), the solution generated by the

SDR approach cannot be *arbitrarily bad regardless problem instance*. As we will see later in our numerical results, the practical performance of the algorithm is much better than the derived worst-case bound (16).

IV. JOINT TRANSMIT NODE SCHEDULING AND VB

The previous section considers the case where a subset of nodes are selected for transmission. Such a scheme may not be fair to all the nodes, as the ones that are being excluded from the cooperative set do not get served. In this section we study a generalized formulation that provides fairness among the transmit nodes.

A. Problem Formulation and Complexity Status

Suppose there are *two* orthogonal time slots available for transmission. The problem is to effectively schedule Q transmit nodes to the first slot and the rest $M - Q$ nodes to the second one, in a way that the *minimum* SNR among these two time slots is maximized. In this case, effectively there are two virtual transmitters in the network, and the scheduling scheme promotes fairness among the virtual transmitters.

Let $\mathbf{w}_k \in \mathbb{C}^M$ denote the virtual beamformer used in the k-th time slot, and let $w_{k,i} \in \mathbb{C}$ denote node i's antenna gain in slot k. Suppose that in both time slots the channel matrices $\mathbf{R}$ remains the same. Mathematically, the problem is given by

$$v_2^{\mathrm{CP}} = \max_{\{\mathbf{w}_1, \mathbf{w}_2, \boldsymbol{\alpha}\}} \min_{k=1,2} \quad \mathbf{w}_k^H \mathbf{R} \mathbf{w}_k \tag{CP2}$$
$$\text{s.t.} \ |w_{1,i}|^2 \le a_i P, \ |w_{2,i}|^2 \le (1 - a_i)P, \ i = 1, \cdots, M$$
$$\sum_{i=1}^{M} a_i = Q, \quad a_i \in \{0, 1\}, \ i = 1, \cdots, M.$$

In the following, we will use $v_2^{\mathrm{CP}}(\mathbf{w}_1, \mathbf{w}_2)$ to denote the objective achieved by a feasible tuple $(\mathbf{w}_1, \mathbf{w}_2)$. Note that it is possible to extend (CP2) to the multiple time slot case by using more discrete variables (M discrete variables per slot). However, the resulting analysis will become quite involved. In the remainder of this paper, we will consider the 2-slot case only.

Similar to the case of (R1), let us introduce a homogenizing variable $\ell \in \{-1, 1\}$. Let us define $\mathbf{B}_0$, $\mathbf{C}_{i,0}$ and $\mathbf{C}_{i,1}$ the same way as in (6b)–(6g). Let us further define

$$\tilde{\mathbf{C}}_{i,0} \triangleq \frac{1}{4} \left(\mathbf{e}_i \mathbf{e}_i^T + \mathbf{e}_{M+1} \mathbf{e}_{M+1}^T + \mathbf{e}_i \mathbf{e}_{M+1}^T + \mathbf{e}_{M+1} \mathbf{e}_i^T \right) \in \mathbb{R}^{(M+1) \times (M+1)}$$
$$\mathbf{A}_{i,1} \triangleq \mathrm{blkdg}[\mathbf{C}_{i,0}, \mathbf{C}_{i,1}, \mathbf{0}] \in \mathbb{C}^{(3M+1) \times (3M+1)},$$
$$\mathbf{A}_{i,2} \triangleq \mathrm{blkdg}[\tilde{\mathbf{C}}_{i,0}, \mathbf{0}, \mathbf{C}_{i,1}] \in \mathbb{C}^{(3M+1) \times (3M+1)}$$
$$\widetilde{\mathbf{B}} \triangleq \mathrm{blkdg}[\mathbf{B}_0, \mathbf{0}, \mathbf{0}] \in \mathbb{R}^{(3M+1) \times (3M+1)}$$
$$\mathbf{x} \triangleq [\mathbf{a}^T, \ell, \mathbf{w}_1^T, \mathbf{w}_2^T]^T, \ \mathbf{x}_0 \triangleq [\mathbf{a}^T, \ell], \ \mathbf{x}_1 \triangleq \mathbf{w}_1, \ \mathbf{x}_2 \triangleq \mathbf{w}_2.$$

Then problem (CP2) can be equivalently written as

$$\max_{\mathbf{x}} \min_{k=1,2} \quad \mathbf{x}_k^H \mathbf{R} \mathbf{x}_k \tag{R2}$$
$$\text{s.t.} \ \mathbf{x}^H \mathbf{A}_{i,1} \mathbf{x} \le 1, \ i = 1 \cdots, M$$
$$\mathbf{x}^H \mathbf{A}_{i,2} \mathbf{x} \le 1, \ i = 1 \cdots, M$$
$$\mathbf{x}^H \widetilde{\mathbf{B}} \mathbf{x} = 4Q, \quad \mathbf{x}[i] \in \{-1, 1\}, \ i = 1, \cdots, M+1.$$

The max-min scheduling problem is at least as difficult as its admission control counterpart, as when fixing the group membership, the subproblem of maximizing the per-group SNR is the same as (QP1). To see this, we again fix an index set $\mathcal{S}_1 \subset \mathcal{M}$ with $|\mathcal{S}_1| = Q$, and let $\mathcal{S}_2 = \mathcal{M} \setminus \mathcal{S}_1$. Then the problem (CP2) reduces to two QPs, one for each slot k:

$$\max_{\widetilde{\mathbf{w}}_1 \in \mathbb{C}^{|\mathcal{S}_k|}} \quad \widetilde{\mathbf{w}}_k^H \mathbf{R}[\mathcal{S}_k] \widetilde{\mathbf{w}}_k \tag{17}$$
$$\text{s.t.} \ |\widetilde{\mathbf{w}}_k[i]|^2 \le P, \ i = 1, \cdots, |\mathcal{S}_k|$$

One may observe that each of these problems has the same structure as problem (QP1). It follows from Proposition 1 that solving either one of them is difficult for general $\mathbf{R}$. Interestingly, unlike the admission control problem, the scheduling problem is difficult even when $\mathbf{R}$ is diagonal or is of rank 1. The following result summarizes the complexity status, the proof of which can be found in [35].

Proposition 2 *Solving problem* (CP2) *is strongly NP-hard for general channel matrix* $\mathbf{R}$*, as well as for the special cases when* $\mathbf{R}$ *is either rank 1 or diagonal.*

B. The Proposed Algorithm

The scheduling algorithm we propose below is similar to the one for the admission control problem—we use the solutions of a relaxation of (CP2) to construct approximate solutions. To proceed, define $\mathbf{X}_0 \in \mathbb{R}^{(M+1) \times (M+1)}$, $\mathbf{X}_1, \mathbf{X}_2 \in \mathbb{R}^{M \times M}$, and let $\mathbf{X} = \mathrm{blkdg}[\mathbf{X}_0, \mathbf{X}_1, \mathbf{X}_2]$. The SDR of problem (R2) is given by

$$v_2^{\mathrm{SDP}} = \max_{\mathbf{X} \succeq 0} \min_{k=1,2} \quad \mathrm{Tr}[\mathbf{R} \mathbf{X}_k] \tag{SDP2}$$
$$\text{s.t.} \ \mathrm{Tr}[\mathbf{A}_{i,1} \mathbf{X}] \le 1, \ i = 1 \cdots, M \tag{18a}$$
$$\mathrm{Tr}[\mathbf{A}_{i,2} \mathbf{X}] \le 1, \ i = 1 \cdots, M \tag{18b}$$
$$\mathrm{Tr}[\widetilde{\mathbf{B}} \mathbf{X}] = 4Q \tag{18c}$$
$$\mathbf{X}[i, i] = 1, i = 1, \cdots, M+1.$$

Similarly, for a fixed index set $\mathcal{S}_k$, the SDR of problem (17) is

$$\max_{\mathbf{Y}_k \succeq 0} \quad \mathrm{Tr}[\mathbf{R}[\mathcal{S}_k] \mathbf{Y}_k] \tag{19a}$$
$$\text{s.t.} \ \mathbf{Y}_k[i, i] \le P, \ i = 1, \cdots, |\mathcal{S}_k|. \tag{19b}$$

To formally describe the proposed algorithm, we need to introduce a few definitions that are similar to those in Section III-C. Let $\mathcal{S}_k \subseteq \mathcal{M}$ be any index set, and let $\mathbf{Y}_k^*$ denote the corresponding solution for problem (19a). Decompose $\mathbf{Y}_k^*$ by $\mathbf{Y}_k^* = \boldsymbol{\Delta}_k^H \boldsymbol{\Delta}_k$, for $k = 1, 2$. Define the following

$$\mathbf{E}_{i,k} \triangleq \boldsymbol{\Delta}_k \mathbf{C}_{i,1}[\mathcal{S}_k] \boldsymbol{\Delta}_k^H, \ \mathbf{E}_k \triangleq \boldsymbol{\Delta}_k \mathbf{R}[\mathcal{S}_k] \boldsymbol{\Delta}_k^H.$$

Let us decompose $\mathbf{E}_k$ using its eigendecomposition: $\mathbf{E}_k = \mathbf{U}_k \boldsymbol{\Sigma}_k \mathbf{U}_k^H$.

The proposed algorithm for joint scheduling and VB follows almost identical steps of the admission control algorithm in Table II, with only minor changes. Below we list the main steps of the proposed algorithm.

1) Compute the optimal solution $\mathbf{X}^*$ of problem (SDP2), such that all the constraints in (18a) and (18b) are tight

^{1}.

2) Find the set $\mathcal{S}_1$ with $|\mathcal{S}_1| = Q$ by $\mathcal{S}_1 = \{j : \mathbf{X}_0^*[j, M + 1] \geq \underline{\mathbf{x}}_{(Q)}\}$; Set $\mathcal{S}_2 = \mathcal{M} \setminus \mathcal{S}_1$.
3) For $k = 1, 2$, compute the solution $\mathbf{Y}_k^*$ of problem (17) with index set $\mathcal{S}_k$.
4) Perform twice the randomization steps identical to those in Step 5)–Step 8) in Table II, replacing $\{\mathbf{U}, \boldsymbol{\Delta}, \mathbf{E}, \mathbf{E}_i\}$ with $\{\mathbf{U}_k, \boldsymbol{\Delta}_k, \mathbf{E}_k, \mathbf{E}_{i,k}\}$, $k = 1, 2$; obtain samples $\{\mathbf{w}_1^{(\ell)}, \mathbf{w}_2^{(\ell)}\}_{\ell=1}^L$.
5) Select the best sample by $\ell^* = \arg\max_\ell \min_{k=1,2}\{(\mathbf{w}_k^{(\ell)})^H \mathbf{R} \mathbf{w}_k^{(\ell)}\}$.

Let us pause to discuss the differences between the above SDR algorithm and its counterpart for admission control. After deciding the set $\mathcal{S}_1$ and $\mathcal{S}_2$, *two* separate randomization procedures are needed, one for each set $\mathcal{S}_1$ and $\mathcal{S}_2$. Intuitively, after deciding $\mathcal{S}_1$ and $\mathcal{S}_2$, we have completed the scheduling task. What remains to be done is to perform VB for the nodes allocated to each slot. This is the goal of the randomization procedure. Moreover, the best sample ℓ^* is selected according to the max-min SNR criteria, which promotes fairness among the two virtual beamformers.

Using the argument identical to that presented in Claim 4 and Claim 5, we can verify that for all ℓ, $[\mathbf{x}_0, \mathbf{w}_1^{(\ell)}, \mathbf{w}_2^{(\ell)}]$ must be feasible for problem (R2). Moreover, the optimal value $v_2^{\text{CP}}(\mathbf{w}^*)$ can be expressed in closed form

$$(\mathbf{w}_k^*)^H \mathbf{R} \mathbf{w}_k^* = \frac{1}{\min_\ell (t_k^{(\ell)})^2} \text{Tr}\left[\mathbf{R}[\mathcal{S}]\mathbf{Y}_k^*\right], \quad k = 1, 2.$$

Let α_2 be the approximation ratio for a solution $(\mathbf{w}_1^*, \mathbf{w}_2^*)$, defined as $v_2^{\text{CP}}(\mathbf{w}_1^*, \mathbf{w}_2^*) \geq \frac{1}{\alpha_2} v_2^{\text{CP}}$. Using techniques similar to the proof of Theorem 1, one can show that α_2 can be bounded by

$$\alpha_2 \leq \frac{8M\lambda_1(\mathbf{R})}{\min\{Q, M-Q\}\lambda_M(\mathbf{R})} \ln(12\max\{Q, M-Q\}). \tag{20}$$

However, compared to Theorem 1, the above bound is less powerful since it is *dependent* on the channel realization. The proof of this result can be found in [35]

V. JOINT RELAY GROUPING AND VB

In this section we show that the approaches developed in the previous sections are also applicable in relay networks. The problem here is to select a set of relays to form a virtual multi-antenna system, and at the same time design their virtual beamformers for signal relaying.

A. Problem Formulation and Complexity Status

Suppose Q out of M relays are to be selected for transmission. Using the system model described in Section II, we can formulate the joint relay grouping and beamforming problem

as follows:

$$v_3^{\text{CP}} = \max_{\mathbf{w}, \mathbf{a}} \quad \frac{\mathbf{w}^H \mathbf{S} \mathbf{w}}{\sigma_n^2 + \mathbf{w}^H \mathbf{F} \mathbf{w}} \tag{CP3}$$
$$\text{s.t.} \quad |w_i|^2\left(P_0 \mathbb{E}[|f_i|^2] + \sigma_\nu^2\right) \leq a_i P, \quad i = 1, \cdots, M,$$
$$\sum_{i=1}^M a_i = Q, \quad a_i \in \{0, 1\}, \quad i = 1, \cdots, M.$$

where the objective is the receive SNR. This problem can be compactly written as

$$v_3^{\text{QP}} = \max_{\mathbf{x}} \quad \frac{\mathbf{x}^H \widetilde{\mathbf{S}} \mathbf{x}}{\sigma_n^2 + \mathbf{x}^H \widetilde{\mathbf{F}} \mathbf{x}} \tag{R3}$$
$$\text{s.t.} \quad \mathbf{x}^H \mathbf{D}_i \mathbf{G}_i \mathbf{x} \leq 1, \quad i = 1, \cdots, M, \tag{21a}$$
$$\mathbf{x}^H \mathbf{B} \mathbf{x} = 4Q, \quad \mathbf{x}[i] \in \{-1, 1\}, \quad i = 1, \cdots, M+1. \tag{21b}$$

where $\mathbf{D}_i$, $\mathbf{B}$ and $\mathbf{x}$ are given in (6b)–(6g), and the $(2M + 1) \times (2M + 1)$ matrices $\widetilde{\mathbf{F}}$, $\widetilde{\mathbf{S}}$ and $\mathbf{G}_i$ are defined as

$$\widetilde{\mathbf{S}} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S} \end{bmatrix}, \quad \widetilde{\mathbf{F}} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{F} \end{bmatrix},$$
$$\mathbf{G}_i \triangleq \begin{bmatrix} \mathbf{I_{M+1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{e}_i \mathbf{e}_i^T\left(P_0\mathbb{E}[|f_i|^2] + \sigma_\nu^2\right) \end{bmatrix}, \quad i = 1, \cdots, M.$$

As always, we first analyze the computational complexity of joint relay grouping and VB problem (CP3). The following theorem shows that solving this problem is generally NP-hard. We refer the readers to Appendix C for proof details.

Proposition 3 *Solving* (CP3) *is NP-hard in general.*

It is worth noting that problem (CP3) is easy when there is no correlation between the channels, or equivalently when both $\mathbf{S}$ and $\mathbf{F}$ are diagonal. The reason is that for a fixed value of $t \geq 0$, solving the following feasibility problem is easy

$$\frac{\mathbf{w}^H \mathbf{S} \mathbf{w}}{\sigma_n^2 + \mathbf{w}^H \mathbf{F} \mathbf{w}} \geq t$$
$$|w_i|^2\left(P_0\mathbb{E}(|f_i|^2) + \sigma_v^2\right) \leq a_i P, \quad \forall i$$
$$\sum_{i=1}^M a_i = Q, \quad a_i \in \{0, 1\}.$$

By performing a bisection on t, we can obtain the optimal solution for (CP3).

Similar to problem (CP2), one can consider the relay scheduling problem over two time slots. Mathematically, this problem can be formulated as

$$\max_{\{\mathbf{w}_1, \mathbf{w}_2, \boldsymbol{\alpha}\}} \min_{k=1,2} \quad \frac{\mathbf{w}_k^H \mathbf{S} \mathbf{w}_k}{\sigma_n^2 + \mathbf{w}_k^H \mathbf{F} \mathbf{w}_k} \tag{CP4}$$
$$\text{s.t.} \quad |w_{k,i}|^2\left(P_0\mathbb{E}[|f_i|^2] + \sigma_v^2\right) \leq a_i P, \quad i = 1, \cdots, M, \ k = 1$$
$$\sum_{i=1}^M a_i = Q, \quad a_i \in \{0, 1\}, \quad i = 1, \cdots, M.$$

It turns out that this problem is NP-hard even for diagonal channel matrices (see [35] for the proof).

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^{1}To find a solution required by step 1), we can start by any optimal solution $\mathbf{X}^*$ of (SDP2), and increase the diagonal elements of $\mathbf{X}_1^*$ or $\mathbf{X}_2^*$ until all the constraints in (18a) and (18b) are satisfied.

Proposition 4 *For diagonal channel matrices, problem* (CP4) *is NP-hard.*

B. The SDR Algorithm

Again let us define $\mathbf{X} \triangleq \mathbf{x}\mathbf{x}^H$. The SDR of the reformulated problem (R3) is given by

$$v_3^{\text{SDP}} \triangleq \max_{\mathbf{X} \succeq 0} \quad \frac{\text{Tr}[\widetilde{\mathbf{S}}\mathbf{X}]}{\sigma_n^2 + \text{Tr}[\widetilde{\mathbf{F}}\mathbf{X}]} \qquad (\text{SDP3})$$

$$\text{s.t.} \quad \text{Tr}[\mathbf{D}_i\mathbf{G}_i\mathbf{X}] \leq 1, \; i = 1, \cdots, M, \qquad (23a)$$

$$\text{Tr}[\mathbf{B}\mathbf{X}] = 4Q, \qquad (23b)$$

$$\mathbf{X}[i,i] = 1, \; i = 1, \cdots, M+1. \qquad (23c)$$

Let $\mathbf{X}^*$ denote the optimal solution of this problem. Clearly, we must have $\text{Tr}[\widetilde{\mathbf{S}}\mathbf{X}^*] = v_3^{\text{SDP}}(\sigma_n^2 + \text{Tr}[\widetilde{\mathbf{F}}\mathbf{X}^*])$. Moreover, $\mathbf{X}^*$ must be the optimal solution of the following problem, with an optimal objective value $v_3^{\text{SDP}}\sigma_n^2$

$$\max_{\mathbf{X} \succeq 0} \quad \text{Tr}[(\widetilde{\mathbf{S}} - v_3^{\text{SDP}}\widetilde{\mathbf{F}})\mathbf{X}] \qquad (\text{SDP4})$$

$$\text{s.t.} \quad (23a) - (23c).$$

This problem is a relaxation of the following QP

$$\max_{\mathbf{x}} \quad \mathbf{x}^H(\widetilde{\mathbf{S}} - v_3^{\text{SDP}}\widetilde{\mathbf{F}})\mathbf{x}$$

$$\text{s.t.} \quad (21a) - (21b).$$

The similarity between problem (SDP4) and the relaxed node selection problem (SDP1) suggests a natural two-step approach to obtain a feasible solution $\mathbf{x}^*$ of (R3):

1) Solve problem (SDP3), obtain v_3^{SDP}.
2) Obtain $\mathbf{x}^* = [\mathbf{x}_0^*, \mathbf{w}^*]$ by applying the algorithm in Table II, with the matrices $\widetilde{\mathbf{R}}$, $\mathbf{D}_i$ replaced by $\widetilde{\mathbf{S}} - v_3^{\text{SDP}}\widetilde{\mathbf{F}}$, $\mathbf{D}_i\mathbf{G}_i$, respectively.

Using the same argument given in Claim 5, we can show that the resulting vector $\mathbf{x}^*$ must be feasible for our relay selection problem.

Unfortunately, at this point we are still unable to derive a finite (data independent) approximation bound for this SDR algorithm. The main difficulty is that, unlike the case of (SDP1), the coefficient matrix $\widetilde{\mathbf{S}} - v_3^{\text{SDP}}\widetilde{\mathbf{F}}$ in the objective of (SDP4) is no longer a positive definite matrix (in fact it is indefinite). This implies that some key properties (e.g., Claim 2) no longer hold true in this case. Nevertheless, in our numerical experiments, we did observe that the proposed SDR algorithm generates high quality approximate solutions for problem (CP3).

VI. NUMERICAL RESULTS

In this section, we present numerical results to evaluate the proposed algorithms. For all simulations presented, we choose the total number of randomization to be $L = 200$.

A. Grouping for Admission Control and Scheduling

We evaluate the performance of the proposed algorithm for joint admission control and beamforming. Let $N > 0$ be a constant, which represents the number of antennas at the receiver. We generate a single cell network with radius 500 meters, and with the BS/receiver located in the cell center. The location for the transmit nodes are randomly generated within the cell, and are at least 100 meters away from the receiver. Let d_i denote the distance between node i and the receiver and let $\mathbf{h}_n \in \mathbb{C}^{M \times 1}$ denote the channel vector for the path between the i-th node and the n-th receive antenna. We model the i-th entry of $\mathbf{h}_n$ as a zero mean circularly symmetric complex Gaussian variable with variance (per real/imaginary dimension) given by $(200/d_i)^{3.5} L_i$, where $10 \log 10(L_i) \sim \mathcal{N}(0, 64)$ is a real Gaussian random variable modeling the shadowing effect. Set $\mathbf{R} = \sum_{n=1}^{N} \mathbf{h}_n\mathbf{h}_n^H$. Suppose the network has $M = 50$ transmit nodes, with each node having the same transmit power $P = -10$ dBW. The proposed algorithm in Section III (abbreviated as Alg.1) is compared with a sparse PCA (SPCA) based algorithm, whose main steps are listed below: i) Approximately find a *sparse* principal component of $\mathbf{R}$ with Q non-zero entries (denoted as $\widehat{\mathbf{w}}$), using the backward-forward algorithm proposed in [36]; ii) normalize $\widehat{\mathbf{w}}$ by a constant ϵ so that all the individual power constraints are satisfied. In its first step, this algorithm tries to find the set of nodes that, when replacing the individual power constraints with a total power constraint, can provide the (approximately) best averaged receive SNR.

Table III demonstrates the maximum, the minimum and the averaged ratios achieved for running the algorithms over 500 independent random generations of the channel. Note that the approximation ratio for a solution $\mathbf{w}^*$ is calculated by $\alpha = \frac{v_1^{\text{CP}}(\mathbf{w}^*)}{v_1^{\text{SDP}}(\mathbf{X}^*)}$.

In Fig. 4–5 we plot the performance of the algorithms for different sizes of the network. For a given network size, we choose $Q = 10$ and let $N = 5$. For each network (Q, M) pair, the algorithm is run for 500 independent realizations of the network. We again plot the maximum, the minimum and the averaged approximation ratios achieved among those 500 realizations. We see that the proposed algorithm achieves very low worst-case approximation ratio, which suggests that high SNR performance is obtained for almost all Monte Carlo runs. In Fig. 6, a similar experiment is conducted with the number of transmit nodes fixed at $M = 50$, but with varying cooperative group size $Q \in [6, 15]$.

In Table IV, we show the performance of the proposed algorithm in a network of $M = 30$ transmit nodes, for the max-min scheduling problem (abbreviated as Alg.2). The proposed algorithm is compared with the following two heuristic benchmarks: 1) randomly partition the nodes into two groups of size Q and $M - Q$, for each of which we solve a PCA followed by a normalization step (abbreviated as R-PCA); 2) randomly partition the nodes into two groups of size Q and $M - Q$, and obtain a solution $\mathbf{w}_1^*$ and $\mathbf{w}_2^*$ following steps S4)–S8) in Table II (abbreviated as R-SDR). Fig. 7 illustrates the effectiveness of Alg.2 in balancing the receive SNRs for different slots. We plot the receive SNRs computed by Alg.2 in both slots (referred to the *favorable/unfavorable* slot in the figure) during 20 Monte Carlo runs of the algorithm. For comparison, the plots are overlaid with those obtained by running Alg.1 in the same network. For the latter case, the

TABLE III
APPROXIMATION RATIO OF THE PROPOSED AND SPCA ALGORITHMS

	Alg.1 Min	Alg.1 Mean	Alg.1 Max	SPCA Min	SPCA Mean	SPCA Max
$N = 5, Q = \lceil M/3 \rceil$	1.12	1.20	1.31	2.88	5.85	9.51
$N = 10, Q = \lceil M/3 \rceil$	1.11	1.21	1.51	2.48	6.74	10.62
$N = 15, Q = \lceil M/3 \rceil$	1.11	1.20	1.43	2.71	6.79	11.27
$N = 5, Q = M/2$	1.09	1.14	1.49	3.04	6.97	12.13
$N = 10, Q = M/2$	1.08	1.18	1.72	2.81	7.94	14.08
$N = 15, Q = M/2$	1.07	1.17	1.87	3.86	8.28	13.74
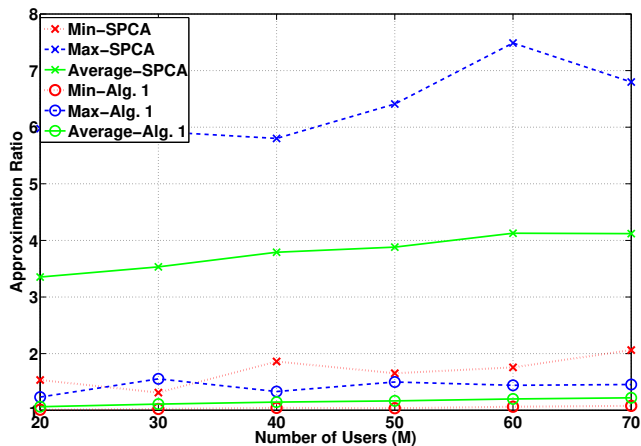
Fig. 4. Approximation ratio for admission control with different network sizes. $M \in [10, 20, 30, 40, 50, 60, 70]$, $Q = 10$, $P = -10$dBW, $N = 5$.
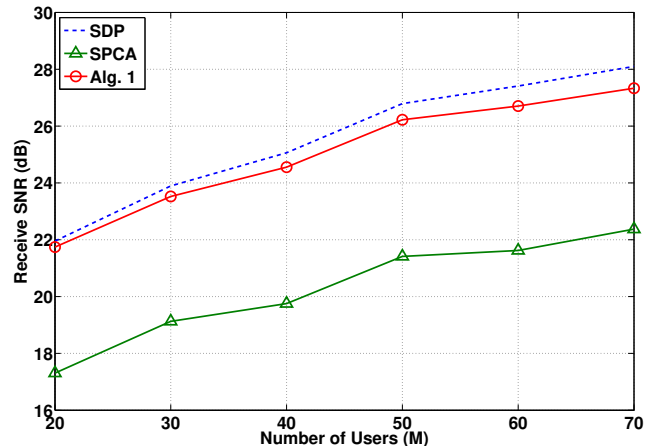
Fig. 5. Receive SNR for admission control with different network sizes. $M \in [10, 20, 30, 40, 50, 60, 70]$, $Q = 10$, $P = -10$dBW, $N = 5$.
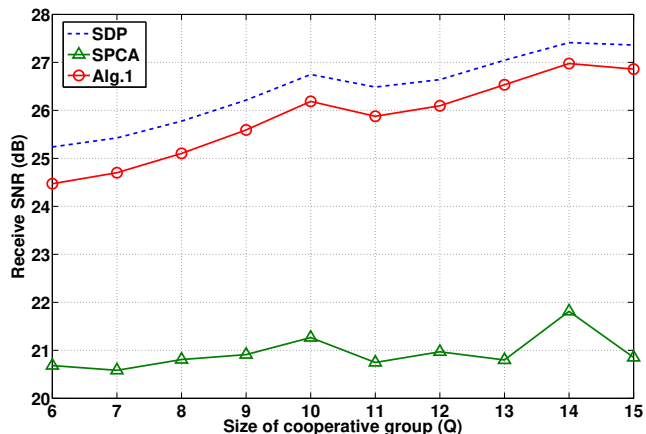

Fig. 6. Receive SNR for admission control with different cooperative group sizes. $Q \in [6, 15]$, $M = 50$, $P = -10$dBW, $N = 5$.
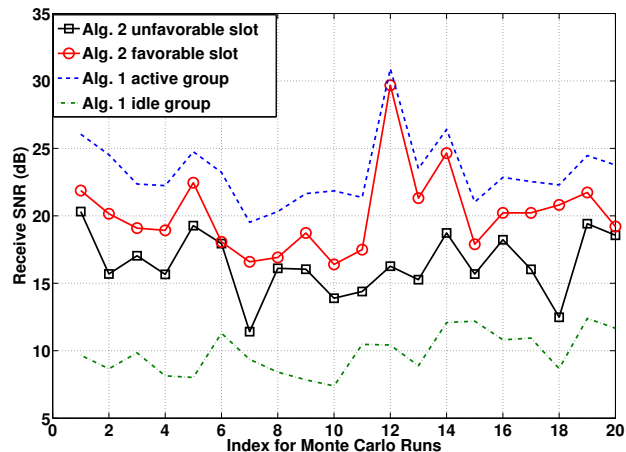

Fig. 7. Receive SNRs in different slots.

receive SNRs are shown for both the active nodes and the *idle nodes*. For the set of idle nodes that are excluded from the cooperative group, their virtual beamformer is computed using steps S4)–S8) in Table II, with $\mathcal{S}$ replaced by $\bar{\mathcal{S}}$, and with all the matrices $\mathbf{U}$, $\mathbf{E}_i$ and $\boldsymbol{\Delta}$ computed using $\bar{\mathcal{S}}$. Fig. 7 shows that for the admission control formulation, when the idle nodes are offered a chance of being served, they can only achieve very low SNR. On the contrary, the scheduling formulation results in more balanced SNRs for both slots.

In the aforementioned numerical results, the proposed SDR algorithms can clearly deliver high quality approximate solutions (in terms of both the averaged and the worse case performance) to the joint admission control/scheduling and VB problem.

B. Grouping for Relay Selection in Relay Networks

In this section, we numerically evaluate the performance of the SDR algorithm for solving the joint relay selection and VB problem (abbreviated as Alg.3).

For the relay network, the channel covariances are generated as follows [11], [12]. Assume f_i can be written by $f_i = \hat{f}_i + \tilde{f}_i$, where $\hat{f}_i$ is the mean of f_i and $\tilde{f}_i$ is a zero-mean random variable. Assume that $\tilde{f}_i$, $\tilde{f}_j$ are independent for $i \neq j$, and choose $\hat{f}_i = e^{j\theta_i}/\sqrt{\eta_{f_i}}$ and $\text{var}(\tilde{f}) = \eta_{f_i}/(1 + \eta_{f_i})$, where θ_i is a uniform random variable on the interval $[0, 2\pi]$, and η_{f_i} is a parameter that determines the level of uncertainty in the channel coefficient f_i. Similarly, let $g_i = \hat{g}_i + \tilde{g}_i$, with $\hat{g}_i$ and $\tilde{g}_i$ defined similarly as $\hat{f}_i$ and $\tilde{f}_i$. The justification of this channel

TABLE IV
APPROXIMATION RATIO OF THE ALGORITHM FOR SCHEDULING

	Alg. 2 Mean	Alg. 2 Max	R-PCA Mean	R-PCA Max	R-SDR Mean	R-SDR Max
N=5, $Q = \lceil M/4 \rceil$	1.88	4.06	9.76	24.84	3.23	10.14
N=10, $Q = \lceil M/4 \rceil$	2.77	9.55	10.87	28.56	5.68	15.74

model can be found in [11], [12]. In our simulations, $\{\eta_{f_i}\}$ and $\{\eta_{g_i}\}$ are generated randomly from -10 dB to 10 dB, accounting for different uncertainty levels for different nodes.

Our SDR algorithm is compared against the following three algorithms:

- *Random-GED*: This is a variant of the generalized eigenvalue decomposition (GED) based algorithm proposed in [11, Section IV-B]. In its original form, all the relays are utilized, and the algorithm computes an approximate solution of the max-SNR problem by performing a PCA for the matrix $\mathbf{S}^{-1}\mathbf{F}$, followed by a normalization step to ensure the individual power constraints. To incorporate the selection of relays, we first *randomly* select Q out of M relays, and then perform the PCA and normalization steps.
- *Random-SDR*: In this algorithm, we first *randomly* select Q out of M relays in the network, and then perform the two-step SDR algorithm with the fixed node grouping (see the algorithm description in Section V).
- *Greedy algorithm*: This algorithm largely follows from the one proposed in [23], in which nodes are added to the cooperation set successively and greedily as long as it can improve the received SNR level, or the required group size is less than Q.

In Fig. 8–9, the approximation ratio and the achieved receive SNR of different algorithms are plotted against the size of the network. Fig. 10–11 show the performance of the algorithms when we increase the relays' transmission powers. In these two sets of figures we set $P_0 = 0$ dBW, $Q = \frac{M}{2}$, and set the total transmit power of the relays $P_{\text{tot}} = 10$ dBW. The curves labeled by "SDP" in Fig. 9 and Fig. 11 represent the upperbounds of the achievable receive SNR which are obtained by solving the relaxed problem (SDP3). We see that the proposed SDR algorithm is close to the optimal solution across all power levels and all network sizes, while the other three algorithms perform notably worse in these experiments. Furthermore, the proposed algorithm has significantly better "worst case" performance than the Random-SDR algorithm (see Fig. 8 and Fig. 10).

VII. FINAL REMARKS

This paper addresses the joint node grouping and the virtual beamformer design problem under different network setup. We formulate this problem as a cardinality constrained quadratic optimization problem. Unfortunately, the resulting optimization problem is computationally intractable as shown in Table V below.

Given the NP-hardness of the problem, we develop several novel approximation algorithms based on the semidefinite programming relaxation (SDR) technique. The quality of the approximate solutions is evaluated both theoretically and via numerical simulation. Somewhat surprisingly, we are able to

prove that the resulting SDR algorithm for admission control has a guaranteed approximation performance in terms of the gap to global optimality, regardless of channel realizations. Such theoretical analysis lends strong support to the practical performance of the proposed algorithms. Indeed, their effectiveness has been confirmed in our extensive numerical experiments.

In closing, we suggest several directions of future research. First, it will be interesting to extend the theoretical analysis for the approximation bounds to the scheduling and the relay selection problems. Second, we expect that our SDR approach can be applied and extended to some new application scenarios. For example, in the admission control problem, to mitigate the interference to a neighboring co-existing system, one could further impose a *total interference* constraint of the form $\mathbb{E}[|\mathbf{w}^H\mathbf{g}|^2] \leq I$ to the selected group of nodes, where $\mathbf{g}$ represents the channel between the transmit nodes and the BS of the neighboring system. Additionally, the proposed SDR approach may prove to be useful in designing future multi-cell cellular systems, where not all, but a subset of BSs can jointly transmit and receive signals for a given mobile user (see the recent work [19], [20], [37] on the partial CoMP technique).

APPENDIX

A. Proof of Claims 4 and 5

Claim 4 is due to the following chain of equalities

$$v_1^{\text{CP}}(\mathbf{w}^{(\ell)}) = (\mathbf{w}^{(\ell)})^H \mathbf{R} \mathbf{w}^{(\ell)} = \frac{1}{(t^{(\ell)})^2}(\boldsymbol{\xi}^{(\ell)})^T \mathbf{U}^H \boldsymbol{\Delta} \mathbf{R}[\mathcal{S}] \boldsymbol{\Delta}^H \mathbf{U} \boldsymbol{\xi}^{(\ell)}$$

$$\stackrel{(a)}{=} \frac{1}{(t^{(\ell)})^2}(\boldsymbol{\xi}^{(\ell)})^T \boldsymbol{\Sigma} \boldsymbol{\xi}^{(\ell)} \stackrel{(b)}{=} \frac{1}{(t^{(\ell)})^2}\text{Tr}[\boldsymbol{\Sigma}] \stackrel{(c)}{=} \frac{1}{(t^{(\ell)})^2}\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*] \tag{24}$$

where (a) and (c) are from (14); (b) is from the fact that $\boldsymbol{\Sigma}$ is diagonal, and the diagonal elements of $\boldsymbol{\xi}^{(\ell)}(\boldsymbol{\xi}^{(\ell)})^T$ are all 1.

To argue the feasibility condition claimed in Claim 5, we first show that the constraint $(\mathbf{x}^{(\ell)})^H \mathbf{D}_i \mathbf{x}^{(\ell)} \leq 1$ is always satisfied for all $i \in \mathcal{M}$. To see this, we consider the following two cases (we omit the superscript (ℓ) for notational simplicity).

Case i): If $i \in \mathcal{S}$, then we have

$$\mathbf{x}^H \mathbf{D}_i \mathbf{x} = (\mathbf{w})^H \mathbf{C}_{i,1} \mathbf{w} + (\mathbf{x}_0)^H \mathbf{C}_{i,0} \mathbf{x}_0$$

$$= (\mathbf{w}[\mathcal{S}])^H \mathbf{C}_{i,1}[\mathcal{S}] \mathbf{w}[\mathcal{S}] + (\mathbf{x}_0)^H \mathbf{C}_{i,0} \mathbf{x}_0$$

$$= \frac{1}{t^2} \boldsymbol{\xi}^T \mathbf{U}^H \boldsymbol{\Delta} \mathbf{C}_{i,1}[\mathcal{S}] \boldsymbol{\Delta}^H \mathbf{U} \boldsymbol{\xi} + \frac{1}{2}(1 - \mathbf{x}_0[i]\mathbf{x}_0[M+1])$$

$$\stackrel{(a)}{=} \frac{1}{t^2} \boldsymbol{\xi}^T \mathbf{U}^H \mathbf{E}_i \mathbf{U} \boldsymbol{\xi} \stackrel{(b)}{\leq} 1$$

where (a) is from the definition of $\mathbf{E}_i$ and from the fact that $\mathbf{x}_0[M+1] = 1$ and for all $i \in \mathcal{S}$, $\mathbf{x}_0[i] = 1$; (b) is from the definition of t in Step S6) of the algorithm.

Case ii): If $i \in \bar{\mathcal{S}}$, then from Step S7) of the algorithm, we have $w_i = 0$. Using the fact that $\mathbf{C}_{i,1} = \mathbf{e}_i \mathbf{e}_i^T \frac{1}{P}$, we have: $\mathbf{x}^H \mathbf{D}_i \mathbf{x} = (\mathbf{w})^H \mathbf{C}_{i,1} \mathbf{w} = |w_i|^2 \frac{1}{P} = 0$. It is straightforward to
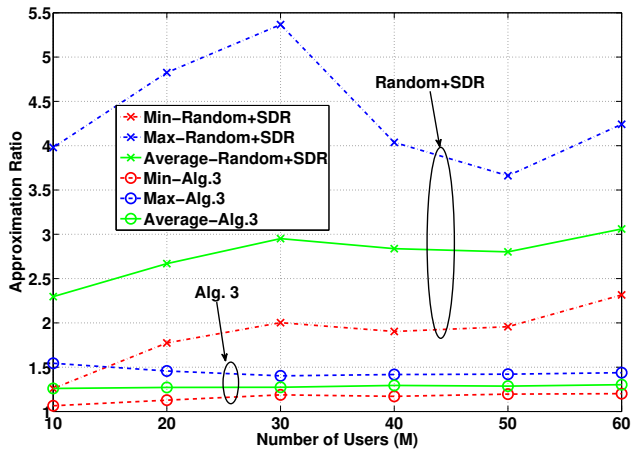
Fig. 8. Approximation ratio with different network sizes. $M \in [10, 20, 30, 40, 50, 60]$, $Q = (\frac{M}{2})$, $P_{\text{tot}} = 10$dBW, 100 independent realizations of channel.
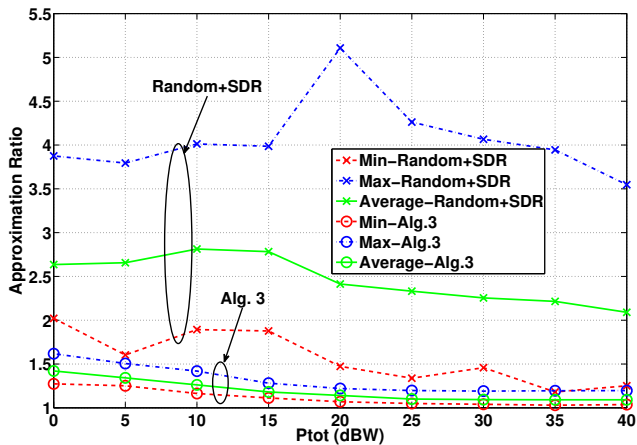

Fig. 9. Averaged received SNR for different algorithms. $M \in [10, 20, 30, 40, 50, 60]$, $Q = (\frac{M}{2})$, $P_{\text{tot}} = 10$dBW, 100 independent realizations of channel.

Fig. 10. Approximation ratio for different algorithms. $M = 30$, $Q = (\frac{M}{2})$, $P_{\text{tot}} = [0, 5, 10, 15, 20, 25, 30, 35, 40]$ dBW, 100 independent realizations of channel.
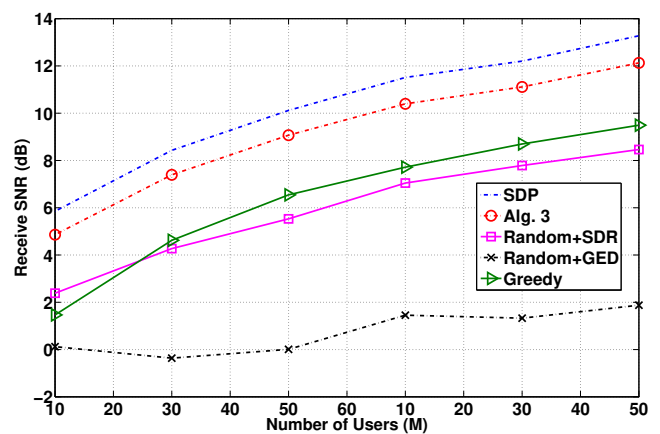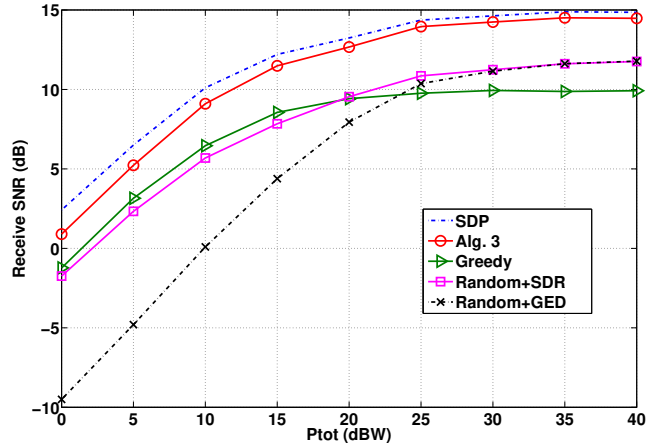

Fig. 11. Averaged receive SNR for different algorithms. $M = 30$, $Q = (\frac{M}{2})$, $P_{\text{tot}} = [0, 5, 10, 15, 20, 25, 30, 35, 40]$ dBW, 100 independent realizations of channel.

TABLE V
SUMMARY OF COMPLEXITY RESULTS

	Admission Control			Scheduling (2-Slot)		
Channel R	General	Diagonal	Rank 1	General	Diagonal	Rank 1
VMIMO	NP-Hard	Ploy. Time Solvable	Ploy. Time Solvable	NP-Hard	NP-Hard	NP-Hard
VMIMO-Relay	NP-Hard	Ploy. Time Solvable	Unknown	NP-Hard	NP-Hard	Unknown

see that the cardinality constraint $\mathbf{x}^H \mathbf{B} \mathbf{x} = 4Q$ is satisfied for each solution $\mathbf{w}^{(\ell)}$, as this constraint mandates that $M - Q$ nodes do not transmit.

In summary, we conclude that $\mathbf{x}$ is feasible for the problem (R1). Due to the equivalence between problems (R1) and (CP1), $\mathbf{w}$ is also feasible for the latter problem.

B. Proof of Theorem 1

Proof: Observe that it is sufficient to show that with high probability $v_1^{\text{CP}}(\mathbf{w}^{(\ell)}) \geq \frac{1}{\alpha_1} v_1^{\text{SDP}}$. Below we first show that for any $\ell = 1, \cdots, L$, there exists a finite constant $\alpha_1 > 1$ satisfying: $\text{Prob}\left(v_1^{\text{CP}}(\mathbf{w}^{(\ell)}) \geq \frac{1}{\alpha_1} v_1^{\text{SDP}}\right) \geq \delta > 0$, or equivalently,

$$\text{Prob}\left(\frac{1}{(t^{(\ell)})^2}\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*] \geq \frac{1}{\alpha_1}\text{Tr}[\mathbf{R}\mathbf{X}_1^*]\right) \geq \delta > 0. \quad (25)$$

That is, with positive probability, the solution $\mathbf{w}^{(\ell)}$ generated by the proposed algorithm is at least as good as $\frac{1}{\alpha_1}$ fraction of v^{SDP}. If this is indeed true, it follows that the probability that the solution $\mathbf{w}^*$ achieves an objective that is at least $\frac{1}{\alpha_1}$ of v_1^{SDP} is given by

$$\text{Prob}\left(\max_\ell v_1^{\text{CP}}(\mathbf{w}^{(\ell)}) \geq \frac{1}{\alpha_1}v_1^{\text{SDP}}\right)$$
$$= 1 - \text{Prob}\left(\max_\ell v_1^{\text{CP}}(\mathbf{w}^{(\ell)}) < \frac{1}{\alpha_1}v_1^{\text{SDP}}\right)$$
$$= 1 - \prod_{l=1}^{L}\text{Prob}\left(v_1^{\text{CP}}(\mathbf{w}^{(\ell)}) < \frac{1}{\alpha_1}v_1^{\text{SDP}}\right) \geq 1 - (1-\delta)^L.$$

Clearly, this probability approaches 1 exponentially as L becomes large.

Below we will show (25). The analysis is divided into the following three steps.

Step 1): Let $\beta > 0$ be a constant. We first show that when $\frac{\beta}{\alpha_1}$ is small enough, with zero probability the event $\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*] \leq \frac{\beta}{\alpha_1}\text{Tr}[\mathbf{R}\mathbf{X}_1^*]$ happens.

To this end, we first lower bound $\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*]$. We have the following two cases.

Case i) Suppose $\mathcal{S} = \mathcal{T} = \{j : \mathbf{X}_0^*[j, M+1] \geq \underline{\mathbf{x}}_Q\}$, then we have

$$v_2^{\text{SDP}} = \text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*] \geq \text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{X}_1^*[\mathcal{S}]]$$

where the inequality is from that fact $\mathbf{X}_1^*[\mathcal{S}]$ is a feasible solution to the problem (11a), and that $\mathbf{Y}^*$ is the optimal solution for that problem. From Step S2) of the algorithm, we must have

$$\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*] \geq \frac{QP}{M}\text{Tr}[\mathbf{R}]. \quad (26)$$

Case ii) Suppose $\mathcal{S} = \{j : \mathbf{R}[i,i] \geq \underline{\mathbf{r}}_Q\}$. Then we have

$$\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*] \overset{(i)}{\geq} P\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{I}_Q] \overset{(ii)}{\geq} \frac{QP}{M}\text{Tr}[\mathbf{R}] \quad (27)$$

where (i) is again from that fact $P\mathbf{I}_Q$ is a feasible solution to the problem (11a), and that $\mathbf{Y}^*$ is the optimal solution for that problem; (ii) is from the fact that each $i \in \mathcal{S}$ is among the largest Q elements in the set $\{\mathbf{R}[i,i]\}_{i=1}^Q$, which leads to $\sum_{i \in \mathcal{S}} \mathbf{R}[i,i] \geq \frac{Q}{M}\sum_{i=1}^M \mathbf{R}[i,i] = \frac{Q}{M}\text{Tr}[\mathbf{R}]$.

We then upper bound $\text{Tr}[\mathbf{R}\mathbf{X}_1^*]$. By a trace inequality for the product of two semi-definite matrices, we have that [38]

$$\text{Tr}[\mathbf{R}\mathbf{X}_1^*] \leq \sum_{i=1}^M \lambda_i(\mathbf{X}_1^*)\lambda_i(\mathbf{R}) \leq \lambda_1(\mathbf{R})\sum_{i=1}^M \lambda_i(\mathbf{X}_1^*) = \lambda_1(\mathbf{R})\text{Tr}[\mathbf{X}_1^*]. \quad (28)$$

Utilizing this result, we have

$$\text{Tr}[\mathbf{R}\mathbf{X}_1^*] \leq \lambda_1(\mathbf{R})\text{Tr}[\mathbf{X}_1^*] \overset{(i)}{=} P\lambda_1(\mathbf{R})\sum_{i=1}^M \left(\frac{1}{2} + \frac{1}{2}\mathbf{X}_0^*[i, M+1]\right)$$

$$= P\lambda_1(\mathbf{R})\left(\frac{M}{2} + \frac{1}{2}\sum_{i=1}^M \mathbf{X}_0^*[i, M+1]\right)$$

$$\overset{(ii)}{=} P\lambda_1(\mathbf{R})\left(\frac{2Q-M}{2} + \frac{M}{2}\right) = QP\lambda_1(\mathbf{R}) \quad (29)$$

where (i) is from the tightness of the first set of constraints of the problem (SDP1) (cf. (12)) and (ii) is due to Claim 2 and Claim 3. Comparing (27) and (29), we see that choosing $\frac{\beta}{\alpha_1} \leq \frac{\text{Tr}[\mathbf{R}]}{M\lambda_1(\mathbf{R})}$ ensures

$$\text{Prob}\left(\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*] \leq \frac{\beta}{\alpha_1}\text{Tr}[\mathbf{R}\mathbf{X}_1^*]\right) = 0.$$

Step 2): For fixed α_1, we bound the probability that $\text{Prob}\left(\frac{1}{(t^{(\ell)})^2} \leq \frac{1}{\beta}\right)$ as follows (omitting (ℓ) for simplicity, and defining $\widehat{\mathbf{E}}_i \triangleq \mathbf{U}^T\mathbf{E}_i\mathbf{U}$)

$$\text{Prob}\left(\frac{1}{t^2} \leq \frac{1}{\beta}\right) = \text{Prob}(t^2 \geq \beta) = \text{Prob}\left(\max_{i \in \mathcal{S}}(\boldsymbol{\xi})^T\widehat{\mathbf{E}}_i\boldsymbol{\xi} \geq \beta\right)$$

$$\leq \sum_{i \in \mathcal{S}}\text{Prob}\left((\boldsymbol{\xi})^T\widehat{\mathbf{E}}_i\boldsymbol{\xi} \geq \beta\right) < 4Q\mu\exp\left(-\frac{\beta}{8}\right)$$

where $\mu = \min[Q, \max_i \text{Rank}(\widehat{\mathbf{E}}_i)]$. The last inequality is obtained by slightly generalizing the existing result in [29, Propo-

sition 1] [2]. To explicitly compute the value for μ, note that by definition, we have: $\widehat{\mathbf{E}}_i = \mathbf{U}^H\mathbf{E}_i\mathbf{U} = \mathbf{U}^H\Delta\mathbf{C}_{i,1}[\mathcal{S}]\Delta^H\mathbf{U}$. As a result, $\text{Rank}(\widehat{\mathbf{E}}_i) \leq 1$ as by definition $\text{Rank}(\mathbf{C}_{i,1}) = 1$, and any one of its principal submatrices must have rank at most 1. We conclude that $0 \leq \mu \leq 1$.

Step 3): Utilizing the above result, choose

$$\beta = 8\ln(5Q), \quad \alpha_1 = \frac{8M\lambda_1(\mathbf{R})}{\text{Tr}[\mathbf{R}]}\ln(5Q),$$

we can bound the left hand side of (25) as follows

$$\text{Prob}\left(\frac{1}{t^2}\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*] \geq \frac{1}{\alpha_1}\text{Tr}[\mathbf{R}\mathbf{X}_1^*]\right)$$

$$\geq \text{Prob}\left(\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*] \geq \frac{\beta}{\alpha_1}\text{Tr}[\mathbf{R}\mathbf{X}_1^*], \frac{1}{t^2} \geq \frac{1}{\beta}\right)$$

$$\geq 1 - \text{Prob}\left(\text{Tr}[\mathbf{R}[\mathcal{S}]\mathbf{Y}^*] \leq \frac{\beta}{\alpha_1}\text{Tr}[\mathbf{R}\mathbf{X}_1^*]\right) - \text{Prob}\left(\frac{1}{t^2} \leq \frac{1}{\beta}\right)$$

$$= 1 - \text{Prob}\left(\frac{1}{t^2} \leq \frac{1}{\beta}\right) > 1 - 4Q\exp\left(-\ln(5Q)\right) = 1 - 4Q/(5Q) = \frac{1}{5}.$$

In conclusion, the final approximation ratio is given by

$$\alpha_1 = \frac{8M\lambda_1(\mathbf{R})}{\text{Tr}[\mathbf{R}]}\ln(5Q) = \frac{8M\lambda_1(\mathbf{R})}{\sum_{i=1}^M \lambda_i(\mathbf{R})}\ln(5Q) \leq 8M\ln(5Q).$$

This completes the proof. ∎

C. Proof of Proposition 3

Proof: It suffices to show that solving (CP3) is NP-hard even when the active relays are known. In other words, it is sufficient to show that solving the problem

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H\mathbf{S}\mathbf{w}}{\sigma_n^2 + \mathbf{w}^H\mathbf{F}\mathbf{w}} \quad (30)$$
$$\text{s.t.} \quad |w_i|^2\left(P_0\mathbb{E}[|f_i|^2] + \sigma_\nu^2\right) \leq P, \ i = 1, \cdots, M,$$

is NP-hard. We prove by using a polynomial time reduction from the integer equal partitioning problem. To this end, let us consider the following system parameters:

$$P_0 = 1, \quad P = 2, \quad \sigma_v^2 = 1, \quad f_i = 1, \forall i.$$

Then the problem (30) can equivalently be written as

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H\mathbb{E}[\mathbf{g}\mathbf{g}^H]\mathbf{w}}{\sigma_n^2 + \mathbf{w}^H\text{diag}\left(\mathbb{E}[\mathbf{g}\mathbf{g}^H]\right)\mathbf{w}} \quad (31)$$
$$\text{s.t.} \quad |w_i|^2 \leq 1, \ i = 1, \cdots, M.$$

Let t denote the objective value of the above problem. In order to check the achievability of a particular value t, we need to check the feasibility of the following set of inequalities:

$$\mathbf{w}^H\left(\mathbb{E}[\mathbf{g}\mathbf{g}^H] - t\text{diag}\left(\mathbb{E}[\mathbf{g}\mathbf{g}^H]\right)\right)\mathbf{w} \geq t\sigma_n^2$$
$$|w_i|^2 \leq 1, \quad \forall i. \quad (32)$$

Therefore, it suffices to show the NP-hardness of checking the achievability of (32). To this end, we first claim that for given a positive definite matrix $\mathbf{A}$, the following set is non-empty

$$\mathcal{T} = \left\{(t, \mathbf{X}) \mid \mathbf{X} - t\text{diag}(\mathbf{X}) = \mathbf{A}, \mathbf{A} \succ 0, t > 0, t \in \mathbb{R}, \mathbf{X} \in \mathbb{R}^{M \times M}\right\}$$

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[2] The cited result can be generalized from the real case to the complex case using the complex form of the large deviation results from Bernstein, see e.g., [39, Theorem 4], and the recent results by Zhang and So [30].

To justify this claim, consider the mapping $\phi_{\mathbf{A}}(t) : \mathbb{R}^+ \mapsto \mathbb{R}^{M \times M}$, where $\phi_{\mathbf{A}}(t) = \mathbf{B}$ with

$$\mathbf{B}_{ij} = \begin{cases} \mathbf{A}_{ij} & i \neq j \\ \frac{\mathbf{A}_{ii}}{1-t} & i = j \end{cases}.$$

Note that $\phi_{\mathbf{A}}(t)$ is continuous over $[0,1)$ and $\phi_{\mathbf{A}}(0) = \mathbf{A}$. Therefore, for small enough $t' > 0$, we have $\phi_{\mathbf{A}}(t') = \mathbf{B}' \succ 0$. In other words, $(t', \mathbf{B}') \in \mathcal{T}$. The above claim implies that there exists a positive definite matrix $\mathbf{R} = \mathbb{E}[\mathbf{g}\mathbf{g}^H]$ and a positive scalar t such that

$$\mathbb{E}[\mathbf{g}\mathbf{g}^H] - t\mathrm{diag}\left(\mathbb{E}[\mathbf{g}\mathbf{g}^H]\right) = 2C\mathbf{I} - \mathbf{c}\mathbf{c}^T,$$

with $C = \|\mathbf{c}\|^2$. Therefore, using a similar argument to the one in the proof of Proposition 1 completes the proof. ∎

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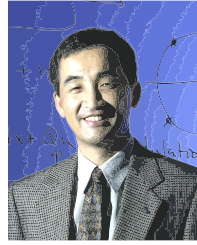
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