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K-VISIT ATTRIBUTE GRAMMARS

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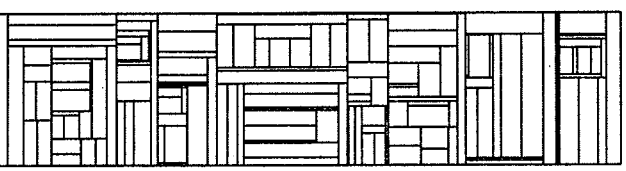

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Abstract

It is shown that any well-defined attribute grammar is k-visit for some k. Furthermore it is shown that given a well-defined grammar G and an integer k, it is decidable whether G is k-visit. Finally we show that the k-visit grammars specify a proper hierarchy with respect to translations.

1. INTRODUCTION

Knuth introduced attribute grammars as a formalism for associating meanings with strings of context-free languages ([7]). Because of the generality of the formalism there have been defined several subclasses of attribute grammars, which are often easier to handle (see e. g. [1], [5], [6], and [4]).

One of these subclasses is the one-visit (or re-ordered, [8]) attribute grammars of [2].

We define an extension of this class - the class of k-visit attribute grammars. The definition of a k-visit attribute grammar is a generalization of that of a one-visit attribute grammar: An attribute grammar G is k-visit if for any derivation tree t of G it is possible to evaluate all the attributes associated with t by walking through t in such a way that no node in t is visited more than k times.

While it is possible to find a local property for the productions, which is equivalent to the global one-visit property (see [2]), there seems to be no such local property corresponding to the k-visit property for an attribute grammar.

We show in this paper that any well-defined (see [7]) attribute grammar G is k-visit for some k. Furthermore it is shown using a pumping argument, that given a well-defined grammar G and an integer k it is decidable whether G is k-visit. Thus we can effectively for any well-defined attribute grammar G find the minimal k such that G is k-visit. Finally we show that the k-visit attribute grammars specify a proper hierarchy with respect to translations.

2. NOTATION AND DEFINITIONS

In this section we will recall the definition of an attribute grammar and define some related concepts. We use a definition of an attribute grammar similar to that given in [2].

A semantic domain $\mathcal{D}$ is a pair $(\underline{D}, \underline{F})$ where $\underline{D}$ is a set of sets and $\underline{F}$ is a set of mappings of functionality $D_1 \times D_2 \times \ldots \times D_m \rightarrow D$ $(m \geq 0)$, where D and D_j for $1 \leq j \leq m$ are sets in $\underline{D}$. The elements of $\underline{D}$ will be called domains whereas the elements of $\underline{F}$ will be called semantic functions.

An attribute grammar (AG) G over a semantic domain $\mathcal{D} = (\underline{D}, \underline{F})$ can be considered as an extension of a context-free grammar (CFG) $G_u = (N, T, P, Z)$.

To each nonterminal X of G_u there is associated a fixed set A(X) called the attributes of X. The set A(X) is divided into two disjoint sets, I(X), the set of inherited attributes, and S(X), the set of synthesized attributes. To each element in A(X) there is associated a domain from $\underline{D}$. We require without loss of generality that S(Z) is a singleton and that $I(Z) = \emptyset$.

To each production p: $F ::= w_0 F_1 w_1 F_2 w_2 \ldots w_{n-1} F_n w_n$, $w_j \in T^*$ $(0 \leq j \leq n)$, $F, F_j \in N$ $(1 \leq j \leq n)$ of G_u there is associated so-called semantic rules. There will be exactly one semantic rule for each attribute b in the set $S(F) \cup I(F_1) \cup I(F_2) \cup \ldots \cup I(F_n)$. A semantic rule for b consists of a semantic function f: $D_1 \times D_2 \times \ldots \times D_m \rightarrow D$ from $\underline{F}$ together with m attributes $a_1, a_2, \ldots, a_m$ from the set $I(F) \cup S(F_1) \cup S(F_2) \cup \ldots \cup S(F_n)$. The attributes b and a_j for $1 \leq j \leq m$ must satisfy that the domain D is associated with b whereas the domain D_j is associated with a_j for $1 \leq j \leq m$. We say that b depends on a_j for $1 \leq j \leq m$.

Example 2.1

Let us define an AG G over a semantic domain $\mathcal{D} = (\underline{D}, \underline{F})$.

$\underline{D}$ will consist of the single set $D = \{A, B\}^*$, i.e. sequences of A's and B's. In $\underline{F}$ we have five semantic functions:

$1_\lambda : \rightarrow D$	$1_\lambda( ) = \lambda,$	the empty string
$1_A : \rightarrow D$	$1_A( ) = A,$	the constant string A
$1_B : \rightarrow D$	$1_B( ) = B,$	the constant string B
$\iota : D \rightarrow D$	$\iota(a) = a,$	the identity function, $a \in D$
$\&: D \times D \rightarrow D$	$\&(a,b) = ab,$	the concatenation function, $a, b \in D.$

The underlying grammar G_u of the AG G will be

Z ::= X
X ::= X X | Y
Y ::= A | B

To each attribute of any symbol we associate the domain D. We give the productions of the AG and their semantic rules below in an affix-like notation. The inherited attributes will be prefixed by downward arrows (↓) whereas synthesized attributes will be prefixed by upward arrows (↑).

p1: $\langle Z \uparrow \iota(a) \rangle ::= \langle X \downarrow 1_\lambda( ) \uparrow a \rangle$

p2: $\langle X \downarrow a \uparrow \iota(c) \rangle ::= \langle X \downarrow \iota(b) \uparrow c \rangle \langle X \downarrow \iota(a) \uparrow b \rangle$

p3: $\langle X \downarrow a \uparrow \&(a,b) \rangle ::= \langle Y \uparrow b \rangle$

p4: $\langle Y \uparrow 1_A( ) \rangle ::= A$

p5: $\langle Y \uparrow 1_B( ) \rangle ::= B$

□

To each node in a derivation tree t defined by the CFG G_u we will associate attributes similar to those of the symbol labeling the node. Let n be a node labeled with a nonterminal X in t. If p: $X ::= X_1 X_2 \ldots X_m$ is the production used to expand n in t then the semantic rules for the attributes in S(X) associated with p are used to give values to the synthesized attributes of n (denoted S(n)). On the other hand if X occurs on the right hand side of the production q: $Y ::= Y_1 Y_2 \ldots Y_h$ and n is introduced in t by an application of q then the semantic rules for the attributes in I(X) associated with q are used to give values to the inherited attributes of n (denoted I(n)).

In the following a derivation tree will have root labelled Z and leaves labelled by symbols from T.

If t is a derivation tree and n a node in t then t_n denotes the subtree of t with root n,and t^n denotes the tree t with the subtree of n, except n itself, removed.

We want to give values to (or evaluate) all the attributes associated with (the nodes of) the tree t. This is done during a walk through the tree. When arriving at a node we may evaluate some of the inherited or synthesized attributes of the node and after that continue to either the father, a son or a brother of the node or the node itself.

A <u>walk</u> through a tree t is a sequence

$$s = \langle n_1, A_1 \rangle \langle n_2, A_2 \rangle \ldots \langle n_r, A_r \rangle$$

where

1) n_j is an interior node in t for $1 \leq j \leq r$
2) If $n_j \neq n_{j+1}$ then n_{j+1} is the father, a son or a brother of n_j, $1 \leq j < r$
3) A_j is a subset of the inherited or synthesized attributes of n_j for $1 \leq j \leq r$

For each node n in t we define

$$s(n) = A_{i_1} A_{i_2} \ldots A_{i_m}$$

if $i_1 < i_2 < \ldots < i_m$ and $\{ i_j : 1 \leq j \leq m \} = \{ h : n_h = n, \ 1 \leq h \leq r \}$

A computation sequence for a derivation tree t is a walk

$$cs = \langle n_1, A_1 \rangle \langle n_2, A_2 \rangle \ldots \langle n_r, A_r \rangle$$

through t satisfying

4) $n_1 = n_r$ is the root of t

5) the attributes of A_j do not depend on those of A_i for $i \geq j$, $1 \leq j \leq r$

6) for each interior node n' in t:

if $cs(n') = B_1 B_2 \ldots B_h$ then

a) $B_j \subseteq I(n')$ if j is odd, $1 \leq j \leq h$

b) $B_j \subseteq S(n')$ if j is even, $1 \leq j \leq h$

c) $\bigcup_{j=1}^{h} B_j = I(n') \cup S(n')$

d) $B_i \cap B_j = \emptyset$ if $i \neq j$

e) let $cs = cs_1 \langle n', B_j \rangle \langle n_1', C_1 \rangle \langle n_2', C_2 \rangle \ldots$
$\langle n_k', C_k \rangle \langle n', B_{j+1} \rangle cs_2$;
if j is odd (even) then n_i is a node in t_n (t^n) for $1 \leq i \leq k$

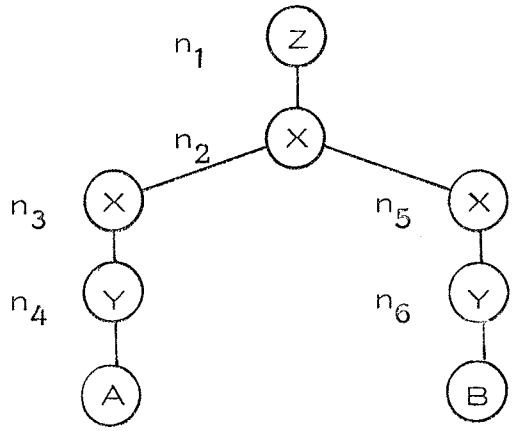
(Note, that 4) implies that h is even.)

□

Thus when walking 'down' in a subtree we evaluate some of the inherited attributes of the root of the subtree. When returning from the subtree we evaluate some of the synthesized attributes of the root of the subtree.

Example 2.2

Consider the following derivation tree t of the AG of Example 2.1.



Let I denote the inherited attributes of a node and S the synthesized.
A computation sequence for t which traverses t twice from left to right is

1. pass $\left\{ \begin{array}{l} \langle n_1,\emptyset\rangle \langle n_2,I\rangle \langle n_3,\emptyset\rangle \langle n_4,\emptyset\rangle \langle n_4,S\rangle \langle n_3,\emptyset\rangle \\ \langle n_5,I\rangle \langle n_6,\emptyset\rangle \langle n_6,S\rangle \langle n_5,S\rangle \langle n_2,\emptyset\rangle \langle n_1,\emptyset\rangle \end{array} \right.$

2. pass $\left\{ \begin{array}{l} \langle n_1,\emptyset\rangle \langle n_2,\emptyset\rangle \langle n_3,I\rangle \langle n_4,\emptyset\rangle \langle n_4,\emptyset\rangle \langle n_3,S\rangle \\ \langle n_5,\emptyset\rangle \langle n_6,\emptyset\rangle \langle n_6,\emptyset\rangle \langle n_5,\emptyset\rangle \langle n_2,S\rangle \langle n_1,S\rangle \end{array} \right.$

□

Let t be a derivation tree and n an interior node in t. Consider a computation sequence cs for t of the form

$$cs = cs_1 \langle n, B_1\rangle cs_2 \langle n, B_2\rangle cs_3$$

where $|cs_1(n)|$ (the length of $cs_1(n)$) is even and $cs_2(n) = \lambda$ (the empty string).

The subsequence cs_2 will walk through the subtree t_n of t with root n. We say that $\langle n, B_1\rangle cs_2 \langle n, B_2\rangle$ specifies a visit to t_n and that the node n is visited once. Thus if $|cs(n)| = 2h$ then n is visited h times.

t is said to be k-visit if there exists a computation sequence cs for t satisfying that for each node n in t: $|cs(n)| \leq 2k$.

An AG G is k-visit if any derivation tree of G_u is k-visit.

Knuth introduced a subclass of AGs called well-defined AGs ([7]). If an AG is well-defined then the attributes do not depend circularly on each other in any derivation tree. The test for the well-definedness property of an AG has exponential time complexity (see [3]). It is easy to show that if the AG is well-defined then there exists a computation sequence for any derivation tree.

Let t be a derivation tree of a well-defined AG. Given a computation sequence for t we can determine the value of the attribute at the root of t. This value will be denoted the meaning of t in the semantic domain $\mathcal{D}$: meaning $(t, \mathcal{D})$. This value is independent of the choice of computation sequence (see [9]).

The translation $T(G, \mathcal{D})$ specified by the well-defined AG G is defined by

$$T(G, \mathcal{D}) = \{(t, \text{meaning}(t, \mathcal{D})) : t \text{ is a derivation tree for } G_u\}$$

Example 2.3

For the derivation tree t of Example 2.2 we have meaning$(t, \mathcal{D})$ = BA. The AG of Example 2.1 specifies the translation

$$T(G, \mathcal{D}) = \{(t, v^R) : t \text{ is a derivation tree for } v \text{ in } G_u\}$$

(v^R is the string v reversed.)

□

3. THE k-VISIT PROPERTY

In this section we will show that for any well-defined AG G there exists a k such that G is k-visit. This is done in two steps. First we show that any computation sequence can be transformed into a special kind of computation sequence called a reduced computation sequence. In the second step we use these reduced computation sequences to determine a k such that the AG is k-visit.

Definition 3.1
A reduced computation sequence for a derivation tree t is a computation sequence cs for t satisfying:

if n is an interior node in t and

$$cs = cs_1 \langle n, \emptyset \rangle\, cs_2 \langle n, \emptyset \rangle\, cs_3$$

where $|cs_1(n)|$ is even and $cs_2(n) = \lambda$
then $cs_1(n) = cs_3(n) = \lambda$.

□

Intuitively, if cs is a reduced computation sequence then we will only visit a node n when some of its attributes can be evaluated. And if there are no attributes associated with n then we will visit n at most once. Note that a reduced computation sequence need not be the shortest computation sequence for a tree.

Example 3.2
A reduced computation sequence for the tree in Example 2.2 is:

$$\langle n_1, \emptyset \rangle \langle n_2, I \rangle \langle n_5, I \rangle \langle n_6, \emptyset \rangle \langle n_6, S \rangle \langle n_5, S \rangle$$
$$\langle n_3, I \rangle \langle n_4, \emptyset \rangle \langle n_4, S \rangle \langle n_3, S \rangle \langle n_2, S \rangle \langle n_1, S \rangle$$

□

Any computation sequence for a derivation tree t can be transformed into a reduced computation sequence. To see this we will define two transformations T_1 and T_2 on computation sequences.

The purpose of both of these transformations will be to remove what we might call superfluous visits to subtrees.

The transformation T_1 can be applied to a computation sequence cs of the form

$$cs = cs_1 \langle n, \emptyset \rangle\ cs_2 \langle n, \emptyset \rangle\ cs_3 \langle n, B \rangle\ cs_4$$

where $|cs_1(n)|$ is even and $cs_2(n) = cs_3(n) = \lambda$.

Thus there will be a visit to n where no attributes of n become evaluated and there will be at least one more visit to n (note that B might be empty). The result of applying T_1 to cs is the sequence

$$cs' = cs_1\ cs_3 \langle n, B \rangle\ cs_2\ cs_4$$

It can be proved that cs' is a computation sequence for t.

Example 3.3

In the computation sequence cs of Example 2.2 we have a superfluous visit to the node n_3. We can apply the transformation T_1 to cs and get

$$\begin{array}{ll} cs_1 & \{\ \langle n_1, \emptyset \rangle \langle n_2, I \rangle \\ cs_3 & \begin{cases} \langle n_5, I \rangle \langle n_6, \emptyset \rangle \langle n_6, S \rangle \langle n_5, S \rangle \langle n_2, \emptyset \rangle \langle n_1, \emptyset \rangle \\ \langle n_1, \emptyset \rangle \langle n_2, \emptyset \rangle \end{cases} \\ \langle n, B \rangle & \langle n_3, I \rangle \\ cs_2 & \{\ \langle n_4, \emptyset \rangle \langle n_4, S \rangle \\ cs_4 & \begin{cases} \langle n_4, \emptyset \rangle \langle n_4, \emptyset \rangle \langle n_3, S \rangle \\ \langle n_5, \emptyset \rangle \langle n_6, \emptyset \rangle \langle n_5, \emptyset \rangle \langle n_2, S \rangle \langle n_1, S \rangle \end{cases} \end{array}$$

□

The transformation T_2 can be applied when the computation sequence cs has the form

$$cs = cs_1 \langle n, B \rangle\ cs_2 \langle n, \emptyset \rangle\ cs_3 \langle n, \emptyset \rangle\ cs_4$$

where $|cs_4(n)|$ is even and $cs_2(n) = cs_3(n) = \lambda$.

Thus we have a visit to n where no attributes become evaluated but there has been at least one more visit to n. The result of applying the transformation T_2 to cs is the sequence

$$cs' = cs_1\ cs_3\ \langle n, B\rangle\ cs_2\ cs_4$$

It can again be proved that cs' will be a computation sequence for t.

Using the transformations T_1 and T_2 repeatedly we get the following lemma.

Lemma 3.4

For each computation sequence for a derivation tree t there exists a reduced computation sequence for the tree.

□

Theorem 3.5

For any well-defined AG G there exists an integer k such that G is k-visit.

Proof

Let K (≥ 1) be the maximal number of attributes associated with any non-terminal of G. Let cs be a reduced computation sequence for a derivation tree t. From the definition of a reduced computation sequence it follows that $|cs(n)| \leq 2K$ for any node n of t. Thus G must be K-visit.

□

4. DECIDABILITY OF THE k-VISIT PROPERTY

Let G be a well-defined AG and let k be an integer. Is G k-visit? From Theorem 3.5 it follows that there is a K such that G is K-visit. If therefore $k \geq K$ then obviously G is k-visit. We will in this section see how to test whether G is k-visit for arbitrary $k > 0$ and thereby how to find the least k such that G is k-visit.

The following lemma is the key for proving the decidability of the k-visit property.

Lemma 4.1

For any well-defined AG G there exists a (computable) integer q such that for all derivation trees t of G_u there exists a derivation tree $\bar{t}$ of height at most q such that t is k-visit if $\bar{t}$ is k-visit.

□

Before proving the lemma we need some technical definitions.

Definition 4.2

Let t be a derivation tree and n a node in t. An outside(n) computation sequence for t is a walk

$$cs = \langle n_1, A_1\rangle \langle n_2, A_2\rangle \ldots \langle n_r, A_r\rangle$$

through t^n satisfying

4') $n_1 = n_r$ is the root of t.

5') If $n_j \neq n$ or $A_j \subseteq I(n)$ then the attributes of A_j do not depend on those in A_i for $i \geq j$, $1 \leq j \leq r$.

6') For each node n' in t^n 6a)-6e) are satisfied.

An inside(n) computation sequence for t is a walk

$$cs = \langle n_1, A_1\rangle \langle n_2, A_2\rangle \ldots \langle n_r, A_r\rangle$$

through t_n satisfying

4'') $n_1 = n_r = n$.

5'') If $n_j \neq n$ or $A_j \subseteq S(n)$ then the attributes of A_j do not depend on those of A_i for $i \geq j$, $1 \leq j \leq r$.

6'') For each node n' in t_n 6a)-6e) are satisfied.

□

Note that any pair of an outside(n) and an inside(n) computation sequence cs_o and cs_i for t, where n is an interior node, and $cs_o(n) = cs_i(n)$ can be combined into a computation sequence for t and any computation sequence is such a combination.

The concept of a reduced computation sequence can easily be generalized to outside(n) and inside(n) computation sequences.

Example 4.3

For the derivation tree t of Example 2.2 we have an outside(n_3) computation sequence

$$\langle n_1, \emptyset\rangle \langle n_2, I\rangle \langle n_3, \emptyset\rangle \langle n_3, S\rangle \langle n_5, I\rangle \langle n_6, \emptyset\rangle \langle n_6, S\rangle$$
$$\langle n_5, S\rangle \langle n_3, I\rangle \langle n_3, \emptyset\rangle \langle n_2, S\rangle \langle n_1, S\rangle$$

And we have an inside(n_3) computation sequence

$$\langle n_3, I\rangle \langle n_4, \emptyset\rangle \langle n_4, S\rangle \langle n_3, \emptyset\rangle \langle n_3, \emptyset\rangle \langle n_3, S\rangle$$

□

Let t be a derivation tree and n a node in t. We define for a reduced inside(n) computation sequence cs for t

$$visit^{(i)}(n, cs) = \langle m, cs(n) \rangle$$

where $m = \max\{\frac{1}{2} |cs(n')| : n' \text{ in } t_n\}$.

We define for a reduced outside(n) computation sequence cs for t

$$visit^{(o)}(n, cs) = \langle m, cs(n) \rangle$$

where $m = \max\{\frac{1}{2} |cs(n')| : n' \text{ in } t^n\}$

Furthermore, let

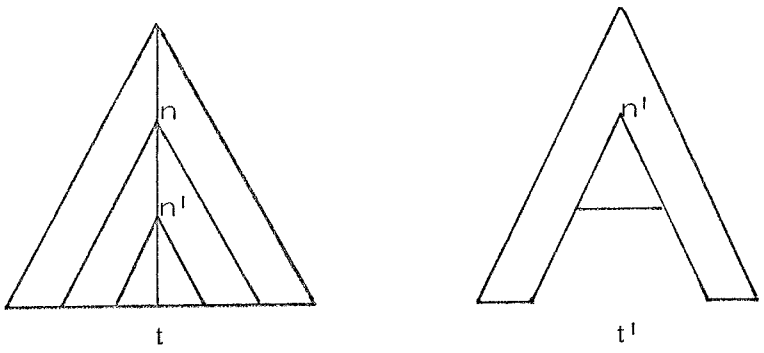
$$VISIT_t^{(i)}(n) = \{visit^{(i)}(n, cs) : cs \text{ is a reduced inside(n) computation sequence for } t\}$$
$$VISIT_t^{(o)}(n) = \{visit^{(o)}(n, cs) : cs \text{ is a reduced outside(n) computation sequence for } t\}$$

We know that the AG G is K-visit where K is the maximal number of attributes associated with any nonterminal of G. Therefore in any tuple $\langle m, B_1, B_2 \ldots B_h \rangle$ in $VISIT_t^{(i)}(n)$ (resp. $VISIT_t^{(o)}(n)$) we have $m \leq K$ and $h \leq 2m$. This means that the number of elements in $VISIT_t^{(i)}(n)$ (resp. $VISIT_t^{(o)}(n)$) is bounded by some (computable) constant V_G, which depends only on the AG G.

Consider a derivation tree t of G_u. Assume that in t we have two nodes n and n' satisfying

i) n' is a descendant of n
ii) n and n' have the same nonterminal X as label
iii) $VISIT_t^{(i)}(n) = VISIT_t^{(i)}(n')$.

Let t' be the derivation tree obtained by replacing (in t) the subtree t_n with the subtree $t_{n'}$.



We will prove that if t' is k-visit then t is k-visit. Assume that t' is k-visit. Then there exist a reduced computation sequence cs' for t' and elements $\langle m_1, cs'(n')\rangle$ in $VISIT^{(i)}_{t'}(n')$ and $\langle m_2, cs'(n')\rangle$ in $VISIT^{(o)}_{t'}(n')$ such that $k \geq \max\{m_1, m_2\}$. Since $VISIT^{(i)}_{t}(n) = VISIT^{(i)}_{t}(n') = VISIT^{(i)}_{t'}(n')$ and $VISIT^{(o)}_{t}(n) = VISIT^{(o)}_{t'}(n')$ there exist an inside(n) computation sequence cs_i for t, such that $cs_i(n) = cs'(n')$ and $\langle m_1, cs_i(n)\rangle$ is in $VISIT^{(i)}_{t}(n)$, and an outside(n) computation sequence cs_o for t such that $cs_o(n) = cs'(n')$ and $\langle m_2, cs_o(n)\rangle$ is in $VISIT^{(o)}_{t}(n)$. t is then k-visit because cs_i and cs_o can be combined to a computation sequence, which visits each node at most $\max\{m_1, m_2\} \leq k$ times.

To complete the proof of the Lemma observe that if the height of t is greater than $q = N_G \cdot 2^{V_G} + 1$ where N_G is the number of nonterminals in G_u, then there exist nodes n and n' in t satisfying i)-iii) above. By removing parts of the tree as above repeatedly we can finally obtain a derivation tree $\bar{t}$ with the required property.

□

Theorem 4.4

For any well-defined AG G and any integer k it is decidable whether G is k-visit or not.

Proof

Lemma 4.1 shows that we only have to check whether all derivation trees of height bounded by some (computable) constant are k-visit.

□

Corollary 4.5

Given a well-defined AG G it is possible to find the minimal k for which G is k-visit.

□

5. A k-VISIT HIERARCHY

In this section se will give an example (from [9]) of a translation that can be specified by a k-visit AG but which cannot be specified by any (k-1)-visit AG over the same semantic domain.

The AG will be over a semantic domain $\mathfrak{D} = (\underline{D}, \underline{F})$. $\underline{D}$ consists of a single domain $D = A^*$, i.e. sequences of A's. In $\underline{F}$ we have two semantic functions

$1_\lambda : \rightarrow D$ $\qquad$ $1_\lambda( ) = \lambda$, $\qquad$ the empty string

$\mathcal{E}_A : D \rightarrow D$ $\qquad$ $\mathcal{E}_A(a) = Aa$, $\qquad$ concatenation with A, $a \in D$

First we specify the productions of the underlying grammar G_u:

$Z ::= X$

$X ::= A\ X \mid X$

We will define an AG G_k with underlying grammar G_u which specifies the translation

$$\tau(k) = \{(t, v^{2k}) : t \text{ is a derivation tree for } v \text{ in } G_u\}.$$

The nonterminal X has associated $2 \cdot k$ attributes, k inherited and k synthesized, and Z has one synthesized attribute. Let G_k have the productions:

p1: $\langle Z \uparrow \mathcal{E}_A(a_k)\rangle ::=$

$\qquad \langle X \downarrow 1_\lambda( ) \downarrow \mathcal{E}_A(a_1) \downarrow \mathcal{E}_A(a_2) \ldots \downarrow \mathcal{E}_A(a_{k-1}) \uparrow a_1 \uparrow a_2 \ldots \uparrow a_k\rangle$

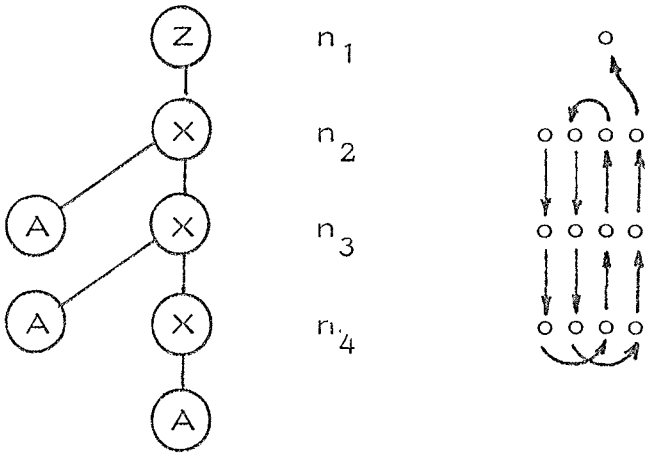
p2: $\langle X \downarrow a_1 \downarrow a_2 \ldots \downarrow a_k \uparrow \mathcal{E}_A(b_1) \uparrow \mathcal{E}_A(b_2) \ldots \uparrow \mathcal{E}_A(b_k)\rangle ::=$

$\qquad A \langle X \downarrow \mathcal{E}_A(a_1) \downarrow \mathcal{E}_A(a_2) \ldots \downarrow \mathcal{E}_A(a_k) \uparrow b_1 \uparrow b_2 \ldots \uparrow b_k\rangle$

p3: $\langle X \downarrow a_1 \downarrow a_2 \ldots \downarrow a_k \uparrow \mathcal{E}_A(a_1) \uparrow \mathcal{E}_A(a_2) \ldots \uparrow \mathcal{E}_A(a_k)\rangle ::= A$

It is easy to see that $T(G_k, \mathfrak{D}) = \tau(k)$.

As an example consider the following derivation tree (k = 2) and its "dependency graph"

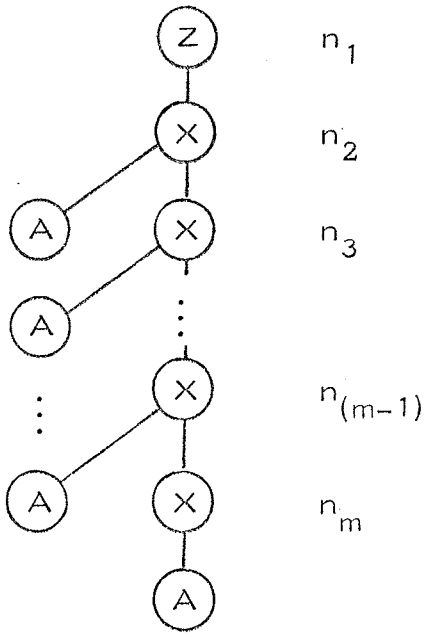


A (reduced) computation sequence for this tree is

$$\langle n_1, \emptyset\rangle \langle n_2, I_1\rangle \langle n_3, I_1\rangle \langle n_4, I_1\rangle \langle n_4, S_1\rangle \langle n_3, S_1\rangle \langle n_3, S_1\rangle$$
$$\langle n_2, I_2\rangle \langle n_3, I_2\rangle \langle n_4, I_2\rangle \langle n_4, S_2\rangle \langle n_3, S_2\rangle \langle n_2, S_2\rangle$$
$$\langle n_1, S_1\rangle$$

where I_j is the j'th inherited attribute of a node and S_j is the j'th synthesized, $1 \leq j \leq k$.

In general consider a derivation tree t of G_k.



The following will be a reduced computation sequence for t:

$$cs = \langle n_1, \emptyset\rangle\ cs_1\ cs_2 \cdots cs_k\ \langle n_1, S_1\rangle$$

where

$$cs_j = \langle n_2, I_j\rangle \langle n_3, I_j\rangle \cdots \langle n_m, I_j\rangle$$
$$\langle n_m, S_j\rangle \langle n_{(m-1)}, S_j\rangle \cdots \langle n_2, S_j\rangle.$$

Since $|cs(n)| = 2k$ for $n \neq n_1$ and $|cs(n_1)| = 2$ it is obvious that G_k is k-visit.

We claim that the translation $\tau(k)$ cannot be specified by a (k-1)-visit AG over the semantic domain $\mathfrak{D}$.

To see this, assume that G_k' is a (k-1)-visit AG over the semantic domain $\mathfrak{D}$ such that $T(G_k', \mathfrak{D}) = \tau(k)$ and let t be the derivation tree for A^m as above. The length of any reduced computation sequence for t in G_k' will be less than or equal to $2(k-1) \cdot m+2$ since each internal node is visited at most k-1 times and the root of t only once. Each application of a semantic function from $\underline{F}$ can at most increase the length of the value of the "new" attribute by one compared with the "old" one. Thus the length of meaning(t, $\mathfrak{D}$) is at most $2(k-1) \cdot m+1$ (we must start by applying $1_\lambda(\ )$) which contradicts the fact that the length equals $2k \cdot m$.

Altogether this proves the following theorem.

Theorem 5.1

There exists a semantic domain $\mathfrak{D}$ such that translations specified by k-visit AGs over $\mathfrak{D}$ define a proper hierarchy.

□

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