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Joint $\overline{X}$ and R Charts with Variable Sample Sizes and Sampling Intervals

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ABSTRACT

Recent studies have shown that the $\overline{X}$ chart with variable sampling intervals (VSI) and/or with variable sample sizes (VSS) detects process shifts faster than the traditional $\overline{X}$ chart. This article extends these studies for processes that are monitored by both, the $\overline{X}$ and the R charts. A Markov chain model is used to determine the properties of the joint $\overline{X}$ and R charts with variable sample sizes and sampling intervals (VSSI). The VSSI scheme improves the joint $\overline{X}$ and R control chart performance (in terms of the speed with which process mean and/or variance shifts are detected.)

Keywords: *Adjusted Average Time to Signal, Control Charts, Statistical Process Control, Variable Sampling Intervals, Variable Sample Sizes, Joint $\overline{X}$ and R Charts.*

Dr. Costa is an Assistant Professor in the Department of Production. This research was carried out while he was on sabbatical at the Center for Quality and Productivity Improvement, University of Wisconsin-Madison.

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Recent studies have shown that the $\overline{X}$ chart with variable sampling intervals (VSI) and/or with variable sample sizes (VSS) detects process shifts faster than the traditional $\overline{X}$ chart. This article extend these studies for processes that are monitored by both, the $\overline{X}$ and the R charts. A Markov chain model is used to determine the properties of the joint $\overline{X}$ and R charts with variable sample sizes and sampling intervals (VSSI). The VSSI scheme improves the joint $\overline{X}$ and R control chart performance (in terms of the speed with which process mean and/or variance shifts are dectected.

INTRODUCTION

The idea of varying the $\overline{X}$ chart design parameters (sample size and/or sampling interval) between minimum and maximum values as a function of what is observed from the process has been explored extensively in recent years. Basically, the position of the current sample point on the chart establishes the size of the next sample and/or the instant of its sampling. If the sample point falls within the central region (see Figure 1) the next sample should be small and/or should be sampled after a long interval of time. On the other hand, if the sample point falls within the warning region the next sample should be large and/or it should be sampled after a short interval of time. The properties of the $\overline{X}$ chart with variable sampling intervals (VSI) were studied by Reynold, Amin, Arnold and Nachlas (1988). Their paper has been the inspiration of several other papers (see Reynolds and Arnold (1989), Reynolds, Amin and Arnold (1990), Runger and Pignatiello (1991), Saccucci, Amin and Lucas (1992), Runger and Montgomery (1993), Amin and Miller (1993), Baxley (1995) and Reynolds, Arnold and Baik (1996)).

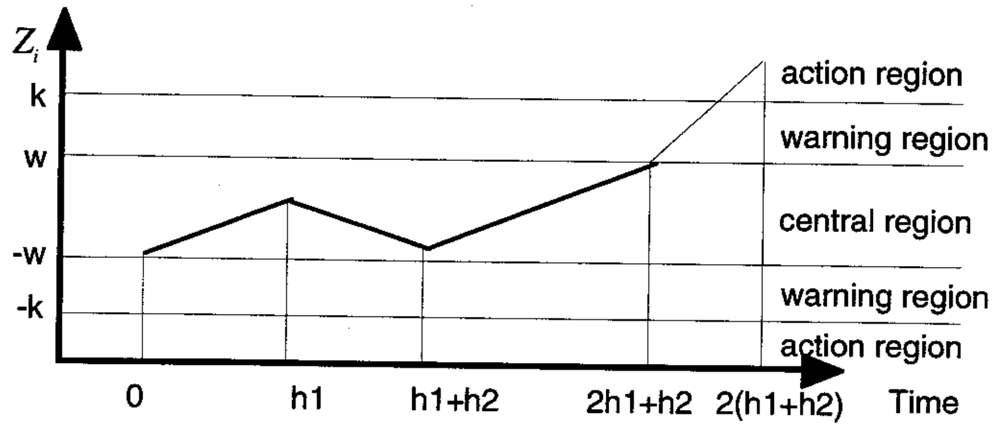
The properties of the $\overline{X}$ chart with variable sample sizes (VSS) were studied by Prabhu, Runger and Keats (1993) and Costa (1994). Finally, the properties of the $\overline{X}$ chart with variable sample sizes and sampling intervals (VSSI) were studied by Prabhu, Montgomery and Runger (1994) and Costa (1995).

In a physical environment, two charts are usually employed together to monitor the process. The $\overline{X}$ chart for monitoring the shift in the process mean and the R chart for monitoring the change in the process variance. Therefore, it seems reasonable to investigate the performance of the joint $\overline{X}$ and R charts with variable sample sizes and sampling intervals (VSSI $\overline{X}$ and R charts). In this paper a single assignable cause process model is considered. The occurrence of the assignable cause shifts the process mean and/or change the process variance. This model has been adopted to study the joint economic design of $\overline{X}$ and R charts (see Saniga (1977, 1979 and 1989), Saniga and Montgomery (1981), Rahim (1989) and Rahim, Lashkari and Banerjee (1988)).

Description of the VSSI $\overline{X}$ and R Charts

Throughout this article, it is assumed that the joint $\overline{X}$ and R charts with variable sample sizes and sampling intervals (VSSI $\overline{X}$ and R charts) are used to monitor a process that is subject to a single assignable cause. The process is assumed to start in a state of statistical

Figure 1: VSSI $\overline{X}$ Chart with the Standardized Sample Mean Z_i Plotted



control with mean $\mu = \mu_0$ and standard deviation $\sigma = \sigma_0$, where μ_0 denotes the target value of the mean. Besides, the measurable quality characteristic X is assumed to be normally distributed. The occurrence of the assignable cause results in a shift in the process mean from μ_0 to $\mu_0 \pm \delta\sigma$, $\delta > 0$ and/or in a change in the process variance from σ_0^2 to $\gamma^2\sigma_0^2$, with $\delta \geq 0$ and $\gamma \geq 1$. The time before the assignable cause occurs has an exponential distribution with parameter λ (this assumption has been widely used, see Ho and Case (1994)). Thus λ^{-1} is the mean time that the process remains in the statistical control state, denoted by in-control state. A steady state analysis (for the VSSI $\overline{X}$ and R charts) is performed since the shift in the process mean and/or the change in the process variance do not occur at the beginning when the process is just starting but at some random time in the future.

Rational subgroups of variable size are taken from the process. The standardized sample means Z_i, that is $Z_i = (\overline{X}_i - \mu_0)/\sigma_{\overline{X}}$, are plotted on a control chart with warning limits $\pm w$ and action limits $\pm k$ where $0 \leq w < k$ and $\sigma_{\overline{X}}$ is the standard deviation of the sample means (see Figure 2). The standardized sample ranges r_i, that is $r_i = R_i/\sigma_0$, are plotted on a chart with warning limit $w_R(n)$ and action limit $k_R(n)$ where $0 \leq w_R(n) < k_R(n)$ ($w_R(n)$ and $k_R(n)$ depend on sample size n).

Let the sample points plotted on the standardized $\overline{X}$ chart be the sample mean points and the sample points plotted on the standardized R chart be the sample range points. The search for the assignable cause is undertaken when the sample mean point falls outside the interval $(-k, k)$ and/or when the sample range point falls outside the interval $(0, k_R(n))$, that is when the $\overline{X}$ and/or R charts produce a signal. When $\mu = \mu_0$ and $\sigma = \sigma_0$ this signal is a false alarm and when $\mu = \mu_0 \pm \delta\sigma$ and/or $\sigma = \gamma\sigma_0$ this signal is a true alarm. After each signal the process is stopped for the assignable cause searching. If the signal is a true alarm the assignable cause is eliminated and the process is brought back to the state of statistical control.

The position of the sample mean and range points (obtained from the current sample) in each chart establish the size of the next sample and the instant of its sampling (see Figure 2).

If the sample mean point falls within the interval $(-w, w)$ and the sample range point falls within the interval $[0, w_R(n))$ the next sample should be small, that is n_1, and it should be sampled after a long time interval, that is h_1. On the other hand, if the sample mean falls within the interval $(-k, w]$ or $[w, k)$ and/or the sample range point falls within the interval $[w_R(n), k_R(n))$ the next sample should be large, that is n_2, and it should be sampled after a short time interval, that is h_2. This criteria is not followed when a false alarm occurs.

The size of the first sample that is taken from the process when it is just starting, or after a false alarm,

Figure 2: VSSI $\overline{X}$ and R Charts with the Standardized Sample Mean Z_i and the Standardized Sample Range r_i Plotted

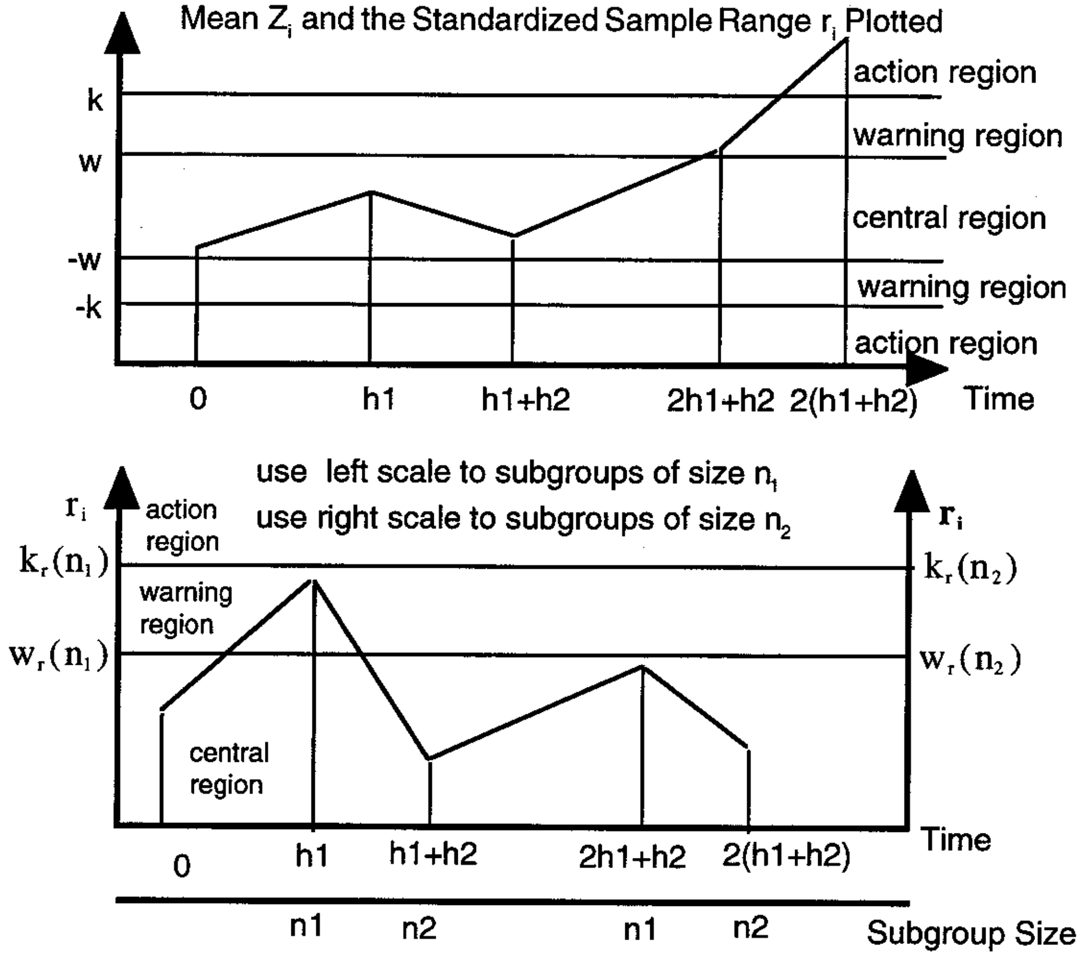


TABLE 1 The states of the Markov chain

i th subgroup		(i+1) st subgroup	
Sample mean position (region)	Sample range position (region)	Process status (in or out-of-control)	State of the Markov chain
Central	Central	in	1
Warning	Warning	in	2
Central	Warning	in	2
Warning	Central	in	2
Central	Central	out	3
Warning	Warning	out	4
Central	Warning	out	4
Warning	Central	out	4

is chosen at random. If the sample was chosen to be large (small) it should be sampled after a short (large) time interval. During the in-control period all samples, including the first one, should have probablility P_o of being small and (1- P_o) of being large, where

$$P_o = P_r\left[|Z| < w \,\middle|\, |Z| < k\right] \times Pr\left[r < w_R(n) \,\middle|\, r < k_R(n), \sigma = \sigma_o\right]$$

To facilitate the computation, w, k, w_R(n) and k_R(n) will be specified with the contstraint that the probability of a sample point to fall in the central region (or warning region), when the process is in-control, is the same for both charts, $\bar{X}$ and R. This way

$$Pr[\,|Z| < w \,|\, |Z| < k\,] = Pr[\,r < w_R(n) \,|\, r < k_R(n), \sigma = \sigma_R\,] \quad (2)$$

when $w = w_R(n) = 0$, $n_1 = n_2 = n_0$ and $h_1 = h_2 = h_0$, we have the joint $\bar{X}$ and R charts with fixed sample sizes and sampling intervals (FSSI $\bar{X}$ and R charts). When $n_1 = n_2 = n_0$ and $h_1 > h_0 > h_2$ the VSSI $\bar{X}$ and R charts are called VSI $\bar{X}$ and R charts. When $h_1 = h_2 = h_0$ and $n_2 > n_0 > n_1$ the VSSI $\bar{X}$ and R charts are called VSS $\bar{X}$ and R charts.

Performance Measure

The speed with which a control chart detects process shifts measures its statistical efficiency. When the interval between sampling is variable the speed is measured by AATS (Adjusted Average Time to Signal). AATS is the average time from the process shift until the $\bar{X}$ and/or R charts produce a signal. In other words, the AATS is the mean time the process remains out-of-control.

If the assignable cause occurs according to an exponential distribution with parameter λ then

$$\text{AATS} = \text{ATC} - 1/\lambda \quad (3)$$

The average time of the cycle (ATC) is the average time from the start of production until the first signal after the process shift.

The memoryless property of the exponential distribution allows the computation of the ATC using the Markov chain approach. This way, at each sampling, one of four transient states is reached according to the status of the process (in or out-of-control) and the size of the sample (large or small). That is:

state 1: the process is in-control and the sample is small;

state 2: the process is in-control and the sample is large;

state 3: the process is out-of-control and the sample is small;

state 4: the process is out-of-control and the sample is large.

Alternatively, the status of the process when the i+1st sample is taken, the position of the ith sample mean on the $\bar{X}$ chart and the position of the ith sample range on the R chart define the transient states of the Markov chain (see Table 1). The joint $\bar{X}$ and R charts produce a signal when the sample mean point and/or sample range point fall in the action region. If the current state is the state 1 or 2 (3 or 4) the signal is a false alarm (true alarm). The absorbing state (state 5) is reached when the true alarm occurs.

The transition probability matrix is given by

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & 0 \\ p_{21} & p_{22} & p_{23} & p_{24} & 0 \\ 0 & 0 & p_{33} & p_{34} & p_{35} \\ 0 & 0 & p_{43} & p_{44} & p_{45} \\ 0 & 0 & 0 & 0 & p_{55} \end{bmatrix}$$

Where p_{ij} denotes the transition probability that i is the prior state and j is the current state, that is,

$$p_1 = p_{11} = p_{21} = \left\{\Pr\left[|Z| < w \,\middle|\, |Z| < k\right]\right\}^2 e^{-\lambda h_1}$$

$$p_2 = p_{12} = p_{22} = \left\{1 - \left\{\Pr\left[|Z| < w \,\middle|\, |Z| < k\right]\right\}^2\right\} e^{-\lambda h_2}$$

$$p_3 = p_{13} = p_{23} = \left\{\Pr\left[|Z|< w \,\middle|\, |Z|< k\right]\right\}^2 \left(1 - e^{-\lambda h_1}\right)$$

$$p_4 = p_{14} = p_{24} = \left\{1 - \left\{\Pr\left[|Z|< w \,\middle|\, |Z|< k\right]\right\}^2\right\}\left(1 - e^{-\lambda h_2}\right)$$

$$p_{33} = \Pr\left[|Y|< w \,\middle|\, Y \sim N(\delta\sqrt{n_1}, \gamma)\right] \times Pr\left[r < w_R(n_1) \,\middle|\, \sigma = \gamma\sigma_0, n = n_1\right]$$

$$p_{34} = Pr\left[|Y|< k \,\middle|\, Y \sim N(\delta\sqrt{n_1}, \gamma)\right] \times Pr\left[w_R(n_1) < r < k_R(n_1) \,\middle|\, \sigma = \gamma\sigma_0, n = n_1\right] + Pr\left[w < |Y|< k \,\middle|\, Y \sim N(\delta\sqrt{n_1}, \gamma)\right] \times \Pr\left[w_R(n_1) < r < k_R(n_1) \,\middle|\, \sigma = \gamma\sigma_0, n = n_1\right]$$

$$p_{35} = Pr\left[|Y| > k \,\middle|\, Y \sim N(\delta\sqrt{n_1}, \gamma)\right] + Pr\left[|Y|< k \,\middle|\, Y \sim N(\delta\sqrt{n_1}, \gamma)\right] \times Pr\left[r < k_R(n_1) \,\middle|\, \sigma = \gamma\sigma_0, n = n_1\right]$$

$$P_{43} = Pr\left[|Y|< w \,\middle|\, Y \sim N(\delta\sqrt{n_2}, \gamma)\right) \times Pr\left[r < w_R(n_2) \,\middle|\, \sigma = \gamma\sigma_0, n = n_2\right]$$

$$P_{44} = Pr\left[|Y|< k \,\middle|\, Y \sim N(\delta\sqrt{n_2}, \gamma)\right] \times Pr\left[w_R(n_2) < r < k_R(n_2) \,\middle|\, \sigma = \gamma\sigma_0, n = n_2\right] + Pr\left[w < |Y|< k \,\middle|\, Y \sim N(\delta\sqrt{n_2}, \gamma)\right] \times Pr\left[r < w_R(n_2) \,\middle|\, \sigma = \gamma\sigma_0, n = n_2\right]$$

$$p_{45} = Pr\left[|Y| > k \,\middle|\, Y \sim N(\delta\sqrt{n_2}, \gamma)\right] + \Pr\left[|Y|< k \,\middle|\, Y \sim N(\delta\sqrt{n_2}, \gamma)\right] \times Pr\left[r > k_R(n_2) \,\middle|\, \sigma = \gamma\sigma_0, n = n_2\right]$$

From the elementary properties of Markov chains (see, e.g., Çinlar(1975))

$$\text{ATC} = b'(I - Q)^{-1} t \quad (4)$$

where $b' = (p_1, p_2, p_3, p_4)$ is the vector starting probabilities. I is the identity matrix of order 4, Q is the transition probability matrix where the elements associated with the absorbing state have been deleted, and $t' = (h_1, h_2, h_1, h_2)$ is the vector of the sampling intervals.

COMPARING CONTROL CHARTS

One sampling scheme is better than another when it allows the joint $\overline{X}$ and R charts to detect changes in the process mean and/or variance faster (in this paper the VSSI, VSS and VSI schemes will be compared with the FSSI scheme). Nevertheless, the sampling schemes should be compared under equal conditions, that is, they should demand the same average quantity of samples (ANS) and the same average quantity of items (ANI) to be inspected during the in-control period. The reason for such a choice is that the process is supposed to be under control most of the time (λ is small).

From the elementary properties of Markov chains

$$\text{ANS} = b'(I-Q)^{-1} u \quad (5)$$

$$\text{ANI} = b'(I-Q)^{-1} v \quad (6)$$

where $u' = (1,1,0,0)$ and $v' = (n_1, n_2, 0, 0)$. Hence

$$\text{ANS} = \frac{p_1 + p_2}{1 - p_1 - p_2} \quad (7)$$

$$\text{ANI} = \frac{p_1 n_1}{1 - p_1 - p_2} + \frac{p_2 n_2}{1 - p_1 - p_2} \quad (8).$$

The ANS, ANI and ATC for the FSSI $\bar{X}$ and R charts are easily determined because in this case, if c_1 and c_2 represent the number of samples before and after the process shift, then c_1+1 and c_2 are two random variables geometrically distributed with parameters q and p respectively, where $q=\exp(-\lambda h)$ and

$$p=\Pr[|Y|>k|Y\sim N(\delta\sqrt{n_o},\gamma)]+\Pr[|Y|<k|Y\sim N(\delta\sqrt{n_0},\gamma)]\cdot Pr\left[r>k_R(n_0)\middle|\sigma=\gamma\sigma_0,n=n_0\right].$$

Hence for the FSSI $\bar{X}$ and R charts

$$\text{ANS}=\frac{q}{1-q} \quad (9)$$

$$\text{ANI}=\left(\frac{q}{1-q}\right)n \quad (10)$$

$$\text{ATC}=\left(\frac{q}{1-q}+\frac{1}{p}\right)h \quad (11).$$

To compare the VSSI $\bar{X}$ and R charts with the $\bar{X}$ FSSI and R charts, the warning limits $\pm w$ should be set so that

$$w=\Phi^{-1}\times\left\{1/2+[\Phi(k)-1/2]\times\sqrt{1-\left(\frac{n_1-n}{n_1-n_1}\right)\exp[-\lambda(h-h_2)]}\right\} \quad (12).$$

When $n_1=n_2=n_0$ and $h_1>h_0\geq h_2$ the VSSI $\bar{X}$ and R charts are called the VSI $\bar{X}$ and R charts. The expression of w to this particular case is

$$w=\Phi^{-1}\times\left\{1/2-[\Phi(k)-1/2]\sqrt{\frac{1-\exp[-\lambda(h-h_2)]}{1-\exp[-\lambda(h_1-h_2)]}}\right\} \quad (13).$$

The expression (12) was obtained matching the ANS (given by expressions (7) and (9)) and ANI (given by expressions (8) and (10)), where $\Phi(\cdot)$ denotes the standard normal cumulative function.

Considering the set of parameters (n_1,n_2,h_1,h_2), the practitioner can fix only three of them. The last one is a function of the others. Following the recommendation of Prabhu, Montgomery and Runger(1994), we have fixed n_1, n_2 and h_2. The last parameter h_1 is determined by equating equations (7) and (9), that is,

$$h_1=h_2-\frac{1}{\lambda}\ln\times\left\{\frac{[\Phi(k)-1/2]^2\exp\left[-\lambda(h-h_2)\right]}{[\Phi(w)-1/2]^2}+\frac{[\Phi(k)-\Phi(w)]-\left[\Phi(k)^2-\Phi(w)^2\right]}{[\Phi(w)-1/2]^2}\right\} \quad (14).$$

Tables 3 and 4 provide the adjusted average time to signal (AATS) for the VSSI, VSS and VSI $\bar{X}$ and R charts designed to match a FSSI $\bar{X}$ and R charts with h_0=1.00, n_0=3 (Table 3) and n_0=5 (Table 4), k=3.25, $k_R(3)$ =5.00 and $k_R(5)$=5.42. The Table 2 brings the VSSI, VSS and VSI scheme parameters. The AATS in Table 3 and 4 were obtained with λ =0.0001 (the choice of λ has minor influence on the AATS if λ>0.01). To the VSSI and VSS schemes n_2 is equal 9 (Table 3) or 10 (Table 4). Bigger values for n_2 improves the efficiency of the VSSI and VSS $\bar{X}$ and R charts but it is not recommended when the R chart is in use. In this situation the R chart should be changed by the S^2 chart. Table 3 and 4 show that all schemes VSSI, VSS and VSI improves the Joint $\bar{X}$ and R charts efficiency. At this point, seems to be interesting to compare the VSSI $\bar{X}$ and R charts with the joint $\bar{X}$ and R charts with supplementary run rules (in terms of their statistical efficiencies). Champ and Woodall (1987) developed a simple and efficient method to obtain the properties of an Shewhart control chart with supplementary run rules. The extension of this study to consider the joint $\bar{X}$ and R charts is not trivial but it is an interesting topic for research.

ILLUSTRATIVE EXAMPLE

Consider a filling bottle process where the quantity of liquid in each bottle is the quality characteristic of interest. The process fills the bottle with 600 ml (μ_0=600). The process standard deviation is know (σ_0=2 ml). Occasionally the liquid brings some impurities that clog partially the feeder line shifting the process mean. To set back the mean (μ) onto target (600ml) the feeder line should be cleaned up. Since the feeder line cleaning requires shut down the process, unnecessary cleanings must be avoided.

The R chart has been used jointly with an $\overline{X}$ chart for monitoring the bottle contents because historical data have shown that the process variance increases when the process mean shifts. The low speed with which the joint $\overline{X}$ and R charts detects moderate shifts in the process (δ around 0.75 and γ around 1.50) has led the quality manager to innovate.

Presently the company is sampling 3 items at each 20 minutes (n_0=3 and h_0=0:20 hours) The standardized $\overline{X}$ values are plotted on the chart with control limits placed at $\pm$ 3.25 and the standardized R values are plotted on the chart with control limit placed at 5.00. This way, approximately 2 false alarm are expected at each 1000 samplings. The new proposal consists in building the standard $\overline{X}$ and R charts with warning regions (see Figure 3) and in adopting the VSSI (B) scheme (see Table 2).

If the current subgroup average and/or range fall in the warning region it is reasonable to wait less (2 minutes, that is $h_2=0.10\times h_0$=0:02 hours) to obtain the next subgroup (which should have more items, n_2=9) because the process can be out-of-control. On other hand, if both the current subgroup mean and range fall in the central region it is reasonable to wait more (23 minutes, that is $h_1=1.15\times h_0$=0:23 hours) to obtain the next subgroup (which should have fewer items, n_1=2), because there is no evidences that the process is out-of-control.

When the process starts, a random procedure decides if the first subgroup (i=1) should be obtained immediately (at 0:02 hours) or later (at 0:23 hours). Consider the first subgroup was scheduled at time 0:23 hours using a sample size of 2. If $\overline{X}_1$=601.2 and R_1= 7.76, then Z_1= 0.85 and r_1= 3.88. This means the sample mean point falls in the central region but the sample range point falls in the warning region. The second subgroup (i=2) will be observed immediately (i.e., at 0:23+0:02=0:25 hours) using a sample size of 9. Suppose this subgroup has mean and range values equal 599.4 and 7.02 respectively. Since Z_2 = -0.90 and r_2= 3.51 the sample mean and the sample range points fall in the central region. The next subgroup is now scheduled at time 0:25+0:23 = 0:48 hours using a sample of size 2. The process is stopped when the sample mean point and/or sample range point fall in the action region.

The VSSI scheme improves the sensitivity of the joint $\overline{X}$ and R charts. For example, one can see from the Table 3 that in order to detect a shift in the process mean (δ=0.75) accompanied by a change in the process variance (γ=1.25), the average time from the process shift until the $\overline{X}$ and/or R charts produce a signal (AATS) has been reduced from 4:29 hours (13.44×20 minutes) to only 1:39 hours (4.94×20minutes).

TABLE 2 VSSI, VSS and VSI Scheme parameters

Scheme	h_1	h_2	n_1	n_2	k=3.25 w	$w_R(n_1)$	$k_R(n_1)$	$w_R(n_2)$	$k_R(n_2)$
VSSI (A)	1.17	0.01	2	9	1.78	2.53	4.58	4.19	5.84
VSSI (B)	1.15	0.10	2	9	1.78	2.53	4.58	4.19	5.84
VSS (A)	1.00	1.00	2	9	1.78	2.53	4.58	4.19	5.84
VSI (A)	2.70	0.01	3	3	0.82	1.80	5.00	1.80	5.00
VSI (B)	2.70	0.10	3	3	0.82	1.80	5.00	1.80	5.00
VSSI (C)	1.40	0.01	3	10	1.42	2.61	5.00	3.89	5.91
VSSI (D)	1.36	0.10	3	10	1.42	2.61	5.00	3.89	5.91
VSS (B)	1.00	1.00	3	10	1.42	2.61	5.00	3.89	5.91
VSI (C)	2.20	0.01	5	5	0.94	2.62	5.42	2.62	5.42
VSI (D)	2.20	0.10	5	5	0.94	2.62	5.42	2.62	5.42

TABLE 3 Values of AATS for the VSSI, VSS, VSI AND FSSI $\overline{X}$ and R charts, when n_0=3

δ	Scheme	γ 1.00	1.25	1.50	1.75	2.00
0.00	FSSI (A)	424.5	44.34	12.42	5.59	3.25
	VSSI (A)	424.5	33.94	6.22	2.42	1.54
	VSSI (B)	424.5	34.61	6.50	2.55	1.61
	VSS (A)	424.5	41.39	9.33	3.86	2.33
	VSI (A)	424.5	29.18	6.54	3.00	2.06
	VSI(B)	424.5	30.33	7.01	3.21	2.16
0.50	FSSI(A)	101.8	23.61	9.04	4.67	2.92
	VSSI(A)	55.95	11.87	3.90	2.02	1.42
	VSSI(B)	56.66	12.27	4.09	2.12	1.50
	VSS(A)	63.86	16.28	6.11	3.19	2.12
	VSI(A)	77.78	13.57	4.66	2.61	1.94
	VSI(B)	79.59	14.36	5.01	2.77	2.02
0.75	FSSI(A)	36.95	13.44	6.55	3.85	2.57
	VSSI(A)	9.72	4.73	2.57	1.70	1.31
	VSSI(B)	10.05	4.94	2.70	1.78	1.37
	VSS (A)	13.31	7.12	4.08	2.62	1.92
	VSI (A)	20.76	6.83	3.38	2.27	1.83
	VSI (B)	22.03	7.36	3.64	2.39	1.89
1.00	FSSI (A)	14.74	7.57	4.57	3.05	2.21
	VSSI (A)	2.97	2.35	1.80	1.42	1.20
	VSSI (B)	3.10	2.46	1.88	1.49	1.24
	VSS (A)	4.44	3.61	2.77	2.12	1.70
	VSI (A)	6.08	3.58	2.48	1.96	1.71
	VSI (B)	6.78	3.90	2.65	2.05	1.75

TABLE 4 Values of AATS for the VSSI, VSS, VSI AND FSSI $\overline{X}$ and R charts, when n_0=5

δ	Scheme	γ 1.00	1.25	1.50	1.75	2.00
0.00	FSSI (B)	424.5	35.69	8.86	3.75	2.13
	VSSI (C)	424.5	22.28	3.48	1.52	1.11
	VSSI (D)	424.5	23.21	3.78	1.64	1.16
	VSS (B)	424.5	32.69	6.88	2.85	1.73
	VSI (C)	424.5	21.20	3.97	1.84	1.37
	VSI(D)	424.5	22.25	4.35	1.99	1.43
0.50	FSSI(B)	56.02	15.64	6.07	3.08	1.89
	VSSI(C)	24.14	6.01	2.20	1.33	1.06
	VSSI(D)	24.99	6.44	2.39	1.41	1.10
	VSS(B)	33.68	10.85	4.39	2.35	1.57
	VSI(C)	37.10	7.61	2.69	1.62	1.31
	VSI(D)	38.49	8.22	2.96	1.74	1.36
0.75	FSSI(B)	16.45	7.78	4.15	2.48	1.66
	VSSI(C)	3.72	2.38	1.55	1.18	1.00
	VSSI(D)	4.01	2.57	1.66	1.24	1.03
	VSS (B)	6.70	4.58	2.89	1.92	1.41
	VSI (C)	7.20	3.33	1.94	1.44	1.26
	VSI (D)	7.92	3.69	2.12	1.53	1.29
1.00	FSSI (B)	5.90	3.98	2.74	1.92	1.41
	VSSI (C)	1.50	1.39	1.20	1.05	0.95
	VSSI (D)	1.60	1.47	1.26	1.09	0.97
	VSS (B)	2.63	2.37	1.95	1.55	1.25
	VSI (C)	2.12	1.79	1.48	1.30	1.21
	VSI (D)	2.43	1.97	1.59	1.35	1.22

CONCLUSIONS AND EXTENSION

In most cases, in which $\overline{X}$ chart is being used to monitor the process mean, some type of chart such as an R chart should be used to monitor the process variance. Adopting the same approach proposed by Costa (1995) to study the VSSI $\overline{X}$ chart this paper has shown that the VSSI scheme improves substantially the performance of the joint $\overline{X}$ and R charts (in terms of the speed with which process mean and/or variance shifts are detected). Table 3 and 4 have shown that the VSSI $\overline{X}$ and R charts always work better than the FSSI $\overline{X}$ and R charts when the aim is to detect moderate shifts in the process mean (δ around 0.75) and/or moderate changes in the process variance (γ around 1.50) This observation has practical importance, because the estimation of δ and γ is not a simple task.

A single assignable cause process model was adopted in this paper. The study of the VSSI $\overline{X}$ and R charts considering the two independent assignable cause process model, where one assignable cause shifts the process mean and the other changes the process variance (see Costa (1993)) is an interesting topic for future research.

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