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# K-means Clustering to Study How Student Reasoning Lines Can Be Modified by a Learning Activity Based on Feynman's Unifying Approach

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**ABSTRACT**
Research in Science Education has shown that often students need to learn how to identify differences and similarities between descriptive and explicative models.
The development and use of explicative skills in the field of thermal science has always been a difficult objective to reach. A way to develop analogical reasoning is to use in Science Education unifying conceptual frameworks.
In this paper we describe a 20-hour workshop focused on Feynman's Unifying Approach and the two-level system. We measure its efficacy in helping undergraduate chemical engineering students explain phenomena by applying an explanatory model. Contexts involve systems for which a process is activated by thermally overcoming a well-defined potential barrier. A questionnaire containing six open-ended questions was administered to the students before instruction. A second one, similar but focused on different physical content was administered after instruction. Responses were analysed using k-means Cluster Analysis and students' inferred lines of reasoning about the analysed phenomena were studied. We conclude that students reasoning lines seem to have clearly evolved to explicative ones and it is reasonable to think that the Feynman Unifying Approach has favoured this change.

**Keywords:** Boltzmann Factor, evaluation, quantitative data analysis in education, k-means clustering, thermally-activated phenomena

## INTRODUCTION

Students' explanatory skill development and use is a relevant aim of university programs, for a meaningful understanding of science as well as for the development of professional competencies. In particular, the development and use of these skills in the field of thermal science has always been a difficult objective to reach, because of the difficulties in understanding the everyday experiences governed by the intrinsic properties of matter,

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## State of the literature

- Research in Science Education has shown that often students need to learn how to identify differences and similarities between descriptive and explicative models, as well as the way these are related to understanding reality and perceiving science.
- In particular, the development and use of explicative skills in the field of thermal science has always been a difficult objective to reach, because of the difficulties in understanding everyday experiences governed by intrinsic properties of matter faced by students at every level of education. In the construction of explicative models inductive reasoning is involved, but an important role is also played by analogical reasoning.
- A way to develop analogical reasoning is to use in Science Education unifying conceptual frameworks for the description and interpretation of natural phenomena concerning fields of science to be considered only apparently as different, like the one that was first proposed by Feynman in his famous Lectures book.
- Several methods of analysis of data coming from student surveys are today available to the researcher in Science Education. In the last years a quantitative analysis method (Cluster Analysis) has proven useful to highlight common patterns in student responses to a questionnaire items and to allow the researchers to infer the student lines of reasoning related to the creation and use of explanations.

## Contribution of this paper to the literature

- A study aimed at understanding how a unifying framework to the description and interpretation of natural phenomena can actually modify undergraduate student lines of reasoning, and help them to develop explicative skills was designed and experimented during a 20-hour workshop. The learning activities engaged the students in dealing with situations and experiments related to different phenomena related to a common conceptual framework. The students were asked to make sense of them by means of group work and laboratory and modelling activities.
- Data analysis was done by means of a well-known Clustering Analysis method, the so-called k-means one that clearly highlighted the typical lines of reasoning deployed by the students both before and after instruction. This was done by finding well distinct clusters of students that show different behaviour in tackling the pre- and post-instruction questionnaires. Each cluster is characterized by a "virtual student" that resumes the answers most frequently given by the cluster students.
- The pre-instruction results show that the students reasoning lines were mainly oriented to the use of lines of reasoning based on the use of memory of past studies and on an application of mathematics without a search for a proper mechanism of functioning. After instruction, these lines of reasoning seem to have clearly evolved to explicative ones. It is reasonable to think that a unifying approach to science has favoured this change, probably due to the analysis of different situation that can be modelled by means of a common conceptual framework.

faced by students at every level of education (Jasien and Oberem, 2002; Streveler et al, 2003; Streveler et al, 2008).

Research has shown the relevance of characterizing the mental models (Johnson-Laird, 1983; Johnson-Laird, 2006; Greca and Moreira, 2000) students use when asked to create or use explanations, and their dependence on the context. It was shown (Bao and Redish, 2006; Clough and Driver, 1986; Maloney and Siegler, 1993) that students are often inconsistent in their use of mental models in situations that an expert would consider equivalent.

In a previous research (Fazio et al, 2013), the lines of reasoning[1] applied by a group of 34 undergraduate students in the second semester of their freshman year of the Undergraduate Program in Chemical Engineering at the University of Palermo, Italy, during the Academic Year 2010-2011, were analysed, by means of a specially designed and validated questionnaire and by interviews taken with some voluntary students. It was found that 30 out of 34 students showed lines of reasoning mainly rooted in common-sense (everyday-like reasoning) or focused on the mere application of previously learnt facts and mathematical formulas (descriptive-like reasoning). Only 4 students tackled the analysis of phenomena by applying explicative lines of reasoning at acceptable significance levels.

Research also highlighted the need for the students to better identify differences and similarities between descriptive and explicative procedures and models, as well as the way these are related to understanding reality and perceiving science (Sperandeo-Mineo et al, 2006; Duit et al, 2005; Fazio et al, 2008; Fazio et al, 2007; Tarantino et al, 2010). As it is well known, in the process of construction of explicative models inductive reasoning is involved, but an important role is also played by analogical reasoning (Duit et al, 1996). This involves the ability to see similarities and differences between a "source" (something perceived as similar to what we are going to analyse) and the "target" (the real phenomena that we are studying), and to generalize ideas and concepts already developed in a given context to different ones. This point is particularly relevant for science education, which aims to develop generalization skills in students by supplying them with unifying frameworks for the description and interpretation of natural phenomena to be considered only apparently as different.

It would be interesting to understand how a unifying framework to the description and interpretation of natural phenomena can actually modify the student lines of reasoning, and help them to develop explicative skills. For this reason, during Academic Year 2013-14 it was designed and developed a study involving a sample of students attending the same Undergraduate Program of the ones that participated to our previous study (Fazio et al, 2013).

Particularly, we want to evaluate the effects on student reasoning lines of a workshop based on the well know Feynman's unifying approach (FUA) to different phenomena (Feynman, Leighton & Sands, 1963). According to this approach, phenomena traditionally considered as different can be described and explained by using a same conceptual framework, i.e. the idea of two-level system and the mathematical description resumed by Boltzmann Factor. Following this, we designed a workshop in which students deal with different phenomena related to this conceptual framework and must make sense of them by means of various laboratory and modelling activities.

Building on Duit's arguments (1996) and on the strong link he points out between analogical reasoning and a constructivist approach to knowledge, we want also to highlight the link between FUA, that is from our point of view a particular analogical reasoning, and the constructivist approach that students should deploy in order to deal with a problem on the basis of explicative lines of reasoning.

In order to collect data and evidences to analyse, a questionnaire containing six open-ended questions on thermally activated phenomena was administered to the students before instruction. A second one, similar but focused on different physical content was administered after the workshop.

A quantitative analysis of the answers to the questionnaires was done by using k-means method (Everitt, 2011; Battaglia and Di Paola, 2015; Battaglia et al, 2016). This method is aimed at evidencing common patterns in the student responses to the questions and at allowing the researchers to infer the student lines of reasoning related to the creation and use of explanations.

In the following sections we provide a background of the general framework at the basis of our proposed learning environment, and present the research question addressed in this paper, the analysis methods used, and a detailed description of our project. The procedure followed to collect and analyse the data and the results is described and discussed in the following sections. Final comments about the implications of our results for the physics education of undergraduate students and suggestions for further developments are provided at the end of the paper.

## THEORETICAL FRAMEWORK

### Analogic reasoning and constructivist approach

In this paper we focus on some relevant aspects useful to explain the behaviour of students in the modelling workshop, based on Feynman’s Unifying Approach. As we said above, we consider this approach as a form of analogical thinking.

Empirical studies on analogical reasoning have been carried out both in Psychology and Math and Science Education. The role of analogies in the learning processes has been analysed from different theoretical perspectives. To clarify what we mean with the term 'analogy' we can say that it is a process of identifying similarities (and differences) between two or more concepts or different meanings of the same concept (Day et al., 2010).

According to Duit (1991, p. 666) an analogy is "a statement of comparison on the basis of similarities between the structures of two or more domains".

Many researchers have also provided different perspectives on the functions of analogies (Chiu and Lin, 2005; Glynn, 1989; Aubusson et al., 2006; Aubusson et al., 2009) in learning Math and Science. Gentner & Gentner (1983) and Black & Solomon (1987) investigated the successful students' use of analogies to make sense of electric current. Ugur et al. (2012) studied the effects of analogy on students' understanding of direct current circuits and their implication in Science Education.

There are also several studies suggesting that analogies often only work in particular learning settings, structured and defined in order to favour the development of critical thinking and possible student free argumentation (Gilbert, 1989; Nageri, 1980).

Shapiro (1985) interpreted the successful use of an analogical approach by students, discussing how analogies help them to modify their existing cognitive structure and, in some case to correct them on the basis of experience. This kind of approach can be actually considered as a constructivist based one.

According to Duit (1991, p. 666), "Although analogical reasoning appears to be quite common both in daily life and in other contexts, spontaneous use of analogies provided by teachers or learning media seldom happens. Analogical reasoning in learning situations requires considerable guidance".

There are many examples of possible teaching models centred on the use of analogy in the literature: Brown's and Clement's (1989) bridging-analogies, Dupin's and Johsua's (1989) analogy teaching model, Glynn's (1989) Teaching-With-Analogies (TWA) model, and Zeitoun's (1984) general model of analogy teaching (GMAT).

According to Brown and Clement (1989) and Dupin and Joshua (1989), teachers should favour student analogical thinking in order to help them to compare similarities and detect differences, not only between reality and more formal concept studied at school, but also to create a bridge, a very strong one, between complex concepts, often too abstract for them.

According to Duit (1991), the advantages of analogies are due to their significance within a constructivist learning framework. In other words, analogical reasoning can be important because it represents one of the key skills for the development of a constructivist approach. In particular, Duit highlights that these advantages include the understanding of abstract concepts. Moreover, according to Duit to stimulate the students to apply analogical reasoning can make evident "everyday/practical" reasoning lines that should be redirected towards more effective explicative ones.

## The Boltzmann Factor

The Boltzman Factor (BF) is an example of a unifying expression that describes many physics, chemistry and biology systems. It is useful for describing the behaviour of natural systems that exchange energy with their environment. A simple interpretation of BF can be given by considering a system of particles in thermodynamic equilibrium at temperature $T$, which can exist in two different states characterized by an energy difference $\Delta E$. In this case, the Boltzmann factor is the weighting factor giving the fraction of particles that stay in the higher energy state (Sturge and Toh, 1999). So, the BF can be considered useful to join the microscopic mechanical world with the macroscopic thermodynamic world, by connecting the particle system energy with the environmental temperature.

It was inspired by the well-known Feynman's Unifying Approach to phenomena traditionally considered as different (Feynman et al, 1963) and is aimed at making clear to university and secondary school students the role played by the BF in describing and explaining phenomena involving systems "characterized by having to borrow energy from somewhere (Prentis et al, 1999)", through the unifying mechanism of functioning described by a two-level system. In all these systems we can focus on physical quantities that can be directly expressed in terms of BF, referring to the population ratio $^{n_2}/_{n_1}$ between particles distributed in the two energy levels:

$$\frac{n_2}{n_1} = c \cdot e^{-\frac{\Delta E}{kT}}$$

The Boltzmann Factor, $e^{-\frac{\Delta E}{kT}}$, describes and explains the behaviour of natural systems (physical, chemical, or biological) at thermal equilibrium. It links the microscopic mechanical world with the macroscopic thermodynamical world by connecting the energy of the system molecules with the temperature of the environment.

Books for undergraduate students use different approaches and arguments for its derivation: most of them, following Feynman (Feynman et al, 1963), heuristically justify it by referring to the "exponential atmosphere", others analyse the quasi-continuous states of the heat bath (Feynman, 1974; Reif, 1965) or use the method of the most probable distribution (Schrödinger, 1967).

As it was already highlighted by Duit, the advantages of analogies are due to their significance within a constructivist framework, and in this sense FUA can be considered as a way to implement a constructivist pedagogical environment, suitable to improve the explicative skills.

As it is well known (Hestenes, 1987; Wells et al, 1995; Hestenes, 1992; Salmon et al, 1990; Ericsson et al, 1998; Hestenes, 2006), in Physics an explicative model is different from a descriptive one because it supposes a system has properties which are not directly observable, but play a role in the observed regularities. Indeed, the model construction and validation process requires the building of several hypothesis typologies: empirical law hypothesis, synthesis of regularities (arising from phenomenological observations and condensed into rules), and hypothesis for the construction of explicative models introducing theoretical representations and often containing non observable entities.

In the construction process of explicative models, inductive reasoning is involved, but an important role is also played by analogical reasoning (Duit et al., 1996), i.e., the ability to see similarities and differences between a ''source'' (something perceived as similar to what we are going to analyse) and the ''target'' (the real phenomena we are studying).

## PEDAGOGICAL EXPERIMENT AND RESEARCH QUESTION

Based on the ideas discussed above, we chose to focus our workshops on the physics underlying the complex world of thermally activated phenomena, because it offers a good opportunity to understand and use unifying frameworks for the description and explanation of different (apparently) natural phenomena. In particular, we focused on physics and chemistry systems that can exist in two different states characterized by an energy difference $\Delta E$ (Boltzmann, 1909; Boltzmann, 1909) where the state transition is thermally activated by overcoming the potential barrier $\Delta E$. They are described by a unifying expression containing the Boltzmann factor, $e^{-\frac{\Delta E}{kT}}$, where T is the system temperature and k is the Boltzmann constant.

### Context and sample

Our research sample consists of 37 freshmen attending the Undergraduate Program in Chemical Engineering during the Academic Year 2013/2014 at the University of Palermo (UniPA), Italy. Many of them attended secondary schools where physics is usually taught by following a traditional, teacher-centred approach, and where physics teaching is mainly based on the transmission of general concepts to students. In some cases, the lessons are integrated with laboratory activities, but these are often performed by the teachers themselves, who follow a confirmatory/demonstrative approach.

During the 1st semester of their Degree Program the students attended general mathematics, physics and inorganic chemistry courses, and passed the exams. When selected to participate in our study, they were attending a 2nd semester Physics course dealing with the fundamentals of electromagnetism.

### Methodology

The reasoning deployed by the students when asked to explain phenomena, and relate them to the physics and chemistry they had already studied in previous courses, was analysed before instruction by using a specially designed and previously validated (Fazio et al, 2013) questionnaire. In the questionnaire, students were asked: 1) to discuss a real life situation (the evaporation of a water puddle at different environmental temperatures); 2) to describe the physical quantities contained in Arrhenius' Law; 3) to clarify the role of a catalyst in a chemical reaction; 4) to give a microscopic interpretation of the Arrhenius' Law; to show generalization skills by finding other natural phenomena that exhibit temperature dependencies similar to the one highlighted by the chemical reaction speed (question 5), and evidencing the similarities among these phenomena, particularly with respect to common physical quantities characterizing all the described systems (question 6). The questionnaire can be found in Appendix A.

The students then took a 20-hour workshop based on Feynman's Unifying Approach. The workshop dealt with a physical content different from the one addressed by the questionnaire, but strictly related to the framework of thermally activated phenomena. At the end of the workshop, a new questionnaire, validated by following a procedure similar to the one used for the pre-instruction questionnaire, and again focused on the study of student lines of reasoning about the use of descriptions/explanations in science, was administered to the students. This questionnaire was similar to the pre-instruction one, but was focused on physical/chemical contents (fluidity) not explicitly discussed before and/or in the workshop (see Appendix B for details).

## Workshop description

The workshop was conducted by one of the authors (as the instructor)[2] and involved the activities described below, which were identified as helpful to students for identifying questions, collecting evidence and data, building models and developing generalization skills. The content dealt with the study of electric current in vacuum systems (thermionic tubes). In particular, situations where the Boltzmann Factor (BF), $e^{-\frac{\Delta E}{kT}}$, can be used to describe electric conduction were analysed.

A full understanding of the physical meaning of the BF goes beyond the mathematical derivations typically dealt with in undergraduate statistical mechanics courses, and it often requires a link to concrete situations. It is easy to find teaching methods aimed at understanding the BF, as many have been published up to now (Feynman et al, 1963; Horne et al, 1973; Prentis, 2000). It was presented (Battaglia et al, 2009) a pedagogical approach to BF based on the comparison between the experimental data and results of a simulation based on a simple mechanical model of a two-level system. Other experiments dealing with chemical kinetics, electrodynamics and fluid dynamics have been presented (Battaglia et al, 2010; Fazio et al, 2012), in order to extend the basis of experimental data on physical situations that can be modelled by using the unifying approach of the BF.

The workshop attended by the students, focused on the Feynman Unifying Approach, was organized in a series of sessions for a total of 20 hours, during which the students had often to ask their questions and search for sources of information to obtain a solution, in some cases even proposing and conducting possible experiments and simulations. Sharing and contrasting the obtained results in great-group discussions was also a requested activity. The students had already studied electric conduction during the regular lessons of the electrodynamics course. During these lessons they also performed voltamperometric experiments on ohmic conductors, like metals, and also studied the resistivity, $\rho$, vs. temperature, $T$, relationship in these materials. Different interactions between the charge carrier and the lattice were discussed in order to make sense of the linear dependence of $\rho$ on

$T$ in metals. Typical models of electrical conduction in solids, like the Drude and the Sommerfeld's (Griffiths, 1998), were also presented during the regular course, and their predictions were compared with the experimental evidence.

The workshop was divided into six phases. These phases are described in some detail below.

*Phase 1 – Definition of the workshop activity's aim*

In this phase, after the first administration of the questionnaire, the instructor presented the project to the students, providing a brief description of the context in which their work would take place and the reasons why they should participate actively. Particularly, it was highlighted that this activity is dedicated to the study of thermal activated phenomena when there is a phase transition. It is also explicated that they have already studied these phenomena but now we want to analyse those from another point of view. During the first two hours of the workshop, conduction in ohmic conductors was recalled and discussed with students. Students were asked to think about other materials that can show behaviour different from the Ohm law and to the $\rho$ vs. $T$ relationship and to search for evidence of this in physics handbooks of their choice and other resources freely available on the internet.

*Phase 2 – Planning of the workshop activity*

During the second phase of the workshop, which was three hours long, students acquired information and planned their activities in small groups, trying to pose questions they would answer during the following activities. Students were told that they could use all the campus libraries and internet resources to gather appropriate literature, if needed. They founded two type of devices which show a non-ohmic behaviour: semiconductor and vacuum tube. Since that electron dynamics in semiconductors was a difficult subject to deal with in depth, students chose to address in detail the electrical conduction process in vacuum tubes, which is easier to discuss. Furthermore, conduction in vacuum tubes is in some ways analogous to the subjects dealt with in the initial questionnaire, i.e. the Arrhenius law in chemical kinetics and the related concept of the activation energy of a chemical reaction (See Appendix A for more detail about the questionnaire). In order to better understand vacuum tube, the instructor introduced the Richardson Law that is at the basis of Thermionic Emission in the vacuum tube. It is worth noting here that Arrhenius law was studied by students during their previous chemistry course, which they attended during the first semester of their freshman year. This study, however, was developed mainly from a macroscopic point of view, without many references to a "functioning mechanism" to explain it[3].

*Phase 3 –Measurement activity*

In the third phase of the workshop, the students carried out their research investigations, designed on the basis of the hypotheses and questions they had formulated during the explorative phase. All groups of students were invited to carry out their own experimental work, by taking into account the physics behind the process of electric conduction in the chosen systems.

The instructor asked them to organize their work in advance and to write down the details of all the experiments that they were planning to carry out. In this phase, students were introduced to the laboratory and were encouraged to explore the measurement facilities and materials available, in order to understand how to design and perform their own experiences.

They dedicated six hours to completing their laboratory activities, collecting and doing a preliminary processing of data. The different groups discussed the possible measurements to be taken in vacuum tube diodes and shared their results. The instructor encouraged the students to find other physical quantity similar to that already met during the questionnaire (for example pressure in the evaporation or velocity reaction). As a result of the discussion, the whole group decided, with reference to similar measurements presented in the manuals, to study the anodic current vs. the filament temperature, which can give information about the values of concentration of electrons emerging from the filament.

*Phase 4 –Modelling and simulation activity*

In this phase, which was three hours long, students are encouraged to build a model of the phenomena previously studied. After a group discussion, where similarities and differences between the functioning mechanisms of the various phenomena were discussed, the instructor encouraged students to focus on the idea of a "two-level" system.

In order to better study the "two-level" system, the end of this phase was devoted to the analysis of a dynamic computer model (Battaglia et al, 2009) related to the subject, built by using the NetLogo[5] simulation environment, which can easily simulate the interactions between a large number of elements. The instructor shows how it could be possible to build a simulation of a "two-level" mechanical system by using NetLogo. Students discussed the following simulated mechanical model of a two-level system with the instructor:

"A large number of balls free to move in a box, on two connected planes, placed at different heights and linked by a chute. Each ball can only hit other balls or the wall (that define the box) in a perfectly elastic way. "

Using the NetLogo simulation, it was possible to study the equilibrium distribution of the balls at the two levels and discuss the factors that influence this distribution. A comparison between the simulation, the experimental results and the models explaining them concluded this part of the Workshop.

*Phase 5 –Data analysis and Report writing*

During this phase of the Workshop, which was four hours long, students discussed some mathematical modelling procedures and searched for a law to describe the concentration vs. temperature trend, which was found to contain the general BF expression. In this procedure students were helped by the instructor. Some groups spent some time discussing the most suitable law and relating it to other possible cases in which BF is used to describe a phenomenon. Students searched for suitable models to make sense to their experimental evidence. They found the specific form of the suitable function and in particular they tried to give meaning to the quantity "energy" contained in the law's exponential term, i.e. in the BF. They found that Richardson's law is the expression analogous to the mathematical function best fitting their experimental data. It contains the BF and students founded that the "energy" reported in Richardson's Law's exponential term is called, in the specialized literature, the "work function"[4], something conceptually identical to the activation energy. Following this, the instructor also suggested to analyse in some detail the energy band model in the metal and the energy gap concept, by comparing this concept with the activation energy and work function concepts, discussed before.

A final scientific report was written by each group, with students sharing their ideas and preliminary results with the other participants. Peer to peer discussion also played an important part in the activities of the previous phases of the project.

*Phase 6 – Discussion of the reports*

In the last two hours of the workshop, the students presented the most significant findings obtained as a result of their experimental work and held a class discussion aimed at comparing and contrasting the results obtained by different groups of students.

## Research Question

Our study is centred on the implementation of a learning environments in which undergraduate students are involved in works specifically oriented towards the construction and use of explanations of thermally activated phenomena. These learning environments are also both aimed to favour students' individual reflection on the role of explanations in the modelling processes in different contexts.

The general aim of the study is to investigate the effect that a Feynman's Unifying approach can have in developing explicative skills in undergraduate students also based on laboratory and modelling activities. Taking into account these considerations, and the theoretical framework previously discussed, we formulated the following research question for this study:

*How are the lines of reasoning applied by undergraduate students when asked to make sense to real-life situations modified by a learning environment focused on Feynman's Unifying Approach in the field of thermally-activated phenomena?*

The general idea at the basis of our research question is that the FUA, a particular case of analogical thinking, may improve in a laboratory and modelling environment the constructivist approach, and therefore the explicative skills.

## DATA ANALYSIS

The quantitative analysis methods that we use in this study are based on clustering techniques. They allow us to partition the students in sub-groups on the basis of their typical behaviour with respect to the way they tackle the questionnaire.

Cluster Analysis (*ClA*) (Everitt, 2011) aims at classifying subject behaviours in different groups, or clusters. These can be analysed in order to deduct their distinctive characteristics and to point out similarities and differences between them. The clustering techniques can be divided in two main families: hierarchical and not-hierarchical ones (Everitt, 2011). Here we will only use a specific not-hierarchical clustering method, called k-means.

In order to apply ClA method it is necessary to codify the answers obtained from the questionnaire in numerical form. Due to the open-ended nature of the questions, after the questionnaires were submitted to the student samples the researchers independently read the students' answers in order to empirically identify the main characteristics of the different student records (the raw data). They agreed to independently construct a coding scheme by means of a Phenomenographic approach (Marton, 1986) to the student answer analysis, and through the identification of keywords that were relevant for the understanding of these records. During a first meeting, the selected keywords were compared and contrasted, and then grouped into categories based on epistemological and linguistic similarities[6] that are actually the typical answering strategies deployed by the students when tackling with the questions. As a third step, each researcher read the student records again and applied the new coding scheme, by assigning each student to a given category for each question.

At the end of this coding procedures, two shared list of $M$ answering strategies to be used for the subsequent analyses was obtained. More specifically, $M_{pre}$ = 59 answering strategies were obtained for the pre-instruction test analysis and $M_{post}$ = 61 ones were obtained for the post-instruction test analysis. Each of the $N$ = 37 students was identified by two arrays, $a_i$ and $a'_i$ ($i = 1, 2, ..., N$) composed by $M_{pre}$ and $M_{post}$ components 1 and 0, respectively. In each of these arrays, 1 was assigned when the related student used a given answering strategy to respond to a question, and 0 when he/she did not use it. Then, two $M$ x $N$ binary matrix (the "matrixes of answers") were built, for the pre- and post-instruction analyses, respectively. They are modelled like that shown in **Table 1**, where the columns report the $N$ student arrays, $a_i$, and the rows represent the $M$ components of each array, i.e. the $M$ answering strategies. We would remark that, as a consequence of the approach above described, the list of the answering strategies we use is, only, the result of answers actually given by the students.

**Table 1.** Example of matrix of answers: the $N$ students are indicated as $S_1$, $S_2$, …, $S_N$, and the $M$ answer strategies as $AS_1$, $AS_2$, …, $AS_M$

| **Strategy** | **Student** | | | |
|---|---|---|---|---|
| | $S_1$ | $S_2$ | … | $S_N$ |
| $AS_1$ | 1 | 0 | … | 0 |
| $AS_2$ | 1 | 0 | … | 1 |
| … | 0 | … | … | … |
| $AS_5$ | 1 | 1 | … | 0 |
| … | 0 | … | … | … |
| $AS_M$ | 0 | 1 | … | 0 |

For example, let us say that student $S_1$ used answering strategies $AS_1$, $AS_2$ and $AS_5$ to respond to the questionnaire questions. Therefore, column $S_1$ in **Table 1** contain the binary digit 1 in the three cells corresponding to these strategies, while all the other cells are filled with 0.

The matrix depicted in **Table 1** contains all the information needed to describe the sample behaviour with respect to the questionnaire answers.

## K-means clustering method

Like other clustering algorithms, k-means requires that a metric be defined. We note that, in our research, each point represents one student, so the definition of metric gives us a measure of the likeness between two elements (the students). This likeness is defined by starting from the $M$ x $N$ binary matrix discussed above.

For the first thing, it is need to perform the calculation of the correlation coefficient $R_{mod}$, (Battaglia et al, 2016) between students, by starting from the $M$ x $N$ binary matrix discussed above. Therefore, the likeness between students $i$ and $j$ can be defined by calculating their "distance" through the metric defined as $d_{ij} = \sqrt{2(1 - R_{mod})}$ (Battaglia et al, 2016).

The k-means clustering method (MacQueen, 1967) is used to obtain clusters from the data. In it, the starting point is the choice of the number of clusters one wants to populate and of an equal number of "seed points", randomly selected. The subjects are then grouped on the basis of the minimum distance between them and the seed points.

Starting from an initial classification, subjects are transferred from one cluster to another or swapped with subjects from other clusters, until no further improvement can be made. The subjects belonging to a given cluster are used to find a new point, representing the average position of their spatial distribution. This is done for each cluster and the resulting points are defined as the cluster *centroids* (Leisch, 2006). This process is repeated and ends when the new centroids coincide with the old ones.

A remarkable feature of the centroid $C_k$ is that it contains the answering strategies most frequently given by students belonging to $Cl_k$ as shown in Di Paola et al (2016) and Battaglia et al (2016).

It is worth noting that if some answering strategies are only slightly more frequent than other ones all those with similar frequencies must also be considered.

Students can be represented in a Cartesian plane according to their mutual distances. As we said before, for each student, $i$, we know the $N$ distances, $d_{ij}$ between such a student and all the students of the sample (being $d_{ii} = 0$). It is, then, necessary to define a procedure to find two Cartesian coordinates for each student, starting from these $N$ distances. This procedure consists in a linear transformation between a N-dimensional vector space and a 2-dimensional one and it is well known in the specialized literature as Multidimensional Scaling (Borg and Groenen, 1997).

## Results

All the clustering calculations were performed using a custom software, written in C language.

In order to define the number $q$ of clusters that best partitions our samples in both pre- and post-instruction tests, the values of the Silhouette function $S_i(q)$ and the relate average value $< S(q) >$, have been calculated for different numbers of clusters (Struyf et al, 1997; Rouseeuw, 1987; Saxena et al., 2013). Actually, the individual value, $S_i(q)$ for each student, $i$, of the sample gives a measure of how similar student $i$ is to the other students in its own cluster when compared to students in other clusters. It ranges from -1 to +1. A value near +1 indicates that student $i$ is well-matched to its own cluster, and poorly-matched to neighboring clusters. If the majority of students have a high silhouette value, then the clustering solution is appropriate. If many students have a low or negative silhouette value,

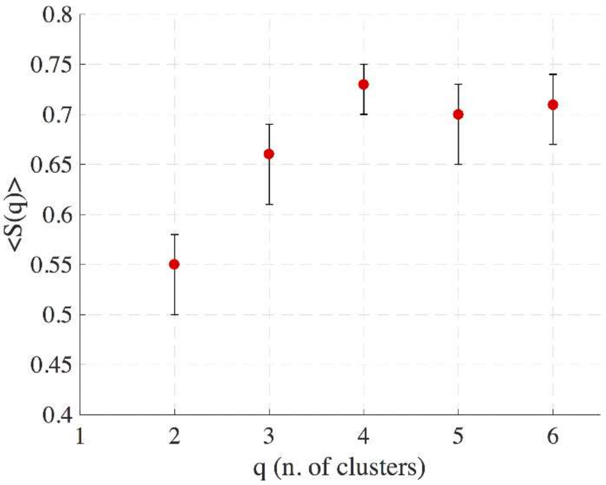

**Figure 1.** Silhouette average values and related 95% confidence intervals (CI) for different cluster partitions of our sample in the pre-instruction test

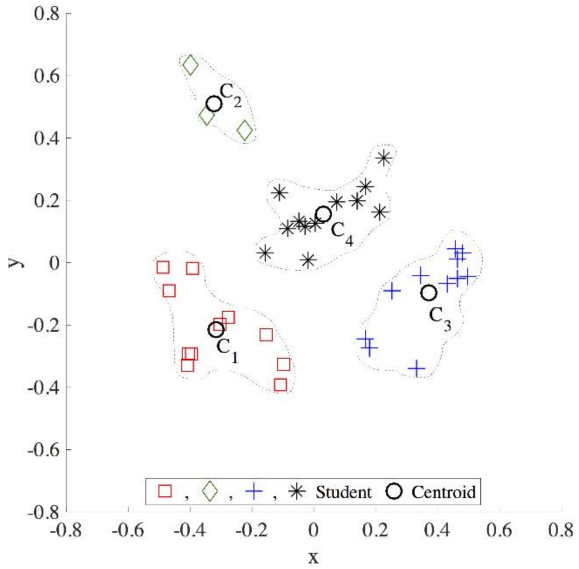

**Figure 2.** k-means graphs of the pre-instruction test results. Each point in this Cartesian plane represents a student. Points labeled $C_1$, $C_2$, $C_3$ and $C_4$ are the cluster centroids. The values in the x and y axis are used to place the different students according to their relative distance

then the clustering solution could be inappropriate.

**Figure 1** shows the silhouette average values (with confidence interval[7]) for each q, in the pre-instruction test. We note that the clustering solutions with 4, 5 and 6 clusters can be all considered equivalent whereas clustering solutions with 2 and 3 clusters can be rejected. However, we must consider here that taking into account clusters with too few students may not be relevant from a pedagogical point a view. In our case the clustering solutions with 5 and 6 clusters give student groups populated with too few elements. Moreover, we analysed the related centroids for both these solutions and noted that they do not give more details (from a pedagogical point of view) than those obtained by using the four cluster solution.

Therefore, we will discuss here the pre-instruction test results that can be obtained by selecting the four cluster solution. We also note that the silhouette average value obtained in this case ($\langle S(4) \rangle = 0.73$, CI = (0.70 - 0.75)) is higher than 0.6. According to the specialized literature, this shows that a reasonable cluster structure has been found (Struyf et al, 1997).

**Figure 2** shows the representation of this partition in a 2-dimensional graph for the pre-instruction test.

The clusters $Cl_k$ ($k$ = 1, 2, 3, 4) can be characterized by their related centroids, $C_k$. As we clarified previously, these are the points in the graphs whose arrays $\overline{a}_k$, contain the answering strategies most frequently applied by students in the related clusters (see **Table 2**). The codes used refer to the answering strategies for the questionnaire items described in Appendix A. The table also shows the number of students in each cluster.

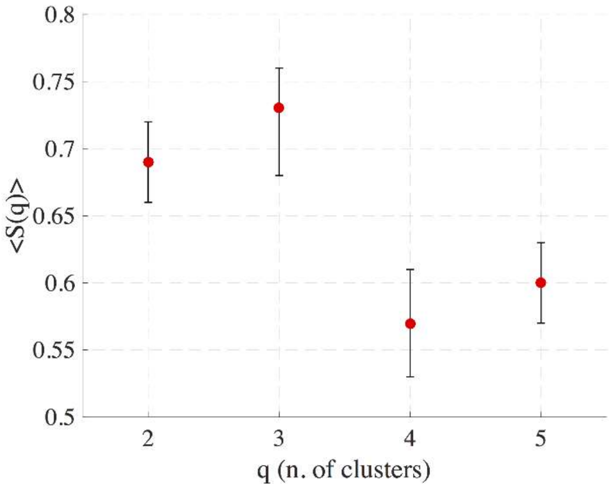

**Figure 3.** Average Silhouette values and related 95% confidence intervals (CI) for different cluster partitions of our sample in the post-instruction test

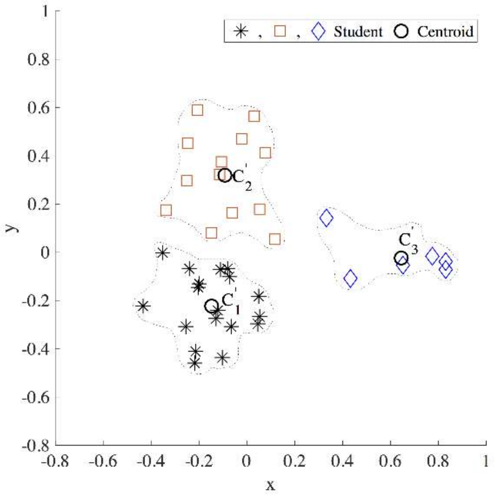

**Figure 4.** k-means graphs of the post-instruction test results. Each point in this Cartesian plane represents a student. Points labeled *C'$_1$*, *C'$_2$*, *C'$_3$* are the cluster centroids. The values in the x and y axis are used to place the different students according to their relative distance

**Figures 3** and **4** and **Table 3** show the results related to the post-instruction test. In this case, as it is shown in **Figure 3**, partitions in 2 and 3 clusters are the best partition and can be

**Table 2.** An overview of pre-instruction test results. The codes used for the most frequently given answers refer to the answering strategies to the questionnaire items described in Appendix A

| **Cluster centroid** | **C$_1$** | **C$_2$** | **C$_3$** | **C$_4$** |
|---|---|---|---|---|
| Most frequently given answers | 1F-G[8], 2B, 3C, 4C, 5A, 6G | 1K, 2B, 3I, 4C-G | 1C, 2C, 3C-3G, 4C, 5D, 6C | 1J-K, 2B, 3H, 4C, 5G, 6G |
| Number of students | 11 | 3 | 11 | 12 |

**Table 3.** An overview of the post-instruction test results. The codes used for the most frequently given answers refer to the answering strategies to the questionnaire items described in Appendix B

| Cluster centroid | C'$_1$ | C'$_2$ | C'$_3$ |
|---|---|---|---|
| More frequently given answers | 1H, 2E, 3J, 4F, 5E, 6H | 1K, 2F, 3I, 4H, 5H, 6G | 1K, 2F, 2G, 3L, 3M, 4I, 5I, 6K |
| Number of students | 18 | 13 | 6 |

considered equivalent. We choose to analyse one of these solutions (the 3 cluster one) because it gives us more details on the students' behaviour than the other (a greater number of clusters allows us to obtain a finer grain detail on the student behaviour), and the student groups are still large enough to coherently do pedagogical considerations on them. This preliminary consideration will be corroborated in the Discussion Section, where we will see that the centroids of our 3-cluster solution are well differentiated. The silhouette average value for this solution is <S(3)> = 0.73, CI = (0.68 - 0.76).

## DISCUSSION

The interpretation of *ClA* results mainly involves the identification of the typical features characterizing answers of students belonging to the same cluster as well as differences and similarities in answering strategies of students belonging to different clusters.

If we look closely to the pre-test results (**Figure 2** and **Table 2**) we note that:

- Cluster $Cl_3$ is mainly composed by students that can find the relevant variables of a phenomenon but are not able to use them to build an explanation (1C) and can name the relevant quantities in Arrhenius law but do not give them a physical meaning (2C). These students mainly describe a catalyst simply in terms of its effect on the speed of a chemical reaction, not supplying additional explanation (3C) or generically citing the energy concept as a reason for this (3G) and simply give a mathematical description of Arrhenius law (4C). Moreover, they highlight low-level explicative skills, as answer to questions 5 is limited to the context of the studied subjects (5D) and in question 6 they find some correct similarities among the cited phenomena but name physical quantities nor really relevant for a common explanation (6C). In conclusion, the most frequent answering strategies of clusters $Cl_3$ do not seem to be driven by a real understanding of thermally activated mechanisms. The virtual student represented by this centroid seems to answer to the questions more on the basis of memories of studied subjects than as a result of mastery of the related concepts.
- Cluster $Cl_1$ mainly contain students that only macroscopically describe the real-life situation (1F-1G), cite some relevant quantity in Arrhenius law, but do not provide a discussion on their meaning, rather referencing to real-life experience (2B), describe a catalyst only in terms of its effect on the speed of a chemical reaction, without further clarification (3C), and simply give a mathematical

description of Arrhenius law (4C). These students mainly answer question 5 naming some phenomena coming from real-life experience and not related to the model, without any further explanation (5A) and can find some similarities among the phenomena, considering *T* and *E* relevant, but do not give any further clarification (6G). In some aspects it appears that the most frequent answering strategies used by students in cluster $Cl_1$ are of a lower level than the ones found for $Cl_3$ centroid, as they seem to be in some cases, driven by a sort of everyday-type reasoning (in answers to questions 2 and 5).

- Students in clusters $Cl_4$ seem to mainly use answering strategies in some cases of a higher level than what we have seen before. The real-life phenomenon of question 1 is described and explained not adequately (1J) by giving a roughly microscopic "functioning mechanism" (1K) and some of the relevant quantities in Arrhenius law are cited, although a proper description of their meaning still lacks (2B). A catalyst is discussed as a substance that speeds-up a chemical reaction and name the concept of "energy", without further clarification (3H) and Arrhenius law is described as a mathematical function of *T* and *E*, without a proper clarification of the meaning of these quantities (4C). However, many of these students our sample can cite, in their answers to question 5, phenomena related to the Arrhenius model also not related to the chemical context, but do not give a clear explanation (5G), while students from the control group are mainly able to only recall phenomena typical of the chemical context, but try to explain them outlining a common microscopic model (5G). In both groups, however, the similarities found among the phenomena cited in answers to question 5 are not properly clarified (6G).
- Students in Cluster $Cl_2$ are quite similar to the students in $Cl_4$ with regard to the first four questions but fail to respond to questions 5 and 6 (the highest level). These are then weaker in issues that require more explanatory skills.

These results are clearly global, as they are related to the most frequent lines of reasoning deployed by the students during the pre-instruction test. For this reason, we also individually checked the answers of each student, and did not find excessive deviations with respect to the one highlighted by the centroid of the cluster he/she belongs to.

Summarizing, the pre-test results highlight that our sample mostly shows a descriptive-like behaviour (1f, 1G, 3G, 3H, 3I, 4C, 4G, 5D, 6G), with some examples of everyday-like one (2B, 5A, 3C). However, a few examples of Explicative-like behaviour are found (1K, 4G). In summary, according to the framework previously discussed (Duit et al, 1996) we can infer that students that hardly find analogies between different phenomena are not able to highlight skills oriented to explanation.

Data reported in **Figures 4** and in **Table 3** show that the results of the post-instruction test are markedly different than the ones of the pre-instruction one.

We note that:

- Cluster $Cl'_1$ is mainly composed of students that mathematically describe the proposed real life situation, but do not find a microscopic explicative model (1H) and are able to cite the relevant quantities in the $\eta(T)$ law, describing their physical meaning (2E). They present the additive role in gas oil by using a formal definition of catalyst in chemical reactions ("*shift of the chemical equilibrium towards the products*") (3J) and give a quantitative explanation of the $\eta(T)$ law in terms of $T$ and $E$, outlining the physical meaning of these quantities (4F). Moreover, some phenomena related to the model, also not related to a chemical context are found, but a proper explanation is not given (5E) and some similarities among the phenomena are found, clarifying the meaning of E and T (6H). All in all, the answering strategies most frequently used by students of cluster $Cl'_1$, although of a general higher level of what we have seen in the pre-instruction results seem to be still anchored to memories of past studies and a search for explanation based on a mechanism of functioning is not clearly evident.
- Students in Cluster $Cl'_2$ mainly verbally describe the real-life situation and give a simple microscopic interpretation (1K), are able to find the relevant quantities in the $\eta(T)$ law and discuss their physical meaning (2F), recognize an additive as a substance that influences the flow speed, naming the energy gap concept, without further clarification (3I), and give an explanation in terms of molecular interaction, outlining the concept of activation energy (4H). Students in this cluster show an explicative behaviour answering to item 5. In fact, they mention some phenomena related to the model and give an explanation outlining a common microscopic model, but energy and temperature are not clearly interrelated (5H). Moreover, they find similarities among the phenomena, considering T and E as characteristic and common to them, without further clarifications (6G). A first approach to analogical thinking can be identified with respect to questions 5 and 6 answers. The answering strategies used by $Cl'_2$ "virtual student" are surely of a higher level than the ones most frequently used by students in cluster $Cl'_1$ and also with respect to the strategies most frequently used by students answering the pre-instruction questionnaire.
- Cluster $Cl'_3$ is mainly composed by students that, like the ones in cluster $Cl'_2$, mainly verbally describe the real life situation and give for it a simple microscopic interpretation (1K) and are able to find the relevant quantities in the $\eta(T)$ law, discussing their physical meaning (2F-G). However, this time the additive role in the flow process is clarified by considering the energy gap concept and relating it to interaction between molecules (3L), or linking the energy gap to molecular energy (3M), and an explanation of the $\eta(T)$ law is found that takes into account interaction between fluid molecules and clarifies the meaning of the activation energy (4I). Many of cluster $Cl'_3$ students are able to name phenomena related to the model, also not related to the chemical context, and to frame them in a

common microscopic model (5I). Moreover, they can find similarities among these phenomena and discuss them in term of the common idea of activation energy (6K). The most common strategies used by students in this cluster are definitely of a high level and may show that these students (although they are only 6) are able to explain the subjects discussed in the questionnaire relating them to a functioning mechanisms based on the idea of thermal activation. During this explanation, an important role is played by analogical reasoning.

Again, we also individually checked the answers of each student, and did not find excessive deviations with respect to the one highlighted by the centroid of the cluster he/she belongs to.

We can summarise that many of the answering strategies used by the students in the post-instruction answers (1K, 2F, 2G, 3L, 3M, 4I, 5H, 5I, 6K) highlight the presence of explicative-like skills and a good ability in building analogies among different phenomena. However, we note that some descriptive-like answering strategies (1H, 2E, 3I, 4F, 5E, 6G) are still present, even if they can be considered of a "higher level" than the ones used by the students before instruction. No everyday-like answering strategies are used after instruction. We can conclude that at the end of Workshop activities answering strategies of a higher conceptual level than the one used before instruction are used by our students.

## CONCLUSION

On the basis of the previous considerations, we can try to answer to our research question that is focused on the modifications that a learning environment based on Feynman Unifying approach could have produced to the lines of reasoning used by undergraduate students when asked to make sense to real-life situations involving thermally activated phenomena. The pre-instruction test results show that the students initially mainly highlight the use of lines of reasoning based on the use of memory of past studies and on an application of mathematics, without a search for a proper mechanism of functioning. In some cases, everyday-like reasoning is also highlighted.

The post-instruction test results, show that 6 of the 37 students in our sample (cluster $Cl'_3$) are clearly able to explain the situations and problems proposed in the questionnaire relating them to a functioning mechanisms based on the idea of thermal activation. The 13 students in clusters $Cl'_2$ share some high level skills with the group cited above, but they not clearly find the relationship between the fundamental quantity like $T$ and $\Delta E$. Although they can cite some microscopic models, these are often not clear. The 18 students in $Cl'_1$, although in some cases still anchored to memories of past studies, show to be able to link the flow process and the role of an additive by considering the energy gap concept and relating it to interaction between molecules, i.e. to a functioning mechanism.

We can conclude that students reasoning lines seem to have clearly evolved to explicative ones. It is reasonable to think that the Feynman Unifying Approach has favoured this change, probably due to the study of different phenomena that can be modelled by means of a common conceptual framework. According to the theoretical framework discussed above, the idea to favour students analogical thinking in order to help them to compare similarities and detect differences between phenomena, and so to create a strong bridge between complex and abstract concepts, was appropriate. Looking to both the pre- and post-instruction tests we can in fact detect that in the construction of students' explicative models an important role is played by analogical reasoning.

On the other hand, the laboratory and modelling approach we followed during the workshop, also supported by the use of computer simulation environments (NetLogo), was based on active student work in small groups. They were allowed to pose their questions and find the way to give them answers, even by freely searching for information and resources from several sources and had the possibility to share their results and compare them, alternating small and great group discussion. We acknowledge that this approach is in many aspects similar to a pedagogical one that in the recent years has become very popular in Science Education, i.e. the so-called Inquiry Based Science Education (IBSE) approach (Bybee, 1993; Olson & Loucks-Horsley (eds.), 2000). It has been proven to be very useful to promote authentic learning and to help students to develop high-level cognitive skills (Karelina & Etkina, 2007; Etkina, 2014; Pizzolato et al, 2014).

For this reason, we plan to extend the research discussed in this paper developing a study that tries to understand if the IBSE approach alone can be held responsible for the development of explicative-like skills similar to the ones we found can be put into action by the Feynman's Unifying Approach we discussed here. This study will be discussed in a forthcoming paper.

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# APPENDICES

## Appendix A

Pre-Instruction Questionnaire and related answering strategies used by students, for each item.

1) A puddle dries more slowly at 20°C than at 40°C. Assuming all other conditions (except temperature) equal in the two cases, explain the phenomenon, pointing out what are the quantities needed for the description of the phenomenon and for the construction of an interpretative model of the phenomenon itself.

| | |
|---|---|
| *1A* | *No explanation is given* |
| *1B* | *The relevant quantities are not identified, but a description/explanation based on common sense is given.* |
| *1C* | *The relevant quantities are identified, but they are not used to give an explanation.* |
| *1D* | *Only temperature is identified as relevant, but the phenomenon is not correctly described.* |
| *1E* | *Only temperature is identified as relevant. It is used to give a rough description of the phenomenon.* |
| *1F* | *The phenomenon is described by means of the macroscopic variables pressure and volume, but a microscopic model is not identified.* |

1G *The phenomenon is described by means of the macroscopic variables temperature, energy and heat, but a microscopic model is not identified.*

1H *The phenomenon is described by means of a mathematical formula, but a microscopic model is not identified.*

1I *The phenomenon is not adequately described (by means of a mathematical formula or verbally), but "molecular collisions" are cited as responsible for it, without further deepening.*

1J *The phenomenon is not adequately described (by means of a mathematical formula or verbally), but a microscopic "functioning mechanism" is presented in terms of energy exchange between molecules.*

1K *The phenomenon is verbally described and a microscopic "functioning mechanism" is roughly sketched.*

1L *The phenomenon is described by means of mathematical relations between macroscopic quantities and a microscopic "functioning mechanism" is found.*

2) In chemical kinetics it is well known that the rate of a reaction, $u$, between two reactants follows the Arrhenius law:

$$u = Ae^{-\frac{E}{kT}}$$

Describe each listed quantity, clarifying its physical meaning and the relations with the other quantities.

2A *The fundamental quantities are not described.*

2B *Some quantities are mentioned, sometimes with reference to real-life experience, but no description of their meaning is given.*

2C *The relevant quantities are found, and some of them are described in terms of their physical meaning.*

2D *The relevant quantities are found. They are only described in mathematical terms.*

2E *The relevant quantities are found and correctly described in terms of their physical meaning. No relationship between them is identified.*

2F *The relevant quantities are found and correctly described in terms of their physical meaning. Some relations between them are identified.*

2G *The relevant quantities are found and correctly described in terms of their physical meaning. The relations between them are correctly identified.*

3) What do you think the role of a catalyst is, in the development of a chemical reaction?

3A *A definition of catalyst is given, which does not conform to the scientifically accepted one.*

*3B A definition of catalyst based on an analogy with the concept of enzyme is given. The analogy is given without providing additional motivation.*

*3C The catalyst is simply described as a substance which speeds up a chemical reaction. No additional explanation is supplied.*

*3D The catalyst is described as a substance which shifts the chemical equilibrium towards the products. No additional explanation is supplied.*

*3E The catalyst is described as a substance which speeds up a chemical reaction. An explanation is given using common language.*

*3F The catalyst is described as a substance which speeds up a chemical reaction. No explanation is given.*

*3G The catalyst is presented as a substance which speeds up a chemical reaction. The concept is generically described in terms of energy.*

*3H The catalyst is presented as a substance which shifts the chemical equilibrium towards the products. The concept is generically described in terms of energy.*

*3I The catalyst is presented as a substance which speeds up a chemical reaction. The concept is described by simply citing the energy gap concept, without any explanation.*

*3K The role of a catalyst in a chemical reaction is discussed referring to the energy gap concept, but only in macroscopic terms.*

*3L The role of a catalyst in a chemical reaction is discussed taking into account the energy gap concept. The concept is explained considering a microscopic model regarding collisions between molecules.*

*3M The role of a catalyst in a chemical reaction is discussed taking into account the energy gap concept. The concept is explained considering a microscopic model which links the energy gap concept with the molecular energy.*

4) Can you give a microscopic interpretation of the Arrhenius law?

*4A A microscopic interpretation of Arrhenius law is not given and only generic references to the quantities in the law are given.*

*4B Scientific concepts, such as energy, temperature or molecular thermal agitation, are mentioned, but they are not correctly related to the Arrhenius law.*

*4C Arrhenius law is described as a mathematical function of T or E. No explanation of the meaning of these quantities is given.*

*4D Arrhenius law is described as a mathematical function of both T and E. No explanation of the meaning of these quantities is given.*

*4E Arrhenius law is described as a function of both T and E and the meaning of these two quantities is outlined mainly in mathematical terms.*

*4F Arrhenius law is described as a function of T or E. The meaning of these two quantities is outlined in mathematical terms.*

4G *Arrhenius law is described outlining the physical quantities involved. Collision theory is sometimes mentioned, but a clear reference to a microscopic model is not always present.*

4H *A generic explanation based on a microscopic model of collisions between molecules is given. The activation energy concept is outlined but its relation with kT is not clearly presented.*

4I *A quantitative explanation in terms of the "collision theory" is given. A correct microscopic model is presented and the role of the activation energy and of kT is clearly expressed.*

5) Can you think of other natural phenomena which can be explained by a similar model?

5A *A few phenomena, coming from real-life experience and not really related to the model, are mentioned. No explanation is given.*

5B *A few phenomena not related to the model are mentioned. An explanation is given using common language.*

5C *A few phenomena not related to the model are mentioned. An explanation is given using mathematical formulas.*

5D *Some phenomena related to the model are mentioned, but these are limited to the context of the attended graduation program (chemical engineering). An explanation is given using mathematical formulas.*

5E *Some phenomena related to the model are mentioned, and non-chemical phenomena are also taken into account, but a clear explanation is not given.*

5F *Some phenomena related to the model are mentioned, and non-chemical phenomena are also taken into account. An explanation is given using mathematical formulas.*

5G *Some phenomena related to the model are mentioned, but these are limited to the context of the attended graduation program (chemical engineering). An explanation is given outlining a common microscopic model.*

5H *Some phenomena related to the model are mentioned, and non-chemical phenomena are also taken into account. An explanation is given outlining a common microscopic model, but energy and temperature are not clearly interrelated.*

6) Which similarities can be identified in the previous phenomena? Is it possible to find a common physical quantity which characterizes all the systems you discussed in the previous questions?

6A *No relevant similarities are detected.*

6B *No similarities are detected and questions 1) and 2) are identified as being related to a different context. An explanation is given, mentioning physical quantities which are not really relevant to the correct explanation of the questions.*

6C *A few correct similarities are found, but physical quantities are given which are not really relevant to the correct explanation of the questions.*

6D *Incorrect similarities are found on the basis of a mathematical formula.*

6E *A few correct similarities are found on the basis of a mathematical formula.*

6F *Correct similarities are found, but E and T are not always considered common to all phenomena.*

6G *Some correct similarities are found. E or T is considered to be characteristic of the various phenomena, but a clear justification is not given.*

6H *Some correct similarities are found. E or T is considered to be characteristic of the various phenomena, clearly explaining why.*

6I *Some correct similarities are found. E or T is considered to be characteristic of the various phenomena, but the relevance of their ratio in explaining the energy threshold processes is not clearly presented.*

6J *Some correct similarities are found. E or T is considered to be characteristic of the various phenomena. The activation energy role is correctly discussed in all the mentioned phenomena, but only in macroscopic terms.*

6K *Some correct similarities are found. E or T is considered to be characteristic of the various phenomena. The activation energy role is correctly discussed in all the mentioned phenomena, on the basis of a microscopic model.*

## Appendix B

Post-Instruction Questionnaire and related answering strategies used by students, for each item.

1) In modern oil mills olive oil flows inside metallic pipes. These pipes are often enclosed in bigger pipes in which hot water flows. Explain the possible reason of this, pointing out what are the quantities needed for a description of the proposed situation and for the construction of an explicative model.

1A *No clear answer is given.*

1B *The relevant quantities are not identified, but a description/explanation based on common sense is given.*

1C *The relevant quantities are identified, but they are not used to give an explanation.*

1D *Only temperature is identified as relevant, but the phenomenon is not correctly described.*

1E *Only temperature is identified as relevant. It is used to give a rough description of the phenomenon.*

1F *The phenomenon is described by means of the macroscopic variables temperature and energy, but a microscopic model is not identified.*

1G *The phenomenon is described by means of the macroscopic variables temperature, energy and fluidity, but a microscopic model is not identified.*

1H *The phenomenon is described by means of a mathematical formula, but a microscopic model is not identified.*

1I *The phenomenon is not adequately described (by means of a mathematical formula or verbally), but "molecular energy" is cited as responsible for it, without further deepening.*

1J *The phenomenon is not adequately described (by means of a mathematical formula or verbally), but a microscopic "functioning mechanism" is presented in terms of energy exchange between water and oil.*

1K *The phenomenon is verbally described and a microscopic "functioning mechanism" is roughly sketched.*

1L *The phenomenon is described by means of mathematical relations between macroscopic quantities and a microscopic "functioning mechanism" is found.*

2) In chemistry it is well known from Eyring's absolute rate theory that the viscosity of a fluid follows the following law:

$$\eta = Ae^{\frac{E_{vis}}{kT}}$$

Describe each listed quantity, clarifying its physical meaning and the relations with the other quantities.

2A *No clear description is given.*

2B *Some quantities are mentioned, sometimes with reference to real-life experience, but no description of their meaning is given.*

2C *The relevant quantities are found, and some of them are described in terms of their physical meaning.*

2D *The relevant quantities are found. They are only described in mathematical terms.*

2E *The relevant quantities are found and correctly described in terms of their physical meaning. No relationship between them is identified.*

2F *The relevant quantities are found and correctly described in terms of their physical meaning. Some relations between them are identified.*

2G *The relevant quantities are found and correctly described in terms of their physical meaning. The relations between them are correctly identified.*

3) In petroleum industry additives are often added to gas oil to work as catalysts. What do you think can the role of these additives be in the flowing of gas oil in a pipe?

3A *No clear answer is given.*

3B *The additives are recognized as substance which improve fluidity, but no reference to a catalysis process is done.*

3C *The role of additives as catalysts is sketched. No additional clarifications are given.*

3D *The additives are simply described in chemical terms, as substances which shift chemical equilibrium towards the products. No additional explanation is supplied.*

3E *The additives are described as substances which speed up the flow. An explanation is given using common language.*

3F *The additives are presented in chemical terms, as substances which shift chemical equilibrium towards the products. An explanation is given using common language.*

3G *The additives are presented as substances which speed-up the flow. The concept is generically described in terms of energy.*

3H *The additives are presented in chemical terms, as substances which shift the chemical equilibrium towards the products. The concept is generically described in terms of energy.*

3I *The additives are presented as substances which speed up the flow. This concept is described by citing the energy gap quantity, but without a clear explanation.*

3J *The additives are presented in chemical terms, as substances which shift the chemical equilibrium towards the products. The concept is described citing the energy gap concept, but without a clear explanation.*

3K *The role of a the additives in the flow process is discussed referring to the energy gap concept, but only in macroscopic terms.*

3L *The role of a the additives in the flow process is discussed taking into account the energy gap concept. The concept is explained considering a microscopic model regarding collisions between molecules.*

3M *The role of a the additives in the flow process is discussed taking into account the energy gap concept. The concept is explained considering a microscopic model which links the energy gap concept with the molecular energy.*

4) Can you give a microscopic interpretation of the $\eta(T)$ law seen in question 2)?

4A *No clear answer is given.*

4B *Scientific concepts, such as energy, temperature or molecular thermal agitation, are mentioned, but they are not correctly related to the law.*

4C $\eta(T)$ *law is described as a mathematical function of T or E. No explanation of the meaning of these quantities is given.*

4D *The law is described as a mathematical function of both T and E. No explanation of the meaning of these quantities is given.*

4E *The law is described as a function of both T and E and the meaning of these two quantities is outlined mainly in mathematical terms.*

4F *The law is described as a function of both T and E. The physical meaning of these two quantities and/or of their ratio in the law is outlined.*

4G *The law is described outlining the physical quantities involved. Interaction between fluid molecules is mentioned, but a clear reference to a microscopic model is not always present.*

*4H* *A generic explanation based on a microscopic model of interaction between molecules is given. The activation energy concept is outlined but its relation with $kT$ is not clearly presented.*

*4I* *A quantitative explanation in terms of interaction between fluid molecules is given. A correct microscopic model is presented and the role of the activation energy and of $kT$ is clearly expressed.*

5) Can you think of other natural phenomena which can be explained by a similar model?

*5A* *A few phenomena, coming from real-life experience and not really related to the model, are mentioned. No explanation is given.*

*5B* *A few phenomena, coming from real-life experience and not really related to the model, are mentioned. Some verbal explanation is given.*

*5C* *A few phenomena not related to the model are mentioned. An explanation is given using mathematical formulas.*

*5D* *Some phenomena related to the model are mentioned, but these are limited to the context of the attended graduation program (chemical engineering). An explanation is given using mathematical formulas.*

*5E* *Some phenomena related to the model are mentioned, and non-chemical phenomena are also taken into account, but a clear explanation is not given.*

*5F* *Some phenomena related to the model are mentioned, and non-chemical phenomena are also taken into account. An explanation is given using mathematical formulas.*

*5G* *Some phenomena related to the model are mentioned, but these are limited to the context of the attended graduation program (chemical engineering). An explanation is given outlining a common microscopic model.*

*5H* *Some phenomena related to the model are mentioned, and non-chemical phenomena are also taken into account. An explanation is given outlining a common microscopic model, but energy and temperature are not clearly interrelated.*

*5I* *Some phenomena related to the model are mentioned, and non-chemical phenomena are also taken into account. An explanation is given outlining a common microscopic model. The role of energy and temperature in the model is clearly discussed.*

6) Which similarities can be identified in the previous phenomena? Is it possible to find a common physical quantity which characterizes all the systems you discussed in the previous questions?

*6A* *No clear answer is given.*

*6B* *No similarities are detected and questions 1) and 2) are identified as being related to a different context. An explanation is given, mentioning physical quantities which are not really relevant to the correct explanation of the questions.*

6C *A few correct similarities are found, but physical quantities are given which are not really relevant to the correct explanation of the questions.*

6D *Incorrect similarities are found on the basis of a mathematical formula.*

6E *A few correct similarities are found on the basis of a mathematical formula.*

6F *Correct similarities are found, but E and T are not always considered common to all phenomena.*

6G *Some correct similarities are found. E or T is considered to be characteristic of the various phenomena, but a clear justification is not given.*

6H *Some correct similarities are found. E or T is considered to be characteristic of the various phenomena, clearly explaining why.*

6I *Some correct similarities are found. E or T is considered to be characteristic of the various phenomena, but the relevance of their ratio in explaining the energy threshold processes is not clearly presented.*

6J *Some correct similarities are found. E or T is considered to be characteristic of the various phenomena. The activation energy role is correctly discussed in all the mentioned phenomena, but only in macroscopic terms.*

6K *Some correct similarities are found. E or T is considered to be characteristic of the various phenomena. The activation energy role is correctly discussed in all the mentioned phenomena, on the basis of a microscopic model.*

## NOTES

1. Modelling literature analyses the mental model properties and often describes people's reasoning as the "running" of the procedures present in their mental models (Redish, 1994; Gilbert et al, 1998). Gilbert and Boulter (Gilbert et al, 1998) define expressed models as the external representations expressed by an individual through actions, speech or writing. According to such definitions, we understand "lines of reasoning" as the external representations of the mental models used by an individual when he or she tries to describe, predict, or explain the physical world.
2. The other two authors were not directly involved in the workshop. However, they acted as independent observers, taking note of the various situations taking place during the student activities. One of them also conducted interviews with the students, making clear that these were in no way connected to an evaluation of their participation in the workshop. These interviews and the logbooks of the other two researchers will be discussed in another paper.
3. Actually, during the chemistry course, some references to the "collision theory" (Pauling, 1998)) were made, but almost all students said that they had never really thought about its real microscopic meaning.

4. The work function of a metal can be considered as the minimum energy electrons need to escape the metal (Jacobs et al, 1948; Kittel, 1966)
5. http://ccl.northwestern.edu/netlogo/
6. For example, students that defined models as simple phenomena or experiments or reproductions of an object on a small scale have been put on the same category since the three definitions have been intended as giving a ontological reality to models.
7. Confidence interval was obtained by using Bootstrap method (Di Ciccio & Efron, 1996).
8. When two strategies are reported for a given question, it means that they both have the same maximum frequency among the centroid's students.

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