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# Josephson Phase Coherence and Superposition of Amplitudes in Macroscopic Systems

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In isolated quantum mechanical systems, solutions to the Schrödinger equation exist such that amplitude interference is maintained (in time) between states with widely different macroscopic properties. An engineering criteria for the required isolation is given in terms of a generalized Josephson law, valid for an arbitrary question in the sense of quantum measurement theory.

Among the more interesting engineering problems associated with quantum measurement theory is that of observing superposition of amplitude interference between states with widely different macroscopic properties.[1] For a quantum mechanical object in total isolation, solutions to the Schrödinger equation exist which maintain (in time) such coherence. The problem is then to find engineering criteria for the required isolation. Among the requirements is certainly the condition that the quantum object be placed in a classical (apparatus) environment with sufficiently low noise levels so as not to scramble the phases of the interfering amplitudes. Our purpose is to consider this criteria using a generalized Josephson law, valid for any "question" in the sense of conventional theoretical descriptions of quantum measurements.

A "question" is a division of possible quantum states into two orthogonal subspaces[2] (projection operators $P$ and $Q$). These might be "spin up" and "spin down", or (in the hisotrical context of the problem at hand) "live cat" and "dead cat". For a statistical ensemble of quantum objects in a state $\rho$, interactions with classical environments[3] capable of yielding a measured answer to the question are conventionally described[4] by a change in statistical state

$$\rho \to P\rho P + Q\rho Q\,. \tag{1}$$

More refined measurements require more questions, i.e., subdivisions of states with projection operators providing spectral decompositions of physical quantities. For a physical quantity $A$, the mean recoil due to the measurement interaction follows from Eq. (1) as

$$\Delta A = \mathrm{Tr}\,\rho(QAP + PAQ), \tag{2}$$

i.e., the recoil in $A$ is due to the destruction of amplitude interference between the possibilities $P$ or $Q$, when the posed question is experimentally answered.

Given the mean connecting matrix elements of the Hamiltonian

$$\mathrm{Tr}\,\rho(QHP) = -(\hbar\nu/2)e^{i\theta}\,, \tag{3}$$

the energy associated with coherent amplitude superposition between $P$ and $Q$ is evidently

$$\Delta E = -\hbar\nu\cos\theta\,. \tag{4}$$

Equation (3) also rigorously determines the ensemble transition rate from $Q$ to $P$, i.e.,

$$\begin{aligned}\dot{W} &= (d/dt)\mathrm{Tr}\,\rho P \\ &= -(d/dt)\mathrm{Tr}\,\rho Q\,. \end{aligned} \tag{5}$$

If the system were completely isolated, then the transition rate would be exactly the Josephson probability ensemble flow

$$\dot{W} = \nu \sin \theta . \qquad (6)$$

Equations (4) and (6) constitute a generalized Josephson effect[5] valid for an arbitrary quantum mechanical measurement, answering any simple posed question. The Josephson law remains valid if both quantum object and classical low noise environment are considered together as a larger total system. The physical significance of the phase $\theta$ depends on the nature of the experimentally posed question. The dissipative measurement destruction of the amplitude interference energy, in Eq. (4), is related to the noise levels in the Josephson bias frequency

$$\omega_b = (d\theta/dt). \qquad (7)$$

A gentle (below critical drive) quantum object-classical apparatus interaction can maintain amplitude interference between different macroscopic states if the bias frequency has sufficiently small noise fluctuations.

Two engineering examples will suffice to illustrate our point. Consider two bulk superconductors connected by a constricted region weak link. If the question posed is in which superconductor does an electron pair reside, then the bias frequency for the answer is determined by the voltage difference between the superconductors

$$\omega_v = (qV/\hbar), \qquad (8a)$$

where $q = 2e$. If the question posed is on which side of the weak link does a quantized Faraday bundle of magnetic flux reside,[6] then the bias frequency for the answer is determined by the current through the weak link construction[7]

$$\omega_I = (\Phi_0 I/\hbar c), \qquad (8b)$$

where $\Phi_0 = (\Pi \hbar c/e)$. Classical noise processes in Eqs. (8) are associated with dissipative circuit elements at electrical noise temperatures. This appears to be typical of bias frequency noise effects connecting dissipation to quantum measurements.

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