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We present a complete explicit $N = 1$, $d = 4$ supergravity action in an arbitrary Jordan frame with nonminimal scalar-curvature coupling of the form $\Phi(z, \bar{z})R$. The action is derived by suitably gauge fixing the superconformal action. The theory has a modified Kähler geometry, and it exhibits a significant dependence on the frame function $\Phi(z, \bar{z})$ and its derivatives over scalars, in the bosonic as well as in the fermionic part of the action. Under certain simple conditions, the scalar kinetic terms in the Jordan frame have a canonical form. We consider an embedding of the next-to-minimal supersymmetric standard model (NMSSM) gauge theory into supergravity, clarifying the Higgs inflation model recently proposed by Einhorn and Jones. We find that the conditions for canonical kinetic terms are satisfied for the NMSSM scalars in the Jordan frame, which leads to a simple action. However, we find that the gauge singlet field experiences a strong tachyonic instability during inflation in this model. Thus, a modification of the model is required to support the Higgs-type inflation.

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I. INTRODUCTION

Supersymmetry imposes certain restrictions on the non-supersymmetric models of particle physics and cosmology. A well-known example of such restrictions is the fact that the supersymmetric version of the standard model (SM) of particle physics requires at least two Higgs superfields. Meanwhile, for cosmology Einstein equations have to be solved; therefore the supersymmetry embedding of the Higgs model inflation requires local supersymmetry, i.e. supergravity. Thus, one can try to see how the potential discovery of supersymmetry may affect various models of inflation, derived in the past in the context of general relativity coupled to scalar fields without supersymmetry. It would be interesting to find general restrictions, as well as to study particular models.

Here we are motivated by a particular issue in cosmology, the so-called $\xi\phi^2R$ coupling, which attracted a lot of attention starting from the early days of inflation [1]. Recently, it also became quite important in the context of SM inflation [2].

Until now, the $N = 1$, $d = 4$ supergravity action in an arbitrary Jordan frame described by the frame function $\Phi(z, \bar{z})$, with arbitrary Kähler potential $\mathcal{K}(z, \bar{z})$, holomorphic superpotential $W(z)$, and holomorphic function $f_{ab}(z)$, was not known. Here we will derive this action, which is the first goal of this paper. This will be achieved by starting with the superconformal theory developed in [3], and by gauge fixing all extra symmetries in order to get a general supergravity action in a Jordan frame.

Our results generalize the formulation of $N = 1$ supergravity in a Jordan frame for the particular case in which the Kähler potential \mathcal{K} and the frame function are related by $\mathcal{K}(z, \bar{z}) = -3 \log(-\frac{1}{3}\Phi(z, \bar{z}))$. The corresponding ac-

tion in a Jordan frame was derived in components in [4,5], and in superspace in [6,7]. In our treatment, we will also specify the conditions required for the frame function to make the kinetic terms of the scalar fields canonical in the Jordan frame.

The nonminimal coupling of scalar fields to curvature is allowed by all known symmetries of the SM and general relativity. If one tries to describe the early universe using the particle physics SM coupled to gravity in the Einstein frame, one finds the following: (1) the coupling λ of the Higgs field has to be of the order 10^{-13} ; (2) the mass of the Higgs field has to be of the order 10^{13} GeV. These conditions may be satisfied in a general theory of a scalar field, but not in the simplest version of the standard model. However, if the $\xi\phi^2R$ coupling is included, i.e. if the embedding of the particle physics SM into the Jordan frame gravity is considered, a satisfactory description of cosmology for the Higgs mass in the interval between 126 and 194 GeV can be found [2]. This is possible for very large values of the nonminimal scalar-curvature coupling $\xi \sim 10^4$. The model predicts the cosmological parameters $n_s \approx 0.97$, and $r \approx 0.003$, which are consistent with cosmological observations. Thus, this model provides very interesting predictions, which will be testable both at LHC and by a Planck satellite.

When this work was in progress, a very interesting proposal [8] was made for how to generalize the model of Bezrukov-Shaposhnikov [2] in the presence of supersymmetry. Under certain assumptions, it was found that slow regime inflation is not possible within the supergravity embedding of the minimal supersymmetric standard model (MSSM), but rather it is possible for the next-to-minimal supersymmetric standard model (NMSSM; see e.g. [9] for a recent review of NMSSM).

In the present paper we will study the supergravity embedding of the NMSSM and look for consistent cosmological models of the Higgs-type inflation.

First, we will derive the complete $N = 1$ action in the general Jordan frame, where it is very simple and has interesting features. This will help to clarify the meaning of the large nonminimal $\xi\phi^2 R$ coupling in the context of supergravity. In particular, the origin of the canonical kinetic terms of all scalars of the NMSSM in the Jordan frame is explained, whereas in the Einstein frame scalar kinetic terms are generally very complicated.

Second, we will study the theory as a function of all three chiral multiplets, namely, two Higgs doublets and a singlet, and analyze various directions in the space of scalar fields. In particular, in [8] it was shown that a slow-roll inflationary regime is possible in NMSSM when the Higgs fields move in the D -flat direction of the two Higgs doublets H_u and H_v , assuming that the gauge singlet S is small. However, it was not clear whether this last assumption is justified, i.e. whether $S = 0$ corresponds to a minimum of the potential with respect to the field S when inflation takes place in the D -flat direction of the two doublet Higgs fields. We will show that, unfortunately, the potential of the field S has a sharp maximum near $S = 0$ in this regime. This means that the inflationary regime studied in [8] is unstable, and a search for more general models is required to find a supersymmetric version of the Higgs-type inflation.

The paper is organized as follows. In Sec. II we present the complete explicit $N = 1$, $d = 4$ supergravity action in an arbitrary Jordan frame with nonminimal scalar-curvature coupling of the form $\Phi(z, \bar{z})R$. This includes the bosonic as well as fermionic action. In the special case in which the frame function $\Phi(z, \bar{z})$ is related to the Kähler potential by the relation $\mathcal{K}(z, \bar{z}) = -3 \log(-\frac{1}{3}\Phi(z, \bar{z}))$, the action reduces to the one derived in [4,5]. In the case $\Phi = -3$, the action becomes the well-known action of $N = 1$ supergravity in the Einstein frame.

Section III is devoted to a detailed discussion of the bosonic part of the supergravity action, which is especially important for cosmology. In particular, sufficient conditions for the kinetic terms of scalars to be canonical are specified.

Section IV starts with a short description of the Higgs-type inflation with nonminimal scalar-curvature coupling. Then, we proceed with an attempt to generalize this model to the supersymmetric case. For this purpose, we study the embedding of the NMSSM into supergravity, focusing on the Einhorn-Jones cosmological model [8]. We study this model in the Jordan as well as in the Einstein frame. The dependence of the potential on the singlet gauge field S , as well as at large values of the Higgs fields in a D -flat direction of the two Higgs doublets, is explicitly computed. We find that this potential has a maximum for small values of S near the inflationary trajectory. The resulting insta-

bility disallows the inflationary regime in the model of [8], unless some way of stabilizing the field S is found. Section V provides a detailed derivation of the Jordan frame supergravity action presented in Sec. II, by gauge fixing the extra symmetries of the superconformal action. Finally, the Appendix contains a discussion of the cosmological behavior of the angle β between the two components of the Higgs field.

II. COMPLETE $N = 1$ SUPERGRAVITY ACTION IN A JORDAN FRAME

The $N = 1$, $d = 4$ supergravity action in a Jordan frame with arbitrary scalar-curvature coupling is uniquely defined by the frame function $\Phi(z, \bar{z})$, Kähler potential $\mathcal{K}(z, \bar{z})$, holomorphic superpotential $W(z)$, holomorphic kinetic gauge matrix $f_{AB}(z)$, and momentum map¹ P_A . It is given by² ($e \equiv \sqrt{-g}$)

$$e^{-1} \mathcal{L} = -\frac{1}{6}\Phi[R(e) - \bar{\psi}_\mu R^\mu] - \frac{1}{6}(\partial_\mu \Phi)(\bar{\psi} \cdot \gamma \psi^\mu) + \mathcal{L}_0 \\ + \mathcal{L}_{1/2} + \mathcal{L}_1 - V + \mathcal{L}_m + \mathcal{L}_{\text{mix}} + \mathcal{L}_d + \mathcal{L}_{\text{4f}}, \quad (2.1)$$

where the curvature $R(e)$ uses the torsionless connection $\omega_\mu^{ab}(e)$, and the gravitino kinetic term is defined using

$$R^\mu \equiv \gamma^{\mu\rho\sigma}(\partial_\rho + \frac{1}{4}\omega_\rho^{ab}(e)\gamma_{ab} - \frac{3}{2}\mathfrak{i}\mathcal{A}_\rho\gamma_*)\psi_\sigma. \quad (2.2)$$

Here \mathcal{A}_μ is the part of the auxiliary vector field containing only bosons, namely,

$$\mathcal{A}_\mu = \frac{1}{6}\mathfrak{i}(\partial_\mu z^\alpha \partial_\alpha \mathcal{K} - \partial_\mu \bar{z}^{\bar{\alpha}} \partial_{\bar{\alpha}} \mathcal{K}) - \frac{1}{3}A_\mu^A P_A, \quad (2.3)$$

where A_μ^A is the Yang-Mills gauge field.

The kinetic terms of spin 0, $\frac{1}{2}$, 1 fields in (2.1) are, respectively, given by

$$\mathcal{L}_0 = -\frac{1}{4\Phi}(\partial_\mu \Phi)(\partial^\mu \Phi) + \frac{1}{3}g_{\alpha\bar{\beta}}\Phi(\hat{\partial}_\mu z^\alpha)(\hat{\partial}^\mu \bar{z}^{\bar{\beta}}), \quad (2.4)$$

$$\mathcal{L}_{1/2} = -\frac{1}{2}\tilde{g}_{\alpha\bar{\beta}}\bar{\chi}^{\bar{\beta}}\not{\partial}\chi^\alpha + \frac{1}{2}\Phi\bar{\chi}^\alpha\gamma^\mu\chi^{\bar{\beta}}\hat{\partial}_\mu z^\gamma \\ \times [-\frac{1}{3}g_{\gamma\bar{\beta}}L_\alpha + \frac{1}{4}L_{\alpha\gamma}L_{\bar{\beta}} - \frac{1}{4}L_\alpha L_{\gamma\bar{\beta}}] + \text{H.c.}, \quad (2.5)$$

$$\mathcal{L}_1 = (\text{Re}f_{AB})[-\frac{1}{4}F_{\mu\nu}^A F^{\mu\nu B} - \frac{1}{2}\bar{\lambda}^A \not{\partial}\lambda^B] \\ + \frac{1}{4}\mathfrak{i}(\text{Im}f_{AB})F_{\mu\nu}^A \tilde{F}^{\mu\nu B} + (\hat{\partial}_\mu \text{Im}f_{AB})\bar{\lambda}^A \gamma_* \gamma^\mu \lambda^B. \quad (2.6)$$

The covariant derivatives of scalars and fermions are defined as follows:

¹This is also equivalently named ‘‘Killing potential,’’ and it encodes the Yang-Mills transformations of the scalars (it may include Fayet-Iliopoulos terms, as well).

²A derivation of this action, as well as a detailed notation, is given in Sec. V.

$$\hat{\partial}_\mu z^\alpha \equiv \partial_\mu z^\alpha - A_\mu^A k_A^\alpha, \quad (2.7)$$

$$D_\mu \chi^\alpha \equiv \left(\partial_\mu + \frac{1}{4} \omega_\mu^{ab}(e) \gamma_{ab} + \frac{3}{2} i \mathcal{A}_\mu \right) \chi^\alpha - A_\mu^A \frac{\partial k_A^\alpha(z)}{\partial z^\beta} \chi^\beta + \Gamma_{\beta\gamma}^\alpha \chi^\gamma \hat{\partial}_\mu z^\beta, \quad (2.8)$$

$$D_\mu \lambda^A \equiv \left(\partial_\mu + \frac{1}{4} \omega_\mu^{ab}(e) \gamma_{ab} - \frac{3}{2} i \mathcal{A}_\mu \gamma^* \right) \lambda^A - A_\mu^C \lambda^B f_{BC}^A.$$

The theory has a modified Kähler geometry. In particular, as given by (2.5), the kinetic term of fermions depends on the metric $\tilde{g}_{\alpha\bar{\beta}} \equiv -\frac{1}{3} \Phi g_{\alpha\bar{\beta}} + \frac{1}{4} \Phi L_\alpha L_{\bar{\beta}}$, where $g_{\alpha\bar{\beta}}$ is the Kähler metric and $L_\alpha \equiv \partial_\alpha \ln(-\Phi)$, $L_{\bar{\alpha}} \equiv \partial_{\bar{\alpha}} \ln(-\Phi) = \bar{L}_\alpha$ [see (5.57) and (5.43) further below]. Concerning the kinetic terms of scalars, see Sec. III B. The potential reads

$$V = \frac{1}{9} \Phi^2 [e^{\mathcal{K}} (-3W\bar{W} + \nabla_\alpha W g^{\alpha\bar{\beta}} \nabla_{\bar{\beta}} \bar{W}) + \frac{1}{2} (\text{Re}f)^{-1AB} P_A P_B]. \quad (2.9)$$

The fermion mass terms are given by

$$\begin{aligned} \mathcal{L}_m &= \frac{1}{2} m_{3/2} \bar{\psi}_\mu P_R \gamma^{\mu\nu} \psi_\nu - \frac{1}{2} m_{\alpha\beta} \bar{\chi}^\alpha \chi^\beta - m_{\alpha A} \bar{\chi}^\alpha \lambda^A \\ &\quad - \frac{1}{2} m_{AB} \bar{\lambda}^A P_L \lambda^B + \text{H.c.}, \\ m_{3/2} &= (-\frac{1}{3} \Phi)^{3/2} e^{\mathcal{K}/2} W, \\ m_{\alpha\beta} &= (-\frac{1}{3} \Phi)^{3/2} e^{\mathcal{K}/2} [\nabla_\beta \nabla_\alpha W + 2L_{(\alpha} \nabla_{\beta)} W], \\ m_{AB} &= -\frac{1}{2} (-\frac{1}{3} \Phi)^{1/2} e^{\mathcal{K}/2} f_{AB\alpha} g^{\alpha\bar{\beta}} \bar{\nabla}_{\bar{\beta}} \bar{W}, \\ m_{\alpha A} &= -\frac{1}{3} i \sqrt{2} \Phi [(\partial_\alpha + \frac{1}{2} L_\alpha) P_A - \frac{1}{4} f_{AB\alpha} (\text{Re}f)^{-1BC} P_C]. \end{aligned} \quad (2.10)$$

The remaining terms read

$$\begin{aligned} \mathcal{L}_{\text{mix}} &= \bar{\psi} \cdot \gamma P_L \left[-\frac{1}{6} i \Phi P_A \lambda^A + \frac{1}{\sqrt{2}} \chi^\alpha e^{\mathcal{K}/2} (\partial_\alpha + (\partial_\alpha K)) \right. \\ &\quad \left. \times \left(-\frac{1}{3} \Phi \right)^{3/2} W \right] + \text{H.c.}, \\ \mathcal{L}_d &= \frac{1}{8} (\text{Re}f_{AB}) \bar{\psi}_\mu \gamma^{ab} (F_{ab}^A + \hat{F}_{ab}^A) \gamma^\mu \lambda^B \\ &\quad + \frac{1}{\sqrt{2}} \left\{ \bar{\psi}_\mu \gamma^\nu \gamma^\mu \chi^\alpha \left[\left(-\frac{1}{3} \Phi \right) g_{\alpha\bar{\beta}} \hat{\partial}_\nu z^{\bar{\beta}} + \frac{1}{4} L_\alpha \partial_\nu \Phi \right] \right. \\ &\quad \left. - \frac{1}{4} f_{AB\alpha} \bar{\chi}^\alpha \gamma^{ab} \hat{F}_{ab}^{-A} \lambda^B - \frac{1}{3} \Phi L_\alpha \bar{\chi}^\alpha \gamma^{\mu\nu} D_\mu \psi_\nu \right. \\ &\quad \left. + \text{H.c.} \right\}, \end{aligned} \quad (2.11)$$

where $\hat{F}_{ab}^A \equiv e_a^\mu e_b^\nu (2\partial_{[\mu} A_{\nu]}^A + g f_{BC}^A A_\mu^B A_\nu^C + \bar{\psi}_{[\mu} \gamma_{\nu]} \lambda^A)$ [see (5.6)]. The explicit expression for the 4-fermion terms \mathcal{L}_{4f} will be presented in Sec. V. Remarkably, also \mathcal{L}_{4f} contains a significant dependence on the frame function Φ and its derivatives.

III. BOSONIC ACTION OF $N = 1$ SUPERGRAVITY IN EINSTEIN AND JORDAN FRAMES

A. The Einstein frame

By setting $\Phi = -3$ in (2.1), the general $N = 1$ action in a Jordan frame reduces to the well-known action of $N = 1$ supergravity in the Einstein frame [4,5].

It is here worth recalling some basic facts about the structure of the bosonic sector of $N = 1$, $d = 4$ supergravity. In $M_P = 1$ units, the action of $N = 1$ supergravity coupled to chiral and vector matter multiplets is usually given in the Einstein frame, where the curvature R appears in the action only through the Einstein-Hilbert term $\frac{1}{2} \times \sqrt{-g_E} R(g_E)$, where $g_E^{\mu\nu}$ is the Einstein frame space-time metric. The theory is defined by a real Kähler function $\mathcal{K}(z, \bar{z})$, by an holomorphic superpotential $W(z)$ and by an holomorphic matrix $f_{AB}(z)$ defining the action of the vector multiplets [5]. A particular feature of the theory is the Kähler geometry of the complex scalar fields.

The purely bosonic Lagrangian density reads

$$\mathcal{L}_E^{\text{bos}} = \mathcal{L}_E^{\text{grav}} + \mathcal{L}_E^{\text{scalar}} + \mathcal{L}_E^{\text{vec}}, \quad (3.1)$$

where

$$\mathcal{L}_E^{\text{grav}} + \mathcal{L}_E^{\text{scalar}} = \sqrt{-g_E} \left[\frac{1}{2} R(g_E) - g_{\alpha\bar{\beta}} \hat{\partial}_\mu z^\alpha \hat{\partial}_\nu z^{\bar{\beta}} g_E^{\mu\nu} - V_E \right], \quad (3.2)$$

$$\begin{aligned} \mathcal{L}_E^{\text{vec}} &= \sqrt{-g_E} \left[-\frac{1}{4} (\text{Re}f_{AB}) F_{\mu\nu}^A F^{\mu\nu B} \right. \\ &\quad \left. + \frac{1}{4} i (\text{Im}f_{AB}) F_{\mu\nu}^A \tilde{F}^{\mu\nu B} \right]. \end{aligned} \quad (3.3)$$

Note that the contractions of space-time indices and the definition of the dual field strength are performed using the Einstein frame metric $g_E^{\mu\nu}$. The strictly positive-definite metric $g_{\alpha\bar{\beta}}(z, \bar{z})$ of the nonlinear sigma model of scalars z^α , $\bar{z}^{\bar{\beta}}$ is given by the second derivative of the real Kähler potential

$$g_{\alpha\bar{\beta}}(z, \bar{z}) \equiv \frac{\partial}{\partial z^\alpha} \frac{\partial}{\partial \bar{z}^{\bar{\beta}}} \mathcal{K}(z, \bar{z}) > 0, \quad (3.4)$$

and $\hat{\partial}_\mu z^\alpha$ is the Yang-Mills gauge covariant derivative of a scalar field, defined by (2.7).

Concerning the potential V_E , the F -term potential V_E^F depends on \mathcal{K} and W . On the other hand, the D -term potential V_E^D depends on the values of the auxiliary D fields, obtained by solving the corresponding equations of motion:

$$\begin{aligned} V_E &= V_E^F + V_E^D \\ &= e^{\mathcal{K}} (-3W\bar{W} + \nabla_\alpha W g^{\alpha\bar{\beta}} \nabla_{\bar{\beta}} \bar{W}) + \frac{1}{2} (\text{Re}f)^{-1AB} P_A P_B. \end{aligned} \quad (3.5)$$

Notice that (3.5) yields that

$$V_J = \frac{\Phi^2}{9} V_E, \quad (3.6)$$

where the potential in the Jordan frame $V_J \equiv V$ is given by (2.9). $\nabla^\alpha W$ denotes the Kähler-covariant derivative of the superpotential. The D -term potential can be presented also in the form $V_E^D = \frac{1}{2}(\text{Re} f^{AB})D^A D^B$, where D^A is the value of the auxiliary field of the vector multiplets. This is the standard form of the purely bosonic part of the $N = 1$, $d = 4$ supergravity action in the Einstein frame. Of course, such a bosonic action can be made supersymmetric by adding suitable fermionic terms (see e.g. [4,5]).

B. The Jordan frame

In order to find the action in an arbitrary Jordan frame, one can perform a change of variables from the Einstein to the Jordan frame. Only the metric and the fermions have to be rescaled; the scalars and the vector fields do not change. The metric in a Jordan frame is related to the metric in the Einstein frame as follows (subscripts “ E ” and “ J ,” respectively, stand for Einstein and Jordan frames throughout)

$$g_J^{\mu\nu} = \Omega^2 g_E^{\mu\nu}, \quad \Omega^2 = -\frac{1}{3}\Phi(z, \bar{z}) > 0. \quad (3.7)$$

Within our treatment, we will consider the scale factor Ω^2 as an arbitrary real function of the complex scalar fields (z, \bar{z}) . Its positivity, through (3.7), correspondingly constrains $\Phi(z, \bar{z})$. Since the new action in a Jordan frame is related to the standard one in the Einstein frame by a change of variables, it is supersymmetric, as the original one.

Instead of performing the above change of variables by “brute force,” in Sec. V we use as a starting point an $N = 1$, $d = 4$ superconformal theory [3] with local $SU(2, 2|1)$ symmetry. Such a superconformal theory has a set of local symmetries that includes all $N = 1$ supergravity symmetries and, in addition, a set of extra local symmetries: local dilatation, $U(1)$ symmetry, and special supersymmetry. The superconformal theory has no dimensionful parameters.

In [3] the local dilatation, $U(1)$ symmetry and special supersymmetry were gauge fixed in a way that allowed one to reproduce the standard $N = 1$, $d = 4$ supergravity action in the Einstein frame. In fact, the purely bosonic action of $N = 1$ supergravity in a Jordan frame is already suggested by Eq. (C.5) of [3]. The complete $N = 1$ supergravity action in $d = 4$ in a generic Jordan frame has been presented in Sec. II, and it is thoroughly derived in Sec. V through a suitable gauge fixing of superconformal supergravity theory [3]. This is a symmetry-inspired approach, alternative to the “brute force” computation based on the change of variables (3.7). Here we will just present the results for the purely bosonic part of the supergravity action in a Jordan frame, which is the most relevant one for cosmology.

As mentioned above, the locally supersymmetric action is defined by the choice of four independent functions: a real Kähler potential $\mathcal{K}(z, \bar{z})$, an holomorphic superpotential $W(z)$, and an holomorphic matrix $f_{AB}(z)$, determining the kinetic vector matrix. This suffices to define the $N = 1$, $d = 4$ supergravity in the Einstein frame. When dealing with a Jordan frame, an additional fourth function, namely, the real frame function $\Phi(z, \bar{z})$, has to be specified. Thus, the purely bosonic part of the $N = 1$, $d = 4$ supergravity in a generic Jordan frame reads

$$\begin{aligned} \mathcal{L}_J^{\text{bos}} = \sqrt{-g_J} \left[-\frac{1}{6} \Phi R(g_J) + \left(\frac{1}{3} \Phi g_{\alpha\bar{\beta}} - \frac{\Phi_\alpha \Phi_{\bar{\beta}}}{\Phi} \right) \right. \\ \times \hat{\partial}_\mu z^\alpha \hat{\partial}^{\mu\bar{\beta}} \bar{z} - \frac{(\Phi_\alpha \hat{\partial} z^\alpha - \Phi_{\bar{\beta}} \hat{\partial} \bar{z}^{\bar{\beta}})^2}{4\Phi} - \frac{\Phi^2}{9} V_E \\ \left. + \mathcal{L}_1^{\text{bos}} \right]. \quad (3.8) \end{aligned}$$

Here V_E is the Einstein frame potential defined in (3.5), $\frac{\Phi^2}{9} V_E = V_J \equiv V$ is the Jordan frame potential given by (2.9), and

$$\Phi_\alpha \equiv \frac{\partial}{\partial z^\alpha} \Phi(z, \bar{z}), \quad \Phi_{\bar{\beta}} \equiv \frac{\partial}{\partial \bar{z}^{\bar{\beta}}} \Phi(z, \bar{z}) = \bar{\Phi}_{\bar{\beta}}. \quad (3.9)$$

Notice that (3.8) is implied by (2.1), (2.4), and (2.7), observing that $\partial_\mu \Phi = \hat{\partial}_\mu \Phi$ because in general Φ is gauge invariant. Furthermore, $\mathcal{L}_{1,J}^{\text{bos}} = \mathcal{L}_{1,E}^{\text{bos}} = \mathcal{L}_1^{\text{bos}}$ is conformal invariant (and therefore frame independent), and it is given by the purely bosonic part of (2.6), or equivalently by $[(-g_E)^{-1/2} \text{ times}]$ (3.3):

$$\begin{aligned} \mathcal{L}_{1,J}^{\text{bos}} = \mathcal{L}_{1,E}^{\text{bos}} = \mathcal{L}_1^{\text{bos}} \\ = -\frac{1}{4}(\text{Re} f_{AB}) F_{\mu\nu}^A F^{\mu\nu B} + \frac{1}{4}(\text{Im} f_{AB}) F_{\mu\nu}^A \tilde{F}^{\mu\nu B}. \quad (3.10) \end{aligned}$$

In the Jordan frame, the contractions of space-time indices and the definition of the dual field strength are performed using the Jordan frame metric $g_J^{\mu\nu}$ given by (3.7).

It should be remarked that (3.8) yields that the geometry of the nonlinear sigma model of scalars is of a modified Kähler type: indeed, due to the term proportional to $(\Phi_\alpha \hat{\partial} z^\alpha - \Phi_{\bar{\beta}} \hat{\partial} \bar{z}^{\bar{\beta}})^2$, the metric is not Hermitian; i.e. there are terms of the form $dzd\bar{z}$ and complex conjugate; furthermore, the metric term of $dzd\bar{z}$ is not of the Kähler type.

As a consequence of the previous treatment and computations, by setting $\Phi = -3$ in (3.8) the purely bosonic part of the $N = 1$, $d = 4$ supergravity action in the Einstein frame [4,5] is recovered. With the choice $\Phi = -3e^{-(1/3)\mathcal{K}(z,\bar{z})}$, (3.8) yields to the purely bosonic action of $N = 1$ supergravity in the particular Jordan frame considered in [4,5].

C. Canonical kinetic terms for scalars

In the Einstein frame, the kinetic term of scalar fields is given by $g_{\alpha\bar{\beta}}\hat{\partial}_\mu z^\alpha\hat{\partial}^\mu\bar{z}^\beta$, where $g_{\alpha\bar{\beta}}(z, \bar{z})$ is given by (3.4). Thus, canonical kinetic terms are possible for the following choice of a Kähler potential:

$$\mathcal{K}(z, \bar{z}) = \delta_{\alpha\bar{\beta}}z^\alpha\bar{z}^\beta + f(z) + \bar{f}(\bar{z}), \quad (3.11)$$

where $f(z)$ is a holomorphic function (associated with the considered Kähler gauge). A 1-modulus example of the canonical Kähler potential (3.11) is provided by the shift-symmetric function $\mathcal{K}(z, \bar{z}) = -\frac{1}{2}(z - \bar{z})^2$, often used in cosmology.

As pointed out above, an early version of $N = 1$, $d = 4$ supergravity theory in a (particular) Jordan, as well as in the Einstein, frame was derived in [4,5] on the basis of the superconformal calculus, with Φ and \mathcal{K} related as follows:

$$\mathcal{K}(\Phi(z, \bar{z})) = -3 \log(-\frac{1}{3}\Phi(z, \bar{z})), \quad (3.12)$$

and Ω^2 is given by (3.7).

Within such a framework, the following simpler form of $\mathcal{L}_J^{\text{bos}}$ given by (3.8) is obtained:

$$\begin{aligned} \mathcal{L}_J^{\text{bos}, \mathcal{K}(\Phi)} = & \sqrt{-g_J} \left[-\frac{1}{6} \Phi R(g_J) - \Phi_{\alpha\bar{\beta}} \hat{\partial}_\mu z^\alpha \hat{\partial}^\mu \bar{z}^\beta \right. \\ & \left. + \Phi \mathcal{A}_\mu^2 - \frac{\Phi^2}{9} V_E + \mathcal{L}_1^{\text{bos}, \mathcal{K}(\Phi)} \right]. \end{aligned} \quad (3.13)$$

The kinetic term for the scalar action is partly determined by the value \mathcal{A}_μ of the bosonic part of the auxiliary field of supergravity, entering in the action $\mathcal{L}_J^{\text{bos}}$ (3.8) as $\Phi \mathcal{A}_\mu \mathcal{A}^\mu$. In the case of gauge-invariant \mathcal{K} , \mathcal{A}_μ reads³

$$\begin{aligned} \mathcal{A}_\mu &= \frac{1}{6} i (\hat{\partial}_\mu z^\alpha \partial_\alpha \mathcal{K} - \hat{\partial}_\mu \bar{z}^{\bar{\alpha}} \partial_{\bar{\alpha}} \mathcal{K}) \\ &= -\frac{i}{2\Phi} (\hat{\partial}_\mu z^\alpha \partial_\alpha \Phi - \hat{\partial}_\mu \bar{z}^{\bar{\alpha}} \partial_{\bar{\alpha}} \Phi). \end{aligned} \quad (3.14)$$

The purely bosonic action (3.13) yields to the following statement: within the relation (3.12) between \mathcal{K} and Φ , in order to have canonical kinetic terms in the Jordan frame it is sufficient

(a) to choose the frame function Φ as follows:

$$\Phi(z, \bar{z}) = -3 + \delta_{\alpha\bar{\beta}}z^\alpha\bar{z}^\beta + J(z) + \bar{J}(\bar{z}), \quad (3.15)$$

where $J(z)$ is holomorphic. Note that (3.12) and (3.15) imply \mathcal{K} to read

³When the Kähler potential is not gauge invariant in direction A , the auxiliary pseudovector has an additional contribution depending on a gauge field, $+\frac{1}{6}iA_\mu^A(r_A - \bar{r}_A)$, where r_A is the holomorphic part of the transformation of the Kähler potential under gauge symmetry, $\delta\mathcal{K}(z, \bar{z}) = \theta^A[r_A(z) + \bar{r}_A(\bar{z})]$; see [3].

$$\mathcal{K}(z, \bar{z}) = -3 \log[1 - \frac{1}{3}\delta_{\alpha\bar{\alpha}}z^\alpha\bar{z}^{\bar{\alpha}} - \frac{1}{3}J(z) - \frac{1}{3}\bar{J}(\bar{z})]; \quad (3.16)$$

(b) to consider only (scalar) configurations for which the contribution from the bosonic part of the auxiliary vector field vanishes:

$$\mathcal{A}_\mu = 0. \quad (3.17)$$

The embedding of the NMSSM into supergravity along the lines suggested in [8] requires only the knowledge of the simple case in which the relation (3.12) between \mathcal{K} and Φ holds. Moreover, concerning the canonicity of the kinetic terms of scalars, in the treatment below we will see that condition (a) is always satisfied, and condition (b) given by (3.17) is satisfied during the cosmological evolution, when the system under consideration depends on three real fields: h_1, h_2, s . Thus, apart from the frame function Φ given by (3.15), the action of the NMSSM embedded in supergravity in the Jordan frame (3.12) along the lines of [8] has canonical kinetic scalar terms and a potential $\frac{\Phi^2}{9}V_E$ (see Secs. IV B and IV C for details). In particular, when only the Higgs field h is nonvanishing in the D -flat direction, the Jordan frame supergravity potential is extremely simple and is given by $\frac{\Lambda^2}{4}h^4$, see Eq. (4.29).

IV. SUPERGRAVITY EMBEDDING OF THE NMSSM AND COSMOLOGY

A. Classical approximation of the Higgs-type inflation with nonminimal ξ coupling

The essential reason for the new version of the SM inflation [2] to work successfully is the following. The SM potential with canonical kinetic term for the Higgs field h is coupled to a gravitational field in a suitable Jordan frame. In other words, the Lagrangian density to start with reads

$$\begin{aligned} \mathcal{L}_J = & \sqrt{-g_J} \left[\frac{M^2 + \xi h^2}{2} R(g_J) - \frac{1}{2} \partial_\mu h \partial_\nu h g_J^{\mu\nu} \right. \\ & \left. - \frac{\lambda}{4} (h^2 - v^2)^2 \right]. \end{aligned} \quad (4.1)$$

At present, $h = v \sim 10^{-16}M_P$ and $M_P^2 = M^2 + \xi v^2$. Therefore $M \approx M_P$ for $\sqrt{\xi} < 10^{16}$. In the subsequent investigation we will consider $\xi < 10^6$. In this case $M = M_P$ with a very good accuracy. In our paper we will use the system of units where $M = M_P = 1$.

In general, the cosmological predictions have to be compared with the observations in the Einstein frame, related to the Jordan one through the conformal rescaling (3.7), with

$$\Phi = -3(1 + \xi h^2), \quad \Omega^2 = 1 + \xi h^2. \quad (4.2)$$

By switching to the Einstein frame, (4.1) yields to

$$\mathcal{L}_E = \sqrt{-g_E} \left(\frac{1}{2} R(g_E) - \frac{1}{2} \partial_\mu \psi \partial_\nu \psi g_E^{\mu\nu} - U(\psi) \right), \quad (4.3)$$

where ψ is a canonically normalized scalar in the Einstein frame, defined by

$$d\psi \equiv dh \sqrt{\frac{\Omega^2 + 6\xi^2 h^2}{\Omega^4}}, \quad (4.4)$$

where

$$U(\psi) = \frac{\lambda}{4} \left(\frac{h^2(\psi) - v^2}{1 + \xi h(\psi)^2} \right)^2. \quad (4.5)$$

The relation between the field h and the canonically normalized field ψ looks very different in *three* different ranges of h . At $h \ll \frac{1}{\xi}$ one has $\psi \approx h$. In the interval $\frac{1}{\xi} \ll h \ll \frac{1}{\sqrt{\xi}}$, the relation between h and ψ is more complicated: $\psi \approx \sqrt{\frac{3}{2}} \xi h^2$. Finally, for $h \gg \frac{1}{\sqrt{\xi}}$ (or, equivalently, $\psi \gg 1$) one has $h \sim \frac{1}{\sqrt{\xi}} e^{\psi/\sqrt{6}}$. In this regime, the potential in the Einstein frame is very flat, which leads to inflation:

$$U(\psi)_{\psi \rightarrow \infty} \Rightarrow \frac{\lambda}{4\xi^2} (1 + e^{-(2\psi)/\sqrt{6}})^{-2}. \quad (4.6)$$

As one can see from (4.6), the constant (ψ -independent) term in the potential $U(\psi)$ is $\frac{\lambda}{4\xi^2}$, so nothing would work without the nonminimal scalar-curvature coupling proportional to ξ .

The Hubble constant during inflation in this model is $H \approx \sqrt{\frac{\Lambda}{3}} \frac{1}{2\xi}$. For the nonsupersymmetric standard model, $\lambda = O(1)$, so one could worry that this energy scale is dangerously close to the possible unitarity bound $\Lambda \sim 1/\xi$ discussed in [10–12]. One should note, however, that most of the arguments suggesting the existence of this bound are based on the investigation of the theory in the small field approximation $\psi \approx h$, where one can use an expansion $\psi = h(1 + \xi^2 h^2 + \dots)$. This approximation is valid only for $h \ll \frac{1}{\xi}$, which is parametrically far from the inflationary regime at $h \gg \frac{1}{\sqrt{\xi}}$. We are going to return to this issue in a forthcoming publication; see also a discussion in [13], and especially in [14], where it was noticed that in NMSSM one may consider the regime with $\lambda \ll 1$, where the concerns about the unitarity bound do not seem to appear.

It is worth noting that potentials exponentially rapidly approaching a constant positive value have been proposed in one of the first models of chaotic inflation in supergravity [15], but at that time models of this type were lacking a compelling motivation. Therefore, it is very tempting to use the intuitively appealing and simple model discussed above as a starting point, in order to analyze the Einhorn-

Jones approach [8] to embed NMSSM into $N = 1$, $d = 4$ supergravity, and its relevance for the issue of inflation.

B. Embedding of the NMSSM into supergravity and the Einhorn-Jones cosmological inflationary model

The Higgs field sector of NMSSM has one gauge singlet and two gauge doublet chiral superfields, namely [9],

$$z^\alpha = \{S, H_1, H_2\}, \quad (4.7)$$

with

$$S = s e^{i\alpha}, \quad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}. \quad (4.8)$$

As in [8], the frame function is chosen as follows:

$$\Phi(z, \bar{z}) = -3 + (S\bar{S} + H_u H_u^\dagger + H_d H_d^\dagger) + \frac{3}{2} \chi (H_u \cdot H_d + \text{H.c.}), \quad (4.9)$$

where

$$H_u \cdot H_d \equiv -H_u^0 H_d^0 + H_u^+ H_d^-. \quad (4.10)$$

Note that (4.9) is of the form (3.15), with $J = \frac{3}{2} \chi H_u \cdot H_d$. In this framework, the Kähler potential is related to Φ through (3.12), and the superpotential is chosen to be

$$W = -\lambda S H_u \cdot H_d + \frac{\rho}{3} S^3. \quad (4.11)$$

Thus, the action of such an implementation of NMSSM depends on five chiral superfields. Through explicit computations, we checked that such an action admits a consistent truncation in which the charged superfields, namely, H_u^+ and H_d^- , are absent. Therefore, below we deal with a simplified action of NMSSM, containing only three superfields: S , H_u^0 , and H_d^0 , such that

$$H_1 = \begin{pmatrix} 0 \\ H_u^0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_d^0 \\ 0 \end{pmatrix}. \quad (4.12)$$

Within this truncation, the frame function and the superpotential, respectively, read

$$\Phi(z, \bar{z}) = -3 + (|S|^2 + |H_u^0|^2 + |H_d^0|^2) - \frac{3}{2} \chi (H_u^0 H_d^0 + \bar{H}_u^0 \bar{H}_d^0); \quad (4.13)$$

$$W = \lambda S H_u^0 H_d^0 + \frac{\rho}{3} S^3. \quad (4.14)$$

Thus, by recalling Eqs. (3.6), (3.13), and (3.15) and by disregarding $\mathcal{L}_1^{\text{bos}, \mathcal{K}(\Phi)}$ in (3.13), one obtains the following Jordan frame supergravity scalar-gravity action for this implementation of NMSSM:

$$\frac{\mathcal{L}_J^{\text{NMSSM}}}{\sqrt{-g_J}} = \frac{1}{2}R(g_J) + \frac{1}{6}\left[\delta_{\alpha\bar{\beta}}z^\alpha\bar{z}^{\bar{\beta}} + \frac{3}{2}\chi(H_u^0H_d^0 + \bar{H}_u^0\bar{H}_d^0)\right]R - \delta_{\alpha\bar{\beta}}\hat{\delta}_\mu z^\alpha\hat{\delta}^\mu\bar{z}^{\bar{\beta}} - \Phi\mathcal{A}_\mu^2 - V_J, \quad (4.15)$$

where \mathcal{A}_μ is given by (3.14).

Remarkably, the scalar-curvature coupling exhibited by (4.15) breaks the discrete \mathbb{Z}_3 symmetry of the theory due to the chosen cubic superpotential (4.14) of NMSSM. Such a symmetry may generate domain walls after the spontaneous breaking of a symmetric phase in the early universe. In such a case, unacceptably large anisotropies of CMB may be generated. This is a well-known domain wall problem of NMSSM (see e.g. [9]). The scalar-curvature coupling in (4.13) and in (4.15) breaks the discrete \mathbb{Z}_3 symmetry. This may help to remove the eventual domain wall problem. Thus, it is challenging and interesting to formulate a consistent cosmology within this framework.

As usual, $V_J = V_J^F + V_J^D$. In the present framework, V_J^F has a zero, second, and fourth power of the S field:

$$V_J^F = \lambda^2|H_u^0|^2|H_d^0|^2 + \lambda\rho(\bar{S}^2H_u^0H_d^0 + \text{c.c.}) - \frac{2\lambda^2|S|^2|H_A^0|^2(\chi(H_u^0H_d^0 + \text{c.c.}) - 2)}{4 + 3\chi^2|H_A^0|^2 - 2\chi(H_u^0H_d^0 + \text{H.c.})} + \rho^2|S|^4. \quad (4.16)$$

On the other hand, V_J^D reads

$$V_J^D = \frac{g^{\prime 2}}{8}(|H_u^0|^2 - |H_d^0|^2)^2 + \frac{g^{\prime 2}}{8}((H_u)^{\dagger}\vec{\tau}H_u + (H_d)^{\dagger}\vec{\tau}H_d)^2, \quad (4.17)$$

where $\vec{\tau}$ is the 3-vector of Pauli σ matrices.

In [8] this model was described at the vanishing value of the gauge singlet field S . In order to analyze the theory consistently, in the present treatment we keep the full dependence on S .

C. Cosmology in the Jordan frame

We start by checking that the CP -invariant solution found in [8], in which S , H_u^0 , and H_d^0 are real, corresponds to a (n at least local) minimum of V_J itself. In order to do so, *a priori* we assume that these three fields are complex, namely $(s, h_1, h_2 \in \mathbb{R}^+, \alpha, \alpha_1, \alpha_2 \in [0, 2\pi))$,

$$S = se^{i\alpha}, \quad H_u^0 = h_1e^{i\alpha_1}, \quad H_d^0 = h_2e^{i\alpha_2}. \quad (4.18)$$

By computing (4.15), it follows that the scalar-gravity action depends only on the combination angles $\gamma \equiv \alpha_1 + \alpha_2$ and $\delta \equiv 2\alpha - \alpha_1 - \alpha_2$. More precisely, the dependence on δ enters via $\lambda\rho \cos\delta$ and the dependence on γ is via $\chi \cos\gamma$. In order to study CP -invariant solution(s) with $\alpha = \alpha_1 = \alpha_2 = 0$, one has to analyze the minima of the potential V_J , also taking into account the R -dependent terms in (4.15) (notice that V_J^D does not depend on any phase).

First, we notice that Eqs. (4.16) and (4.17) and the definition of δ yield that the dependence on δ enters only in one term in the potential, namely,

$$V_J(\delta) = 2\lambda\rho|S|^2|H_u^0||H_d^0|\cos\delta. \quad (4.19)$$

This potential has a minimum at $\delta = 0$, under the condition that $\lambda\rho$ is negative: $\lambda\rho = -|\lambda\rho|$.

Second, in order to deal correctly with the dependence on γ , one can look at the expected minimum of the potential at $S = 0$ [8]. Equation (4.16) implies that the Jordan frame potential at $S = 0$ is very simple:

$$V_J^F|_{S=0} = \lambda^2|H_u^0|^2|H_d^0|^2. \quad (4.20)$$

At $S = 0$ the dependence on γ enters only through the frame function Φ given by Eq. (4.13). By switching to the Einstein frame and recalling the relation (3.6), one obtains

$$(V_E(\gamma))_{S=0} = \frac{9\lambda^2|H_u^0|^2|H_d^0|^2}{\Phi^2} = \frac{\lambda^2|H_u^0|^2|H_d^0|^2}{[1 - \frac{1}{3}(|H_u^0|^2 + |H_d^0|^2) + \chi|H_u^0||H_d^0|\cos\gamma]^2}. \quad (4.21)$$

Since during inflation $1 - \frac{1}{3}(|H_A^0|^2) > 0$ [8], it can be checked that during inflation $\gamma = 0$ is a minimum of V_J , under the condition that $\chi > 0$.

Thus, the CP -invariant solution with three real fields is confirmed to be a minimum in the directions of angles δ and γ during inflation. Therefore we can take

$$S = s, \quad H_u^0 = h_1, \quad H_d^0 = h_2, \quad (4.22)$$

provided that the coupling constants of the model under consideration satisfy

$$\lambda\rho < 0, \quad \chi > 0. \quad (4.23)$$

Notice that (4.13) and (4.22) yield that the kinetic scalar terms in the Jordan frame are canonical, since both sufficient conditions (3.15) and (3.17) are satisfied [in particular, $\mathcal{A}_\mu = 0$ on scalar configurations (4.22)]:

$$(\mathcal{L}_J^{\text{NMSSM}})_{\text{kinetic}} = -\sqrt{-g_J}[(\partial_\mu s)^2 + (\partial_\mu h_1)^2 + (\partial_\mu h_2)^2]. \quad (4.24)$$

It is now convenient to switch to the standard mixing of the Higgs fields, defined as

$$h_1 \equiv h \cos\beta, \quad h_2 \equiv h \sin\beta, \quad (4.25)$$

which leaves us with two real fields, h and β , instead of h_1 and h_2 .

Through Eq. (4.17), the D -flat direction, defined by

$$V_J^D = 0, \quad (4.26)$$

requires that

$$\sin(2\beta) = 1; \quad h_1^2 = h_2^2 = h^2/2. \quad (4.27)$$

Thus, along the D -flat direction, the curvature term of (4.15) simplifies to

$$(\mathcal{L}_J^{\text{NMSSM}})_{\text{curv}} = \frac{\sqrt{-g_J}}{2} \left[1 - \frac{1}{3}(s^2 + h^2) + \frac{1}{2}\chi h^2 \right] R(g_J). \quad (4.28)$$

On the other hand, along the D -flat direction (4.26) and (4.27) the F -term potential reads

$$V_J^F = \frac{\lambda^2}{4} h^4 - |\lambda\rho| s^2 h^2 - \frac{2\lambda^2 s^2 h^2 (\chi h^2 - 2)}{4 + 3\chi^2 h^2 - 2\chi h^2} + \rho^2 s^4. \quad (4.29)$$

In [8] the inflationary regime driven by the Higgs within NMSSM was shown to take place for

$$\chi h^2 \gg 1 \gg h^2, \quad s \approx 0, \quad \beta = \frac{\pi}{4}, \quad (4.30)$$

in Planck units $M_P^2 = 1$. For small s , (4.29) can be simplified as follows:

$$V_J^F \sim \frac{\lambda^2}{4} h^4 - \left(|\lambda\rho| + \frac{2\lambda^2}{3\chi} \right) s^2 h^2. \quad (4.31)$$

The effective mass of the s field is negative, but one actually has to take into account an effective contribution from the curvature-scalar coupling. This latter provides a positive contribution; however, it does not remove the tachyonic instability of the system in the s direction. Indeed, for small s , the complete expression of the effective potential is

$$\tilde{V}_J^F \sim \frac{\lambda^2}{4} h^4 - \left(|\lambda\rho| + \frac{\lambda^2}{3\chi} \right) s^2 h^2. \quad (4.32)$$

As we will see in the next section, the instability in the s direction is very strong, corresponding to a large tachyonic mass and a slow-roll parameter $|\eta| \geq 2/3$. As a result, a rapidly developing tachyonic instability does not allow inflation to occur in the regime studied in [8].

Note that in general instead of $\lambda\rho < 0$ one could take $\lambda\rho > 0$. Correspondingly, such a choice of coupling constants would stabilize the real part of the field S , but it would lead to an equally strong instability in the direction of its imaginary part. In other words, independently of the sign of $\lambda\rho$, the potential with respect to the complex field S has a saddle point at $S = 0$, which results in the tachyonic instability in one of the two directions.

D. Switching to the Einstein frame

In the Einstein frame, (3.6) and (4.16) yield that the F -term potential is

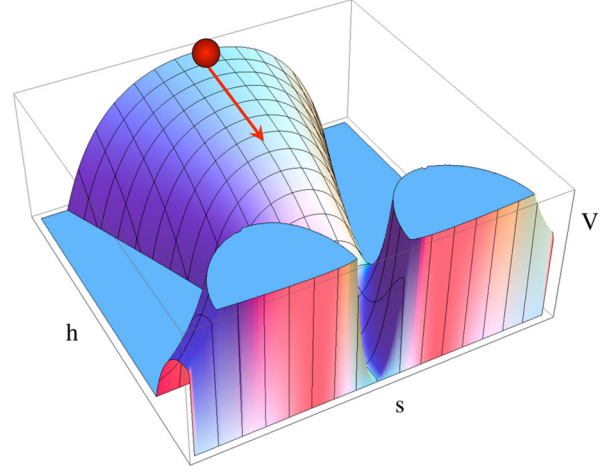


FIG. 1 (color online). The F -term potential V^F in the Einstein frame. The inflationary trajectory $s = 0$ is unstable.

$$V_E^F = \frac{9}{\Phi^2} V_J = \frac{\frac{\lambda^2}{4} h^4 - |\lambda\rho| s^2 h^2 - \frac{2\lambda^2 s^2 h^2 (\chi h^2 - 2)}{4 + 3\chi^2 h^2 - 2\chi h^2} + \rho^2 s^4}{\left[1 - \frac{1}{3}(s^2 + h^2) + \frac{1}{2}\chi h^2 \right]^2}. \quad (4.33)$$

Let us compute the effective mass of the s field also in the Einstein frame, where by definition there is no contribution from the curvature coupling. During the inflationary regime (4.30) [8], the leading behavior of the potential is

$$V_E^F \sim \frac{\lambda^2}{\chi^2} - \left(|\lambda\rho| + \frac{\lambda^2}{3\chi} \right) \frac{4s^2}{\chi^2 h^2} + O(s^4). \quad (4.34)$$

The shape of the potential is shown in Fig. 1. The trajectory with $s = 0$ at large h , which was expected to be an inflationary trajectory in [8], is unstable. It corresponds to the top of the ridge for the potential V_E^F ; see Fig. 1.

In order to find whether this instability is dangerous, one should calculate the tachyonic mass of the s field and compare it to the Hubble constant. This will allow us to check whether the tachyonic instability develops rapidly, or whether it occurs on a time scale much smaller than the cosmological time scale H^{-1} . An alternative way to approach this issue is to find the related value of the relevant slow-roll parameter η .

To find the effective mass of the s field, attention must be paid to the nonminimal normalization of the field $S = s e^{i\alpha}$. At constant α , the kinetic term of field S is given by

$$g_{S\bar{S}} \partial S \partial \bar{S} = \frac{2}{\chi h^2} \partial S \partial \bar{S} = \frac{2}{\chi h^2} (\partial s)^2. \quad (4.35)$$

Thus, in the vicinity of the inflationary trajectory $s \approx 0$ (4.30), the Lagrangian density of the field s is

$$\mathcal{L}_{E,s} = -\frac{2}{\chi h^2}(\partial s)^2 - \frac{\lambda^2}{\chi^2} + \left(|\lambda\rho| + \frac{\lambda^2}{3\chi}\right)\frac{4s^2}{\chi^2 h^2} + \mathcal{O}(s^4). \quad (4.36)$$

In terms of the canonical scalar field $\tilde{s} = \frac{2s}{\sqrt{\chi}h}$, such a Lagrangian at small fields \tilde{s} is

$$\mathcal{L}_{E,\tilde{s}} = -\frac{1}{2}(\partial\tilde{s})^2 - \frac{\lambda^2}{\chi^2} + \left(\frac{|\lambda\rho|}{\chi} + \frac{\lambda^2}{3\chi^2}\right)\tilde{s}^2 + \mathcal{O}(\tilde{s}^4), \quad (4.37)$$

resulting in the mass squared of the \tilde{s} field to be tachyonic:

$$m_{\tilde{s}}^2 \sim -2\left(\frac{\lambda^2}{3\chi^2} + \frac{|\lambda\rho|}{\chi}\right) < 0. \quad (4.38)$$

Taking into account that during inflation $H^2 = V/3 \approx \frac{\lambda^2}{3\chi^2}$, it thus follows that

$$m_{\tilde{s}}^2 \leq -\frac{2\lambda^2}{3\chi^2} = -\frac{2V}{3} = -2H^2 = R/6. \quad (4.39)$$

Interestingly, $m_{\tilde{s}}^2$ resembles the conformal mass $m^2 = -R/6$, but has an opposite sign. Since $|m_{\tilde{s}}^2| > H^2$, the trajectory $\tilde{s} = 0$ is exponentially unstable and unsuitable for inflation. One can also reach the same conclusion by computing the relevant slow-roll parameter η in the \tilde{s} direction:

$$\eta_{\tilde{s}} \equiv \frac{m_{\tilde{s}}^2}{V} = -\frac{2}{3} - \frac{2|\lambda\rho|\chi}{\lambda^2} < -\frac{2}{3}. \quad (4.40)$$

We did not find any way to solve this problem of the Einhorn-Jones model [8].

It should also be clearly stated that there are many other scalar fields in this model, and the field s is not the only one that may experience a tachyonic instability. This is supported by the results obtained in the Appendix, where the dependence of the potential on the angular variable β is studied. Therein, we find that in certain cases the postinflationary cosmological trajectory may experience an additional tachyonic instability and deviate from the value $\beta = \frac{\pi}{4}$ characterizing the D -flat direction (4.30).

We should emphasize, however, that these results are model dependent. We believe that the cosmological models based on $N = 1$, $d = 4$ supergravity in the Jordan frame can be very interesting, and they certainly deserve further investigation. In the past, a systematic study of such models was precluded by the absence of the corresponding formalism, which we presented in a complete form in Sec. II. In the next section we will give a detailed derivation of the complete $N = 1$, $d = 4$ supergravity action in a generic Jordan frame.

V. DERIVATION OF THE COMPLETE $N = 1$ SUPERGRAVITY ACTION IN A JORDAN FRAME

Here we use the superconformal action [3] and gauge fix it to get a complete $N = 1$ supergravity action, including fermions, in an arbitrary Jordan frame. Superconformal invariance means that the action is invariant under the local symmetries of the superconformal algebra. This involves, apart from the super-Poincaré transformations, local dilatations, a local $U(1)$ R symmetry, local special conformal transformations, and an extra special supersymmetry, denoted as S supersymmetry. One first constructs a ‘‘superconformal action,’’ i.e. an action that is invariant under all symmetries of the superconformal algebra. Then one gets rid of the extra symmetries by imposing gauge conditions.

The vierbein e_{μ}^a and gravitino ψ_{μ} are the gauge fields of the translations and Q supersymmetry, which belong to the super-Poincaré algebra. The gauge field of local Lorentz rotations is the spin connection ω_{μ}^{ab} , which is a constrained field; i.e. it has as usual a value that depends on e_{μ}^a and ψ_{μ} . We will write here the expressions in terms of $\omega_{\mu}^{ab}(e)$, which is the usual torsionless spin connection of gravity. Also the gauge fields of special conformal transformations and of S supersymmetry are such composite fields. In the expressions below, they have been substituted by their values. On the other hand, the gauge field of the $U(1)$ R symmetry, A_{μ} , is an auxiliary field. Its value will be given below. Finally, the gauge field of dilatations is a field b_{μ} , which will later be set to zero by a gauge condition for the special conformal symmetry.

The superconformal transformations of the vierbein and gravitino are (apart from general coordinate transformations)

$$\begin{aligned} \delta e_{\mu}^a &= -\lambda^a_b e_{\mu}^b - \lambda_D e_{\mu}^a + \frac{1}{2}\bar{\epsilon}\gamma^a\psi_{\mu}, \\ \delta\psi_{\mu} &= \left(-\frac{1}{4}\lambda^{ab}\gamma_{ab} - \frac{1}{2}\lambda_D + \frac{3}{2}i\lambda_T\gamma_*$$

where λ^{ab} are the parameters of local Lorentz transformations, λ_D are those of dilatations, and λ_T are those of the $U(1)$ R symmetry. ϵ and η are the spinor parameters of Q and S supersymmetry, respectively.

A. The superconformal action

We first repeat the result for the full superconformal action using the notation that we will use in this paper. The action contains three superconformal-invariant terms

$$\mathcal{L} = [\mathcal{N}]_D + [\mathcal{W}]_F + [f_{AB}\bar{\lambda}^A P_L \lambda^B]_F. \quad (5.2)$$

The first one is defined by a Kähler potential $\mathcal{N}(X, \bar{X})$ for the superconformal fields, the second uses a superpotential $\mathcal{W}(X)$, and the third involves the chiral kinetic matrix

$f_{AB}(X)$ (where A are the gauge indices) and gauginos λ^A . The matrix $P_L = \frac{1}{2}(1 + \gamma_*)$ projects on the left-handed fermions. The dilatation symmetry implies that \mathcal{N} should be homogeneous of first order in both X and \bar{X} , \mathcal{W} should be homogeneous of third degree, and $f_{AB}(X)$ is of zeroth order, i.e.

$$X^I \frac{\partial}{\partial X^I} \mathcal{N} = \bar{X}^{\bar{I}} \frac{\partial}{\partial \bar{X}^{\bar{I}}} \mathcal{N} = \mathcal{N}, \quad X^I \frac{\partial}{\partial X^I} \mathcal{W} = 3 \mathcal{W},$$

$$X^I \frac{\partial}{\partial X^I} f_{AB} = 0. \quad (5.3)$$

The superconformal chiral multiplets contain the bosonic fields X^I and fermions $\Omega^I = P_L \Omega^I$. We assume that they transform under the gauge symmetries depending on Killing vectors $k_A^I(X)$,

$$\delta X^I = \theta^A k_A^I, \quad \delta \Omega^I = \theta^A \partial_J k_A^I \Omega^J. \quad (5.4)$$

$$\delta X^I = (\lambda_D + i\lambda_T) X^I + \frac{1}{\sqrt{2}} \bar{\epsilon} \Omega^I,$$

$$\delta \Omega^I = \left(-\frac{1}{4} \lambda^{ab} \gamma_{ab} + \frac{3}{2} \lambda_D - \frac{1}{2} i \lambda_T \right) \Omega^I + \frac{1}{\sqrt{2}} P_L (\not{D} X^I + F^I) \epsilon + \sqrt{2} X^I P_L \eta,$$

$$\delta A_\mu^A = -\frac{1}{2} \bar{\epsilon} \gamma_\mu \lambda^A,$$

$$\delta \lambda^A = \left(-\frac{1}{4} \lambda^{ab} \gamma_{ab} + \frac{3}{2} \lambda_D + \frac{1}{2} i \lambda_T \gamma_* \right) \lambda^A + \left[\frac{3}{2} \lambda_D + \frac{3}{2} i \gamma_* \lambda_T \right] \lambda^A + \left[\frac{1}{4} \gamma^{ab} \hat{F}_{ab}^A + \frac{1}{2} i \gamma_* D^A \right] \epsilon, \quad (5.6)$$

$$\mathcal{D}_\mu X^I = (\partial_\mu - b_\mu - i A_\mu) X^I - \frac{1}{\sqrt{2}} \bar{\psi}_\mu \Omega^I - A_\mu^A k_A^I,$$

$$\hat{F}_{ab}^A = e_a^\mu e_b^\nu (2\partial_{[\mu} A_{\nu]}^A + g f_{BC}^A A_\mu^B A_\nu^C + \bar{\psi}_{[\mu} \gamma_{\nu]} \lambda^A).$$

After elimination of the auxiliary fields, the terms in (5.2) mix. The scalars form a Kähler manifold with metric, connection, and curvature given by

$$G_{I\bar{J}} = \mathcal{N}_{I\bar{J}}, \quad \Gamma_{JK}^I = G^{I\bar{L}} \mathcal{N}_{JK\bar{L}},$$

$$R_{I\bar{K}J\bar{L}} = \mathcal{N}_{I\bar{J}K\bar{L}} - \mathcal{N}_{I\bar{J}\bar{M}} G^{M\bar{M}} \mathcal{N}_{M\bar{K}\bar{L}}. \quad (5.7)$$

The superconformal action⁵ can be split into several parts:

$$e^{-1} \mathcal{L} = \frac{1}{6} \mathcal{N}[-R(e, b) + \bar{\psi}_\mu R^\mu + e^{-1} \partial_\mu (e \bar{\psi} \cdot \gamma \psi^\mu)] \mathcal{L}_0$$

$$+ \mathcal{L}_{1/2} + \mathcal{L}_1 - V + \mathcal{L}_m + \mathcal{L}_{\text{mix}} + \mathcal{L}_d + \mathcal{L}_{4f}. \quad (5.8)$$

The leading kinetic terms of the matter multiplets are

⁴Note that this does not imply that the Kähler potential or superpotential of the Einstein theory should be invariant under the gauge transformations, as we will see below.

⁵There is a possible generalization including a Chern-Simons term (see [16]), which we neglect here.

These Killing vectors should satisfy homogeneity equations due to the conformal symmetry and leave \mathcal{N} and \mathcal{W} invariant.⁴ These statements can be encoded in the following equations:

$$\partial_J k_A^I = 0, \quad X^J \partial_J k_A^I = k_A^I,$$

$$\mathcal{N}_I k_A^I + \mathcal{N}_{\bar{I}} k_A^{\bar{I}} = 0,$$

$$\mathcal{P}_A = \frac{1}{2} i (\mathcal{N}_I k_A^I - \mathcal{N}_{\bar{I}} k_A^{\bar{I}}) = i \mathcal{N}_I k_A^I = -i \mathcal{N}_{\bar{I}} k_A^{\bar{I}},$$

$$\partial_{\bar{I}} \mathcal{P}_A = i \mathcal{N}_{\bar{J}I} k_A^J, \quad \mathcal{W}_I k_A^I = 0. \quad (5.5)$$

We use here the notation that derivatives on \mathcal{N} and \mathcal{W} are denoted by adding indices, similar to (3.4).

The physical fields of the chiral and gauge multiplets transform as follows under the superconformal transformations:

$$\mathcal{L}_0 = -G_{I\bar{J}} D^\mu X^I D_\mu \bar{X}^{\bar{J}},$$

$$\mathcal{L}_{1/2} = -\frac{1}{2} G_{I\bar{J}} [\bar{\Omega}^I \hat{\not{D}} \Omega^{\bar{J}} + \frac{1}{2} \bar{\Omega}^{\bar{J}} \hat{\not{D}} \Omega^I],$$

$$\mathcal{L}_1 = (\text{Re} f_{AB}) \left[-\frac{1}{4} F_{\mu\nu}^A F^{\mu\nu B} - \frac{1}{2} \bar{\lambda}^A \not{D} \lambda^B \right]$$

$$+ \frac{1}{4} i [(\text{Im} f_{AB}) F_{\mu\nu}^A \tilde{F}^{\mu\nu B} + (D_\mu \text{Im} f_{AB}) \bar{\lambda}^A \gamma_* \gamma^\mu \lambda^B]. \quad (5.9)$$

The potential in the conformal form is

$$V = V_F + V_D = G^{I\bar{J}} \mathcal{W}_I \bar{\mathcal{W}}_{\bar{J}} + \frac{1}{2} (\text{Re} f)^{-1AB} \mathcal{P}_A \mathcal{P}_B. \quad (5.10)$$

Bilinear fermion terms can be divided into those that give rise to physical masses, terms relevant for the super-Brout-Englert-Higgs mechanism and terms with derivative couplings to bosonic fields:

$$\begin{aligned}
 \mathcal{L}_m &= \frac{1}{2} \mathcal{W} \bar{\psi}_\mu P_R \gamma^{\mu\nu} \psi_\nu - \frac{1}{2} \nabla_I \mathcal{W}_J \bar{\Omega}^I \Omega^J + \frac{1}{4} G^{IJ} \bar{\mathcal{W}}_J f_{ABI} \bar{\lambda}^A P_L \lambda^B + \sqrt{2} i \left[-\partial_I \mathcal{P}_A + \frac{1}{4} f_{ABI} (\text{Re} f)^{-1BC} \mathcal{P}_C \right] \bar{\lambda}^A \Omega^I \\
 &\quad + \text{H.c.}, \\
 \mathcal{L}_{\text{mix}} &= \bar{\psi} \cdot \gamma P_L \left(\frac{1}{2} i \mathcal{P}_A \lambda^A + \frac{1}{\sqrt{2}} \mathcal{W}_I \Omega^I \right) + \text{H.c.}, \\
 \mathcal{L}_d &= + \frac{1}{8} (\text{Re} f_{AB}) \bar{\psi}_\mu \gamma^{ab} (F_{ab}^A + \hat{F}_{ab}^A) \gamma^\mu \lambda^B + \frac{1}{\sqrt{2}} \left\{ G_{IJ} \bar{\psi}_\mu \not{D} X^J \gamma^\mu \Omega^I - \frac{1}{4} f_{ABI} \bar{\Omega}^I \gamma^{ab} \hat{F}_{ab}^A \lambda^B \right. \\
 &\quad \left. - \frac{2}{3} \mathcal{N}_I \bar{\Omega}^I \gamma^{\mu\nu} D_\mu \psi_\nu + \text{H.c.} \right\}.
 \end{aligned} \tag{5.11}$$

Finally, the 4-fermion terms are

$$\begin{aligned}
 \mathcal{L}_{4f} &= \frac{1}{96} \mathcal{N} [(\bar{\psi}^\rho \gamma^\mu \psi^\nu)(\bar{\psi}_\rho \gamma_\mu \psi_\nu + 2\bar{\psi}_\rho \gamma_\nu \psi_\mu) - 4(\bar{\psi}_\mu \gamma \cdot \psi)(\bar{\psi}^\mu \gamma \cdot \psi)] \\
 &\quad + \left\{ -\frac{1}{4\sqrt{2}} f_{ABI} \bar{\psi} \cdot \gamma \Omega^I \bar{\lambda}^A P_L \lambda^B + \frac{1}{8} \nabla_I f_{ABJ} \bar{\Omega}^I \Omega^J \bar{\lambda}^A P_L \lambda^B + \text{H.c.} \right\} \\
 &\quad + \frac{1}{16} e^{-1} \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_\nu \psi_\rho \left(\bar{\Omega}^{\bar{J}} \gamma_\sigma \Omega^I + \frac{1}{2} i \text{Re} f_{AB} \bar{\lambda}^A \gamma_* \gamma_\sigma \lambda^B \right) - \frac{1}{2} G_{IJ} \bar{\psi}_\mu \Omega^{\bar{J}} \bar{\psi}_\mu \Omega^I + \frac{1}{4} R_{IK\bar{J}\bar{L}} \bar{\Omega}^I \Omega^{\bar{K}} \bar{\Omega}^{\bar{L}} \\
 &\quad - \frac{1}{16} G^{IJ} f_{ABI} \bar{\lambda}^A P_L \lambda^B \bar{f}_{CD\bar{J}} \bar{\lambda}^C P_R \lambda^D + \frac{1}{16} (\text{Re} f)^{-1AB} (f_{ACI} \bar{\Omega}^I - \bar{f}_{AC\bar{I}} \bar{\Omega}^{\bar{I}}) \lambda^C (f_{BDJ} \bar{\Omega}^J - \bar{f}_{BD\bar{J}} \bar{\Omega}^{\bar{J}}) \lambda^D + \mathcal{N} (A_\mu^F)^2.
 \end{aligned} \tag{5.12}$$

This superconformal action contains the bosonic and fermionic parts of the auxiliary field A_μ , which are

$$\begin{aligned}
 \mathcal{A}_\mu &= i \frac{1}{2\mathcal{N}} [\mathcal{N}_{\bar{I}} (\partial_\mu \bar{X}^{\bar{I}} - A_\mu^A k_A^{\bar{I}}) - \mathcal{N}_I (\partial_\mu X^I - A_\mu^A k_A^I)] = i \frac{1}{2\mathcal{N}} [\mathcal{N}_{\bar{I}} \partial_\mu \bar{X}^{\bar{I}} - \mathcal{N}_I \partial_\mu X^I] + \frac{1}{\mathcal{N}} A_\mu^A \mathcal{P}_A, \\
 A_\mu^F &\equiv i \frac{1}{4\mathcal{N}} \left[\sqrt{2} \bar{\psi}_\mu (\mathcal{N}_I \Omega^I - \mathcal{N}_{\bar{I}} \bar{\Omega}^{\bar{I}}) + G_{IJ} \bar{\Omega}^I \gamma_\mu \Omega^{\bar{J}} + \frac{3}{2} (\text{Re} f_{AB}) \bar{\lambda}^A \gamma_\mu \gamma_* \lambda^B \right].
 \end{aligned} \tag{5.13}$$

$R(e, b)$ is defined with the spin connection $\omega_\mu^{ab}(e, b)$, which is intermediate between the full connection $\omega_\mu^{ab}(e, b, \psi)$ and the torsionless one $\omega_\mu^{ab}(e)$:

$$\begin{aligned}
 \omega_\mu^{ab}(e, b, \psi) &= \omega_\mu^{ab}(e, b) + \frac{1}{2} \psi_\mu \gamma^{[a} \psi^{b]} + \frac{1}{4} \bar{\psi}^a \gamma_\mu \psi^b, \quad \omega_\mu^{ab}(e, b) = \omega_\mu^{ab}(e) + 2e_\mu^{[a} e^{b]\nu} b_\nu, \\
 \omega_\mu^{ab}(e) &= 2e^{\nu[a} \partial_{[\mu} e_{\nu]}^{b]} - e^{\nu[a} e^{b]\sigma} e_{\mu c} \partial_\nu e_\sigma^c.
 \end{aligned} \tag{5.14}$$

Fermion terms are extracted from covariant derivatives D_μ , whose superconformal $U(1)$ connection involves only the bosonic part \mathcal{A}_μ . Thus, explicitly,

$$\begin{aligned}
 D_\mu X^I &= \partial_\mu X^I - b_\mu X^I - A_\mu^A k_A^I - i \mathcal{A}_\mu X^I, \\
 \hat{D}_\mu \Omega^I &= (\partial_\mu - \frac{3}{2} b_\mu + \frac{1}{4} \omega_\mu^{ab}(e, b) \gamma_{ab} + \frac{1}{2} i \mathcal{A}_\mu) \Omega^I - A_\mu^A \partial_J k_A^I \Omega^J + \Gamma_{JK}^I \Omega^K D_\mu X^J, \\
 D_\mu \lambda^A &= (\partial_\mu - \frac{3}{2} b_\mu + \frac{1}{4} \omega_\mu^{ab}(e, b) \gamma_{ab} - \frac{3}{2} i \mathcal{A}_\mu \gamma_*) \lambda^A - A_\mu^C \lambda^B f_{BC}^A.
 \end{aligned} \tag{5.15}$$

We also defined in a similar way

$$R^\mu \equiv \gamma^{\mu\rho\sigma} (\partial_\rho + \frac{1}{2} b_\mu + \frac{1}{4} \omega_\rho^{ab}(e, b) \gamma_{ab} - \frac{3}{2} i \mathcal{A}_\rho \gamma_*) \psi_\sigma, \tag{5.16}$$

while $D_\mu \psi_\nu$ contains also ψ torsion in the derivative. The action (5.8) is invariant under the superconformal transformations. We now will break those symmetries that are not required for super-Poincaré supergravity: special conformal transformations, dilatations, and S supersymmetry.

B. Partial gauge fixing and modified Kähler geometry

First, we eliminate the special conformal transformations, by imposing *the special conformal transformations gauge choice*:

$$b_\mu = 0. \tag{5.17}$$

Next, we discuss the gauge choice for dilatations. The *dilatational gauge*, D gauge, that has been chosen in the past is [17]

$$\mathcal{N} = -\frac{3}{\kappa^2}. \quad (5.18)$$

This brings the Einstein-Hilbert term in its canonical form. We further put $\kappa = 1$. To solve such a gauge condition, an appropriate way [3] is to change variables from the basis $\{X^I\}$ to a basis $\{y, z^\alpha\}$, where $\alpha = 1, \dots, n$ using

$$X^I = yZ^I(z). \quad (5.19)$$

We do not specify the $(n + 1)$ functions Z^I of the base space coordinates z^α , so that we keep the freedom of arbitrary coordinates on the base. The Z^I must be non-degenerate in the sense that the $(n + 1) \times (n + 1)$ matrix

$$\begin{pmatrix} Z^I \\ \partial_\alpha Z^I \end{pmatrix} \quad (5.20)$$

should have rank $n + 1$. There are many ways to choose the Z^I . One simple choice, labeling the I index from 0 to n , can be

$$Z^0 = 1, \quad Z^\alpha = z^\alpha. \quad (5.21)$$

Then the gauge condition can be solved for the modulus of y . Its phase is determined by a gauge condition for the R symmetry. The homogeneity properties then determine that

$$\begin{aligned} \mathcal{N} &= y\bar{y}Z^I(z)G_{I\bar{J}}(z, \bar{z})\bar{Z}^{\bar{J}}(\bar{z}), \\ G_{I\bar{J}}(z, \bar{z}) &= \partial_I \partial_{\bar{J}} \mathcal{N}(X, \bar{X}) = \mathcal{N}_{I\bar{J}}. \end{aligned} \quad (5.22)$$

The function that acts as the Kähler potential for this gauge is

$$\mathcal{K}(z, \bar{z}) = -3 \ln[-\frac{1}{3}Z^I(z)G_{I\bar{J}}(z, \bar{z})\bar{Z}^{\bar{J}}(\bar{z})]. \quad (5.23)$$

This defines the Kähler metric

$$g_{\alpha\bar{\beta}} = \partial_\alpha \partial_{\bar{\beta}} \mathcal{K}(z, \bar{z}). \quad (5.24)$$

Note that there is an arbitrariness in the definition (5.19). We may consider redefinitions

$$y' = ye^{f(z)/3}, \quad Z'^I = Z^I e^{-f(z)/3}. \quad (5.25)$$

These redefinitions lead to a different Kähler potential:

$$\mathcal{K}'(z, \bar{z}) = \mathcal{K}(z, \bar{z}) + f(z) + \bar{f}(\bar{z}). \quad (5.26)$$

Hence these can be identified with Kähler transformations for the Kähler potentials defined by (5.23). In view of these Kähler transformations, it is often useful to define Kähler-covariant derivatives. The gauge field for the parameter $f(z)$ is then $\partial_\alpha \mathcal{K}$, while for $\bar{f}(\bar{z})$ it is $\partial_{\bar{\alpha}} \mathcal{K}$. In both cases these are the gauge fields because they transform with a derivative on the parameters. We thus define

$$\begin{aligned} \nabla_\alpha Z^I &= \partial_\alpha Z^I + \frac{1}{3}(\partial_\alpha \mathcal{K})Z^I, & \nabla_{\bar{\alpha}} Z^I &= \partial_{\bar{\alpha}} Z^I = 0, \\ \nabla_{\bar{\alpha}} \bar{Z}^{\bar{J}} &= \partial_{\bar{\alpha}} \bar{Z}^{\bar{J}} + \frac{1}{3}(\partial_{\bar{\alpha}} \mathcal{K})\bar{Z}^{\bar{J}}, & \nabla_\alpha \bar{Z}^{\bar{J}} &= \partial_\alpha \bar{Z}^{\bar{J}} = 0. \end{aligned} \quad (5.27)$$

We now define weights of functions under Kähler trans-

formations. Any object that transforms like Z^I in (5.25) is defined to have weights $(w_+, w_-) = (1, 0)$. Hence, y has weight $(-1, 0)$.

The objects that appear in the superconformal formulation do not transform under Kähler transformations. For any quantity that in the superconformal variables is of the form

$$\mathcal{V}(X, \bar{X}) = y^{w_+} \bar{y}^{w_-} V(z, \bar{z}), \quad (5.28)$$

we define that V has weights (w_+, w_-) , and the Kähler-covariant derivatives are

$$\begin{aligned} \nabla_\alpha V &= [\partial_\alpha + \frac{1}{3}w_+(\partial_\alpha \mathcal{K})]V, \\ \bar{\nabla}_{\bar{\alpha}} V &= [\partial_{\bar{\alpha}} + \frac{1}{3}w_-(\partial_{\bar{\alpha}} \mathcal{K})]V. \end{aligned} \quad (5.29)$$

Remark that Φ does not transform under Kähler transformations, and thus has weights $(0, 0)$. On the other hand $e^{\mathcal{K}/3}$ has weights $(-1, -1)$, and thus $\nabla_\alpha e^{\mathcal{K}/3} = 0$.

The gauge and $U(1)$ transformations on X^I split in those for y and z^α as follows:

$$\delta y = y(\frac{1}{3}\theta^A r_A(z) + i\lambda_T), \quad \delta z^\alpha = \theta^A k_A^\alpha(z), \quad (5.30)$$

where $\frac{1}{3}r_A(z)$ can be considered as the component of the Killing vectors in the direction of y .

Our new setup will assume the following gauge conditions:

$$D \text{ gauge} \quad \mathcal{N} = \Phi(z, \bar{z}), \quad U(1) \text{ gauge} \quad y = \bar{y}, \quad (5.31)$$

with an arbitrary function $\Phi(z, \bar{z})$. We keep the definition of \mathcal{K} as in (5.23), with the associated Kähler transformations and covariant derivatives as in (5.27), and all the above equations remain valid. Furthermore, all the results below will then reduce to those of [3,5] when $\Phi = -3$.⁶

The value for y for the new gauge choice is

$$y = \bar{y} = \sqrt{-\frac{\Phi}{3}} \exp \frac{\mathcal{K}}{6}. \quad (5.32)$$

However, in many equations we will keep the phase of y arbitrary. The $U(1)$ gauge choice can be taken at any time.

The vanishing of the derivative of the D -gauge condition with respect to z^α leads to

$$\mathcal{N}_I \nabla_\alpha Z^I = 0, \quad \mathcal{N}_{\bar{I}} = \bar{y} G_{I\bar{J}} \bar{Z}^{\bar{J}}. \quad (5.33)$$

Note that this equation does not feel the presence of the function Φ . With these equations one can write the matrix identity

⁶Or $\Phi = -3\kappa^{-2}$, where κ is the gravitational coupling constant that has often been set to 1. To restore κ one also replaces $\exp \mathcal{K}$ with $\kappa^6 \exp \kappa^2 \mathcal{K}$, and thus also $g_{\alpha\bar{\beta}}$ with $\kappa^2 g_{\alpha\bar{\beta}}$, and ψ_μ with $\kappa \psi_\mu$.

$$\begin{pmatrix} -3 & 0 \\ 0 & g_{\alpha\bar{\beta}} \end{pmatrix} = e^{\mathcal{K}/3} \begin{pmatrix} Z^I \\ \nabla_\alpha Z^I \end{pmatrix} G_{I\bar{J}} \begin{pmatrix} \bar{Z}^{\bar{J}} \\ \bar{\nabla}_{\bar{\beta}} \bar{Z}^{\bar{J}} \end{pmatrix}. \quad (5.34)$$

Every matrix here is $(n+1) \times (n+1)$, and should be invertible.

This matrix identity is useful to translate quantities in the X^I basis to quantities in the $\{y, z^\alpha\}$ basis. For example, it implies that the inverse of $G_{I\bar{J}}$ is

$$G^{I\bar{J}} = e^{\mathcal{K}/3} \left(-\frac{1}{3} Z^I \bar{Z}^{\bar{J}} + g^{\alpha\bar{\beta}} \nabla_\alpha Z^I \bar{\nabla}_{\bar{\beta}} \bar{Z}^{\bar{J}} \right). \quad (5.35)$$

We assume that Φ is a (Yang-Mills) gauge-invariant function. Hence the dilatation gauge condition is invariant. However, the $U(1)$ gauge is not [see the transformations (5.30)], and it is not invariant under Kähler transformations (5.25) either. It is not invariant under supersymmetry either, but we postpone this for when we have discussed a new basis of the fermions. This implies that we cannot forget the transformations with parameter λ_T , but should relate it to the gauge transformations and Kähler transformations (and later also to supersymmetry)

$$\lambda_T = \frac{1}{6} i \theta^A [r_A - \bar{r}_A] + \frac{1}{6} i [f(z) - \bar{f}(\bar{z})]. \quad (5.36)$$

Taking this into account, and also the gauge invariance of Φ , we find that the Kähler potential \mathcal{K} transforms under gauge transformations as

$$\delta \mathcal{K} = \theta^A [r_A(z) + \bar{r}_A(\bar{z})]. \quad (5.37)$$

The moment map \mathcal{P}_A defined in (5.5) depends on this quantity $r_A(z)$ as

$$\begin{aligned} \mathcal{P}_A &= \left(-\frac{1}{3} \Phi \right) P_A, \\ P_A &= i(k_A^\alpha \partial_\alpha \mathcal{K} - r_A) = -i(k_A^{\bar{\alpha}} \partial_{\bar{\alpha}} \mathcal{K} - \bar{r}_A). \end{aligned} \quad (5.38)$$

Another convenient way to state this, is to write the Killing vectors in the X^I basis as

$$k_A^I = y [k_A^\alpha \nabla_\alpha Z^I + \frac{1}{3} i P_A Z^I]. \quad (5.39)$$

The bosonic part of the value of the auxiliary field \mathcal{A}_μ [see (5.13)] is

$$\begin{aligned} \mathcal{A}_\mu &= \frac{1}{6} i (\partial_\mu z^\alpha \partial_\alpha \mathcal{K} - \partial_\mu \bar{z}^{\bar{\alpha}} \partial_{\bar{\alpha}} \mathcal{K}) - \frac{1}{3} A_\mu^A P_A \\ &= \frac{1}{6} i (\hat{\partial}_\mu z^\alpha \partial_\alpha \mathcal{K} - \hat{\partial}_\mu \bar{z}^{\bar{\alpha}} \partial_{\bar{\alpha}} \mathcal{K}) + \frac{1}{6} i A_\mu^A (r_A - \bar{r}_A). \end{aligned} \quad (5.40)$$

Independent of the gauge conditions, one proves that the kinetic terms of the scalars, \mathcal{L}_0 in (5.9), is

$$\begin{aligned} \mathcal{L}_0 &= -\frac{1}{4 \mathcal{N}} (\partial_\mu \mathcal{N})(\partial^\mu \mathcal{N}) - \mathcal{N} (\hat{\partial}_\mu z^\alpha) (\hat{\partial}^\mu \bar{z}^{\bar{\beta}}) \frac{\partial}{\partial z^\alpha} \\ &\quad \times \frac{\partial}{\partial \bar{z}^{\bar{\beta}}} \ln[Z^I(z) G_{I\bar{J}} \bar{Z}^{\bar{J}}(\bar{z})], \end{aligned} \quad (5.41)$$

$$\hat{\partial}_\mu z^\alpha \equiv \partial_\mu z^\alpha - A_\mu^A k_A^\alpha.$$

After the gauge choice, this is thus

$$\begin{aligned} \mathcal{L}_0 &= -\frac{1}{4 \Phi} (\partial_\mu \Phi)(\partial^\mu \Phi) + \frac{1}{3} g_{\alpha\bar{\beta}} \Phi (\hat{\partial}_\mu z^\alpha) (\hat{\partial}^\mu \bar{z}^{\bar{\beta}}) \\ &= \Phi \left(\frac{1}{3} g_{\alpha\bar{\beta}} - \frac{1}{2} L_\alpha L_{\bar{\beta}} \right) (\hat{\partial}_\mu z^\alpha) (\hat{\partial}^\mu \bar{z}^{\bar{\beta}}) \\ &\quad - \frac{1}{4} \Phi [L_\alpha L_{\bar{\beta}} (\hat{\partial}_\mu z^\alpha) (\hat{\partial}^\mu z^\beta) + \text{H.c.}], \end{aligned} \quad (5.42)$$

which is the same as (2.4), and where we introduced L for $\ln \Phi$ and

$$L_\alpha = \partial_\alpha \ln(-\Phi), \quad L_{\bar{\alpha}} = \partial_{\bar{\alpha}} \ln(-\Phi). \quad (5.43)$$

For the superpotential, we define $\mathcal{W}(X) = y^3 W(z)$. Hence $W(z)$ has Kähler weights (3, 0). This leads to

$$\begin{aligned} \mathcal{W}_I Z^I &= 3y^2 W(z), \\ y^{-2} \mathcal{W}_I \nabla_\alpha Z^I &= \nabla_\alpha W \equiv \partial_\alpha W + (\partial_\alpha \mathcal{K}) W. \end{aligned} \quad (5.44)$$

The F term in the superpotential therefore reduces to (taking only bosonic terms from the field equation of F^I)

$$V_F = \frac{1}{9} \Phi^2 e^{\mathcal{K}} (-3W\bar{W} + \nabla_\alpha W g^{\alpha\bar{\beta}} \bar{\nabla}_{\bar{\beta}} \bar{W}). \quad (5.45)$$

This agrees with what we already expected in (3.6). Because of (5.38), also the D term has the same overall Φ -dependence

$$V_D = \frac{1}{18} \Phi^2 (\text{Re} f)^{-1AB} P_A P_B. \quad (5.46)$$

This agrees with what we found in (3.6).

We introduce now modified Kähler-covariant derivatives, which take the presence of Φ into account. For an object V that has weights (w_+, w_-) , we define

$$\begin{aligned} \tilde{\nabla}_\alpha V &= [\partial_\alpha + \frac{1}{3} w_+ (\partial_\alpha \mathcal{K}) + \frac{1}{2} (w_+ + w_-) L_\alpha] V, \\ \tilde{\nabla}_{\bar{\alpha}} V &= [\partial_{\bar{\alpha}} + \frac{1}{3} w_- (\partial_{\bar{\alpha}} \mathcal{K}) + \frac{1}{2} (w_+ + w_-) L_{\bar{\alpha}}] V. \end{aligned} \quad (5.47)$$

We can also define the covariant derivatives in space-time, using

$$\tilde{\nabla}_\mu = (\hat{\partial}_\mu z^\alpha) \tilde{\nabla}_\alpha + (\hat{\partial}_\mu \bar{z}^{\bar{\alpha}}) \tilde{\nabla}_{\bar{\alpha}}. \quad (5.48)$$

One can evaluate these before or after gauge fixing of the $U(1)$ symmetry. Before the latter is gauge fixed, we have to add the covariantization of the latter. y then has weight $(-1, 0)$, so that we define

$$\tilde{\nabla}_\mu y = (\partial_\mu - i \mathcal{A}_\mu - \frac{1}{3} \hat{\partial}_\mu z^\alpha \partial_\alpha \mathcal{K} + \frac{1}{2} \partial_\mu L - \frac{1}{3} A_\mu^A r_A) y = 0. \quad (5.49)$$

The calculation is modified after the $U(1)$ gauge fixing, but the result is still the same. The $U(1)$ transformation is gone, but due to (5.36) and (5.30), the Kähler transformation of y is [in agreement with its value in (5.32)]

$$y' = y \exp[f(z) + \bar{f}(\bar{z})]/6. \quad (5.50)$$

Thus y now has Kähler weights $(w_+, w_-) = (-\frac{1}{2}, -\frac{1}{2})$, leading again to

$$\begin{aligned} \tilde{\nabla}_\mu y = & (\partial_\mu - \frac{1}{6}\hat{\partial}_\mu z^\alpha \partial_\alpha \mathcal{K} - \frac{1}{6}\hat{\partial}_\mu \bar{z}^{\bar{\alpha}} \partial_{\bar{\alpha}} \mathcal{K} + \frac{1}{2}\partial_\mu L \\ & - \frac{1}{6}A_\mu^A (r_A + \bar{r}_A))y = 0. \end{aligned} \quad (5.51)$$

We thus find that y is invariant under the new covariant derivatives

$$\tilde{\nabla}_\alpha y = \tilde{\nabla}_{\bar{\alpha}} y = 0. \quad (5.52)$$

This will facilitate many calculations.

There are some differences between these modified covariant derivatives and the ordinary covariant derivatives (5.27). Most important is that the antichiral modified covariant derivative does not vanish on Z^I :

$$\tilde{\nabla}_{\bar{\alpha}} Z^I = \frac{1}{2}Z^I L_{\bar{\alpha}}. \quad (5.53)$$

The commutator of the covariant derivatives on scalar functions still satisfies the rule

$$[\tilde{\nabla}_\alpha, \tilde{\nabla}_{\bar{\beta}}]V(z, \bar{z}) = \frac{1}{3}(w_- - w_+)g_{\alpha\bar{\beta}}V(z, \bar{z}). \quad (5.54)$$

This leads also to an expression that we will need below:

$$\tilde{\nabla}_{\bar{\beta}} \tilde{\nabla}_\alpha Z^I = Z^I (\frac{1}{3}g_{\alpha\bar{\beta}} + \frac{1}{2}L_{\alpha\bar{\beta}}) + \frac{1}{2}L_{\bar{\beta}} \tilde{\nabla}_\alpha Z^I. \quad (5.55)$$

The matrix Eq. (5.34) gets modified:

$$\begin{pmatrix} \Phi & \frac{1}{2}\Phi L_{\bar{\beta}} \\ \frac{1}{2}\Phi L_\alpha & \tilde{g}_{\alpha\bar{\beta}} \end{pmatrix} = y\bar{y} \begin{pmatrix} Z^I \\ \tilde{\nabla}_\alpha Z^I \end{pmatrix} G_{IJ} \begin{pmatrix} \bar{Z}^{\bar{J}} \\ \tilde{\nabla}_{\bar{\beta}} \bar{Z}^{\bar{J}} \end{pmatrix}, \quad (5.56)$$

where

$$\tilde{g}_{\alpha\bar{\beta}} \equiv -\frac{1}{3}\Phi g_{\alpha\bar{\beta}} + \frac{1}{4}\Phi L_\alpha L_{\bar{\beta}}. \quad (5.57)$$

To obtain the second holomorphic derivative of Z^I , one can take a covariant derivative on the second line of (5.56) to obtain

$$\begin{aligned} \tilde{\nabla}_\beta \tilde{\nabla}_\alpha Z^I = & -y\Gamma_{JK}^I \tilde{\nabla}_\alpha Z^J \tilde{\nabla}_\beta Z^K + L_{(\alpha} \tilde{\nabla}_{\beta)} Z^I \\ & + Z^I (\frac{1}{2}L_{\alpha\beta} - \frac{1}{4}L_\alpha L_\beta), \end{aligned} \quad (5.58)$$

where

$$L_{\alpha\beta} = \nabla_\alpha L_\beta = \partial_\alpha L_\beta - \Gamma_{\alpha\beta}^\gamma L_\gamma. \quad (5.59)$$

This can be used further to calculate the curvature of the projective manifold. Indeed, acting with $y\bar{y}\tilde{\nabla}_{\bar{\beta}}\tilde{\nabla}_\alpha Z^I G_{IJ}\tilde{\nabla}_{\bar{\alpha}}$ on this equation, and using that on a vector quantity

$$[\tilde{\nabla}_{\bar{\alpha}}, \tilde{\nabla}_{\bar{\beta}}]\tilde{\nabla}_\alpha Z^I = \frac{1}{3}g_{\beta\bar{\alpha}}\tilde{\nabla}_\alpha Z^I + R_{\bar{\alpha}\beta\alpha}{}^\gamma \tilde{\nabla}_\gamma Z^I, \quad (5.60)$$

we obtain after many cancellations of L -dependent terms

$$\begin{aligned} & (-\frac{1}{3}\Phi)[R_{\alpha\bar{\alpha}\beta\bar{\beta}} - \frac{2}{3}g_{\bar{\alpha}(\alpha}g_{\beta)\bar{\beta}}] \\ & = (y\bar{y})^2 R_{I\bar{I}J\bar{J}} \tilde{\nabla}_\alpha Z^I \tilde{\nabla}_\beta Z^J \tilde{\nabla}_{\bar{\alpha}} \bar{Z}^{\bar{I}} \tilde{\nabla}_{\bar{\beta}} \bar{Z}^{\bar{J}}. \end{aligned} \quad (5.61)$$

Observe that the cancellations can be explained due to the fact that the dilatational symmetry of the embedding manifold implies that $Z^I R_{I\bar{I}J\bar{J}} = 0$.

C. The physical fermions

In order to define the physical bosons in the previous section, we changed from the conformal basis $\{X^I\}$ to the basis $\{y, z^\alpha\}$. We now make a similar change of basis from the conformal fermions $\{\Omega^I\}$ to a new basis⁷ $\{\chi^0, \chi^\alpha\}$, using

$$\Omega^I = y(\chi^0 Z^I + \chi^\alpha \tilde{\nabla}_\alpha Z^I). \quad (5.62)$$

Our aim is to have $\chi^0 = 0$ as a gauge condition for the S gauge transformations. We therefore choose

$$S \text{ gauge} \quad \mathcal{N}_I \Omega^I = \frac{1}{2}\Phi_\alpha \chi^\alpha, \quad (5.63)$$

which is equivalent to $\chi^0 = 0$. Hence the gauge-fixed fermions are $\Omega^I = y\chi^\alpha \tilde{\nabla}_\alpha Z^I$.

The covariant derivative of the physical fermion is

$$\begin{aligned} D_\mu \chi^\alpha = & \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}(e)\gamma_{ab} + \frac{3}{2}i\mathcal{A}_\mu \right) \chi^\alpha \\ & - A_\mu^A \frac{\partial k_A^\alpha(z)}{\partial z^\beta} \chi^\beta + \Gamma_{\beta\gamma}^\alpha \chi^\gamma \hat{\partial}_\mu z^\beta. \end{aligned} \quad (5.64)$$

The covariant derivative on the conformal fermions (5.15) can then be rewritten as

$$\begin{aligned} \hat{D}_\mu \Omega^I = & D_\mu (y\chi^\alpha \tilde{\nabla}_\alpha Z^I) + y^2 \Gamma_{JK}^I \chi^\alpha \tilde{\nabla}_\alpha Z^K \tilde{\nabla}_\beta Z^J \hat{\partial}_\mu z^\beta \\ = & y(D_\mu \chi^\alpha) \tilde{\nabla}_\alpha Z^I + y\chi^\alpha (\hat{\partial}_\mu z^\beta \tilde{\nabla}_\beta + \hat{\partial}_\mu \bar{z}^{\bar{\beta}} \tilde{\nabla}_{\bar{\beta}}) \tilde{\nabla}_\alpha Z^I \\ & + y^2 \Gamma_{JK}^I \chi^\alpha \tilde{\nabla}_\alpha Z^K \tilde{\nabla}_\beta Z^J \hat{\partial}_\mu z^\beta \\ = & y(D_\mu \chi^\alpha) \tilde{\nabla}_\alpha Z^I + y\chi^\alpha \hat{\partial}_\mu z^\beta [L_{(\alpha} \tilde{\nabla}_{\beta)} Z^I \\ & + Z^I (\frac{1}{2}L_{\alpha\beta} - \frac{1}{4}L_\alpha L_\beta)] \\ & + y\chi^\alpha \hat{\partial}_\mu \bar{z}^{\bar{\beta}} [Z^I (\frac{1}{3}g_{\alpha\bar{\beta}} + \frac{1}{2}L_{\alpha\bar{\beta}}) + \frac{1}{2}L_{\bar{\beta}} \tilde{\nabla}_\alpha Z^I], \end{aligned} \quad (5.65)$$

using (5.58) and (5.55). This can be inserted in the kinetic fermion terms, $\mathcal{L}_{1/2}$ in (5.9). The contribution of the last line of (5.65) can be complex conjugated such that this leads to

⁷We again use the implicit chiral notation, i.e. $P_L \chi^\alpha = \chi^\alpha$ and $P_R \chi^{\bar{\alpha}} = \chi^{\bar{\alpha}}$.

$$\begin{aligned} \mathcal{L}_{1/2} = & -\frac{1}{2}\tilde{g}_{\alpha\bar{\beta}}\bar{\chi}^{\bar{\beta}}\not{D}\chi^{\alpha} + \frac{1}{2}\Phi\bar{\chi}^{\alpha}\gamma^{\mu}\chi^{\bar{\beta}}\hat{\delta}_{\mu}z^{\gamma} \\ & \times \left[-\frac{1}{3}g_{\gamma\bar{\beta}}L_{\alpha} + \frac{1}{4}L_{\alpha\gamma}L_{\bar{\beta}} - \frac{1}{4}L_{\alpha}L_{\gamma\bar{\beta}}\right] + \text{H.c.} \end{aligned} \quad (5.66)$$

Now we consider the fermion mass terms, \mathcal{L}_m in (5.11). We rewrite them as

$$\begin{aligned} \mathcal{L}_m = & \frac{1}{2}m_{3/2}\bar{\psi}_{\mu}P_R\gamma^{\mu\nu}\psi_{\nu} - \frac{1}{2}m_{\alpha\beta}\bar{\chi}^{\alpha}\chi^{\beta} - m_{\alpha A}\bar{\chi}^{\alpha}\lambda^A \\ & - \frac{1}{2}m_{AB}\bar{\lambda}^A P_L \lambda^B + \text{H.c.}, \end{aligned} \quad (5.67)$$

where the (complex) gravitino mass parameter can be easily recognized as

$$m_{3/2} = \mathcal{W} = y^3 W = (-\frac{1}{3}\Phi)^{3/2} e^{\mathcal{K}/2} W. \quad (5.68)$$

For the mass terms of the chiral fermions, we have

$$m_{\alpha\beta}\chi^{\alpha}\chi^{\beta} = \nabla_I \mathcal{W}_J \bar{\Omega}^I \Omega^J. \quad (5.69)$$

We first observe that in (5.44) we can insert as well the modified covariant derivatives $\tilde{\nabla}$ due to the homogeneity conditions. Then we take a further covariant derivative using again (5.58), which gives

$$\begin{aligned} \tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}W = & y^{-1}\nabla_J \mathcal{W}_I \tilde{\nabla}_{\beta}Z^I \tilde{\nabla}_{\alpha}Z^I \\ & + y^{-2}\mathcal{W}_I (L_{(\alpha}\tilde{\nabla}_{\beta)}Z^I + \frac{1}{2}L_{\alpha\beta}Z^I - \frac{1}{4}L_{\alpha}L_{\beta}Z^I). \end{aligned} \quad (5.70)$$

Therefore,

$$\begin{aligned} m_{\alpha\beta} = & y^2\nabla_I \mathcal{W}_J \tilde{\nabla}_{\beta}Z^I \tilde{\nabla}_{\alpha}Z^I \\ = & y^3\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}W - y\mathcal{W}_I (L_{(\alpha}\tilde{\nabla}_{\beta)}Z^I + \frac{1}{2}L_{\alpha\beta}Z^I \\ & - \frac{1}{4}L_{\alpha}L_{\beta}Z^I) \\ = & y^3[\tilde{\nabla}_{\beta}\tilde{\nabla}_{\alpha}W - L_{(\alpha}\tilde{\nabla}_{\beta)}W - 3W(\frac{1}{2}L_{\alpha\beta} - \frac{1}{4}L_{\alpha}L_{\beta})] \\ = & (-\frac{1}{3}\Phi)^{3/2} e^{\mathcal{K}/2} [\nabla_{\beta}\nabla_{\alpha}W + 2L_{(\alpha}\nabla_{\beta)}W]. \end{aligned} \quad (5.71)$$

For the mass terms involving λ , we first need an equation for the derivative of f_{AB} :

$$f_{AB\alpha} = \nabla_{\alpha}f_{AB} = yf_{ABI}\tilde{\nabla}_{\alpha}Z^I = yf_{ABI}\nabla_{\alpha}Z^I. \quad (5.72)$$

A further derivative on this equation is relevant for the 4-fermion terms. Using (5.58) and the homogeneity of degree zero of f_{AB} so that $f_{ABI}Z^I = 0$, we obtain

$$\begin{aligned} \nabla_{\beta}f_{AB\alpha} = & \tilde{\nabla}_{\beta}f_{AB\alpha} \\ = & y^2\nabla_J f_{ABI}\tilde{\nabla}_{\alpha}Z^I\tilde{\nabla}_{\beta}Z^J + L_{(\alpha}\partial_{\beta)}f_{AB}. \end{aligned} \quad (5.73)$$

For the $\lambda\lambda$ mass term we use the expression for G^{IJ} in (5.35), the same homogeneity equation of f_{AB} , (5.44) and (5.72), to translate

$$\begin{aligned} m_{AB} = & -\frac{1}{2}G^{IJ}\bar{\mathcal{W}}_J f_{ABI} \\ = & -\frac{1}{2}(-\frac{1}{3}\Phi)^{1/2} e^{\mathcal{K}/2} f_{AB\alpha} g^{\alpha\bar{\beta}} \tilde{\nabla}_{\bar{\beta}}\bar{W}. \end{aligned} \quad (5.74)$$

The conformal expression of the $\lambda\chi$ mass term gives

$$m_{\alpha A} = i\sqrt{2}[\partial_I \mathcal{P}_A - \frac{1}{4}f_{ABI}(\text{Re}f)^{-1BC} \mathcal{P}_C] y \tilde{\nabla}_{\alpha} Z^I. \quad (5.75)$$

We first calculate

$$\begin{aligned} \partial_{\alpha}\mathcal{P}_A = & y\tilde{\nabla}_{\alpha}Z^I\partial_I\mathcal{P}_A + \bar{y}\tilde{\nabla}_{\alpha}\bar{Z}^{\bar{I}}\partial_{\bar{I}}\mathcal{P}_A \\ = & y\tilde{\nabla}_{\alpha}Z^I\partial_I\mathcal{P}_A + \frac{1}{2}L_{\alpha}\mathcal{P}_A, \end{aligned} \quad (5.76)$$

due to (5.53) and the homogeneity equation $X^I\partial_I\mathcal{P}_A = \mathcal{P}_A$. Using (5.38) this gives

$$y\tilde{\nabla}_{\alpha}Z^I\partial_I\mathcal{P}_A = -\frac{1}{3}\Phi\partial_{\alpha}\mathcal{P}_A - \frac{1}{6}\mathcal{P}_A\partial_{\alpha}\Phi. \quad (5.77)$$

Using also again (5.72), we obtain

$$m_{\alpha A} = -\frac{1}{3}i\sqrt{2}\Phi[(\partial_{\alpha} + \frac{1}{2}L_{\alpha})\mathcal{P}_A - \frac{1}{4}f_{AB\alpha}(\text{Re}f)^{-1BC}P_C]. \quad (5.78)$$

For \mathcal{L}_d in (5.11) we need only one new calculation:

$$\begin{aligned} G_{I\bar{J}}D_{\mu}\bar{X}^{\bar{J}}y\tilde{\nabla}_{\alpha}Z^I = & y\bar{y}G_{I\bar{J}}(\hat{\delta}_{\mu}z^{\beta}\tilde{\nabla}_{\beta} + \hat{\delta}_{\mu}\bar{z}^{\bar{\beta}}\tilde{\nabla}_{\bar{\beta}})\bar{Z}^{\bar{J}}\tilde{\nabla}_{\alpha}Z^I \\ = & \frac{1}{4}\Phi L_{\alpha}L_{\beta}\hat{\delta}_{\mu}z^{\beta} + \bar{g}_{\alpha\bar{\beta}}\hat{\delta}_{\mu}\bar{z}^{\bar{\beta}}. \end{aligned} \quad (5.79)$$

To calculate the 4-fermion terms, we need the fermionic part of the auxiliary field A_{μ} . Its conformal expression was given in (5.13), which can be evaluated as

$$\begin{aligned} A_{\mu}^F = & \frac{i}{4\sqrt{2}}\bar{\psi}_{\mu}(L_{\alpha}\chi^{\alpha} - L_{\bar{\alpha}}\bar{\chi}^{\bar{\alpha}}) + \frac{i}{4\Phi}\bar{g}_{\alpha\bar{\beta}}\bar{\chi}^{\alpha}\gamma_{\mu}\chi^{\bar{\beta}} \\ & + \frac{3i}{8\Phi}(\text{Re}f_{AB})\bar{\lambda}^A\gamma_{\mu}\gamma_{*}\lambda^B. \end{aligned} \quad (5.80)$$

One term in the square of this expression is the χ^4 term, which combines (after a Fierz transformation) with the curvature term in \mathcal{L}_{4f} , where (5.61) is now convenient. The result is given in the beginning of the paper, in Sec. II. Here we still give the 4-fermion term:

$$\begin{aligned}
\mathcal{L}_{4f} = & \frac{1}{96} \Phi [(\bar{\psi}^\rho \gamma^\mu \psi^\nu)(\bar{\psi}_\rho \gamma_\mu \psi_\nu + 2\bar{\psi}_\rho \gamma_\nu \psi_\mu) - 4(\bar{\psi}_\mu \gamma \cdot \psi)(\bar{\psi}^\mu \gamma \cdot \psi)] \\
& + \left\{ -\frac{1}{4\sqrt{2}} f_{AB\alpha} \bar{\psi} \cdot \gamma \chi^\alpha \bar{\lambda}^A P_L \lambda^B + \frac{1}{8} \bar{\chi}^\alpha \chi^\beta \bar{\lambda}^A P_L \lambda^B [\nabla_\beta f_{AB\alpha} - L_\alpha f_{AB\beta}] + \text{H.c.} \right\} \\
& + \frac{1}{16} e^{-1} \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_\nu \psi_\rho \left(\frac{1}{2} i \text{Re} f_{AB} \bar{\lambda}^A \gamma_* \gamma_\sigma \lambda^B + \tilde{g}_{\alpha\bar{\beta}} \bar{\chi}^{\bar{\beta}} \gamma_\sigma \chi^\alpha \right) + \frac{1}{6} \Phi g_{\alpha\bar{\beta}} \bar{\psi}_\mu \chi^{\bar{\beta}} \bar{\psi}^\mu \chi^\alpha \\
& - \frac{1}{32} \Phi \bar{\psi}_\mu (\chi^\alpha L_\alpha + \chi^{\bar{\alpha}} L_{\bar{\alpha}}) \bar{\psi}^\mu (\chi^\beta L_\beta + \chi^{\bar{\beta}} L_{\bar{\beta}}) - \frac{9}{64\Phi} [(\text{Re} f_{AB}) \bar{\lambda}^A \gamma_\mu \gamma_* \lambda^B]^2 \\
& + \frac{3}{16} g^{\alpha\bar{\beta}} \Phi^{-1} f_{AB\alpha} \bar{\lambda}^A P_L \lambda^B \bar{f}_{CD\bar{\beta}} \bar{\lambda}^C P_R \lambda^D + \frac{1}{16} (\text{Re} f)^{-1AB} (f_{AC\alpha} \bar{\chi}^\alpha - \bar{f}_{AC\bar{\alpha}} \bar{\chi}^{\bar{\alpha}}) \lambda^C (f_{BD\beta} \bar{\chi}^\beta - \bar{f}_{BD\bar{\beta}} \bar{\chi}^{\bar{\beta}}) \\
& \times \lambda^D \left(-\frac{1}{4} g_{\alpha\bar{\beta}} + \frac{3}{16} L_\alpha L_{\bar{\beta}} \right) (\text{Re} f_{AB}) \bar{\chi}^\alpha \lambda^A \bar{\chi}^{\bar{\beta}} \lambda^B - \frac{1}{8\sqrt{2}} \left(g_{\alpha\bar{\beta}} \bar{\chi}^\alpha \gamma^\mu \chi^{\bar{\beta}} + \frac{3}{2} (\text{Re} f_{AB}) \bar{\lambda}^A \gamma_\mu \gamma_* \lambda^B \right) \\
& \times \bar{\psi}_\mu (L_\gamma \chi^\gamma - L_{\bar{\gamma}} \chi^{\bar{\gamma}}) - \frac{1}{12} \Phi \left(R_{\alpha\bar{\gamma}\beta\bar{\delta}} - \frac{1}{2} \kappa^2 g_{\alpha\bar{\gamma}} g_{\beta\bar{\delta}} - \frac{1}{4} L_\alpha L_{\bar{\gamma}} g_{\beta\bar{\delta}} + \frac{3}{32} L_\alpha L_\beta L_{\bar{\gamma}} L_{\bar{\delta}} \right) \bar{\chi}^\alpha \chi^\beta \bar{\chi}^{\bar{\gamma}} \chi^{\bar{\delta}}. \quad (5.81)
\end{aligned}$$

VI. CONCLUSIONS

The main goal of our paper was to derive a complete formulation of $N = 1$, $d = 4$ supergravity in a generic Jordan frame. We found that, in general, this formulation is very nontrivial. It involves modified Kähler geometry (in the sense specified in our treatment), and it gives rise to many new complicated terms in the supergravity Lagrangian.

However, we identified a subclass of theories where the resulting formulation is remarkably simple. This subclass includes the recently proposed model of Einhorn and Jones [8], which was introduced as an $N = 1$ supergravity realization of the Higgs field inflation [2]. We found that the inflationary regime in this model is unstable.

Hopefully, however, the general formalism developed in our paper may allow one to find new realistic inflationary models in supergravity. As a starting approach, one can simply study in the Jordan frame several classes of inflationary models in supergravity, which were found a long time ago in the Einstein frame. As shown by the example of the Higgs inflation, sometimes it is helpful to identify and study various physical features of the cosmological models by switching from one frame to another.

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APPENDIX: STABILITY WITH RESPECT TO THE ANGLE β

As we found in Secs. *IV C* and *IV D*, the inflationary trajectory with $s = 0$ (4.30) of the Einhorn-Jones model [8] is unstable with respect to a rapid generation of the s field. Other scalar fields may also have nontrivial dynamical properties. If after a modification of this model one can find a way to stabilize the s field, one would then need to study the cosmological behavior of all other fields. As an example, in this Appendix we will analyze the behavior of the angle β , ignoring the issue of the s field instability.

For $h^2 \ll 1$ [consistent with (4.30)], the Einstein frame potential V_E of the fields h and β reads

$$V_E(h, \beta) = \frac{2h^4 \lambda^2 \sin^2 2\beta + (g^2 + g'^2) h^4 \cos^2 2\beta}{2(2 + h^2 \chi \sin 2\beta)^2}. \quad (\text{A1})$$

The first term in the numerator originates from the F term, the second term from the D term.

During inflation, in the slow-roll regime at $h^2 \chi \gg 1$ [see (4.30)], the potential with respect to β is minimized by the condition of D flatness, corresponding to $\beta = \pi/4$. In this regime,

$$V_E(h, \beta = \pi/4) = \frac{h^4 \lambda^2}{(2 + h^2 \chi)^2} \approx \frac{\lambda^2}{\chi^2}. \quad (\text{A2})$$

One could contemplate the possibility of an additional slow-roll regime with respect to the slow variation of β [8]. However, the stabilization of β during inflation is very firm. Indeed, when we take into account that the non-canonical kinetic terms in the angular direction near the minimum are proportional to $\frac{1}{\chi}$, we find that the effective mass squared of the fluctuations of the field β is given by

$$m_\beta^2 \sim \frac{h^2[-4\lambda^2 + (g^2 + g'^2)(2 + h^2\chi)]}{(2 + h^2\chi)^2}. \quad (\text{A3})$$

During inflation, in the limit $\chi h^2 \gg 1$,

$$m_\beta^2 = \frac{g^2 + g'^2}{\chi}, \quad (\text{A4})$$

and the slow-roll parameter η with respect to the field β thus reads

$$\eta_\beta \equiv \frac{m_\beta^2}{V} \approx \chi \frac{g^2 + g'^2}{\lambda^2}. \quad (\text{A5})$$

This means that for $\chi(g^2 + g'^2) \gg \lambda^2$, one has $\eta_\beta \gg 1$. Thus, there is no slow-roll regime with respect to the change of β during inflation, because the mass squared of perturbations of the angle β is much greater than H^2 . Therefore, *during inflation* the field β rapidly approaches $\pi/4$ and stays there.

However, the angle β may play an interesting dynamical role *at the end of inflation*. Our calculations show that the potential vanishes at $h = 0$ for all β ; see Fig. 2. However, in our investigation we did not take into account spontaneous symmetry breaking in the SM, as well as soft terms leading to supersymmetry breaking, which are important at an energy scale much smaller than the energy scale relevant for inflation. Clearly, the low-energy scale dynamics of the field β will depend on the above-mentioned effects that we ignored, but also on the value of the field β at the end of inflation.

One could expect that until the low-energy effects become important, the field β remains equal to $\pi/4$. However, this is not always the case. Indeed, (A3) yields that when χh^2 becomes smaller than $O(1)$ and inflation

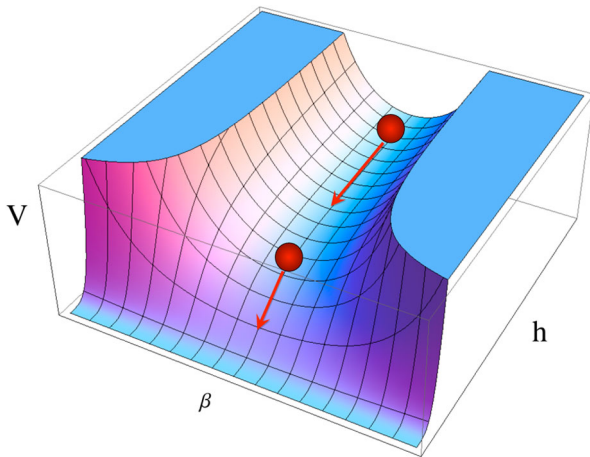


FIG. 2 (color online). During inflation at large h the angular variable β is stabilized at $\beta = \pi/4$, corresponding to $h_1 = h_2$. For $g^2, g'^2 \gg \lambda^2$, this stabilization is preserved even after the end of inflation.

ends, the mass squared of the field β becomes

$$m_\beta^2 \approx h^2 \left(-\lambda^2 + \frac{g^2 + g'^2}{2} \right), \quad (\text{A6})$$

thus affecting the slow-roll parameter η_β as follows [recall Eq. (A5)]:

$$\eta_\beta \approx \chi \left[\chi h^2 \left(-1 + \frac{g^2 + g'^2}{2\lambda^2} \right) \right]. \quad (\text{A7})$$

Note that typically $|\eta_\beta| \sim \chi \gg 1$, and that when χh^2 becomes smaller than $O(1)$ inflation ends. Therefore, for $g^2, g'^2 > 2\lambda^2$, the D term continues to dominate the dynamics of the field β even at the end of inflation, η_β remains large and positive, and β continues to be captured at its original value $\beta = \pi/4$; see Fig. 3. Oscillations of the inflaton field h near the minimum of its potential may lead to perturbative [18,19] as well as nonperturbative [20,21] decay of this field, which can be very efficient because the coupling constants of the corresponding interactions are rather large. A detailed discussion of reheating in the original (nonsupersymmetric) version of this scenario can be found in the second and third references of [2].

The situation is more complicated in the opposite case $g^2, g'^2 < 2\lambda^2$, in which the field moving along the trajectory $\beta = \pi/4$ experiences strong tachyonic instability at the end of inflation, which leads to spontaneous symmetry breaking; see Fig. 3. This effect, which is called ‘‘tachyonic preheating’’ [22,23], is similar to the waterfall regime in the hybrid inflation scenario [24].

The physical meaning of ‘‘tachyonic preheating’’ within the framework under consideration can be understood as follows. As mentioned, the inflationary regime ends when

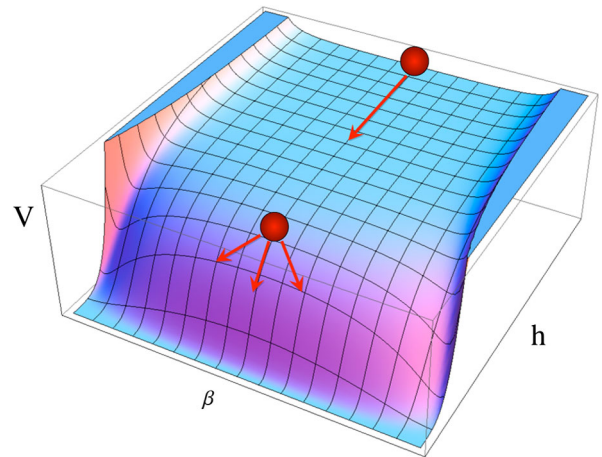


FIG. 3 (color online). During inflation at large h the angular variable β is stabilized at $\beta = \pi/4$, corresponding to $h_1 = h_2$. For $g^2, g'^2 \ll \lambda^2$, at the end of inflation the curvature of the potential in the β direction becomes large and negative, much greater than the curvature in the inflaton direction. This leads to tachyonic instability, generation of large fluctuations of the field β , and spontaneous symmetry breaking.

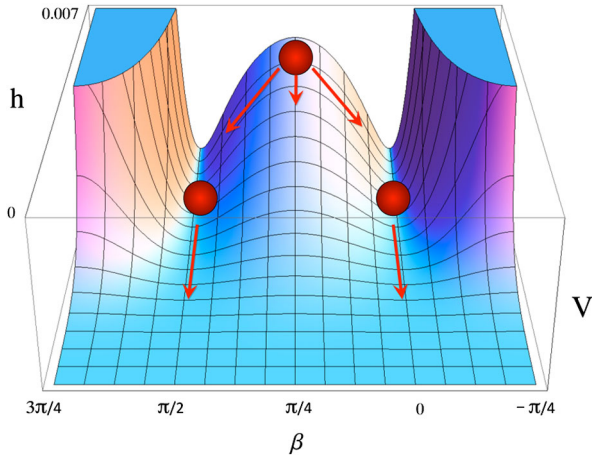


FIG. 4 (color online). “Tachyonic preheating” effect at the end of inflation (for $g^2, g'^2 \ll \lambda^2$).

χh^2 becomes $O(1)$. At that time, the parameter η_h describing the slow-roll in the h direction becomes $O(1)$, which means that the effective mass squared of the field h becomes $O(H^{-1})$. Therefore the field h reaches the minimum of the potential at $h = 0$ within the time $\Delta t = O(H^{-1})$ from the end of inflation. This last, postinflationary, part of the field evolution is shown in Fig. 4.

During that time, quantum fluctuations of the field β start growing, $\delta\beta \sim e^{m\beta t}$, and they rapidly reach the minima of the potential in the β direction, which correspond to the two valleys in Fig. 4, at $\beta \approx 0$ and at $\beta \approx \pi/2$. Spontaneous symmetry breaking occurs within the time m_β^{-1} , which is shorter than H^{-1} by the factor $O(\eta^{-1/2}) \sim 10^{-2}$. In other words, this process occurs almost instantly, on the cosmological time scale. When this happens, the universe becomes divided into domains with the field β taking values in one of the two valleys in Fig. 4. These domains, of initial size m_β^{-1} , will be separated from each other by domain walls corresponding to the ridge of the potential at $\beta = \pi/4$. Then the field h will continue rolling down to smaller values of h , following the two valleys of the potential. A detailed evolution of the field distribution can be studied by the methods developed in [22,23].

In order to find out which of the two regimes ($g^2, g'^2 > 2\lambda^2$ versus $g^2, g'^2 < 2\lambda^2$) occurs in the realistic versions of this scenario, one should perform an investigation of the running of the coupling constants from their present day values to the end of inflation, similar to the investigation performed in [2]. However, prior to such an investigation, one should find a solution to the main problem of this scenario, which is the tachyonic instability with respect to the field s found in Sec. IV.

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