

Kähler–Einstein metrics on Fano manifolds*

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Abstract. This is an expository paper on Kähler metrics of positive scalar curvature. It is for my Takagi Lectures at RIMS in November of 2013. In this paper, I first discuss the Futaki invariants, the K-stability and its relation to the K-energy. Next I will outline my work in 2012 on the existence of Kähler–Einstein metrics on K-stable Fano manifolds. Finally, I will present S. Paul’s work on stability of pairs with some modifications of mine.

Keywords and phrases: Kähler–Einstein, metrics, manifold, Futaki invariant, K-stability, partial C^0 -estimate

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