

$K_{L,S} \rightarrow \pi\pi\nu\bar{\nu}$ decays within and beyond the standard model

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The decays $K_{L,S} \rightarrow \pi^0 \pi^0 \nu\bar{\nu}$ and $K_{L,S} \rightarrow \pi^+ \pi^- \nu\bar{\nu}$ involve the weak transition from a strange to a non-strange quark. Although they have considerably smaller branching ratios than those for the corresponding rare processes involving single pions, some may be more distinguishable experimentally from background processes and thus could provide another probe of the $s \rightarrow d \nu\bar{\nu}$ transition. Using the recent knowledge of the CKM matrix elements and measurements of related processes, we give improved predictions for their branching ratios both within and beyond the standard model.

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I. INTRODUCTION

Historically, rare kaon decays have provided a crucial testing ground in which to study flavor-changing neutral current (FCNC) and CP -violating phenomena. Within the context of the standard model, the observation of such decays gives us further information on the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and provides independent checks on the consistency of our understanding drawn from other measurements, such as those of semileptonic decays and B -meson decays. They also serve as a good place to look for physics beyond the standard model. Even if new physics is first observed outside the kaon sector, we will want to know the footprint it leaves in rare kaon decays and specifically on the $s \rightarrow d$ weak transition.

In this paper we focus on the decays $K_{L,S} \rightarrow \pi^0 \pi^0 \nu\bar{\nu}$ and $K_{L,S} \rightarrow \pi^+ \pi^- \nu\bar{\nu}$. As noted previously [1,2], at the quark level these FCNC processes involve the $s \rightarrow d \nu\bar{\nu}$ transition. In the standard model, this transition is dominated by short-distance contributions involving loop diagrams that contain W and Z bosons and heavy quarks [3]. With the top quark mass much greater than that of the charm quark, the imaginary part of the amplitude for this transition arises almost entirely from loop diagrams with top quarks and the resulting amplitude is then proportional to the CKM factor $\text{Im}(V_{td}^* V_{ts})$.

The decays $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ and $K_L \rightarrow \pi^0 \nu\bar{\nu}$ are both governed by the same $s \rightarrow d \nu\bar{\nu}$ transition. The latter decay is CP -violating and dominated to high accuracy by the short-distance contribution. As noted above, its amplitude is consequently proportional to $\text{Im}(V_{td}^* V_{ts})$ in the standard model. For both decays, the hadronic matrix elements of the relevant weak current can be related to ones which enter charged current semileptonic decays, whose direct measurement bypasses any theoretical uncertainty in the hadronic matrix element as well. The branching ratios that are predicted in the standard model lie roughly between 10^{-11} and 10^{-10} . With negligible long-distance contributions and little hadronic uncertainty, these decays have been pointed to as crucial for

precision experimental tests of the standard model and correspondingly as places to look for new physics if the standard model fails [4,5]. Because of this, they are being pursued experimentally in spite of very difficult experimental backgrounds.

The rare decays $K_{L,S} \rightarrow \pi\pi\nu\bar{\nu}$ that we study in this paper have the same advantage of allowing a theoretical clean study of the $s \rightarrow d \nu\bar{\nu}$ transition, although some of the relevant semileptonic matrix elements are not as accurately measured experimentally. As shown previously [1,2] in the standard model, they unfortunately have the serious disadvantage that their predicted branching ratios are several orders of magnitude smaller than for the decays involving a single pion. It is nevertheless interesting to pursue them because they involve different combinations of the CP -conserving and CP -violating parts of the $s \rightarrow d \nu\bar{\nu}$ transition, and they thus provide additional handles on both the real and imaginary parts of amplitude. Furthermore, from an experimental point of view, some or all of these decays may prove to be more susceptible to the extraction of a signal from the background.

In this paper, we significantly refine the predictions for $K \rightarrow \pi\pi\nu\bar{\nu}$ decays in the standard model using the recent knowledge of the CKM matrix elements and measurements of decay rates for related processes. We also examine how large these branching ratios could be for physics beyond the standard model and find that there are significant, model-independent limits from other measurements. The paper is organized as follows: In Sec. II, we provide the general framework for studying the $s \rightarrow d \nu\bar{\nu}$ transition and the contributions to it both within and beyond the standard model. Section III sets forth its relationship to the $K \rightarrow \pi\pi\nu\bar{\nu}$ decays under discussion, the CP properties of the amplitudes involved in those decays, and the connection of the relevant hadronic matrix elements to measured semileptonic decays. Numerical results are given in Sec. IV, followed by some conclusions in Sec. V.

II. GENERAL FRAMEWORK

The effective Hamiltonian for $s \rightarrow d \nu\bar{\nu}$ transitions takes the form

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$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} W_{ds} [(\bar{s} \gamma_\mu (1 - \gamma_5) d) \times (\bar{\nu} \gamma^\mu (1 - \gamma_5) \nu)] + \text{H.c.}, \quad (1)$$

where the short-distance physics is lumped in W_{ds} . In the standard model, one-loop contributions to W_{ds} are dominated by penguin and box diagrams with intermediate charm and top quarks:

$$W_{ds}^{SM} = \lambda_{sd}^c X(x_c) + \lambda_{sd}^t X(x_t), \quad (2)$$

where $\lambda_{sd}^i \equiv V_{is}^* V_{id}$, with V_{ij} being the appropriate CKM matrix element, and $x_i = \bar{m}_i^2 / M_W^2$. The QCD corrections to the short-distance contributions $X(x_i)$ have been calculated some time ago in leading order [6,7] and then in next-to-leading order [8,9]. Since the top-quark mass is comparable to the weak scale, these corrections are very small for $X(x_t)$, as can be seen explicitly in the values given [8,9] for $X(x_t)$ when written as $X(x_t) = \eta_t X_0(x_t)$, with the QCD-uncorrected top quark contribution [3]

$$X_0(x_t) = \frac{x_t}{8} \left[\frac{x_t + 2}{x_t - 1} + \frac{3x_t - 6}{(x_t - 1)^2} \log(x_t) \right], \quad (3)$$

and the QCD correction factor $\eta_t = 0.994$. On the other hand, these corrections have considerable importance for $X(x_c)$.

The quantity $X(x_t)$ is roughly three orders of magnitude larger than $X(x_c)$, and since $\text{Im} \lambda_{sd}^c = -\text{Im} \lambda_{sd}^t$, the top contribution completely dominates in the imaginary part of W_{ds}^{SM} . However, $\text{Re} \lambda_{sd}^t \ll \text{Re} \lambda_{sd}^c$, allowing the charm contribution, although still smaller in magnitude than that from top, to be roughly comparable and to interfere constructively in the real part of W_{ds}^{SM} .

As illustrative examples of physics that lie beyond the standard model, we consider two very different possibilities:

(i) Effective flavor-changing neutral current (FCNC) interaction: Such an interaction, as formulated by Nir and Silverman [10,11], takes the form of an extra term in the effective Lagrangian of the form:

$$\mathcal{L}^{(Z)} = -\frac{g}{4 \cos \theta_W} U_{ds} \bar{d} \gamma_\mu (1 - \gamma_5) s Z^\mu. \quad (4)$$

When combined with the coupling of the Z boson to neutrino-antineutrino pairs, one finds that

$$W_{ds}^{NP} = \frac{\pi^2}{\sqrt{2} G_F M_W^2} U_{ds} = 0.93 \times 10^2 U_{ds} \quad (5)$$

as the new piece of W_{ds} in the effective Hamiltonian that corresponds to the basic process, $s \rightarrow d \nu \bar{\nu}$.

Upper bounds for W_{ds} have been determined by other processes involving K mesons and were summarized [12] recently to be

$$|\text{Re}(U_{ds})| \leq 10^{-5}, \quad (6)$$

$$|U_{ds}| \leq 3 \times 10^{-5}, \quad (7)$$

$$|\text{Re}(U_{ds}) \text{Im}(U_{ds})| \leq 1.3 \times 10^{-9}, \quad (8)$$

$$|\text{Im}(U_{ds})| \leq 10^{-5}. \quad (9)$$

The bound on $|U_{ds}|$ arises from the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, whose width is proportional to $|W_{ds}|^2$. It can be improved by using the most recent measurement [13], of the branching ratio, $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.5_{-1.2}^{+3.4} \times 10^{-10}$. This value is consistent with what is expected in the standard model and corresponds to $|W_{ds}| = 0.98_{-0.54}^{+0.80} \times 10^{-3}$. If we were to assume that the total branching ratio were due to new physics arising from U_{ds} , then the bound on $|U_{ds}|$ would be reduced from that in Eq. (7) to $|U_{ds}| < 1.6 \times 10^{-5}$.

(ii) Supersymmetry: A dominant supersymmetric effect arises from penguin diagrams involving charged-Higgs plus top-quark intermediate states or squark and chargino intermediate states. These give additional pieces to the effective Hamiltonian of the form [14]

$$W_{ds}^{NP} = \lambda_{sd}^t \frac{m_H^2}{M_W^2 \tan^2 \beta} H(x_{tH}) + \frac{1}{96} \tilde{\lambda}_t, \quad (10)$$

where $\tan \beta$ is the ratio of the two Higgs vacuum expectation values and $x_{tH} = m_t^2 / M_{H^\pm}^2$. The quantity $H(x)$ is given by

$$H(x) = \frac{x^2}{8} \left[-\frac{\log x}{(x-1)^2} + \frac{1}{x-1} \right]. \quad (11)$$

The parameter $\tilde{\lambda}_t$ can be bounded by similar considerations to those that were used for U_{ds} . The observed branching ratios for the decays $K_L \rightarrow \mu^+ \mu^-$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ have been used to set the limits [14]

$$|\text{Re} \tilde{\lambda}_t| \leq 0.21, \quad (12)$$

$$|\tilde{\lambda}_t| \leq 0.35. \quad (13)$$

The most recent branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ could be used to revise the last limit to $|\tilde{\lambda}_t| < 0.16$.

As we will see shortly, the limitations imposed by experiments on the parameters of both these examples of physics beyond the standard model lead to similar restrictions on how large the branching ratios can be for the processes we are studying.

III. $K \rightarrow \pi \pi \nu \bar{\nu}$ DECAYS

When the effective four-fermion operator relevant for the decay we are considering is sandwiched between the initial and final states, it factorizes into a product of matrix elements of the hadronic current and the leptonic current. We will use isotopic spin to relate the hadronic matrix elements relevant to $K_{L,S} \rightarrow \pi \pi \nu \bar{\nu}$ to those for $K^+ \rightarrow \pi \pi e^+ \nu$, where the corresponding branching ratios (and hence squares of matrix elements) have been measured.

We consider first the process $K_L \rightarrow \pi^0 \pi^0 \nu \bar{\nu}$. The $\pi^0 \pi^0$ pair forms a CP -even state, and has total isospin, $I=0$. The $\nu \bar{\nu}$ pair, created by a virtual Z^0 , is CP even as well. Since

there must be one unit of orbital angular momentum to allow the total angular momentum of the final state to be that of the initial K_L , namely zero, the final state is CP -odd. The overall decay process is then CP -conserving for the major (CP -odd) piece of the K_L , and the resulting amplitude is proportional to the real part of W_{ds} . The opposite result holds for the piece of the K_L that is proportional to ϵ and CP -even; the corresponding decay process is CP -violating and the amplitude is proportional to the imaginary part of W_{ds} . It is thus suppressed on two counts and is completely negligible.

Using the relationship,

$$\langle \pi^0 \pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle = \langle \pi^0 \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle, \quad (14)$$

we find that

$$BR(K_L \rightarrow \pi^0 \pi^0 \nu\bar{\nu}) = \frac{3\alpha^2 |\text{Re } W_{ds}|^2}{2\pi^2 \sin^4 \theta_W |V_{us}|^2} \frac{\tau_{K_L}}{\tau_{K^+}} BR(K^+ \rightarrow \pi^0 \pi^0 e^+ \nu), \quad (15)$$

where the factor of 3 accounts for the three species of neutrinos.

By relating the desired branching ratio to a measured one, we have avoided having either to do a calculation of the hadronic matrix elements or to perform a detailed analysis in terms of invariant amplitudes, as was done in previous analyses [1,2]. Of course, the final results for the branching ratio must be consistent, since both approaches agree with the available data on charged-current semileptonic decays, and in particular those for the decay rate for the process $K^+ \rightarrow \pi^0 \pi^0 e^+ \nu$. For the purposes of this paper of discussing the absolute and relative size of the various branching ratios within and beyond the standard model, it is considerably easier to formulate the results directly in terms of relationships to branching ratios for measured semileptonic decays.

A similar formula can be obtained for $BR(K_S \rightarrow \pi^0 \pi^0 \nu\bar{\nu})$, but with $\text{Re } W_{ds}$ replaced by $\text{Im } W_{ds}$ and τ_{K_L} replaced by τ_{K_S} . Since the K_S has a much shorter lifetime and the major part of the K_S corresponds to a transition that is CP violating, this branching ratio is orders of magnitude smaller than that for $K_L \rightarrow \pi^0 \pi^0 \nu\bar{\nu}$. Although the part of the K_S state proportional to ϵ corresponds to a CP -conserving transition, the smallness of ϵ still gives rise to a net decay amplitude that is much smaller than that for the CP -even part of the K_S .

The analysis for the decay $K_L \rightarrow \pi^+ \pi^- \nu\bar{\nu}$ can be carried out analogously. It is convenient to break it up into the cases where the $\pi^+ \pi^-$ pair in the final state has total isospin zero and one, since there is no interference between them in the decay rate. For the isospin zero case, the argument about the CP properties of the final state is the same as given before, and we simply have a factor of two in the rate for the $\pi^+ \pi^-$ final state compared to that for the $\pi^0 \pi^0$ final state discussed above:

$$BR[K_L \rightarrow (\pi^+ \pi^-)_{I=0} \nu\bar{\nu}] = \frac{3\alpha^2 |\text{Re } W_{ds}|^2}{\pi^2 \sin^4 \theta_W |V_{us}|^2} \frac{\tau_{K_L}}{\tau_{K^+}} BR(K^+ \rightarrow \pi^0 \pi^0 e^+ \nu). \quad (16)$$

This situation is slightly more complicated for the case where the $\pi^+ \pi^-$ pair has isospin, $I=1$. The $\pi\pi$ pair is still CP -even, but it must be in a p -wave. There are two possible ways in which the total angular momentum of the final state can be zero, which correspond to the relative orbital angular momentum of the $\pi\pi$ and $\nu\bar{\nu}$ pairs being zero or one. As is expected when there is such limited phase space, the latter amplitude is strongly suppressed by centrifugal barrier effects compared to the former [2]. So we are left with a single amplitude where the relative orbital angular momentum is zero. The CP of the final state is even and the transition involving the major part of the K_L is CP -violating. Using the relationship,

$$\sqrt{2} \langle (\pi^+ \pi^-)_{I=1} | (\bar{s}d)_{V-A} | K^0 \rangle = \langle (\pi^- \pi^0)_{I=1} | (\bar{s}u)_{V-A} | K^0 \rangle, \quad (17)$$

we find that

$$BR[K_L \rightarrow (\pi^+ \pi^-)_{I=1} \nu\bar{\nu}] = \frac{3\alpha^2 |\text{Im } W_{ds}|^2}{4\pi^2 \sin^4 \theta_W |V_{us}|^2} BR(K_L \rightarrow \pi^+ \pi^- \nu\bar{\nu}). \quad (18)$$

The total branching ratio for $K_L \rightarrow \pi^+ \pi^- \nu\bar{\nu}$ is then found by simply adding the two results above: $BR(K_L \rightarrow \pi^+ \pi^- \nu\bar{\nu}) = BR[K_L \rightarrow (\pi^+ \pi^-)_{I=0} \nu\bar{\nu}] + BR[K_L \rightarrow (\pi^+ \pi^-)_{I=1} \nu\bar{\nu}]$.

The corresponding formula for $K_S \rightarrow \pi^+ \pi^- \nu\bar{\nu}$ can again be obtained by the interchange of $\text{Re } W_{sd}$ and $\text{Im } W_{sd}$ and multiplication of the right-hand side by τ_{K_S}/τ_{K_L} .

IV. NUMERICAL CALCULATION

To obtain numerical predictions we have used a set of parameters taken from the Particle Data Group [15], including the fine-structure constant at the weak scale, $\alpha=1/129$; $M_W=80.3$ GeV; $\sin^2 \theta_W=0.23$; and the measured semileptonic branching ratios needed in Eqs. (15), (16) and (18). We correspondingly find that

$$\begin{aligned} BR(K_L \rightarrow \pi^0 \pi^0 \nu\bar{\nu}) &= [(3.1 \pm 0.6) \times 10^{-7}] |\text{Re } W_{ds}|^2, \\ BR[K_L \rightarrow (\pi^+ \pi^-)_{I=0} \nu\bar{\nu}] &= [(6.2 \pm 1.2) \times 10^{-7}] |\text{Re } W_{ds}|^2, \\ BR[K_L \rightarrow (\pi^+ \pi^-)_{I=1} \nu\bar{\nu}] &= [(0.93 \pm 0.05) \times 10^{-7}] \\ &\quad \times |\text{Im } W_{ds}|^2, \end{aligned} \quad (19)$$

where the error bars come from those of the experimental measurements of the relevant semileptonic branching ratios. We have not taken into account radiative corrections or isotopic-spin violating differences in form factors and phase space, as has been done for the case of the decays involving a single pion [16], given the (larger) uncertainties in other

TABLE I. Branching ratios of various $K_{L,S} \rightarrow \pi \pi \nu \bar{\nu}$ modes within and beyond the standard model.

| Processes | Br($\times 10^{-13}$) | |
|---|--------------------------------|-----------------------|
| | SM | New physics (maximal) |
| $K_L \rightarrow \pi^0 \pi^0 \nu \bar{\nu}$ | 1.4 ± 0.4 | 10 |
| $K_S \rightarrow \pi^0 \pi^0 \nu \bar{\nu}$ | $(1.9 \pm 0.8) \times 10^{-4}$ | 8×10^{-3} |
| $K_L \rightarrow \pi^+ \pi^- \nu \bar{\nu}$ | 2.8 ± 0.8 | 20 |
| $K_S \rightarrow \pi^+ \pi^- \nu \bar{\nu}$ | $(11 \pm 2) \times 10^{-4}$ | 2×10^{-2} |

parts of input at this stage of the analysis of these decays. Formulas similar to Eq. (19) hold for the related decays of the K_S to the same final states.

For the specific calculation of W_{ds} in the standard model we need the values of $X(x_t)$ and of $X(x_c)$ in next-to-leading order [8,9], and that of the CKM matrix elements [17]

$$\begin{aligned} \text{Re } V_{td} &= 0.0076 \pm 0.0015, \\ \text{Im } V_{td} &= 0.0031 \pm 0.0008, \end{aligned} \quad (20)$$

and $V_{ts} = -V_{cb} = -0.040 \pm 0.002$, aside from the well-known matrix elements connecting the first and second generations. Using this and with $\bar{m}_t = 166 \pm 5$ GeV, we find that

$$W_{ds}^{SM} = [(-6.7 \pm 1.0) + i(1.9 \pm 0.5)] \times 10^{-4}, \quad (21)$$

and the branching ratios for the various processes shown in Table I.

In K_L decays the contribution of the isospin one $\pi^+ \pi^-$ final state is negligible, since it is already suppressed compared to that with isospin zero from Eq. (19) and the magnitude of the real part of W_{ds}^{SM} is considerably greater than that of the imaginary part. Thus the ratio between $\pi^+ \pi^-$ and $\pi^0 \pi^0$ rates is very close to the factor of two characteristic of isospin zero.

Our results are given in Table I, and both K_L branching ratios lie between 10^{-13} and 10^{-12} . Our predictions in the standard model for $BR(K_L \rightarrow \pi^0 \pi^0 \nu \bar{\nu})$ are consistent with the previous calculation [2] of $1 - 3 \times 10^{-13}$ and those [1,2] of $1.1 - 5 \times 10^{-13}$ and $2 - 5 \times 10^{-13}$ for $BR(K_L \rightarrow \pi^+ \pi^- \nu \bar{\nu})$, but the allowed range is now considerably restricted. The K_S branching ratios are in the 10^{-17} range in the standard model. The decay $K_S \rightarrow \pi^+ \pi^- \nu \bar{\nu}$ gets important contributions from both $I=0$ and $I=1$ $\pi\pi$ final states since the suppression of the $I=1$ final state is compensated by the ratio of $(\text{Re } W_{ds} / \text{Im } W_{ds})^2$ in the standard model.

For the representative examples of physics beyond the standard model, we also show in Table I values for the branching ratios that correspond to the maximal values one could obtain consistent with the bounds in Eqs. (6)–(9) and (12) and (13), respectively. These maximal values are similar in both cases and arise when the parameters of the new physics are chosen to maximize W_{ds}^{NP} consistent with the constraints coming from known K physics. Among these constraints is the recent branching ratio measurement for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, which is equally sensitive to both the real and imaginary parts of the total W_{ds} with minimal theoretical assumptions. Hence, similar maximal values of W_{ds}^{NP} are obtained in any model of new physics. Note that when taking ratios to the standard model branching ratios, a much bigger factor is possible when the new physics enters the imaginary part of W_{sd}^{NP} , and is CP -violating, since the imaginary part of W_{sd}^{SM} is considerably smaller than the real part.

V. SUMMARY

We have used recent information on the CKM matrix to narrow the range of the predicted branching ratios for $K_{L,S} \rightarrow \pi^0 \pi^0 \nu \bar{\nu}$ and $K_{L,S} \rightarrow \pi^+ \pi^- \nu \bar{\nu}$ decays in the standard model. These branching ratios are in the neighborhood of $1 - 4 \times 10^{-13}$ for the K_L decays, which make them possibly observable at the few event level in the next round of experiments that are setting out to see the CP -violating decays with a single π^0 in the final state. These branching ratios could be larger by up to about an order of magnitude in theories that go beyond the standard model.

The branching ratios for the K_S decays are in the neighborhood of $10^{-17} - 10^{-16}$ in the standard model and seem unlikely to ever be observed. Here new physics could boost the branching ratios by more than an order of magnitude, although even then the maximum branching ratio of around 10^{-15} is still beyond the limits of observation. For both K_S and K_L decays, increased experimental accuracy in the measurement of the branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, assuming it remains consistent with the standard model, will put more stringent restrictions on non-standard-model physics in the $s \rightarrow d \nu \bar{\nu}$ transition and limit the deviations from the standard model that can be observed in the decays under discussion here as well.

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