Jumps in Bond Yields at Known Times[*]

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Abstract

We construct a no-arbitrage term structure model with jumps in the entire state vector at deterministic times but of random magnitudes. Jump risk premia are allowed for. We show that the model implies a closed-form representation of yields as a time-inhomogeneous affine function of the state vector, and derive other theoretical implications. We apply the model to the term structure of US Treasury rates, estimated at the daily frequency, allowing for jumps on days of employment report announcements. Our model can match the empirical fact that the term structure of interest rate volatility has a hump-shaped pattern on employment report days (but not on other days). The model also produces patterns in bond risk premia that are consistent with the empirical finding that much of the time-variation in excess bond returns accrues at times of important macroeconomic data releases.

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1   Introduction

Macroeconomic news comes out in a lumpy manner via scheduled news announcements, especially the monthly employment report that includes both nonfarm payrolls and the unemployment rate. These announcements are important for all asset prices, but especially for bond yields (Andersen, Bollerslev, Diebold, and Vega, 2007). Nevertheless, term structure models mostly assume that the factors driving the term structure of interest rates are continuous diffusions, and so that news comes out continuously. Some models allow for jumps, but these are typically jumps at random times, following a Poisson arrival process (Das, 2002; Duffie, 2001; Feldhutter, Schneider, and Trolle, 2008; Jiang and Yan, 2009; Johannes, 2004). Researchers using this approach find that many—though not all—of the jumps occur at times of news announcements (Andersen, Bollerslev, and Diebold, 2007). But, if we are thinking of the jumps as reflecting scheduled news announcements, then they are perhaps better viewed as jumps at deterministic times but of random magnitudes.[1] In this perspective, depending on how big the surprise component of a particular announcement is, the jump may be big or small. But every announcement leads to some jump, and its timing is known *ex ante*.

Further impetus for taking a closer look at the role of deterministic jumps in term structure models is provided by recent studies that document differences in risk-return characteristics over announcement versus non-announcement periods. In particular, Faust and Wright (2008) find that excess returns in bond futures market accrue at times of scheduled data announcements, suggesting that the well-known failure of expectations hypothesis is mainly due to announcement reactions. Analogous phenomena occur in other asset markets as well: Mueller, Tahbaz-Salehi, and Vedolin

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[1]Throughout, when we refer to jumps of random size/magnitude, the *sign* of the jumps is random too: they can be either positive or negative.

(2015) find that FOMC announcement days contribute in large part to the failure of the UIP (uncovered interest rate parity) hypothesis in foreign exchange markets, and Lucca and Moench (2015) and Savor and Wilson (2013) document interesting equity market risk premium patterns associated with scheduled announcements. Though Faust and Wright (2008) considered different bond maturities separately in a regression setup, it is useful to build dynamic term structure models that can parsimoniously capture this effect for the entire yield curve. If investors perceive macro announcements like the employment report as major risk events, then this model can tell us the implications of these risk events for the pricing of bonds and the dynamics of bond yields, and bond risk premia and term premia. In this paper, we develop such a model.

Our model can be viewed as an extension of standard affine Gaussian model to the case of deterministic jumps, with two key features: (1) all elements of the state vector are allowed to jump (with a general correlation structure), and (2) the pricing kernel also jumps, i.e., jump risks are *priced*, and jump risk premia are *state-dependent*. As in standard affine models like Dai and Singleton (2000) and Duffee (2002), the state variables are all latent factors, but we think of them as being driven in part by macroeconomic data. Changes in the state variables that are driven by news about the economy might consequently have different implications for future expected rates and term premia than other shifts in the state variables. Our model allows for this possibility, while a conventional model with latent factors that follow a diffusion does not. We show that the model has a closed-form affine representation for yields, although one in which the loadings have a deterministic dependence on time. After discussing the theoretical properties of the model, we fit the model to daily data on the term structure of US Treasury yields, assuming that there are jumps on the days of employment reports.

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Our paper is related to three strands of research on the effects of scheduled announcements on bond yields. One is a small literature on term structure models with deterministic jumps. The best known examples are Piazzesi (2001, 2005), but these papers' focus is on jumps in the target fed funds rate, and jump risk is not priced.[2] Another strand consists of papers that study news announcement effects using term structure models but without explicit modeling of jumps or jump risk premia, including Fleming and Remolona (1999), Hördahl, Remolona, and Valente (2015) and Bauer (2015). The term structure models in these studies are derived under the implicit assumption that data generating process is the same for announcement and non-announcement periods. A third strand, which includes studies by Beber and Brandt (2006, 2009), is a literature on studying the implications of scheduled macro announcements for bond option prices. The present paper is the first to specify and estimate a multifactor term structure model for bond yields allowing for deterministic jumps in the entire state vector and allowing for state-dependent jump risk premia. Our model gives implications for bond risk premia (expected excess returns) on jump days (employment report days) and on non-jump days that are not available in existing work. These implications are our main focus.

Our key empirical results are as follows:

i We find that bond risk premia implied by our flexibly specified jump model are notably bigger in absolute value on jump days than on non-jump days. The employment report day bond risk premia are also bigger in absolute value than those that we obtain from estimation of a homogeneous (no-jump) model. These results are consistent with aforementioned findings of Faust and Wright (2008).

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[2]Piazzesi (2005) considers Poisson jumps in the federal funds target rate alone, where the jump intensity is high and state-dependent within windows bracketing scheduled FOMC meeting events, and low at other times. The jump intensity is nonzero at other times to allow for the small probability of an unscheduled FOMC move. Piazzesi (2001) considers a model that also includes state variables corresponding to nonfarm payroll employment and CPI, that also jump at deterministic times.

ii Our model captures the hump shape of the term structure of yield volatility on employment report days (low at the short maturities and peaking at about two-year maturity) documented in Fleming and Remolona (1999) and Piazzesi (2005). We also examine the model-implied term structure of the volatility of the changes in expectations and term premium components of yields on employment report days, and find that the term structure of the volatility due to the expectations component has a different shape for jump versus non-jump (diffusion) movements in yields.

iii A restricted version of jump model (with jumps in the short rate only) produces implied bond risk premia and a term structure of employment-day volatilities that are very different from those from the flexible model.

iv A time-homogeneous term structure model (standard affine Gaussian model) produces bond risk premia for daily holding periods that differ notably from the flexibly specified jump model, but bond risk premia for longer holding periods are more similar between the two models. These results suggest that time-homogeneous models, which are simpler to implement, may still give sensible results in *some* applications.

The plan for the remainder of this paper is as follows. In section 2, we report some empirical facts about the behavior of yields on announcement and non-announcement days. In section 3, we describe the model with jumps at deterministic times, derive an expression for bond prices in the model, and examine its theoretical implications. In section 4, we discuss the methodology for model estimation, and section 5 discusses empirical results. Section 6 concludes.

2 Yields and Macro Announcements

First, we briefly examine some empirical facts about bond yields and their relationship to macroeconomic announcements that our model is designed to capture. Table 1 shows the standard deviation of three-month, two-year and ten-year zero-coupon US Treasury daily yield changes on the days of certain macro announcements, and on non-announcement days.[3] We see that employment report days show substantially higher volatility of interest rates than non-announcement days, or indeed days of any other types of macroeconomic announcements, consistent with earlier studies (Andersen, Bollerslev, Diebold, and Vega, 2007; Balduzzi, Elton, and Green, 2001; Fleming and Remolona, 1997). The difference between the volatility on employment report days and non-announcement days is overwhelmingly statistically significant.

That market participants are aware of these patterns can be seen clearly from data on very-short-maturity options. Starting in 2011, the Chicago Mercantile Exchange has traded options on five-, ten- and thirty-year Treasury futures that expire each Friday afternoon.[4] As of the Thursday close, these are options with one day left to maturity. We obtained settlement prices on these options on Thursdays, and computed the Black implied volatility of the call option that is closest to being at-the-money. Employment reports are nearly always released on Fridays. We computed the average options implied volatility on the Thursdays that preceded employment reports and on all other Thursdays. The results are reported in Table 2. The one-day Treasury implied volatility is nearly twice as big ahead of employment reports than on other Thursdays. This confirms that the market anticipates jumps

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[3]The 2-year and 10-year yields are from the dataset of Gürkaynak, Sack, and Wright (2007), while the 3-month yield is the 3-month T-bill yield. By non-announcement days, we mean days that have no employment report, CPI, durable goods, FOMC, GDP, PPI or retail sales announcement.

[4]Though new, these very short-term Treasury options have become reasonably liquid. The May 2016 CME options review reported that the average daily trading volume, aggregating across maturities, is 87,000 contracts.

associated with employment report announcements, as earlier found by Beber and Brandt (2006, 2009), using longer-maturity options.

Figure 1 plots the standard deviation of yield changes on employment report and non-announcement days against the maturity. On non-announcement days, the volatility curve is fairly flat in maturity, reflecting the well-known fact that the vast majority of yield curve shifts are level shifts (Litterman and Scheinkman, 1991). But on employment-report days, the level of volatility is higher, but is also hump-shaped in maturity—the most volatile yields on employment report release days are intermediate-maturity yields, and the volatility is notably lower at the short end of the yield curve (such as three months). This was earlier found by Fleming and Remolona (1999) and Piazzesi (2001). It can also be seen in Table 1 for each of the announcement types separately. This is an empirical fact that a standard diffusive term structure model cannot capture. It represents the effects of news today on expectations of future monetary policy, and also on risk premia. A more stark way of documenting this stylized fact is to look at the volatility in yield changes caused by employment report announcements, assuming that the only difference between employment report and non-announcement days is the existence of the employment report news[5]. We also show this in Figure 1. The jump-induced volatility has a particularly pronounced hump shape.

Apart from the volatility associated with announcements, risk-return characteristics also differ over announcement versus non-announcement periods. Faust and Wright (2008) regress excess returns in bond futures market over both announcement and non-announcement windows onto yield curve factors, and find that most of the

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[5]If $\sigma_{m,A}$ and $\sigma_{m,NA}$ are the standard deviations of bond yields at maturity m on employment report and non-announcement days, respectively, then we define the standard deviation of yields owing to the employment report as $\sqrt{\sigma_{m,A}^2 - \sigma_{m,NA}^2}$.

evidence for time-varying predictability of excess bond returns occurs over the announcement windows. This implies that the well-known failure of the expectations hypothesis is mainly due to announcement reactions.

3 Term structure model with jumps at known times: Theory

3.1 The model

Our model specifies that x_t is an n-dimensional latent state vector. Under the physical measure, x_t, follows the jump-diffusion:

$$dx_t = K(\theta - x_t)dt + \Sigma dW_t + \xi_t dN_t \tag{3.1}$$

where W_t is an n-dimensional vector of independent standard Brownian motions, N_t is a counting process with jumps at deterministic times $t = T_i$, $i = 1, 2, 3, ...$ ($dN_t = 1$ for $t = T_i$, 0 at other times), and ξ_{T_i} is an n-dimensional vector of random jump sizes.[6]

The random jump size vector, ξ_{T_i}, is assumed to be normally distributed with a state-dependent mean: $\xi_{T_i} \sim \mathcal{N}(\mu(x_{T_i-}), \Omega)$, where $\Omega = \Upsilon\Upsilon'$, and $\mu(.)$ is an affine function of the state vector right before the jump, i.e.,

$$\mu(x_{t-}) = \gamma + \Gamma x_{t-}. \tag{3.2}$$

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[6]For a nice pedagogical discussion of jumps at deterministic times, see Piazzesi (2009). Piazzesi notes in Chapter 3.5.2 that jumps in deterministic times lead to bond yields that are nonstationary (time-inhomogeneous).

The short-term interest rate is:

$$r_t = \rho_0 + \rho' x_t. \tag{3.3}$$

Assume that the pricing kernel is

$$\frac{dM_t}{M_t} = -r_t dt - \lambda'_t dW_t + J(\xi_t, x_{t-})dN_t \tag{3.4}$$

$$J(\xi_t, x_{t-}) = \exp(-\psi'_{t-}\Upsilon^{-1}(\xi_t - \mu(x_{t-})) - \frac{1}{2}\psi'_{t-}\psi_{t-}) - 1, \tag{3.5}$$

where $\lambda_t = \lambda + \Lambda x_t$ and $\psi_{t-} = \psi + \Psi x_{t-}$.

Under the risk-neutral measure, x_t, follows the jump diffusion:

$$dx_t = K_Q(\theta_Q - x_t)dt + \Sigma dW_t^Q + \xi_t^Q dN_t$$

where the jump size vector $\xi_{T_i}^Q$ has the distribution $\mathcal{N}(\mu_Q(x_{T_i-}), \Omega)$, $\mu_Q(x_{t-}) = \gamma_Q + \Gamma_Q x_{t-}$, $K_Q = K + \Sigma\Lambda$, $\theta_Q = K_Q^{-1}(K\theta - \Sigma\lambda)$, $\gamma_Q = \gamma - \Upsilon\psi$ and $\Gamma_Q = \Gamma - \Upsilon\Psi$.

Our model nests the standard (time-homogeneous) essentially affine term structure model (model $EA_0(3)$ in the terminology of Duffee (2002)). Indeed, it can be viewed as a natural generalization of the $EA_0(3)$ model to include deterministic jump effects and risk premia associated with these jumps; our flexible affine specification of market price of jump risk ψ_t (equation (3.5)) can be viewed as an analogue of the affine specification of market price of diffusion risk in the $EA_0(3)$ model.

A key feature of our model is that it allows for jumps in all elements of the state vector. While many existing term structure models with jumps have focused on specifications in which only the short rate (or the target federal funds rate) has

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jumps,[7] it is important to allow for jumps in other state variables. Indeed, the hump shaped response pattern we have seen in Figure 1 suggests that, although a good employment report may affect very short term rates, it can have a bigger impact on expectations about future rate hikes.[8] However, jumps in the expected path of short rate are not the only possible source of jumps in bond yields. A significant part of jumps in bond yields could come from jumps in term premia. This feature is also incorporated into our model as jumps in the market price of risk.[9]

A few other remarks are in order. First, in this paper we do not model stochastic volatility of yields. While the time-varying volatility of yields is well documented, empirical studies such as Jones, Lamont, and Lumsdaine (1998) find that the volatility associated with macroeconomic announcements effects are short-lived; therefore, our model with homoskedastic yields during non-announcement periods and jumps at announcements can still be expected to capture some essential features of the yield curve response to data releases.

Second, we envision our model as capturing yield response to monthly employment report. Other macroeconomic data announcements, such as retail sales, CPI, etc., also give rise movements in interest rates that can be thought of as jumps (Andersen, Bollerslev, Diebold, and Vega, 2007). For these, we could in principle extend the model to allow for jumps of multiple types, by introducing different kinds of jumps corresponding to different types of announcements:

$$dx_t = K(\theta - x_t)dt + \Sigma dW_t + \xi_t^A dN_t^A + \xi_t^B dN_t^B + ... \qquad (3.6)$$

[7]These include Das (2002), Johannes (2004), Jiang and Yan (2009) and Piazzesi (2005). Exceptions include Piazzesi (2001) and Feldhutter, Schneider, and Trolle (2008).

[8]This "path shock" is in the same sense as in Gürkaynak, Sack, and Swanson (2005), who have discussed the patterns of yield/futures curve responses to FOMC announcements.

[9]Fleming and Remolona (1999) explain the hump-shaped yield response pattern purely in terms of changes in the expected path of the short rate (revisions in "central tendencies" in their model), as they assume that term premia are constant.

However, given the especially strong effects of employment report as discussed in Section 2, it seems reasonable to first try to study the effects of employment report as we do in this paper. Other scheduled announcement responses are in effect approximated as a part of diffusion dynamics. We expect many of the key conclusions of our model to be preserved in richer models.

3.2 Expression for bond prices

Let the time t price of a zero-coupon bond maturing at time T be $P(t,T)$. Then

$$P(t,T) = E_t^Q \left(\exp(-\int_t^T r_s ds) \right) \tag{3.7}$$

Proposition 1 provides an expression for this price.

Proposition 1. Suppose that between time t and T there are p jumps at $T_1, T_2, ..., T_p$, and that the risk-neutral dynamics of state variables and the short rate are given by equations (3.1) and (3.3), respectively. Then

$$P(t,T) = \exp(a(t,T) + b(t,T)'x_t) \tag{3.8}$$

where

$$b(t,T) = \exp(-K_Q'(T_1 - t))((I + \Gamma_Q')b_1 + K_Q^{-1'}\rho) - K_Q^{-1'}\rho \tag{3.9}$$

$$a(t,T) = a_1 + b_1'\gamma_Q + \frac{1}{2}b_1'\Omega b_1 + A(T_1 - t; (I + \Gamma_Q')b_1) \tag{3.10}$$

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and[10]

$$a_{i-1} = a_i + b_i'\gamma_Q + \frac{1}{2}b_i'\Omega b_i + A(T_i - T_{i-1}; (I + \Gamma_Q')b_i) \quad (3.11)$$

$$b_{i-1} = B(T_i - T_{i-1}; (I + \Gamma_Q')b_i) \quad (3.12)$$

iterating backwards from the "initial" conditions

$$a_p = A(T - T_p; 0_{n\times 1}) \quad (3.13)$$

$$b_p = B(T - T_p; 0_{n\times 1}), \quad (3.14)$$

and $A(\tau; \eta), B(\tau; \eta)$ given by

$$B(\tau; \eta) \equiv \exp(-K_Q'\tau)(\eta + K_Q'^{-1}\rho) - K_Q'^{-1}\rho \quad (3.15)$$

$$A(\tau; \eta) \equiv \int_0^\tau [(K_Q\theta_Q)'B(s; \eta) + \frac{1}{2}B(s; \eta)'\Sigma\Sigma'B(s; \eta) - \rho_0]ds \quad (3.16)$$

$$= (K_Q\theta_Q)' \left[\int_0^\tau \exp(-K_Q's)ds\right] (\eta + K_Q'^{-1}\rho) - (\rho_0 + \theta_Q'\rho)\tau$$

$$+ \frac{1}{2}(\eta + K_Q'^{-1}\rho)' \left[\int_0^\tau \exp(-K_Qs)\Sigma\Sigma'\exp(-K_Q's)ds\right] (\eta + K_Q'^{-1}\rho)$$

$$- \rho'K_Q^{-1}\Sigma\Sigma' \left[\int_0^\tau \exp(-K_Q's)ds\right] (\eta + K_Q'^{-1}\rho)$$

$$- (\eta + K_Q'^{-1}\rho)' \left[\int_0^\tau \exp(-K_Qs)ds\right] \Sigma\Sigma'K_Q'^{-1}\rho + \tau\rho'K_Q^{-1}\Sigma\Sigma'K_Q'^{-1}\rho.$$

The proofs of the propositions, including Proposition 1, are collected in the Appendix.[11] Even with jumps, prices are an exponential affine function of the state vector, and consequently yields are an affine function of the state vector, but the

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[10]Throughout this paper, we define $\exp(A) = I + A + A^2/2 + A^3/6 + \cdots$ for any square matrix A.

[11]Note that the integrals in (3.16) can be computed analytically. We have $\int_0^\tau \exp(-K_Q's)ds = K_Q'^{-1}(I - \exp(-K_Q'\tau))$ and $vec(\int_0^\tau \exp(-K_Qs)\Sigma\Sigma'\exp(-K_Q's)ds) = ((I \otimes K_Q) + (K_Q \otimes I))^{-1}vec(\Sigma\Sigma' - \exp(-K_Q\tau)\Sigma\Sigma'\exp(-K_Q'\tau))$.

loadings depend not only on the time-to-maturity, but also on time itself.

When there is no state dependence in jumps ($\Gamma_Q = 0$), the expressions for $a(t,T)$ and $b(t,T)$ are particularly simple. As shown in the Appendix, in this case:

$$
\begin{aligned}
a(t,T) &= \tilde{a}(T-t) + \sum_{t<T_i<T} \bigg(-\rho'K_Q^{-1}(I-e^{-K_Q(T-T_i)})\gamma_Q \\
&\quad + \frac{1}{2}\rho'K_Q^{-1}(I-e^{-K_Q(T-T_i)})\Omega(I-e^{-K_Q'(T-T_i)})K_Q'^{-1}\rho \bigg), \qquad (3.17) \\
b(t,T) &= \tilde{b}(T-t), \qquad (3.18)
\end{aligned}
$$

where $\tilde{a}$ and $\tilde{b}$ are factor loadings for the standard affine-Gaussian model (without jumps), and the sum in equation (3.17) denotes summation over all T_i's between t and T. When $\Gamma_Q = 0$, $b(t,T)$ is a continuous function of $T - t$ (as can be seen from equation (3.18)), thus the factor loading right after a jump, $b(T_i, T)$ and the factor loading right before the jump, $b(T_{i-}, T)$ are the same.[12] But in general ($\Gamma_Q \neq 0$) they differ.

3.3   Properties of the model

We now explore the theoretical implications of the model.

3.3.1   Yield curve: cross-section and dynamics

The bond pricing formulas derived above imply that the yield curve is continuous. Using the law of iterated expectations:

$$
\begin{aligned}
P(t,T_i) &= E_t^Q(e^{-\int_t^{T_i} r_s ds}) = E_t^Q(e^{-\int_t^{T_i-} r_s ds} E_{T_i-}(e^{-\int_{T_i-}^{T_i} r_s ds})) \\
&= E_t^Q(e^{-\int_t^{T_i-} r_s ds} \cdot 1) = P(t, T_i-). \qquad (3.19)
\end{aligned}
$$

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[12]Here and elsewhere in this paper, we denote $T_i - 0^+$ by T_i-, and $T_i + 0^+$ by T_i.

and hence the yield curve is continuous as the maturity T approaches one of the jump dates:

$$\lim_{T \to T_i-} P(t,T) = P(t,T_i). \qquad (3.20)$$

However, the yield curve has kinks at maturity points corresponding to deterministic jump dates, as we show in proposition 2.

Proposition 2. The yield curve has a discontinuous first derivative at $T = T_1, T_2, ...$:

$$\lim_{T \to T_i-} \frac{\partial}{\partial T} P(t,T) \neq \frac{\partial}{\partial T} P(t,T_i). \qquad (3.21)$$

The presence of kinks in the yield curve is theoretically interesting, though for realistic parameter values the yield curves implied by the model still look smooth, which we know to be the case empirically.

We now turn to the discussion of yield dynamics. In order to simplify the exposition and the empirical analysis of the model, we assume that employment reports are equally spaced $(T_2 - T_1 = T_3 - T_2 = ... = 1/12 \equiv \bar{\delta})$. Then the bond yield at time t with remaining time-to-maturity τ can be represented as:

$$y_{\tau,t} = a_y(\tau; \delta(t)) + b_y(\tau; \delta(t))'x_t, \qquad (3.22)$$

where $\delta(t) = T_1 - t$ is the time to the next jump (employment report), $a_y(\tau; \delta(t)) \equiv -a(t, t+\tau)/\tau$ and $b_y(\tau; \delta(t)) \equiv -b(t, t+\tau)/\tau$.

Equation (3.22) has the implication that, due to the time-dependence of a_y and b_y, the change in yield between time t and time $t+h$, Δy, is not exactly $b_y(\tau; \delta(t))'\Delta x$. Still, for small h, $\Delta y \approx b_y(\tau; \delta(t))'\Delta x$, except when the interval $[t, t+h]$ contains a jump. Meanwhile, the yield change over an infinitessimal period containing the jump

at time T_i is given by:

$$
\begin{aligned}
\Delta y &= (b_y + \Delta b_y)'(x_{T_i-} + \Delta x) - b_y' x_{T_i-} + \Delta a_y \\
&= b_y(\tau;0)'\Delta x + \Delta b_y'\Delta x + [\Delta b_y' x_{T_i-} + \Delta a_y],
\end{aligned}
\tag{3.23}
$$

where $\Delta a_y = a_y(\tau;\bar{\delta}) - a_y(\tau;0)$ and $\Delta b_y = b_y(\tau;\bar{\delta}) - b_y(\tau;0)$. In addition to the usual term $(b_y(\tau;0)'\Delta x)$, this expression now contains a $\Delta b_y'\Delta x$ term, as well as the term in square brackets which represents a predictable change in yields.[13] Using equations (A.3) and (A.4) in the Appendix, the jump discontinuity in b_y is given by $\Delta b_y = -\Gamma_Q' b_y(\tau;\bar{\delta})$. Therefore, if jump risk is not priced ($\Gamma_Q = 0_{n \times n}$), then $\Delta b_y = 0$. But in general, Δb_y is nonzero, and changes in bond yields at deterministic jumps contain a state-dependent predictable component. This gives rise to distinct patterns in bond risk premia, to which we now turn.

3.3.2 Bond risk premia and term premia

Consistent with the standard usage of the term, we refer to the expected excess return on bonds as bond risk premia. The annualized bond risk premium at time t for a holding period of h and a time-to-maturity of τ is given by:

$$
\begin{aligned}
\rho_{\tau,t,h} &= \frac{1}{h}E_t\left(\log(\frac{P(t+h,t+\tau)}{P(t,t+\tau)}) - \log(\frac{1}{P(t,t+h)})\right) \\
&= \frac{1}{h}[b(t+h,t+\tau)E_t(x_{t+h}) - (b(t,t+\tau) - b(t,t+h))x_t \\
&\quad + a(t+h,t+\tau) - a(t,t+\tau) + a(t,t+h)].
\end{aligned}
\tag{3.24}
$$

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[13]We are assuming here that $\gamma = 0$ and $\Gamma = 0$, so then $E_{T_i-}(\Delta x) = 0$. Otherwise, the first two terms in equation (3.23) would also have a predictable part.

Because $E_t(x_{t+h})$ is affine in x_t, this expression is affine in x_t. At non-jump times[14]:

$$
\begin{aligned}
d \log P(t,T) &= \frac{dP(t,T)}{P(t,T)} - \frac{1}{2}\|b(t,T)'\Sigma\|^2 dt \\
&= \left[r_t + b(t,T)'\Sigma(\lambda + \Lambda x_t) - \frac{1}{2}\|b(t,T)'\Sigma\|^2\right]dt + b(t,T)'\Sigma dW_t. \quad (3.25)
\end{aligned}
$$

Meanwhile, the expected log return at jumps, $E_{T_i-}(\Delta^J \log P)$, is given by:

$$
E_{T_i-}(\log P(T_i,T)) - \log P(T_i-,T) = E_{T_i-}(\log P(T_i,T)) - \log E_{T_i-}^Q(P(T_i,T))
$$

$$
\approx E_{T_i-}(\log P(T_i,T)) - E_{T_i-}^Q(\log P(T_i,T)) = b(T_i,T)'(\mu(x_{T_i-}) - \mu^Q(x_{T_i-})), \quad (3.26)
$$

where we have used the result (A.2) from the Appendix in the first line. Thus, neglecting the convexity term $\frac{1}{2}\|b(t,T)'\Sigma\|^2$ in equation (3.25), for small h,

$$
\rho_{\tau,t,h} \approx \begin{cases} \varrho^D(t,t+\tau), & T_i \notin [t,t+h] \\ \varrho^D(t,t+\tau) + (1/h)\varrho^J(t,t+\tau), & T_i \in [t,t+h] \end{cases} \quad (3.27)
$$

with

$$
\begin{aligned}
\varrho^D(t,T) &= b(t,T)'\Sigma(\lambda + \Lambda x_t) \\
\varrho^J(t,T) &= b(t,T)'\Upsilon(\psi + \Psi x_t).
\end{aligned}
$$

Therefore, the bond risk premium $\rho_{\tau,t,h}$ fluctuates slowly, except on jump days where it is enhanced by the $(1/h)\varrho^J$ term.

Turning to term premia, we define the term premium for the τ-period bond yield

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[14]This formula is obtained by applying Ito's Lemma to $P(t,T)$ under both the P and Q measures, and noting that the drift term for $\frac{dP}{P}$ under the Q measure has to be equal to $r_t dt$.

at time t, $\mathrm{TP}_{\tau,t}$ in the conventional way as: [15]

$$\mathrm{TP}_{\tau,t} \equiv y_{\tau,t} - \mathrm{E}_{\tau,t}, \qquad (3.28)$$

where $\mathrm{E}_{\tau,t}$ is the expectations component of the bond yield:

$$\mathrm{E}_{\tau,t} = \frac{1}{\tau} E_t \left[\int_t^{t+\tau} r_s ds \right]. \qquad (3.29)$$

When jumps have a zero mean in the physical measure ($\gamma = 0, \Gamma = 0$), the expression for $\mathrm{E}_{\tau,t}$ is the same as in the model without jumps, i.e.,

$$\mathrm{E}_{\tau,t} = \rho_0 + \rho'\theta - \frac{1}{\tau}\rho' K^{-1}(I - e^{-\tau K})\theta + \frac{1}{\tau}\rho' K^{-1}(I - e^{-\tau K})x_t. \qquad (3.30)$$

4 Estimating the Model

4.1 Estimation approach

The model described in section 3 implies that yields are time-inhomogeneous affine functions of the latent state vector. Treating observed yields as being contaminated with small measurement error, the model can easily be estimated by maximum likelihood on daily data via the Kalman filter.

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[15]This defines the term premium as the deviation from the expectations hypothesis, and includes the Jensen's inequality effect in the term premium.

Specifically, we have the following observation equation and state equation:

$$y_{\tau,t} = a_y(\tau; \delta(t)) + b_y(\tau; \delta(t))' x_t + e_{\tau,t} \tag{4.1}$$

$$x_t = x_{t-1} + K(\theta - x_{t-1})\Delta t + \epsilon_t + \xi_t \tag{4.2}$$

$$\epsilon_t \sim \mathcal{N}(0_{n\times 1}, \Sigma\Sigma'\Delta t), \tag{4.3}$$

$$\xi_t \sim \mathcal{N}(0_{n\times 1}, \Omega) \text{ for } t = T_1, T_2, ... \tag{4.4}$$

The first equation is the same as equation (3.22), except that we have added a measurement error $e_{\tau,t}$ that is assumed to be i.i.d. over time and maturities. In order to simplify the implementation, we assume that the time to next employment report, $\delta(t)$, takes on 22 values only (approximately corresponding to the number of trading days in a month), ranging between 0 and $\bar{\delta} = 1/12$. In the state equation, Δt is one business day (1/250). We restrict the ξ_t vector to have zero mean in the physical measure, as the more general version $\xi_t \sim N(\gamma + \Gamma x_{t-1}, \Omega)$ becomes too unwieldy for estimation (ξ_t is still allowed to have a non-zero mean under the risk-neutral measure). Equation (4.4) implies that the conditional variance of x_t has a deterministically varying pattern: it is $\Sigma\Sigma'\Delta t + \Omega$ on announcement days ($t = T_1, T_2, ...$) and $\Sigma\Sigma'\Delta t$ on non-announcement days. We fit the model to daily data on 3-month, 6-month and 1, 2, 4, 7 and 10-year zero-coupon US Treasury yields, using the dataset of Gürkaynak, Sack, and Wright (2007) for maturities of one year or greater, and T-bill yields for 3-month and 6-month maturities. The data span 1990-2007 inclusive. At the zero lower bound, short- and even intermediate-term yields become insensitive to news (Swanson and Williams, 2014), but our model does not incorporate the zero lower bound. For this reason, we omit recent data from our estimation. In order to help pin down the parameters related to physical dynamics, we augment our Kalman filter based estimation with survey forecast data, as in Kim and Orphanides (2012).

The survey forecast data are Blue Chip forecasts of three-month Treasury yields at 6 month and 12 month horizons, that are available monthly, and the long range Blue Chip forecasts at the 5-10 year horizon that are available twice a year. When observed, they are assumed to equal to the true expectations plus a zero-mean iid measurement error.

4.2 Estimated specifications

As is standard in the literature, the number of factors, n, is set to 3. We first allow for jumps in all elements of the state vector and adopt the following normalizations for identification: we restrict ρ to be $[0, 0, 1]'$, specify K as lower triangular and θ as a vector of zeros, and let[16]

$$
\Sigma = \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix}.
$$

We shall denote this specification as the "J-Full" model.

This specification looks somewhat different from the usual specification in which K is lower-triangular, Σ is an identity matrix (or diagonal matrix of n free parameters), and ρ is a vector of n free parameters (or vector of ones). We choose our normalization to make the third element of the state vector directly interpretable as the short rate.[17]

In order to compare with a specification in which there is jump in the short rate only, we also estimate a restricted model in which the Ω matrix is zero except

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[16]c is a scale constant which we choose to be 0.01.

[17]No loss of generality is incurred here, since our specification can be obtained from the "usual" specification by applying the invariant transformation $x_t = L\tilde{x}_t$, where $\tilde{x}_t$ is the state vector in the "usual" specification, x_t is our specification, and $L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \rho_1 & \rho_2 & \rho_3 \end{bmatrix}$.

for Ω_{33}. We shall denote this model as the "J-Short" model. Lastly, in order to compare models with and without jumps, we also estimate the homogeneous model (affine-Gaussian model), which shall refer to as the "No-Jump" model.

5   Empirical Results

The parameter estimates for the three specifications (J-Full, J-Short, No-Jump) are given in Table 3. Figure 2 plots the b_y factor loadings against the time-to-maturity τ for the J-Full and J-Short model. (To save space, we omit the graph for the No-Jump model). Because the b_ys also depend on $\delta(t)$ (the time to the next jump), we plot both $b_y(\tau; 0)$ and $b_y(\tau; \bar\delta)$, so that the range of $b_y(\cdot; \delta)$ can be seen. Because $x_3 = r_t$ in our normalization, $b_y(\tau; 0) \to (0, 0, 1)'$ as $\tau \to 0$. The difference between $b_y(\cdot; 0)$ and $b_y(\cdot; \bar\delta)$ is visible, though it is quite small.

5.1   Term structure of volatilities

In examining the empirical content of the estimated models, we first consider their implications regarding the term structure of volatilities. In our model, changes in bond yields on non-announcement days come from the diffusive part of the state variable dynamics; the diffusive contribution to the daily variance of yields is:[18]

$$var(\Delta^D y) \approx b_y(\tau; \bar\delta)' \Sigma\Sigma' b_y(\tau; \bar\delta)\Delta t. \tag{5.1}$$

On announcement days, in addition to the diffusion contribution, there is also the jump contribution. As discussed in Section 3, the change in bond yield at jumps contains a predictable component. It is, however, small in comparison to typical size

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[18]Henceforth, we approximate $b_y(\tau; \delta(t))$ by $b_y(\tau; \bar\delta)$. Figure 2 showed that $b_y(\tau; \delta(t))$ does not depend materially on $\delta(t)$.

of jump in yields. Therefore, the variance of the announcement-induced jump in yield can be well approximated as:

$$var(\Delta^J y) \approx b_y(\tau; \bar{\delta})' \Omega b_y(\tau; \bar{\delta}). \tag{5.2}$$

Figure 3 plots the term structure of interest rate volatility associated with employment report jumps ($\sqrt{var(\Delta^J y)}$), along with the term structure of daily interest rate volatility without jumps ($\sqrt{var(\Delta^D y)}$), implied by our estimated J-Full model. Our model matches the empirical fact, reviewed in Section 2, that these two volatility term structures are different, and that the jump volatility term structure has a hump shape. Matching the volatility term structure of yields on announcement days is important to investors trading bonds and bond derivatives at high frequency around news releases.

Figure 3 also shows the corresponding results from our J-Short model. In this model, the jump volatility term structure looks very different from the case where all three factors are allowed to jump. It does not have a hump shape, and instead slopes down.[19] The model with jumps in the short rate alone implies that employment report announcements should have little impact on ten-year yields, which we know to be counterfactual.

Piazzesi (2001) also estimated a term structure model which has jumps in state variables corresponding to nonfarm payroll employment and CPI at deterministic times (times of data releases). These jumps affect the yield curve through their effect on the intensity for the arrival of Poisson jumps that represent discrete changes in the federal funds target rate. She found that the term structure of the sensitivity of bond yields to nonfarm payroll surprises in her estimated model is monotonically

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[19]This is because the factor loading for the short rate (the only state variable that has jumps in the J-Short model), $[b_y(\tau; \bar{\delta})]_3$, is a monotonically decreasing function of time-to-maturity τ.

downward sloping (Piazzesi 2001, Figure 8, right panel), as in our J-Short model.

The difference between the J-Full and J-Short models can be seen from the estimated Ω matrices. We obtain $\sqrt{\text{diag}(\Omega)} = [0.00082, 0.00063, 0.00036]'$ for the J-Full model and $[0, 0, 0.00054]'$ for the J-Short model. Note that the J-Short model by design has nonzero standard deviation of jump in x_{3t} (short rate) only. On the other hand, in the J-Full model, the standard deviation of jump in x_{3t} (short rate) is the *smallest* of all the state variables.

These results indicate that it is important to allow for jumps in more than just the short rate.

5.2   News: term premia and expectations

To better understand the sources of yield curve movements that underlie the hump-shaped volatility term structure pattern discussed above, we decompose the model-implied term structure of interest rate volatility associated with employment report jumps and the model-implied term structure of the diffusive component of daily interest rate volatility into term premium and expected future short rate components. This is in particular important to central banks who want to parse interest rate volatility around news announcements into term premium and rate expectations components, as discussed in Bauer (2011) and Beechey (2007).

In our model, term premia and expectations component are affine in the state vector (equations (3.28), and (3.30)); thus, the term premium and expectations component of the jump in yields at employment report at time T_i can be expressed as $\Delta^J \text{E} \approx b_y^{\text{E}\prime} \xi_{T_i}$ and $\Delta^J \text{TP} \approx b_y^{\text{TP}\prime} \xi_{T_i}$, respectively.[20]   Hence, the contribution of

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[20]The factor loadings b_y^{E} and b_y^{TP} are implicitly defined as $\text{E}_{\tau,t} \equiv a_y^{\text{E}}(\tau; \bar{\delta}) + b_y^{\text{E}}(\tau; \bar{\delta})' x_t$ and $\text{TP}_{\tau,t} \equiv a_y^{\text{TP}}(\tau; \bar{\delta}) + b_y^{\text{TP}}(\tau; \bar{\delta})' x_t$.

expectations and term premium components to the variance are:[21]

$$var(\Delta^J \mathrm{E}) \approx b_y^{\mathrm{E}\prime} \Omega b_y^{\mathrm{E}}, \tag{5.3}$$

$$var(\Delta^J \mathrm{TP}) \approx b_y^{\mathrm{TP}\prime} \Omega b_y^{\mathrm{TP}}. \tag{5.4}$$

Likewise, the approximate contributions of the expectations and term premium components to the diffusive part of daily yield variance can be approximated as:

$$var(\Delta^D \mathrm{E}) \approx b_y^{\mathrm{E}\prime} \Sigma\Sigma' b_y^{\mathrm{E}} \Delta t, \tag{5.5}$$

$$var(\Delta^D \mathrm{TP}) \approx b_y^{\mathrm{TP}\prime} \Sigma\Sigma' b_y^{\mathrm{TP}} \Delta t. \tag{5.6}$$

Figure 4 shows $\sqrt{var(\Delta^J \mathrm{E})}$, $\sqrt{var(\Delta^J \mathrm{TP})}$, $\sqrt{var(\Delta^D \mathrm{E})}$ and $\sqrt{var(\Delta^D \mathrm{TP})}$ for the J-Full and J-Short models.

The J-Full model results show an interesting difference between jump volatility and diffusion volatility: The expectations component of jump volatility first rises with maturity, peaks around 1-2 years, and then declines. This behavior seems consistent with the "path shock" intuition: employment reports have a stronger effect on policy rate expectations a bit further out than very near-term horizons such as the next few FOMC meeting. In contrast, the expectations component of diffusion volatility declines monotonically as a function of maturity. It can also be seen that, while the contribution of the term premium component to the jump volatility is small at short maturities, it grows with maturity. At maturities of about three years and above, it exceeds the expectations component contribution. In other words, the term premium shock is an important contributor to the yield response to employment reports, especially at longer maturities.

In the J-Short model, jump volatility comes mainly from the expectations com-

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[21]Note that these do not add up to $var(\Delta^J y)$ since expectations and term premium components are generally correlated, so there is an extra term $2b_y^{\mathrm{E}\prime} \Omega b_y^{\mathrm{TP}}$.

ponent at all maturities, with the term premium component contributing relatively little. In further contrast to the J-Full model, the contribution of the expectations component to jump volatility declines monotonically with maturity.

5.3   Bond risk premia

As noted in the Introduction, an important departure from the existing literature on term structure modeling with announcement effects is that in our model jump risk is priced. In Section 3, we discussed theoretical implication of this for bond risk premia. Here we explore the quantitative implications based on our estimated models.

Using daily filtered state variables, Figure 5a plots annualized bond risk premia for a one-day holding period (equation (3.24) with $h = 1$ day $= 1/250$) for ten-year bonds for the J-Full model and the No-Jump model. For a closer look, Figure 5b shows the same plot with a magnified y-axis. It can be seen from Figures 5a,b that expected excess returns for the No-Jump model vary slowly over time, being positive for most of the 1990s and around 2005, but negative in the early 2000s.[22] Incorporating jumps makes the expected excess returns much larger in absolute magnitude on employment report days, as can be seen from sharp spikes on these days. These spikes originate from the discontinuity in $b(t, \cdot)$ in equation (3.24) at jump dates. Equation (3.27) also gives a clear explanation for these spikes. It can be also seen from Figures 5a,b that the average level of spikes moves in ways similar to the expected excess returns on non-announcement days, albeit with a different scale. Furthermore, the expected excess returns on non-employment report days are *smaller* in absolute value than the expected excess return from the homogeneous (No-Jump) model. These results indicate that part of the bond risk premium is earned on employment report days as

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[22]Interestingly, Campbell, Sunderam, and Viceira (2009) have stressed that the CAPM beta of bonds have been generally negative in 2000s.

compensation for jump risk. This is consistent with Faust and Wright (2008) who do not estimate a term structure model, but who do find that bond excess returns on days of macroeconomic news announcements are predictable.

Figure 5c shows the corresponding plot of one-day expected excess returns based on the J-Short model. The pattern of expected excess returns from this model does not have the aforementioned properties associated with Figures 5a and 5b (J-Full). Although the J-Short model still allows for jumps in the pricing kernel (short-rate jump risk is still priced), this is not sufficient to produce the kind of jump risk premia variation that we saw in the J-Full model.

While the spikes in Figure 5 may appear large in magnitude, this is a consequence of the fact that we are measuring annualized risk premia with a jump in a short holding period $[t, t+h]$. Recall from Section 3.3 that annualized bond risk premia for short holding periods can be well approximated by equation (3.27), with diffusion component ϱ_t^D and jump component $(1/h)\varrho_t^J$. As $h \to 0$ (while $[t, t+h]$ still contains jump event), $|\rho_{\tau,t,h}| \to \infty$, due to the factor $1/h$. The expected return at jumps, ϱ_t^J, are not extremely large in magnitude: for example, a spike of typical size, say 0.25, in Figure 5a corresponds to $0.25h = 0.25/250 = 0.001 = 0.1\%$ expected change in bond price. Another angle by which to look at the magnitudes of jump risk premia is through Sharpe ratios, to which we return below.

Table 4 shows the correlation between ϱ_t^D and ϱ_t^J implied by our estimated models and daily filtered state variables, for two- and ten-year bonds.[23] It also shows the correlation of daily changes in ϱ_t^D and ϱ_t^J. It can be seen that in the case of the J-Full model, both the level and difference correlations are positive. This corroborates the visual impression from Figure 5 that bond risk premia associated with jumps move

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[23]Although the jump component is nonzero only on employment report days, we can think of there being an underlying process, and examine how it is related to the diffusion component

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in ways that are similar to bond risk premia associated with diffusions. On the other hand, the J-Short model produces a negative correlation between jump and diffusion contributions, underscoring the fact that the model with jumps in the short rate alone is too restrictive to produce realistic variation in bond risk premia. The positive relationship between the diffusion and jump components of bond risk premia observed in the J-Full model is consistent with findings in Balduzzi and Moneta (2012), who report that a single factor can capture much of predictable returns to macro announcements, including employment report and other announcements: The positive relationship between the diffusion and jump components may be partly reflecting this structure, since responses to announcements other than employment reports are treated as part of diffusion in our implementation.

Next, we compare the *monthly average* of one-day expected excess returns for the J-Full model with the No-Jump model.[24] As can be seen in Figure 6, the monthly averaging removes the spike patterns seen earlier, and produces bond risk premia that are similar to the those of the homogeneous (No-Jump) model. This is especially so for relatively short maturity bonds such as the 2-year (Figure 6a), but even for longer maturity bonds, such as the 10-year (Figure 6b), monthly averages from the two models are quite close. This implies that the homogeneous model can be viewed as a rough approximation of more granular models (such as the J-Full model) *for longer holding periods* (such as a month).

We can decompose the bond risk premia for a one-month holding period for month t approximately as:

$$\rho_{\tau,t,1/12} \approx \overline{\varrho_M^D}(t;\tau) + 12\varrho_M^J(t;\tau) \tag{5.7}$$

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[24]This monthly average of one-day expected excess returns is approximately equal to the one-month holding period bond risk premia based on the state vector right before the beginning of that month, as state variables have half-lives that are greater than a month. Note that, due to the monthly frequency of employment reports, one month is the shortest interval over which we can expect to see relatively smooth behavior of bond risk premia.

where $\overline{\varrho_M^D}(t; \tau)$ is the average of expected daily excess bond returns for month t based on the filtered state variables right before month t, and $\varrho_M^J(t; \tau)$ is also computed with filtered state variables right before month t (the employment report being close to the start of the month).[25] This allows us to assess how much jump risk premia contribute to predictability in one-month expected excess returns. Figure 7 plots the jump and diffusion contributions to one-month holding period bond risk premia, based on the J-Full model, for two- and ten-year maturities. Not surprisingly, $\overline{\varrho_M^D}(t) + 12\varrho_M^J(t)$ in Figure 7 agrees fairly well with monthly averages of daily bond risk premia in Figure 6. At the two-year maturity, the jump risk premium contributes relatively little to the one-month holding period bond risk premium. However, at the ten-year maturity, the jump risk premium contributes substantially to the variability of the one-month holding period bond risk premium. For ten-year bonds (and to a lesser extent for two-year bonds), the jump and diffusive components have a visible positive correlation, consistent with our earlier findings in Table 4.

It is also of interest to examine the annualized Sharpe ratios corresponding to the following three trading strategies:

  i Strategy ("τ-J") of investing (i.e., taking a long position) in a τ-year bond only over a short interval surrounding the employment report: $\sqrt{12}\frac{\varrho_M^J(t)}{\sqrt{b'\Omega b}}$,

  ii Strategy ("τ-D") of investing in a τ-year bond every day of the month, *except* for the day containing the employment report: $\frac{\overline{\varrho_M^D}(t)}{\sqrt{b'\Sigma\Sigma'b}}$, and

  iii Strategy ("τ-all") of investing in a τ-year bond every day of the month: $\frac{\overline{\varrho_M^D}(t) + 12\varrho_M^J(t)}{\sqrt{b'\Sigma\Sigma'b + 12 b'\Omega b}}$.

Table 5 shows the mean, standard deviation, minimum, and maximum of these Sharpe ratios for these strategies implied by the J-Full model over our sample period. The

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[25]We henceforth suppress the dependence of $\overline{\varrho_M^D}$ and ϱ_M^J on τ.

means of the τ-J strategy are modestly negative for both $\tau =$ 2-year and 10-year, reflecting the frequent occurrences of negative jump risk premia during the 2000s as was seen in Figure 7,[26] while the means for τ-D and τ-all strategies are modestly positive. The τ-J Sharpe ratios have substantial variability, especially for ten-year bonds; the ten–year τ-J Sharpe ratios range between -2.0 and 1.6 in our sample period. That said, these magnitudes are not too large relative to some of the annualized Sharpe ratios mentioned in other asset markets or trading strategies, such as algorithmic trading.

Lastly, and in the same spirit as Figure 6, we compared the term premia, (hold-to-maturity bond risk premia, as in equation (3.28)) implied by J-Full and No-Jump models, for two-year and ten-year maturities. The two models' term premium estimates are very similar, especially at the two-year maturity, for the same reasons as discussed in relation to Figure 6, but are not shown, so as to conserve space.

6   Conclusion

Through the prism of a term structure model with deterministic jumps, in this paper we have explored the implications of the presence of influential scheduled data release events (employment reports). While the model could be extended and refined further, the key insights from the present model are expected to be robust. We find that "path shocks"—innovations that shift the *future* expected path of the short rate, and "term premium shocks"—innovations that shift the term premium component of yields, are both important contributors to jumps in bond yields at these events. We also find that deterministic jumps lead to a behavior of bond risk premia for short holding

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[26]Since our model does not have time-varying volatility, the time series of Sharpe ratios are proportional to expected excess returns.

periods that is notably different from the predictions of standard (time-homogeneous) models. Nonetheless, the behavior of bond risk premia for longer holding periods and term permia (hold-to-maturity risk premia) can be reasonably well approximated with time-homogeneous models.

Appendix: Proofs of Propositions

Proof of Proposition 1

Using the Feynman-Kac formula, for any interval $[t, u]$ that doesn't include any jump event, we have:

$$E_t^Q(e^{-\int_t^u r_s ds + \eta' x_u}) = \exp(A(u - t; \eta) + B(u - t; \eta)' x_t) \tag{A.1}$$

where $B(\tau; \eta)$ and $A(\tau; \eta)$ are given by equations (3.15) and (3.16).

Between time t and T there are p jumps at $T_1, T_2, ..., T_p$. Consider $P(T_i, T)$ and $P(T_i-, T)$, where by T_i we mean $T_i + 0^+$, and by T_i- we mean $T_i - 0^+$. Using the law of iterated expectations, we have:

$$P(T_i-, T) = E_{T_i-}^Q(e^{-\int_{T_i-}^{T_i} r_s ds} E_{T_i}^Q(e^{-\int_{T_i}^T r_s ds})) = E_{T_i-}^Q(P(T_i, T)). \tag{A.2}$$

We know from equation (A.1) that at the time of the last jump:

$$P(T_p, T) = \exp(a_p + b_p' x_{T_p}).$$

where $a_p = A(T - T_p; 0)$ and $b_p = B(T - T_p; 0)$. Suppose that $P(T_i, T)$ is of the form:

$$P(T_i, T) = \exp(a_i + b_i' x_{T_i}). \tag{A.3}$$

From this, and equation (A.2), we have:

$$P(T_i-, T) = E^Q_{T_i-}(e^{a_i+b'_i(x_{T_i-}+\xi_{T_i})})$$

$$= e^{a_i+b'_i x_{T_i-}+b'_i(\gamma_Q+\Gamma_Q x_{T_i-})+\frac{1}{2}b'_i\Omega b_i}$$

$$= e^{a_i+b'_i\gamma_Q+\frac{1}{2}b'_i\Omega b_i+[(I+\Gamma'_Q)b_i]'x_{T_i-}},$$

(A.4)

where we have used the fact that the jump vector is normally distributed. For the bond price at the time of jump $i-1$, we have:

$$P(T_{i-1}, T) = E^Q_{T_{i-1}}(e^{-\int_{T_{i-1}}^{T_i-} r_s ds} P(T_i-, T))$$

$$= e^{a_i+b'_i\gamma_Q+\frac{1}{2}b'_i\Omega b_i} E^Q_{T_{i-1}}[e^{-\int_{T_{i-1}}^{T_i-} r_s ds+[(I+\Gamma'_Q)b_i]'x_{T_i-}}],$$

$$= e^{a_i+b'_i\gamma_Q+\frac{1}{2}b'_i\Omega b_i} e^{A(T_i-T_{i-1};(I+\Gamma'_Q)b_i)+B(T_i-T_{i-1};(I+\Gamma'_Q)b_i)'x_{T_i-}}$$

where we have used equation (A.1) and the fact that there are no jumps between T_{i-1} and T_i in the last step. This means that $P(T_{i-1}, T) = e^{a_{i-1}+b'_{i-1}x_{T_{i-1}}}$ where

$$a_{i-1} = a_i + b'_i\gamma_Q + \frac{1}{2}b'_i\Omega b_i + A(T_i - T_{i-1}; (I + \Gamma'_Q)b_i)$$

$$b_{i-1} = B(T_i - T_{i-1}; (I + \Gamma'_Q)b_i).$$

We have thus proved equation (A.3) by induction over $i = p, p-1, p-2, ...1$, where $\{a_i\}$ and $\{b_i\}$ are given by the recursions in equations (3.11)-(3.14). For the bond price at time t, we have:

$$P(t, T) = E^Q_t(e^{-\int_t^{T_1-} r_s ds} P(T_1-, T)).$$

This yields equation (3.8) and completes the proof of the proposition. ∎

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Derivation of bond prices in the $\Gamma_Q = 0_{n \times n}$ case

In this part of the appendix, we derive equations (3.17) and (3.18), that apply just in the case where $\Gamma_Q = 0$. Let $\tilde{a}(t, T)$ and $\tilde{b}(t, T)$ denote the values of $a(t, T)$ and $b(t, T)$ without any deterministic jumps ($\gamma_Q = 0$, $\Omega = 0$ and $\Gamma_Q = 0$). The familiar affine Gaussian bond pricing equation, without deterministic jumps, takes the form:

$$P(t, T) = e^{\tilde{a}(T-t) + \tilde{b}(T-t)' x_t}.$$

Next consider the case where there are jumps, but there is no state dependence ($\Gamma_Q = 0$). Because $b(t, T)$ does not depend anywhere on γ_Q or Ω, $b(t, T) = \tilde{b}(t, T)$, proving (3.18).

From (3.12), (3.14) and (3.15), we can write

$$b_i = -[I - \exp(-K_Q'(T - T_i))] K_Q'^{-1} \rho.$$

From (3.10) and (3.11),

$$
\begin{aligned}
a(t, T) &= \Sigma_{j=1}^p [b_j' \gamma_Q + \frac{1}{2} b_j' \Omega b_j] + \tilde{a}(t, T) \\
&= \tilde{a}(t, T) - \Sigma_{j=1}^p \rho' K_Q^{-1} [I - \exp(-K_Q'(T - T_i))] \gamma_Q \\
&\quad + \frac{1}{2} \rho' K_Q^{-1} [I - \exp(-K_Q'(T - T_i))] \Omega [I - \exp(-K_Q'(T - T_i))] K_Q'^{-1} \rho,
\end{aligned}
$$

proving (3.17).

Proof of Proposition 2

To prove equation (3.21), it is sufficient to show that

$$\frac{\partial}{\partial \epsilon} b(t, T_i + \epsilon)\bigg|_{\epsilon=0} \neq -\frac{\partial}{\partial \epsilon} b(t, T_i - \epsilon)\bigg|_{\epsilon=0}. \tag{A.5}$$

We show this for $T_i = T_1$ (nearest jump date). Since there is no jump between t and T_1, we have

$$b(t, T_1 - \epsilon) = B(T_1 - \epsilon - t; 0), \tag{A.6}$$

where we have used equation (A.1). For $b(t, T_i + \epsilon)$, note that

$$P(t, T_1 + \epsilon) = E_t^Q(e^{-\int_t^{T_1^-} r_s ds} P(T_1, T_1 + \epsilon)) = E_t^Q(e^{-\int_t^{T_1^-} r_s ds} E_{T_1^-}^Q(e^{A(\epsilon;0) + B(\epsilon;0)' x_{T_1}}))$$

$$= E_t^Q(e^{-\int_t^{T_1^-} r_s ds} e^{A(\epsilon;0) + B(\epsilon;0)'(I + \Gamma_Q) x_{T_1^-} + \frac{1}{2} B(\epsilon;0)' \Omega B(\epsilon;0)}). \tag{A.7}$$

Therefore

$$b(t, T_1 + \epsilon) = B(T_1 - t; (I + \Gamma_Q)' B(\epsilon; 0)). \tag{A.8}$$

Expanding equations (A.6) and (A.8) in powers of ϵ, we have (using equation (3.15)),

$$b(t, T_1 - \epsilon) = B(T_1 - t; 0) + e^{-K_Q'(T_1 - t)} \rho \epsilon + O(\epsilon^2) \tag{A.9}$$

$$b(t, T_1 + \epsilon) = B(T_1 - t; 0) - e^{-K_Q'(T_1 - t)} (I + \Gamma_Q') \rho \epsilon + O(\epsilon^2). \tag{A.10}$$

Therefore, equation (A.5) holds for $i = 1$, unless $\Gamma_Q = 0$. The demonstration of equation (A.5) for $i > 1$ is straightforward using the law of iterated expectations $P(t, T_i \pm \epsilon) = E_t^Q(e^{-\int_t^{T_{i-1}} r_s ds} P(T_{i-1}, T_i \pm \epsilon))$.

That shows equation (3.21) for the case $\Gamma_Q \neq 0$. To complete the proof of Proposition 2, we must show equation (3.21) in the $\Gamma_Q = 0$ case. In this case,

the first derivative of $b(t,T)$ is continuous at T_i. However, from equation (3.17):

$$a(t, T_i + \epsilon) = a(t, T_i - \epsilon) + \tilde{a}(t, T_i + \epsilon) - \tilde{a}(t, T_i - \epsilon)$$

$$+ 1(\epsilon > 0) \left(-\rho' K_Q^{-1}(I - e^{-K_Q \epsilon}) \gamma_Q + \frac{1}{2} \rho' K_Q^{-1}(I - e^{-K_Q \epsilon}) \Omega (I - e^{-K_Q' \epsilon}) K_Q'^{-1} \rho \right).$$

The term $1(\epsilon > 0)(\cdot)$ has a nonzero derivative with respect to ϵ, so the first derivative of $a(t,T)$ is discontinuous at T_i. Hence there is again a kink, and equation (3.21) holds. ∎

Table 1: Standard Deviation of Yield Changes on Announcement Days

	Three-month	Two-year	Ten-year
Nonfarm payrolls	6.0*	10.1***	8.9***
Durable Goods	4.7	6.3***	5.8
Retail Sales	4.0**	6.9***	7.0***
PPI	3.8*	6.0	5.9
FOMC	5.9	6.7**	5.4
GDP	5.0	6.3***	6.3***
CPI	6.1	6.8***	6.5**
None	4.9	5.1	5.3

Notes: This table shows the standard deviation of three-month, two-year and ten-year zero-coupon yield changes (in basis points) on days of selected announcements, and on days of no announcements. For each type of announcement, cases in which the volatility is significantly different on that type of announcement day relative to non-announcement days at the 10, 5 and 1 percent significance level are marked with one, two and three asterisks, respectively. Newey-West standard errors are used. The sample period is January 1990 to December 2007.

Table 2: One Day Options Implied Volatilities on Thursdays

	Employment Reports	Other Days	t-stat
Five year	0.36	0.20	10.56
Ten-year	0.60	0.33	11.21
Thirty-year	0.96	0.57	8.14

Notes: This table reports the average one-day volatilities implied by options on five-, ten- and thirty-year Treasury futures as of the day before employment report releases and all other Thursdays. Implied volatilities are in percentage points in price terms; dividing by duration of the cheapest-to-deliver gives the approximate implied volatility in yield terms. In the final column, the t statistic tests the hypothesis that the implied volatilities are equal on employment report and other days. The sample period is February 2011 to February 2015.

	J-Full	J-Short	No-Jump
K_{11}	0.0102 (0.0222)	0.0241 (0.0265)	0.0170 (0.0260)
K_{21}	-0.1463 (0.3120)	-0.8381 (0.4502)	-0.2318 (0.2605)
K_{31}	-0.1173 (0.3216)	0.6595 (0.4261)	-0.0294 (0.2956)
K_{22}	2.2006 (0.5740)	3.2498 (0.4670)	1.6373 (0.6506)
K_{32}	-2.3023 (0.4078)	-3.1530 (0.3993)	-1.9081 (0.4177)
K_{33}	0.6764 (0.2035)	0.3504 (0.0338)	0.6713 (0.2660)
Σ_{31}	0.0022 (0.0010)	0.0019 (0.0007)	0.0012 (0.0011)
Σ_{32}	-0.0051 (0.0006)	-0.0056 (0.0003)	-0.0043 (0.0011)
Σ_{33}	-0.0064 (0.0004)	-0.0061 (0.0001)	-0.0073 (0.0006)
ρ_0	0.0171 (0.0357)	0.0120 (0.0279)	0.0187 (0.0283)
λ_1	0.0635 (1.2979)	-0.9730 (0.5646)	-0.3967 (0.7636)
λ_2	-1.0458 (1.7400)	-1.0731 (1.5471)	0.1693 (0.8939)
λ_3	-4.9772 (6.9200)	-6.1942 (5.5682)	-5.4023 (6.1607)
$[\Sigma\Lambda]_{11}$	-0.1703 (0.2160)	0.1244 (0.0694)	-0.0024 (0.1606)
$[\Sigma\Lambda]_{21}$	0.3357 (0.1682)	0.5048 (0.2575)	0.1762 (0.1965)
$[\Sigma\Lambda]_{31}$	-0.7580 (0.1884)	-0.4027 (0.1804)	-0.7933 (0.1834)
$[\Sigma\Lambda]_{12}$	-0.3564 (0.4816)	-0.2089 (0.2469)	-0.0960 (0.4632)
$[\Sigma\Lambda]_{22}$	-0.6992 (0.3483)	-1.3224 (0.4230)	-0.5947 (0.2552)
$[\Sigma\Lambda]_{32}$	-0.7048 (0.3460)	-0.5794 (0.3664)	-0.9205 (0.2854)
$[\Sigma\Lambda]_{13}$	0.2796 (0.3003)	-0.0426 (0.0174)	0.0445 (0.2148)
$[\Sigma\Lambda]_{23}$	-0.2888 (0.1259)	0.1022 (0.0603)	-0.1466 (0.0155)
$[\Sigma\Lambda]_{33}$	0.5355 (0.3151)	-0.2088 (0.1102)	0.4724 (0.3826)
γ_{Q1}	-0.0006 (0.0009)		
γ_{Q2}	-0.0007 (0.0008)		
γ_{Q3}	-0.0003 (0.0005)	-0.0005 (0.0005)	
Γ_{Q11}	0.0071 (0.0104)		
Γ_{Q21}	0.0109 (0.0072)		
Γ_{Q31}	0.0023 (0.0059)	0.0109 (0.0055)	
Γ_{Q12}	0.0228 (0.0166)		
Γ_{Q22}	0.0149 (0.0136)		
Γ_{Q32}	-0.0245 (0.0102)	-0.0232 (0.0091)	
Γ_{Q13}	-0.0046 (0.0098)		
Γ_{Q23}	-0.0124 (0.0069)		
Γ_{Q33}	0.0079 (0.0058)	0.0011 (0.0031)	
Υ_{11}	0.0008 (0.0001)		
Υ_{21}	0.0005 (0.0001)		
Υ_{31}	0.0001 (0.0001)		
Υ_{22}	0.0004 (0.0001)		
Υ_{32}	0.0001 (0.0002)		
Υ_{33}	-0.0004 (0.0001)	0.0005 (0.0000)	

Notes: Parameter estimates for the J-Full, J-Short, and No-Jump models. Standard errors are given in parenthesis. We impose the following normalization restrictions: $K_{12} = K_{13} = K_{23} = 0$, $\Sigma_{11} = \Sigma_{22} = 0.01$, $\Sigma_{12} = \Sigma_{21} = \Sigma_{13} = \Sigma_{23} = 0$, $\rho = [0, 0, 1]'$, $\theta = [0, 0, 0]'$, $\Upsilon_{12} = \Upsilon_{13} = \Upsilon_{23} = 0$. In addition, for tractability we set $\gamma = [0, 0, 0]'$, and $\Gamma = 0_{3\times3}$.

Table 4: Correlation of jump and diffusion components of bond risk premia

		Two-year	Ten-year
J-Full	$\mathrm{cov}(\varrho^D, \varrho^J)$	0.61	0.57
		(0.04)	(0.04)
	$\mathrm{cov}(\Delta\varrho^D, \Delta\varrho^J)$	0.90	0.96
		(0.02)	(0.02)
J-Short	$\mathrm{cov}(\varrho^D, \varrho^J)$	-0.74	-0.87
		(0.03)	(0.02)
	$\mathrm{cov}(\Delta\varrho^D, \Delta\varrho^J)$	-0.96	-0.98
		(0.02)	(0.02)

Notes: This table shows the simple correlation of the diffusion and jump components of bond risk premia ($\varrho^D \equiv b(t,T)'\Sigma(\lambda + \Lambda x_t)$ and $\varrho^J \equiv b(t,T)'\Upsilon(\psi + \Psi x_t)$) based on estimated parameters and state variables (daily series). The Δ operator denotes daily changes. Standard errors are in parentheses, computed using the Bartlett formula with 8 lags.

Table 5: Summary statistics for Sharpe ratios

|	2Y-J	2Y-D	2Y-all	10Y-J	10Y-D	10Y-all
mean	-0.18	0.30	0.22	-0.31	0.40	0.29
s.d.	0.59	0.74	0.83	0.74	0.24	0.40
min	-1.53	-1.50	-1.81	-2.10	-0.13	-0.63
max	1.15	2.04	2.31	1.61	0.92	1.25

Notes: This table shows summary statistics for the time series of annualized Sharpe ratios implied by the estimated J-Full model for the following trading strategies: "τ-J" is the strategy of taking long position in τ-year bond only over a short interval nesting the employment report (financed by borrowing at the riskless rate r_t); "τ-D" is the strategy of taking long position in τ-year bond, except for a short interval nesting the employment report; "τ-all" is the strategy of taking long position in τ-year bond at all times.

Figure 1: Volatility of Yield Changes
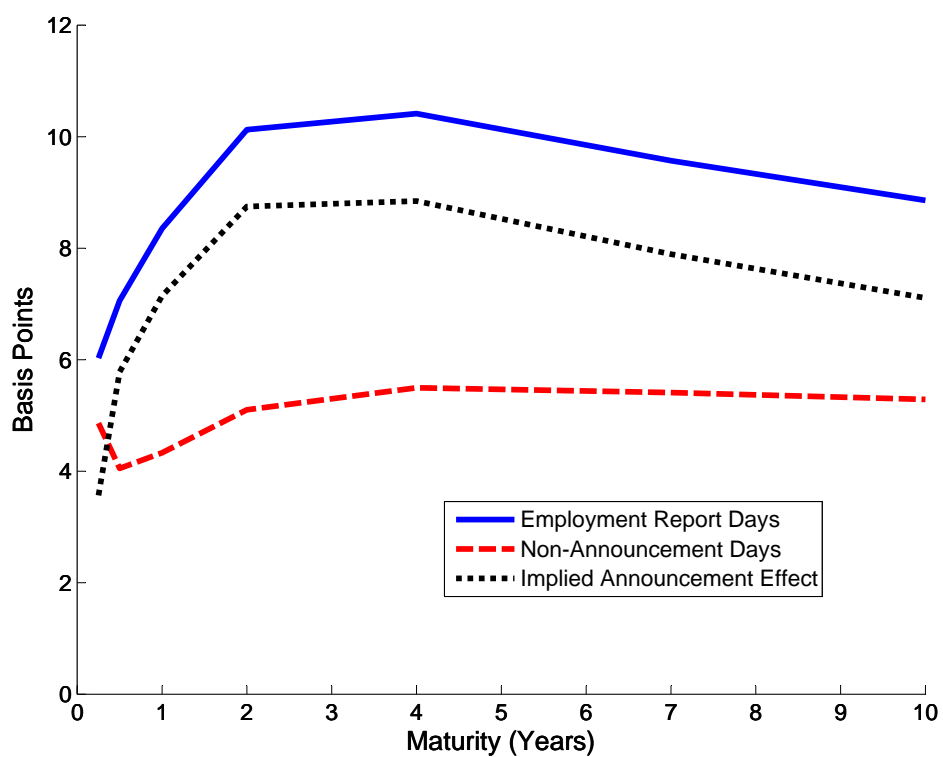
NOTE: This figure plots the standard deviation of daily changes in US Treasury zero-coupon yields on days of employment report releases and on non-announcement days against the bond maturity. The sample period is January 1990 to December 2007.

## Figure 2: Estimated b_y factor loadings
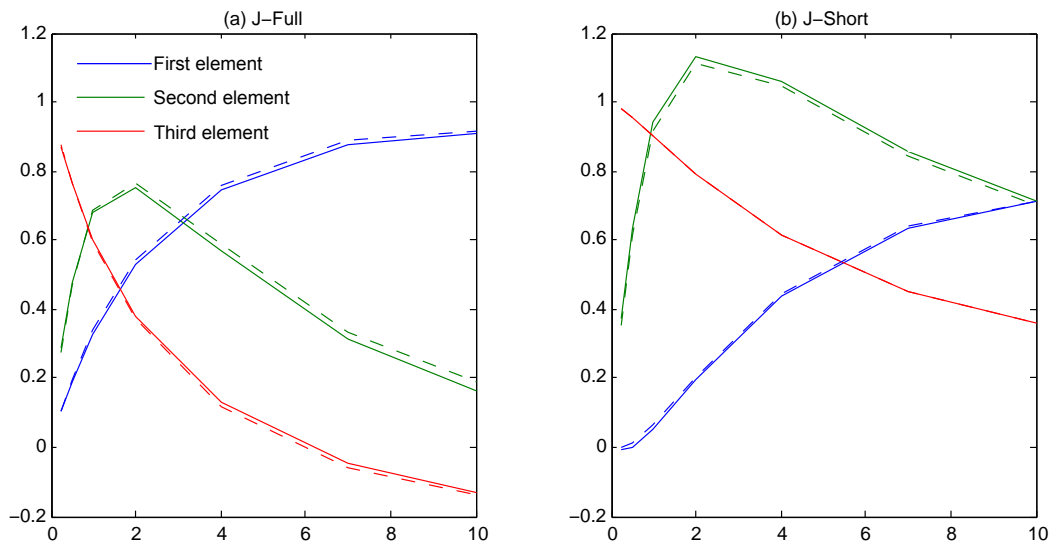
NOTE: This figure plots the three elements of the estimated b_y vector as a function of time-to-maturity τ, for the J-Full model (left panel) and the J-short model (right panel). The solid and dashed lines denote $b_y(\cdot; 0)$ and $b_y(\cdot; \bar{\delta})$, respectively.

**Figure 3: Model-implied term structure of volatility**
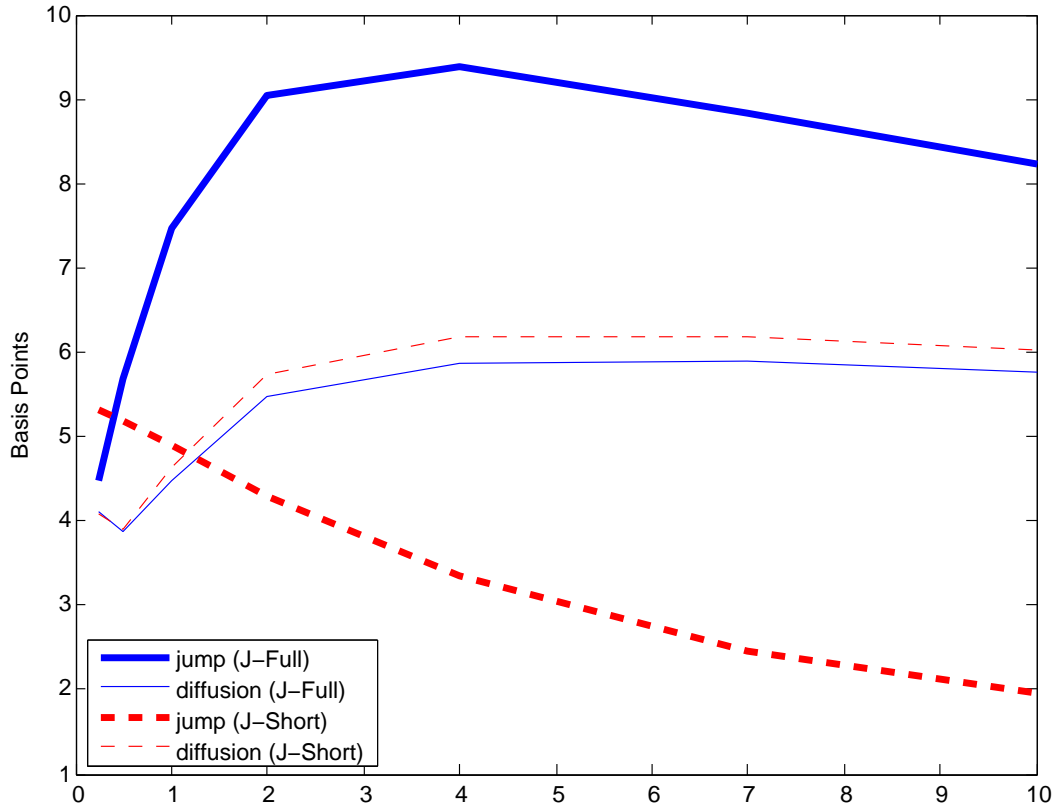
NOTE: This figure plots the model-implied term structure of interest rate volatility associated with employment report jumps (equation (5.2)), along with the model-implied term structure of the diffusion component of daily interest rate volatility (equation (5.1)). Results are shown both for the model with jumps in all state variables and the model with jumps in the short rate alone.

**Figure 4: Model-implied term structure of volatility:
expectations and term premium**
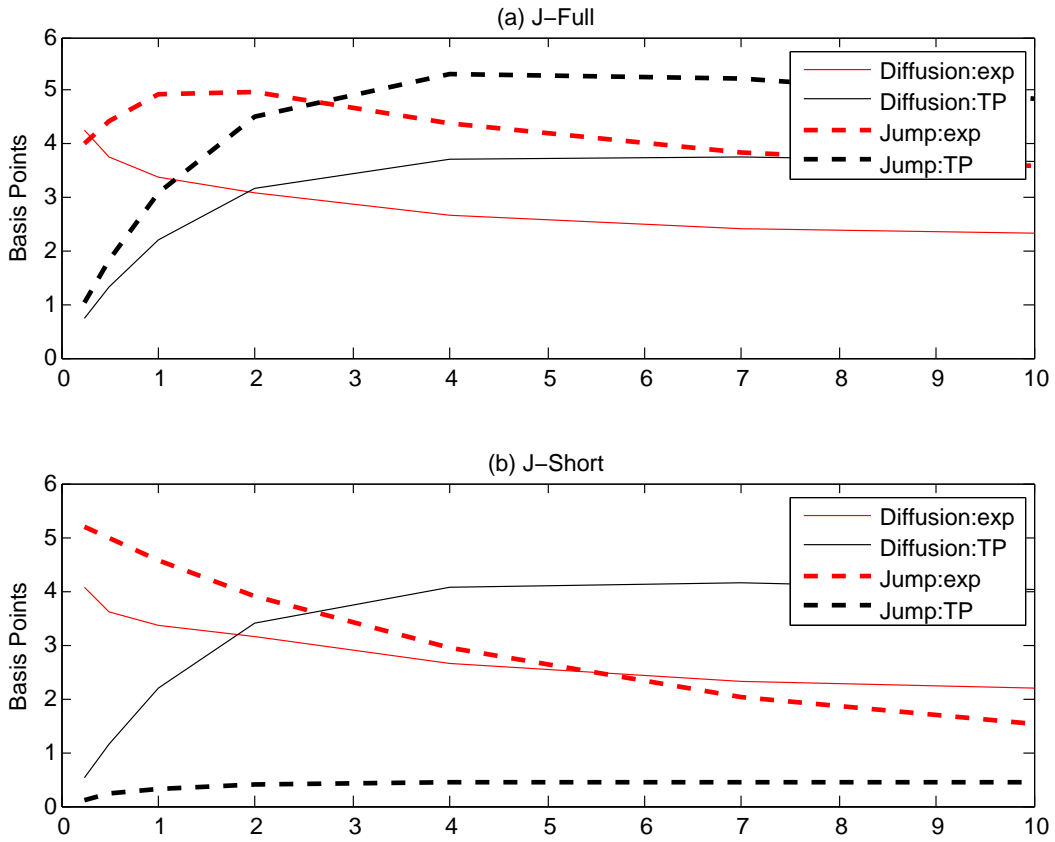
NOTE: This figure plots the term structures of the expectations (exp) and term premium (TP) components of jump and diffusion volatility (equations (5.3)-(5.6)) in both the J-Full and J-short models.

**Figure 5: One day expected excess return on ten-year bond**
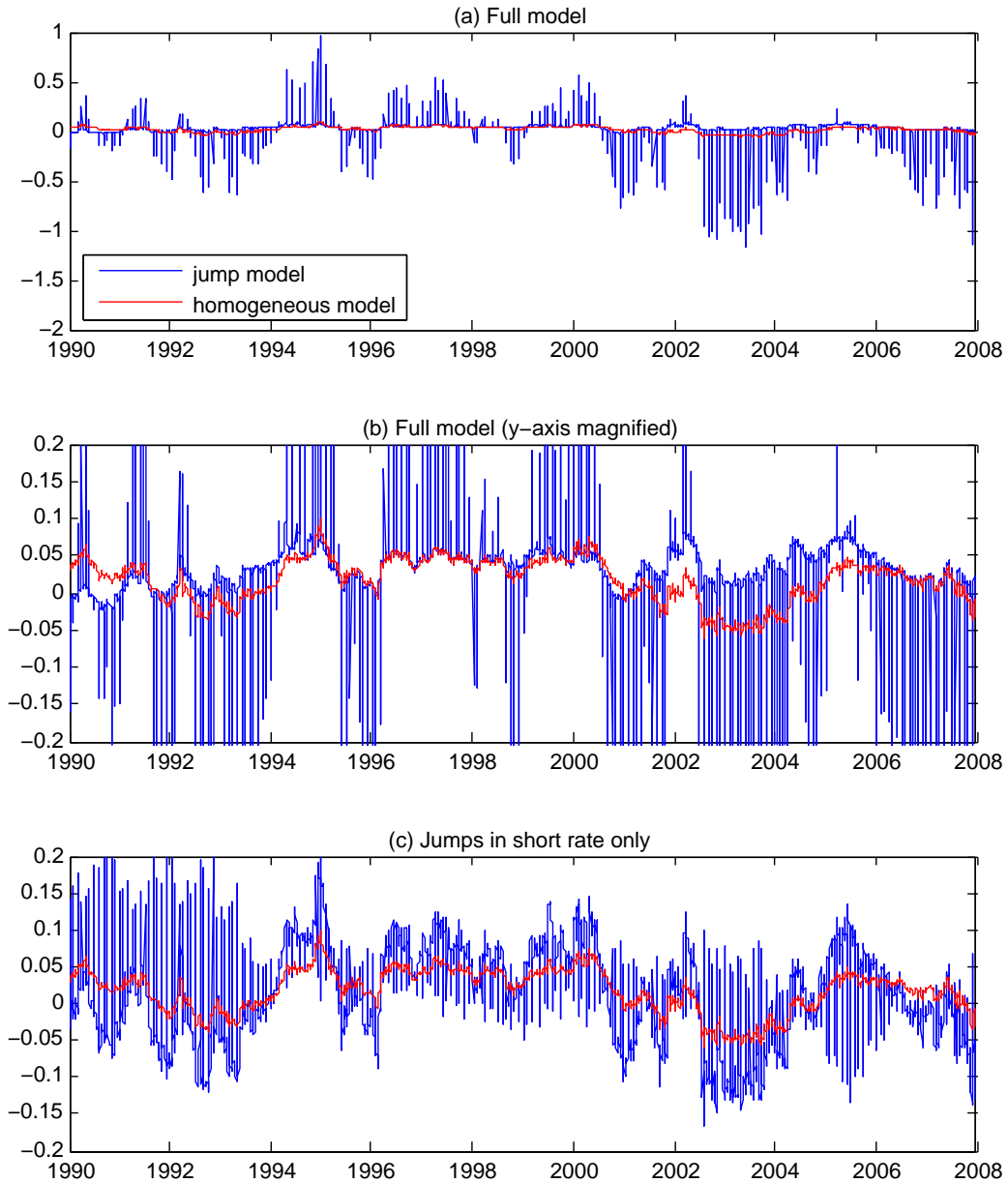
NOTE: This figure plots the one day holding period *ex ante* expected excess returns on holding a ten-year bond over a one-day bond (equation (3.24)). Panels (a) and (b) show the results for the J-Full and No-Jump (homogeneous) models, and Panel (c) shows the results for the J-Short and No-Jump models. Units are annualized returns.

Figure 6: Monthly average of one-day expected excess return on ten-year bond
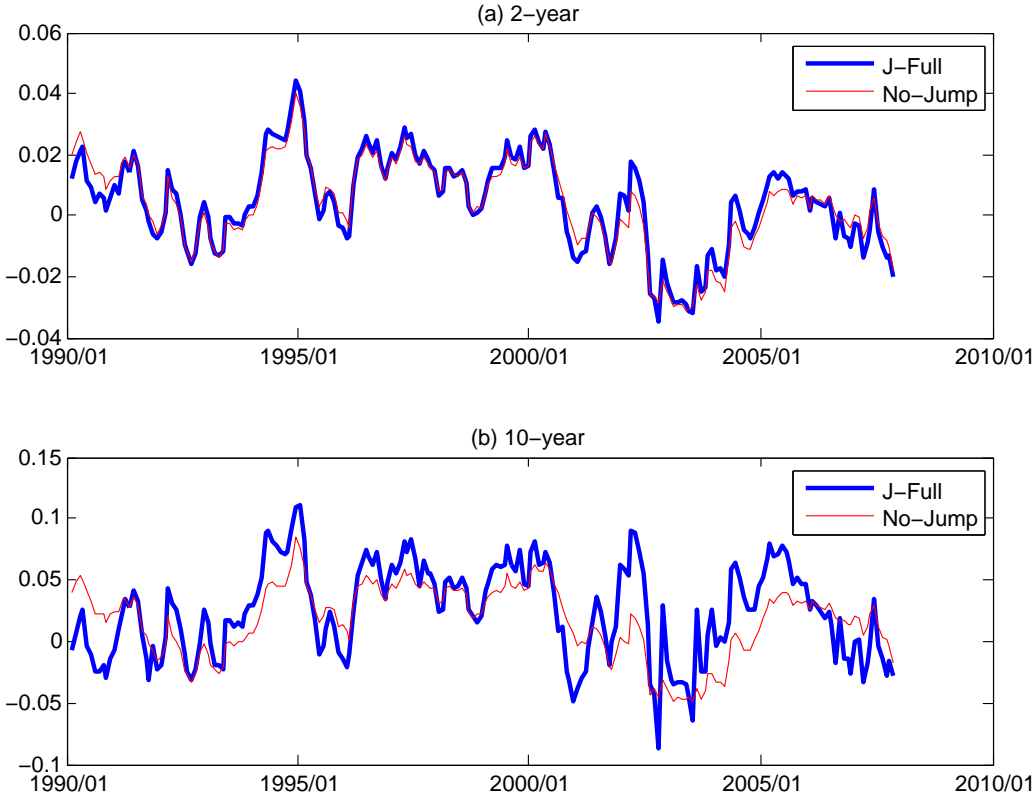
NOTE: This figure plots the monthly average of one-day expected excess returns from the J-Full model and the No-Jump model for two- and ten-year bonds. Units are annualized returns.

Figure 7: Decomposition of the one-month holding period bond risk premia into jump and diffusion components

(a) 2–year

(b) 10–year
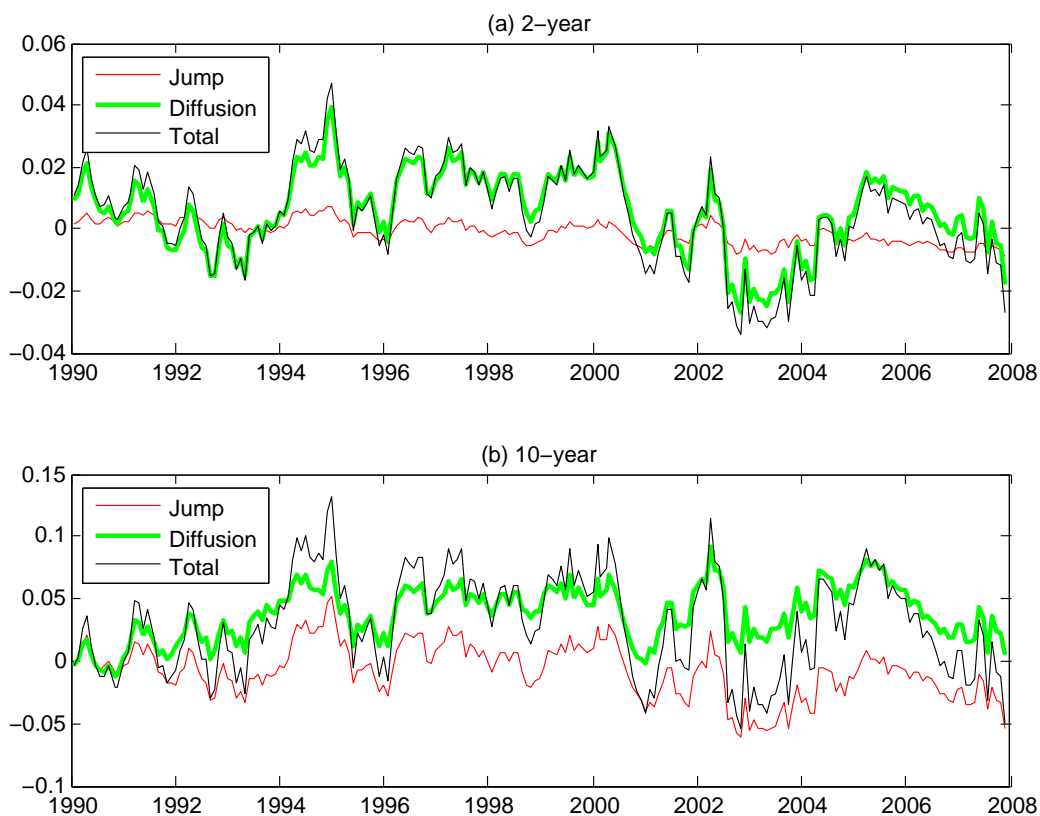
NOTE: This figure plots the decomposition of the one-month holding period bond risk premia into jump and diffusion components, based on the J-Full model, for two- and ten-year bonds. Units are annualized returns.

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