

A Service of

ZBW

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Lavoie, Marc; Godley, Wynne

# Working Paper Kaleckian Models of Growth in a Stock-Flow Monetary Framework: A Neo-Kaldorian Model

Working Paper, No. 302

**Provided in Cooperation with:** Levy Economics Institute of Bard College

*Suggested Citation:* Lavoie, Marc; Godley, Wynne (2000) : Kaleckian Models of Growth in a Stock-Flow Monetary Framework: A Neo-Kaldorian Model, Working Paper, No. 302, Levy Economics Institute of Bard College, Annandale-on-Hudson, NY

This Version is available at: http://hdl.handle.net/10419/186973

#### Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

#### Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



# WWW.ECONSTOR.EU

# Working Paper No. 302

Kaleckian Models of Growth in a Stock-Flow Monetary Framework: A Neo-Kaldorian Model

by Marc Lavoie Departm ent of Econom ics, University of Ottawa, Canada and <u>W ynne Godley</u> Jerom e Levy Econom ics Institute

# June 2000

This paper integrates a stock-flow monetary accounting fram ework, as proposed by Godley and Cripps (1983) and Godley (1993, 1996, 1999), with Kaleckian models of growth, as proposed by Rowthorn (1981), Dutt (1990) and Lavoie (1995). Our stock-flow accounting is related to the social accounting matrices (SAM) originally developed by Richard Stone in Cambridge, with double-entry bookkeeping used to organise national income and flow of funds concepts. We present a consistent set of sectoral and national balance sheets where every financial asset has a counterpart liability; and budget constraints for each sector describe how the balance between flows of expenditure, factor income and transfers generate counterpart changes in stocks of assets and liabilities. These accounts are comprehensive in the sense that everything comes from somewhere and everything goes somewhere, or to put it more form ally, all stocks and flows can be fitted into matrices in which columns and rows all sum to zero. W ithout this arm ature, accounting errors may pass unnoticed and unacceptable in plications may be ignored.

The paper dem onstrates the usefulness of this fram ework when deploying a m acroeconom ic m odel, however simple. The approach was used by Godley (op. cit.) to describe an econom y which tended towards a stationary steady state, with no secular growth. In this paper, the same m ethodology is used to analyse a growing econom y.

A useful starting point for our study is the so-called neo-Pasinettim odel proposed by Kaklor (1966). In Kaklor's model, the budget constraint of the firm plays an important role in determining the macroeconom is rate of profit, for a given rate of accumulation. In addition, through his "valuation ratio", which is very similar to what later became known as Tobin's q-ratio, Kaklor provides a link between the wealth of households and the financial value of the firm s on the one hand, and the replacement value of tangible capital assets on the other.

One drawback to Kablor's 1966 "neo-Pasinetti" model, as Davidson (1968) was quick to point out, is that it does not describe a monetary economy, for Kablor assumed that households hold their entire wealth in the form of equities and hold no money deposits. This assumption gave rise to the bizarre conclusion that households 'propensity to save has no in pact on the steady-state macroeconom is profit rate, a conclusion which gave the model its name.<sup>(1)</sup> To take money into account, Davidson proposed the concept of a "marginal propensity to buy placements out of household savings" (1972, p. 272; cf. 1968, p. 263), while Skott (1981) set out explicit stock-flow norms linking the two components of wealth (money and equities) to the consumption decision. The Skott model in its various incarnations (1988, 1989) is closest to the model used here, since Skott uses explicit budget constraints with money/credit stocks for both firms and households.

Ourm odelextends Kaklor's 1966 m odelby assuming that firms obtain finance by borrowing from banks as well as by issuing equities. It includes an account of households' portfolio behaviour à la Tobin (1969) where the proportion of wealth held in the form of money baknces and equities depends on their relative rates of return. It also includes an investment function, which makes the rate of growth of the economy largely endogenous. The model is Kaleckian because, in contrast both with Cambridge models of growth à la Robinson and Kaldor, and also with classical models of growth (Shaikh 1989, Dum énil and Lévy 1999, Moudud 1999), rates of utilization in the long period are not constrained to their norm alor standard levels.<sup>(2)</sup> Ourm odelm ay be said to be neo-Kaklorian because it includes many features, such as markup pricing, endogenous growth and flexible rates of utilization, as well as endogenous credit money and exogenous interest rates, that Kaklor (1982, 1985) em phasized towards the end of his career.<sup>(3)</sup>

The first section of the paper presents our social accounting matrices while the second section gives the behavioural equations of the model. The third section describes experiments in which we explore the impact of changes in the propensity to consume, liquidity preference, the rate of interest, the rate at which securities are issued, the retention ratio and the real wage on variables such as the rate of accumulation, the rate of profit, the rate of capacity utilization, Tobin's q ratio, and the debt ratio of firm s.

#### The social accounting fram ework

We have made many drastic simplifications in the service of transparency. Our postulated economy has neither a foreign sector nor a government, while banks have zero net worth. Firm s issue no bonds, only equities, and hold no money balances, in plying that whenever firm s sellgoods, they use any proceeds in excess of outlays to reduce their bans. No bans are made to households<sup>(4)</sup> and there is no inflation.

The balance sheet matrix of this economy is presented in Table 1 while Table 2 gives the flow matrix which describes transactions between the three sectors of the economy and which distinguishes, in the case of firms and banks, between current and capital transactions. Sym bols with plus signs describe sources of funds, while negative signs indicate uses of funds. The financial balance of each sector — the gap between its income and expenditure reading each column vertically — is always equal to the total of its transactions in financial assets, so every column represents a budget constraint. The subscripts s and d have been added to relevant variables (denoting, very mughly speaking, "supply" and "dem and"), the purpose of which is to emphasise that each variable must make behavioural sense wherever it appears. The inclusion of these subscripts in no way qualifies the obvious fact that each row of the flow matrix must sum to zero; but we shall be at pains to make explicit the means by which this equivalence comes about .The watertight accounting of the model in plus that the value of any one variable is bgically in plued by all the other variables taken together. It also in plues that any one of the columns in Table 2 is bgically in plued by the sum of the other four.

In writing out our system of equations, each endogenous variable will only appear once on the LHS, facilitating the counting of equations and unknowns and making it easier for the reader

to reconstruct the whole model in his or herm ind. When a variable does appear on the LHS for a second time — therefore in an equation which is logically in plied by other equations that equation will be numbered with the suffixes A, B, etc..

Take the first column of Table 2. The regular income of households,  $Y_{hr}$ , is defined as the sum of all the positive terms of that column, wages  $W_s$ , distributed dividends  $F_D$ , and interest received on money deposits  $r_m M_{d(-1)}$ , where  $r_m$  is the rate of interest on money deposits, and  $M_{d(-1)}$  is the stock of money deposits held at the end of the previous period.

$$Y_{hr} \equiv W_{s} + F_{D} + r_{m} M_{d(-1)}$$

From the first column of Table 1, we know that the wealth, V, of households is equal to the sum of m oney holdings plus the value of equity holdings:

$$(2A) V \equiv M_d + e_d P_e$$

where  $e_d$  is the number of equities and  $p_e$  is the price of equities. We can rewrite (2A) as:

$$\Delta M_{d} \equiv V - \Delta [e_{d} p_{e}]$$

where  $\Delta$  is a first difference operator

The second term on the RHS of equation (2) can be written:

$$(2) \Delta [e_d p_e] \equiv (e_d p_e) - (e_{d(-1)} p_{e(-1)}) \equiv \Delta e_d p_e + \Delta p_e e_{d(-1)}$$

which says that the change in the value of the stock of equities is equal to the value of transactions in equities ( $\Delta e_d p_e$ ) plus capitalgains on equities held at the beginning of the period ( $\Delta p_e e_{d(-1)}$ ). The change in wealth, using column 1 of Table 2 again, can be written as:

$$(3) \Delta V \equiv Y_{hr} - C_{d} + G$$

where  $C_d$  is consumption and G, the value of the capital gains that accrue to households in the period, is given by:

$$(4) G \equiv \Delta p_e e_{d(-1)}$$

Rearranging equation (3) allows us to retrieve the Haig-Sim ons definition of income,  $Y_{hs}$ , according to which income is the sum of consumption and the increase in wealth.

(3A) 
$$Y_{hs} \equiv C_d + \Delta V \equiv Y_{hr} + G$$

The current account of the firm sector, shown in column 2 of Table 2, yields the well-known identity between national product and national income.

(1A)  $C_s + I_s \equiv W_d + F_T$ 

where  $I_s$  is investment and  $F_T$  is total profits. This equation, since it is logically implied by the other four columns of Table 2, was dropped when we came to solve the model.

Total profits  $F_T$  are made up of distributed dividends  $F_D$ , retained earnings  $F_U$ , and interest payments on bank bans  $r_1 L_{d(-1)}$ , where  $r_1$  is the rate of interest on bans  $L_{d(-1)}$  outstanding at the end of the previous period:

 $(5) F_{U} \equiv F_{T} - F_{D} \quad r_{1} L_{d(-1)}$ 

The capitalaccount of the firm sector is given in column 3 of Table 2, which shows the financial constraint of firm s:

(6) 
$$\Delta L_d \equiv I_d - F_U - \Delta e_s p_e$$

Equation (6) says that investment  $I_d$  must be financed by some combination of retained earnings, sale of new equities and additional borrowing from banks. This is the budget constraint of firms that was introduced by Kaldor (1966).

Our banking system is the simplest possible one. There is no government sector, so a fortiori there is no government debt, no high powered money, and no currency. This is a pure W icksellian credit economy, where all money takes the form of bank deposits. As an added simplification, banks do not make profits, so the rate of interest on money deposits and the rate of interest on bans are identical. With these assumptions, the banks' balance sheet is given by:

$$(7) M_{s} = L_{s}$$

while its appropriation account implies:

(8) 
$$r_{m} = r_{1}$$

2.Behavioural relationships

#### Firm s

Firm s have four categories of decision to take. They must decide what the markup on costs is going to be (see Coutts, Godley and Nordhaus 1978, and Lavoie 1992, ch. 3). In the present model, it is assumed that prices are set as a mark-up on unit direct costs which consist entirely of wages. We have a simple markup rule:

 $(9) p = (1 + P)w / \mu$ 

with p the price level, w the nominal wage rate, the markup, and where  $\mu$  is output per unit of labour such that:

(10) N  $_d \equiv S /\mu$ 

where  $N_{d}$  is the dem and for labour and output,  $S_{d}$ , is:

$$(11) S \equiv C_{s} + I_{s}$$

We shall assume that the parameters in the above equations are all constant, in plying constant unit costs and constant returns to scale. The wage rate is also assumed to be exogenous (and constant), and the markup stays the same regardless of the degree of capacity utilization. These are very strong assumptions made in order to bring a limited range of problem s into sharp focus; it will be not be difficult to amend them in a later model. We also define units in such a way that the price level is equal to unity, so that there is no difference between nom inal and real values.

Under these assumptions the main purpose of the pricing decision is to determ ine the share of income between profits and wages. For instance, since the totalwage bill is  $W_d \equiv w N_d$ , while the totalwage income of households is  $W_s \equiv w N_s$ , and since there is assumed to be an infinitely elastic supply of labour,

$$(12) N_{s} = N_{d}$$

totalprofits are given by:

(13) 
$$F_{T} = \{P / (1 + P)\}S$$

Entrepreneurs must next decide how much to produce. It is assumed that firms adapt supply to dem and within each period. This implies that sales are always equal to output, and hence aggregate supply S is exactly equal to aggregate dem and, given by the sum of consumption  $C_d$  and investment  $I_d$ . We thus have the first of our two equilibrium conditions, where equilibrium is achieved by a quantity adjustment (an instantaneous one), as is always the case in standard Keynesian or Kaleckian models:

$$(14) S = C_{s} + I_{s} = C_{d} + I_{d}$$

The third kind of decision m ade by firms concerns the quantity of capital goods that should be ordered and added to the existing stock of capital K — their investment. Because we have a growth m odel, the investment function is defined in growth rates. We shall identify the determ inants of the rate of accumulation of capital g, such that:

$$I_d = \Delta K = g K$$

Investment functions are controversial. In Kaldor (1966) there was no investment function, the growth rate being exogenous. In Joan Robinson (1956) there was an investment function, where the rate of capital accumulation depends on the expected profit rate. Some authors believe that it is more appropriate to take the rate of capacity utilization and the normal rate of profit (rather than the realized one) as the determ inants of the investment function (Bhaduri and Marglin 1990, Kurz 1990). These models usually assume away debt and money. Obviously, in a monetary model, the interest rate and the leverage ratio should play a role. The possibilities are endless.<sup>(5)</sup> We have decided to use the investment function recently tested empirically by Ndikum ana (1999). His model is inspired by the empirical work of Fazzari and Mott (1986-87), which they present as a Kalecki-Steindl-Keynes-Minsky investment function. In the Ndikum ana model, there are four variables which explain the rate of accumulation: the ratio of cash flow to capital, the ratio of interest payments to capital. Tobin's q ratio, and the rate of growth of sales. We shall use the first three of these and replace the fourth by the rate of capacity utilization, which was one of the variables in plicitly used by Fazzari and Mott.<sup>(6)</sup> Before setting out the investment function, we make the following five definitions:

The rate of capacity utilization u, which is the ratio of output to full-capacity output  $S_{fr}$ :

(16) 
$$u \equiv S / S_{fc}$$

where the capital to full capacity ratio f is defined as a constant:

$$(17) S \equiv K / \Box;$$

Tobin's q ratio, which is the financial value of the firm divided by the replacement value of its capital:

(18) 
$$q \equiv V / K \equiv (L_s + e_s p_e) / K$$
;

The leverage ratio 1, which is the debt to capital ratio of the firm s:

$$(19) l \equiv L_{d}/K ;$$

The rate of cash flow r<sub>cf</sub>, which is the ratio of retained earnings to capital:

(20) 
$$r_{cf} \equiv F_{U}/K$$

The investment function, or more precisely the rate of capital accumulation g, is given by equation (21), with  $\Upsilon_0$  comprising exogenous investment ("animal spirits") and all other  $\Upsilon$ 's being (positive) parameters. The parameters are all assumed to take effect after one period, on the assumption that investment goods must be ordered and that they take time to be produced and installed, and that entrepreneurs make their orders at the beginning of the period, when they have in perfect know ledge concerning the current period.

$$(21) g = Y_0 + Y_1 r_{cf(-1)} - Y_2 r_{1(-1)} \cdot I_{(-1)} + Y_3 q_{(-1)} + Y_4 u_{(-1)}$$

In this model, as in the model tested by Ndikum ana (1999), interest payments have two negative effects; they enter the investment function twice, once directly, but also indirectly by reducing cash flow and therefore the ability to finance investment internally. The direct effect of high interest payment commitments is to reduce the creditworthiness of firms and increase the probability of insolvency which may cause firms to slow down their expansion projects; this is because entrepreneurs will be more prudent, to ensure that they stay in business (Crotty 1996, p. 350); and banks will be more reluctant to provide bans to firms with high debt commitments.

Tobin's q- ratio is not usually incorporated into heterodox growth models with financial

variables. For instance, it is not present in the models of Taylor and O 'Connell (1985) and Franke and Semm ler (1989), although these models do have some mainstream features such as a fixed money supply. The valuation ratio, however, is to be found in the investment functions of Rimmer (1993) and of Gatti, Gallegatiand Gardini (1990). The latter refer to their investment function as a Keynes-Davidson-Minsky theory of investment determination, citing Davidson (1972) and Minsky (1975).<sup>(7)</sup> Thus, it is clear that various Post Keynesians have considered the introduction of the valuation ratio (the q -ratio) as a determinant of investment, although Kaldor him self did not believe that such a ratio would have much effect on investment.<sup>(8)</sup>

Introducing the valuation ratio may reduce the rate of accum ulation decided by entrepreneurs whenever households show little desire to save or to hold their wealth in the form of equities. As pointed out by Moore (1973, p. 543), such an effect "leads back to the neoclassical conclusions of the control of the rate of accum ulation by saver preferences, albeit through a quite different mechanism . A reward to property must be paid ... to induce wealth owners to hold voluntarily, and not to spend on current consumption, the wealth accum ulation that results from business investment". We shall see that some of the usual conclusions of Keynesian or Kaleckian models can indeed be overturned, depending on the values taken by the reaction parameters, when the valuation ratio is included as a determ inant of the investment function.

There is nothing in the model to force the q- ratio towards unity. We could have written the investment function by saying that capital accumulation is a function  $\gamma_3$  of the difference (q - 1). But this is like subtracting  $\gamma_3$  from the constant in the investment function; it does not imply q converges to unity in steady state growth. For this to happen, we would need to claim that the *change* in the rate of accumulation is a function of the difference (q - 1). Form ally, we would need to write the difference equation:  $dg = \gamma(q - 1)$ , so that g becomes a constant when q = 1. In stationary neoclassical models, this result is achieved by assuming that I = I (q - 1), as in Sargent (1979, p.10).

One may wonder where expectations enter the investment function, since (nearly) all the determ inants of investment are one-period lagged variables. For instance, in the investment functions of Taylor and O 'Connell (1985) and Franke and Semm ler (1989), the rate of accumulation depends on the current rate of profit augmented by a premium which represents expectations of future rates of profit relative to the current one. As a first step these authors assume the premium to be an unexplained constant; in elaborations of the model, the premium is an inverse function of the debt ratio. In other words, it is assumed that expected future rates of profit decline when debt ratios rise. We have a similar mechanism by virtue of the term  $\gamma_2 r_{1(-1)}$ .  $1_{(-1)}$ , on the grounds that an increase in debt commitments will slow down accumulation. In addition, a change in the exogenous term in the investment function,  $\gamma_0$ , can represent a change in expectations regarding future profitability or future sales relative to current conditions.

Finally, we consider the fourth category of decision which firm s m ust take. Once the investment decision has been taken, firm s m ust decide how it will be financed. Which variable ought to be considered as the residual one? Franke and Sem m ler (1991:336), for instance, take equity financing as a residual. However, they note that the recent literature on credit and financial constraints m ay suggest rather that "debt financing should become the residual term to close the gap between investment and equity finance", and this is exactly what will be done here.<sup>(9)</sup> Firm s borrow from the banks whatever amount is needed once they

have used up their retained earnings and the proceeds from new equity issues. As Godley (1996, p. 4), suggests, bank bans "provide residual buffer finance". This has already been given a form alrepresentation in equation (6) which gave the budget constraint of firm s:  $\Delta L_{d}$ 

$$\equiv I_d - F_U - \Delta e_s P_e.$$

We propose two behavioural equations, one determ ining the split between distributed dividends and retained earnings, and the other determ ining the amount of new equities to be issued. Distributed dividends are a fraction  $(1 - s_f)$  of profits realized in the previous

period (net of interest payments). Again, a lag is introduced on the ground that firm s distribute dividends each period on the basis of the profits earned the previous period, having in perfect know ledge of soon-to-be realized profits. It is assumed how ever that these distributed dividends are upscaled by a factor which depends on the past rate of accumulation, to take into account of the fact that the economy is continuously growing.

$$(22) F_{D} = (1 - s_{f}) (F_{T(-1)} - r_{1(-1)} L_{s(-2)}) (1 + g_{(-1)})$$

This form ulation of the dividend decision, though without the lags, can be found in Kaldor's 1966 model ( $F_D = (1 - s_f) F_T$ ). Similarly, Kaldor assumes that firms finance a percentage x of the investment expenditures, regardless of the price of equities or of the value taken by the valuation ratio.<sup>(10)</sup> This is clearly an oversim plification, but we shall adopt it as an approximation, so that:

$$\Delta e_{s} p_{e} = x.1$$

W ith the above two equations, and rem em bering that Kaklor assum es away bank debt, Kaklor (and W ood (1975)) arrives at the following determ ination of the overall rate of profit:

$$r = g. (1 - x)/s_{f}$$

where  $r = F_{T}/K$  is the overall rate of profit and where g is the exogenous rate of accumulation. This equation is the source of Kaldor's (1966) surprising belief that the rate of household saving has no impact on the rate of profit, for a given rate of growth. By contrast, when there is bank debt and money, the budget constraint (om itting time lags) is telling us that:

$$(I_{d}/K) = g = s_{f}.(F_{T} - r_{1}L_{d})/K + x.I_{d}/K + (\Delta L_{d}/L_{d})(L_{d}/K)$$

In the steady-state case where bank debts are growing at the same rate as the capital stock, i.e., when  $\Delta L_d / L_d = g$ , the equilibrium value of the rate of profit is given by a variant of Kaklor's equation:

$$r = g .(1 - x - 1)/s + r_1.1$$

Thus, in steady-state growth, the rate of profit is positively related to the rate of accumulation g and to the rate of interest on bank bans  $r_1$ .  $\frac{(11)}{(11)}$  The problem here, however, is that the debt ratio of firms, 1, can be considered as a parameter, given by history, only in the short period. In the bng period, the debt ratio is am ong the endogenous variables, to be determ ined by the model and dependent, am ong other things, on the rate of household saving

and the growth rate of the economy, so that the above expression is hardly inform a tive.  $\frac{(12)}{(13)}$ Simulations will allow us to observe the actual relationship between the rate of profit, the rate of growth and the debt ratio. $\frac{(13)}{(13)}$ 

# BANKS

Banks make bans on dem and and obviously they accept and exchange deposits as well as pay and receive interest.

$$(24) L_{s} = L_{d}$$

The equality between ban dem and and ban supply should be interpreted as representing the equality between the *effective* dem and for bans and the supply of bans.  $\frac{(14)}{}$  All credit-worthy dem ands for bans are granted in this system. In the present model, when debt comm itm ents increase, the symptoms of the crum bling credit-worthiness of firms, accompanied by a shift in the effective dem and for bans (and possibly in the notional dem and for bans), appear as a downward shift of the investment function (21), under the negative impact of the r, 1 term representing debt comm itm ents.

It would have been possible to make the rate of interest on bans a positive function of the debt ratio of firms, introducing a kind of Kaleckian effect of increasing risk, but this would have simply compounded the negative effect of high leverage ratios on investment.

# HOUSEHOLDS

Households must decide how much they wish to consume and save, thereby determ ining how much wealth they will accumulate. They must also decide the proportions of their wealth they wish to hold in the form of money and equities. We have already discussed, in the first section, the budget constraint that households face when taking these decisions. Here we focus on behaviour.

Using a modified version of the Haig-Sim ons definition of income, consumption is held to depend on expected regular household income and on capitalgains which occurred in the previous period. W hen they make their spending decisions, households still do not know exactly what their income is going to be  $\frac{(15)}{100}$ . The consumption equation is then:

$$C_{d} = a_{1} Y_{hr} * + (a_{1}/\mathbb{I}) G_{(-1)}$$

with  $0 < a_1 < 1$ , a > 1, and:

$$(26) Y_{hr}^{*} = (1 + g_{y(-1)}) (Y_{hr(-1)})$$

(27) 
$$g_{v} = \Delta Y_{hr} / Y_{hr(-1)}$$

where the \* sym bol represents expected values

Expected regular household income is assumed to depend on the realized regular household income of the previous period, and on the rate of growth,  $g_{v_i}$  of regular household income

the previous period. The implication of such a consumption function is that unexpected income increases are not spent in the current period; rather they are saved, much in line with the *disequilibrium hypothesis* put forth by Marglin (1984, ch. 17) and other non-orthodox authors. This unexpected saving is held entirely in the form of additionalm oney deposits since the allocation of wealth to equities has already been decided on the basis of expected income. Thus actualm oney balances are a residual — they constitute an essential flexible element of the system (Godley 2000, p. 18; Lavoie 1984, p. 789).

Our consumption function is nearly the same as that suggested by Kaklor (1966:318) in a footnote to his neo-Pasinettiarticle, where there is a single savings propensity for the household sector applying equally to wages, dividends and capitalgains. Here the propensity to consume applies uniform by to wages, dividends and interest income. It is doubtful whether, in a world of uncertainty, households would treat accrued capitalgains, i.e., non-realized capitalgains, on the same footing as regular income. Indeed, some empirical studies have found no relationship between consumption and contemporaneous capitalgains. How ever, "studies that have included lagged measures of capitalgains have often found a significant impact" (Baker, 1997, p. 67). As result, we have assumed that only lagged capitalgains enter the consumption function, and that a sm aller propensity to consum e applies to these gains.

It would have been possible to introduce a third element in the consumption function, namely the stock of wealth accumulated previously,  $V_{(-1)}$ , with a certain propensity to consume out of it, say  $a_2$ , an addition akin to the mainstream models of consumption (the life-cycle and the permanent income hypotheses). In models dealing with stationary steady states without growth, such an addition is a necessary requirement, because if the  $a_1$  coefficient is less than one wealth must be rising relative to income, without limit (Godley 1999, p. 396). However, in a growth model wealth is continuously growing, and hence the standard Keynesian consumption function, with  $a_1 < 1$  and  $a_2 = 0$ , is adequate. In a growing economy, equation (25), where consumption only depends on flows of regular or accrued income, stillmakes it possible to incorporate the theory of credit, money and asset allocation into that of income determination in a coherent way. We shall therefore stick with the Kaldorian consumption function for the time being.  $\frac{(16)}{2}$ 

Coming to households' portfolio choice, we follow the methodology developed by Godley (1999), and inspired by Tobin (1969).  $\frac{(17)}{2}$  It is assumed that households wish to hold a certain proportion  $\lambda_0$  of their expected wealth V \* in the form of equilies (and hence a proportion (1 -  $\lambda_0$ ) in the form of money deposits), but that this proportion is modulated by the relative rates of return on bank deposits and equilies, and by the transactions dem and form oney (related to expected household income). The two asset dem and functions are thus:

(28) 
$$(p_{e} e_{d})^{*} / V^{*} = \lambda_{0} - \lambda_{1} r_{m} + \lambda_{2} r_{e(-1)} - \lambda_{3} (Y_{hr}^{*} / V^{*})$$
  
(28A)  $M_{d}^{*} / V^{*} = (1 - \lambda_{0}) + \lambda_{1} r_{m} - \lambda_{2} r_{e(-1)} + \lambda_{3} (Y_{hr}^{*} / V^{*})$ 

where the  $\lambda$ s are parameters, where the \* symbol again represents expected values, and where  $r_{e(-1)}$  is the rate of return obtained on equities in the previous period. The rate of return on equities of the current period is defined as the ratio of dividends received plus capitalgains over the value of the stock of held equities in the previous period. (29)  $r_{e} \equiv (F_{D} + G)/(p_{e(-1)} \cdot e_{d(-1)})$ 

The two asset dem and functions are hom ogeneous in wealth, i.e., the proportions of the two assets being held does not vary in the long run with the absolute size of wealth although, by virtue of the final term in each function, there is a transactions dem and form oney which can make a temporary difference. The two asset functions sum to one because households are assumed to make consistent plans, symmetric to the adding-up condition of equation (2A). Portfolio plans, under the adding-up assumption, are thus:

$$(30) M_{d} * \equiv V * - (e_{d} p_{e}) *$$

Equation (30) in plies that one of the two asset dem and functions must be dropped for the model to solve. And this is indeed what is done in the simulations; equation (28A) describing the money dem and function has been dropped and replaced by (30).

Expected regular household income was defined by equation (26). Expected capitalgains are assumed to depend on past capitalgains and the rate of accumulation of capital in the previous period, so that:

$$(31) G^* = (1 + g_{(-1)}) (G_{(-1)})$$

On the other hand, for households to have consistent plans, the expected level of wealth must be in line with its expected budget constraint. The realized budget constraint of households was already defined by equation (3). The following equation is its equivalent, within the realm of expectations:

$$(32) V * \equiv V_{(-1)} + Y_{hr} * + G * C_{d}$$

W hen expectations and plans are fulfilled, the ratios targeted in equations (28) and (28A) will be exactly realized. In this case, the only element of flexibility resides in the price of equities  $p_e$ , since all the other elements, including e —the number of equities — are predeterm ined. The price of equities will rise until the targeted ratio is attained since there cannot be any discrepancy between the number of shares which have been issued and the number of shares that households hold. In other words, there has to be a price-clearing mechanism in the equity market, such that:

$$(33) e_{d} = e_{s}$$

W hat happens when expectations about regular income are m istaken? As pointed out above, an extra element of flexibility resides in the amount of money balances held by households. On the basis of their expectations, regardless of whether they are realized or not, households invest in the stock market in such a way that:

$$(34) p_{e} e_{d} = (p_{e} e_{d})^{*}$$

# System -wide im plications

We now have the same number of equation as unknowns, including equations in both the "dem and" (equation 2) and the "supply" of money (equation 7). So the whole model is now

cbsed and there is therefore neither a need nor a place for an equilibrium condition such as:

$$(7A) M_s = M_d$$

However, from the balance sheets of Table 1 we know that the equality between the money deposits households find them selves holding and the money deposits supplied by banks — which are equal to the bans they have made — must invariably hold. Indeed this property of the model provides a way in which its accounting bgic can, in practise, be tested. Having solved the model, we can check the accounting , using the simulations, to verify that the numbers do indeed generate  $M_s = M_d$ . It is only when an accounting error has been committed, that the equality given by (7A) will not be realized. With the accounting right, the equality must hold. And in the present model, the equality holds with no need for any asset price or interest rate adjustment.

If household income, and hence household wealth, turn out to be different from expected levels, the adjustment factor is the amount of money left with households,  $M_d$ , compared

with  $M_d * \frac{(18)}{k}$  For instance, suppose that actual household income is higher than its expected level:  $Y_{hs} > Y_{hs} * As a result, because consumption does not depend on actual$ current income, there will be a corresponding gap between the actual and expected change in $wealth: <math>\Delta V > \Delta V * As$  a consequence, the am ount of money held by households will be higher than what they expected to hold by exactly the am ount that income has been underestimated. Form ally, we have:  $\frac{(19)}{k}$ 

(2B)  $M_d = M_d \star + (Y_{hs} - Y_{hs} \star)$ 

Equation (2B) shows that the planned dem and form oney can be different from the realized one. In other words, we know that it is possible to have:  $M_s > M_d^*$ . But this has no bearing on whether or not an excess supply of money can arise. This inequality is due to mistaken expectations; it has no causal significance of its own. In particular, it cannot be said that the excess money supply, defined here as  $M_s M_d^*$ , can be a cause of an excess dem and on the goods market, or of an excess dem and on the equities market (which would push down financial rates of return).

Is it for a moment surprising that the stock of money people fetch up with, whether or not they have made wrong predictions, is identically the same amount as the bans which firms find that they have incurred — although this follows from a distinct set of decisions. Our model is so simple that it reveals with unusual clarity why this must be so. Kaldor's (1982) intuition — that there can never be an excess supply of money — is vindicated.

Kaldor's assertion has often been called into question. Some authors have noted that, because m oney deposits are created as a result of bans being granted to firm s, m oney dem and could exceed m oney supply. Coghlan (1978, p. 17) for instance, says that: "If we accept that advances can be largely exogenous ....then the possibility m ust exist that bank deposits can grow beyond the desires of m oney holders". That claim is wrong, how ever. As shown here, and as explained inform ally by Lavoie (1999), such a m isunderstanding arises as a result of ignoring the overall constraints in posed by double-entry financial bookkeeping.<sup>(20)</sup>

Finally, it should be pointed out that the seeds of our generalization of Kaldor's 1966 m odel

to a monetary economy can already be found in Joan Robinson's works (1956, 1971).<sup>(21)</sup> Robinson endorsed Kaldor's neo-Pasinettitheorem, with the proviso that "the banking system is assumed to be generating a sufficient increase in the quantity of money to offset liquidity preference" (1971, p. 123). She had argued earlier that "banks must allow the total of bank deposits to increase with the total of wealth", and that banks must "lend to entrepreneurs (directly or by taking up second-hand bonds), the difference between rentier saving and rentier lending" (Robinson 1956, p. 277).<sup>(22)</sup>

#### EXPERMENTS

The model presented above was solved numerically and subjected to a series of simulation experiments. First we assigned values to the various parameters using reasonable stylized facts. Then we solved the model, and found a steady state solution through a process of successive approximation. Having found a steady-state, we conducted experiments by modifying one of the exogenous variables or one of the economically-significant parameters of the model at a time. The advantage of this approach is that it is always possible to find out exactly why the model generates the results it does. The disadvantage is that we can only analyse local stability: we do not know if there are other equilibria, or if these other equilibria are stable. What we do show, is that over a reasonable range of parameter values, including obviously the values which we chose, the model does yield a stable solution.

We quickly discovered that the model could be run on the basis of two regimes. In the first regime, the investment function reacts less to a change in the valuation ratio — Tobin's q -ratio — than it does to a change in the rate of utilization. In the second regime, the coefficient of the q -ratio in the investment function is larger than that of the rate of utilization  $(\Upsilon_3 > \Upsilon_4)$ . The two regimes yield a large number of identical results, but when these results differ, the results of the first regime seem more intuitively acceptable than those of the second regime. For this reason, we shall call the first regime a normal regime, while the second regime will be known as the puzzling regime. The first regime also seems to be more in line with the empirical results of N dikum ana (1999) and Franke and Sem m ler (1996), who find very small values for the coefficient of the q-ratio in their investment functions, that is, their empirical results are more in line with the investment coefficients underlying the normal regime.

# CHANGES IN THE PROPENSITY TO CONSUME

Let us first consider changes in the propensity to consume. We shall spend more space on this issue, because it is a particularly touchy one, as indicated in the previous section. The paradox of savings — a higher propensity to consume or a bwer propensity to save leads to faster growth — is a crucial component of the Keynesian/Kaleckian school, in contrast to the classical/Marxian models of growth or to the neoclassical models of endogenous growth, where the opposite occurs. Here, whether the paradox of savings occurs or not depends on the value taken by the coefficient of the q -ratio in the investment function.

In the norm al regime the paradox of savings holds. An increase in the propensity to consume leads to an increase in the rate of accumulation, both in the short period and in the long period, despite the fall in the q-ratio.

The bgic of this result is the following. The increase in the propensity to consume leads to higher rates of utilization and higher rates of profit, both of which encourage entrepreneurs to increase the rate of accumulation. The higher profits of entrepreneurs allow them to reduce their dependence on debt and reduce the leverage ratio *L* All these effects are shown in Figure 1 (a), where, as in all following figures, the various series are expressed as a ratio of the steady-state base case.

On the other hand, the initial fall in savings is accompanied by a falling dem and for equilies, which initially slows down the rate of increase in the price of equilies, and hence reduces the q-ratio and the rate of return on equilies  $r_e$  (see Figure 1 (b)).<sup>(23)</sup> The initial fall in  $r_e$  increases the dem and form oney as a share of wealth. However, as profits and capitalkeep on growing, the rate of return on equilies recovers, and hence, in the new steady-state the money to wealth ratio is bower than in the previous steady-state (Figure 1 (c)). Because entrepreneurs hardly react to the fall in the q-ratio, accumulation keeps going strongly: its steady-state rate is higher than that of the initial steady-state, but it is bower than the previously achieved peak (Figure 1 (a)). The paradox of savings holds in this regime.

In the *puzzling* regime, the paradox of savings does not hold. The faster rate of accumulation initially encountered is followed by a floundering rate, due to the strong negative effect of the falling *q*-ratio on the investment function. The turnaround in the investment sector also leads to a turnaround in the rate of utilization of capacity. All this leads to a new steady-state rate of accumulation which is lower than the rate existing just before the propensity to consume was increased (see Figure 1 (d)). Thus in the *puzzling* regime, although the economy follows Keynesian or Kaleckian behaviour in the short-period, long period results are in line with those obtained in classicalm odels or in neoclassicalm odels of endogenous grow th: the higher propensity to consume is associated with a slower rate of accumulation in the steady state. In the *puzzling* regime, by refusing to save, households have the ability over the long period to undo the short-period investment decisions of entrepreneurs (Moore 1973). On the basis of the *puzzling* regime, it would thus be right to say, as Dum énil and Lévy (1999) claim, that one can be a Keynesian in the short period, but that one must hold classical views in the long period.

# CHANGES IN THE INTEREST RATE ON LOANS AND DEPOSITS

The key difference between the behaviour of the *norm al* and the *puzzling* regimes is the in pact of a change in the (real) interest rate on bans (and deposits). Recall that an increase in the interest rate has two effects on effective dem and. On the one hand, as is shown in mainstream E/LM models, an increase in the rate of interest has a negative in pact on investment. But on the other hand, an increase in interest rates has a favourable effect on consumption dem and and hence on the rate of capacity utilization, since more income is now being distributed to households. This effect is underlined in the models of stationary steady states presented by Godley (1999), where a higher interest rate leads to a higher stationary level of output. The positive effect on effective dem and, for a given level of investment, is also present in Skott (1988), in a model which is closely related to the present one.

In our model, with the chosen parameters, the negative investment effect is initially strongest in both regimes. In the normal regime the negative effect of the higher debt commitments carries over to the long period (Figure 2 (a)). However, in the *puzzling* regime, despite the heavier debt commitments due both to the higher rate of interest and the higher leverage ratio 1, an increase in interest rates eventually *drives up* the steady-state rate of accumulation to a level which exceeds the growth rate associated with the lower rate of interest (Figure 2 (b)) — a rather surprising and counter-intuitive result. This counter-intuitive result justifies the name *puzzling* which we have attributed to this second regime.

In both regimes, despite an initial downward move, the steady-state rate of utilization ends up higher than its starting value (see Figure 2 (a)). In addition, the q -ratio is quickly pushed upwards (see Figure 2 (b)), as more disposable income allows households to spend more on equities. This effect has particularly strong repercussions on capital accumulation in the second regime, which explains why the increase in the rate of interest drives up the steady-state rate of growth.

It may be also noted that in the *norm al* regime the higher lending rates of interest are associated in the long period with lower rates of return on equities, while in the *puzzling* regime there is a positive long-period link between lending rates of interest and rates of return on equities.

#### CHANGES IN THE PROPENSITY TO HOLD EQUITIES

The other experiments show little difference of behaviour between the first and second regimes. For instance, in both regimes, a shift in liquidity preference, out of money deposits and into equilies, symbolized by an increase in the  $\lambda_0$  parameter of the portfolio equations, leads to an increase in the short and the bng period rate of accumulation. The view of liquidity preference in the present model is consistent with that offered by Mott (1985-86, p. 230), according to whom "liquidity preference is a theory of the desire to hold short-versus bng-term assets". Here money deposits are the short-term asset, while equilies are the bng-term one.

Our experiments give considerable support to the Post Keynesian belief that liquidity preference, defined in a broad way, does matter in a monetary economy. The favourable in pact of bwer liquidity preference can be observed independently of any change in the confidence or animal spirits of entrepreneurs or their bankers (as proxied by the  $\lambda_0$  coefficient in the investment equation, or by the level of the real rate of interest). Our model allows us to identify the mechanisms by which pure liquidity effects can affect the real economy.

The favourable in pact of the increasing desire of households to hold equities instead of money can be attributed to two standard effects. On the one hand, the increase in the stock-dem and for equities pulls up the price of equities and creates capitalgains (Figure 3 (a)). These gains are then partly consumed, thus raising the rate of capacity utilization, and hence, in the next period, it shifts up the investment function. On the other hand, the increase in the dem and for equities pushes up the q-ratio, the increase in which also contributes to shift up the investment function. All these effects are accompanied by a lowermoney to wealth ratio and a lower debt ratio, which also contributes to the faster accumulation rate of the economy (all these effects are shown in Figure 3 (b)).

There is a feedback bop which operates as a result of the initial increase in the desire of households to hold securities; there is an acceleration in the rate of growth of the economy and the rate of utilization rises. All this drives up the rate of return on securities  $r_e$ , thus reinforcing the desire of households to reduce their money deposits relative to their overall wealth.

Mott (1985-86, p.231) asserts that "liquidity preference is governed primarily by the profitability of business". In all of our experiments, the steady-state values of the rate of accumulation and the rate of return on equities moved in the same direction. Since the demand

for equities depends on the rate of return on equity, we may say that there is indeed a link between the good performance of the economy and the preference of households for long-term assets.  $\frac{(24)}{}$ 

#### CHANGES IN REAL WAGES

A typicalKaleckian effect is also to be found in the present model Assume that there is a decrease in the markup  $\beta$ , which, *ceteris paribus*, implies that there is an increase in the realwage of workers, relative to their productivity,  $(w/p)/\mu$ .<sup>(25)</sup> This means that the *share* of wages is now higher, while that of profits is bwer. In standard Kaleckian growth models, an increase in the realwage leads to an increase in the long-period rate of accumulation and in the long-period rate of capacity utilization (Rowthorn 1981, Dutt 1990, Lavoie 1995). The same result is obtained here.

The increase in realwages leads to an increase in consumption dem and, because firm s will now be distributing more income to households, while retaining less. As a consequence, the rate of capacity utilization is pushed upwards. Note that the increase in capacity utilization willonly be felt one period later since consumption depends on expected regular household income, rather than on realized regular income.

Initially, in the short period, despite the increase in the rate of utilization, the rate of profit of businesses falls, because of the lower markup. This short-period result is in contrast with the result achieved in time-continuous Kaleckian models, because in these models everything is simultaneous, so that firms react immediately to the higher rate of utilization by speeding up their rate of accumulation, generating higher rates of profit in the process.

In the present model, by contrast, the rate of capital accumulation set by firms depends on the variables of the previous period, and as a result the increase in the rate of utilization induced by rising realwages has no immediate effect on accumulation. In later periods, however, the rate of accumulation starts recovering from the lower rate of profit initially induced by the lower markup. Over time the faster accumulation helps to improve profitability. In the long period, the rate of accumulation is much higher with higher real wages, whatever the regime of the model. In the *normal* regime, the more likely one, the rate of profit does not totally recover.<sup>(26)</sup> This last result, as pointed out above, is in contrast with the time-continuous Kaleckian models of growth. In addition, the lower markup set by firms leads to a higher debt ratio, a not-so obvious result. All these effects are shown in Figure 4.

#### CHANGES IN PARAMETERS CONTROLLED BY THE FIRMS

W hen discussing the behaviour of firms, it was assumed that firms had the ability to set the number of equities that they wished to issue each period — a rule was given according to which firms financed x % of their investment by issuing new shares — and that firms chose a retention ratio on profits (net of interest payments). W hat happens when firms decide to change these percentages? (27)

First consider the case when the x ratio is increased. Firm s issue more securities. This leads to an initial fall in the rate of growth of equity prices, and hence to a fall in the q-ratio. This fall induces a capital bas and hence a slow down in consumption dem and growth. This slow down leads to a fall in the rate of utilization and hence in the cash flow of firm s. The fall in these two determ inants of the rate of accumulation, as well as the fall in

its third determ inant — the q -ratio — lead to a perm anent slow down in the rate of accumulation, as shown in Figure 5 (a). The only positive effect of issuing more securities is that the debt ratio is reduced, but this appears to be a second-order effect.

If the model correctly describes the behaviour of a true economy, the reluctance of companies to issue equities may appear to be well founded. Larger issues of equities have detrimental effects on a monetary economy, leading to a fall in the growth rate, the rate of profit, and the rate of return on equities.

Let us now consider the case of an increase in the retention ratio of firms. This increase has two contradictory effects on effective dem and. On the one hand, it autom atically increases the cash flow which is available to firms to finance their investments, thus pushing up the investment function. In addition, firms have to borrow less, and hence can reduce their debt ratios. On the other hand, households are left with less regular income, and hence the rate of grow th of consumption dem and slows down. With the chosen parameters, the positive effects on the rate of accumulation initially overwhelm the negative ones, but over the long period an increase in the retention ratio does have a negative effect on the rate of growth of the economy. All these effects are shown in Figure 5 (b). In the steady state, there is also a negative in pact on the overall rate of profit and the rate of return on equities.

# OTHER CONSIDERATIONS

W ithin the Post Keynesian tradition, the works of Minsky (1975) have attracted a considerable amount of attention. His financial fragility hypothesis, within the context of the present model, can be represented with the help of a proxy, the leverage or debt ratio 1, which is the ratio of debt to capital (valued at production prices). Minsky has argued at length that he expects the debt ratio of firms to rise in boom s, as firms would attempt to take advantage of the more relaxed borrowing standards set by banks. During recessions, the opposite would occur, as firms would reduce the relative weight of their debt. Thus, according to Minsky and some of his followers, there is a procyclicalm ovem ent between econom is activity and debt or leverage ratios.

Lavoie and Seccameccia (2000) have argued that Minsky makes an error of composition, by confusing intentions and malizations. It may be that, during the boom, all entrepreneurs would like to increase leverage and take advantage of the more relaxed borrowing standards. Whether they actually succeed in doing so is an entirely different story, since the actual increase in debt will depend on realized profits. It may well be that realized debt ratios fall during the boom, when entrepreneurs would like them to rise, and that realized debt ratios increase during a slowdown, just when entrepreneurs would prefer to see them fall. This is what Steindl (1952) calls the paradox of debt.

The empirical work conducted on this issue has not been entirely conclusive. The consensus am ong specialists used to be that debt ratios were essentially counter-cyclical (W olfson 1994), thus favouring Minsky's financial fragility hypothesis, but the more recent econom ic cycles seem to have produced pro-cyclical debt ratios. Table 3 sum marizes the qualitative relationship between the steady state rate of accumulation and the steady state debt ratio. Clearly these relations depend on the parameter that was modified. However, the analysis of the previous subsection has shown that when entrepreneurs reduce the retention ratio  $s_{-f}$ 

and the share issuing ratio x, this leads to a long-period increase in the debt ratio of firm s. Thus, it if entrepreneurs attempt to take advantage of the more relaxed financial standards by reducing those two financial ratios, they will indeed be successful in increasing their bng-period debt ratios. In that case, Minsky is clearly correct.

However, as argued by Kalecki, the long run is simply a succession of short runs. In other words, econom iss are most of the time in a traverse from one (hypothetical) steady state to another, as Hicks would have it. W hat is the relationship between the rate of accumulation and the debt ratio during the transition between two steady states? Table 4 allows us to give some kind of answer to this question. The table was compiled by examining the graphs of various simulations. The first line gives the short-run relationship between the growth rate of capital and the debt ratio right after the parameter change has been inflicted on the economy. In most cases, the rate of accumulation and the debt ratio move in opposite ways.

The second line of Table 4 yields the short-run relationship between the rate of accum ulation and the debt ratio at the end of the transition, when the economy is actually converging towards its new steady state. In all cases, whatever the source of the disturbance, the relationship is a positive one. W henever the convergence process is actually occurring, the rate of accum ulation and the debt ratio are moving in the same direction. For instance, when there is overshooting, the deceleration of the rate of accum ulation towards its steady state position will always be accompanied by a decrease in the debt ratio.<sup>(28)</sup>

The above shows that the relationship between econom is activity and debt ratios in a dem and-led monetary economy is highly complex. With one exception, there are no simple rules, and one cannot assert that debt ratios should move one way or another in a boom. This of course explains why it is so difficult to determ ine whether debt ratios are pro-or counter-cyclical. However, we know for sure that the mechanism of convergence towards a steady state growth equilibrium requires that the rate of accumulation and the debt ratio move in the same direction at the end of the transition path. We also know that, if entrepreneurs decrease the retention ratio and the share issuing ratio in an attempt to take advantage of the more relaxed financial standards in a boom period, the new steady-state debt ratio will indeed be higher.

#### CONCLUSION

Post Keynesian economics, as reported by Chick (1995), is sometimes accused of lacking coherence, formalism, and logic. The method proposed here is designed to show that it is possible to pursue heterodox economics, with alternative foundations, which are more solid than those of the mainstream. The stock-flow monetary accounting framework provides such an alternative foundation which is based essentially on two principles. First, the accounting must be right. All stocks and all flows must have counterparts somewhere in the rest of the economy. The watertight stock flow accounting in poses system constraints which have qualitative implications. This is not just a matter of logical coherence; it also feeds into the intrinsic dynamics of the model.

Secondly, we need only assume, in contrast to neoclassical theory, a very limited am ount of rationality on the part of economic agents. Agents act on the basis of their budget constraints.<sup>(29)</sup> O therw ise, the essential rationality principle is that of adjustment. Agents react to what they perceive as disequilibria, or to the disequilibria that they take note of, by making successive corrections. There is no need to assume optimization, perfect information, rational expectations, or generalized price-clearing mechanisms.

Another feature of the present analysis is the simulation method. With simulations, a full model can be articulated and its properties ascertained and understood, without the need to mesort to meduced form s. The simulation method enables one to penetrate, with one's understanding, dynam ic models of far greater complexity than can be handled by analytic means. Indeed, even practioners of multi-dimensional stability analysis resort to simulations to figure out how their models behave (see for instance Flaschelet al 1997). Non-linearities can be easily introduced. For instance, we can program behaviour to change whenever a variable exceeds or drops below some threshold level, as in the model of Godley (1999). In that model, the steady state was stationary. It is quite possible, how ever to superpose the present model to that previous model, to obtain a growth model with highly complex but coherent features. These would include a government sector, a detailed banking sector, and consumption and production that occurs in realtime, with inventories, and with output supply not being generally equal to output dem and.

Although narration and verbal explanation are in order — indeed essential — we are suggesting a method which has much rigour and demonstrability. In our methodology we can justify every point by reference to a precise system of relationships. If others disagree, they can be challenged to say precisely what simplification or parameter is inappropriate. Every relationship can be changed, and one can find out whether the change makes any difference to the results. This method ought to be helpful to resolve some controversial issues.

The present model has provided two examples. First, our fram ework has allowed us to discuss the relationship between the supply of credit and the dem and form oney. We have shown how and why an excess supply of money can never arise. Second, we have been able to assess the relationship between the rate of growth of firms and their debt ratios, a relationship much discussed in the work of Minsky. We discovered that the relationship between those two variables is not the same depending on the stage of the transition process, and depending on whether a comparison of steady states is being made, thus showing that the behaviour of debt ratios is a complex matter, even in simple growth models.

	Households	Firm s	Banks	Σ
Money	+ M		- M s	0
Equities	+e <sub>d</sub> p <sub>e</sub>	- e , p ,		0
Capital		+ <i>K</i>		+ K
Loans		- L <sub>d</sub>	+ L s	0
Σ (net worth)	+ V	$K - (L_d + e_s p_e)$	0	+ K

Table 1: Balance sheets

# Table 2 : Transactions m atrix

	Households	Firm s		Banks			
		Current	Capital	Current	Capital		
Consum ption	- C	+ C <sub>s</sub>				0	
Investm ent		+ I <sub>s</sub>	-I <sub>d</sub>			0	
W ages	+ W _s	- W <sub>d</sub>				0	
Profits	$+ F_D$	$-(F_{U} +$	+ F <sub>U</sub>			0	
		F )					
Interest on		-		$+ r L_{s(-1)}$		0	
bans		r 1 L d (-1)					
Interest on	+			-		0	
deposits	r M d (-1)			<u>r</u> M <sub>s (-1)</sub>			
∆in bans			$+\Delta L_d$		$-\Delta L_s$	0	
$\Delta$ in money	$-\Delta M_{d}$				+ Δ <i>M</i> <sub>s</sub>	0	
Issue of	- Ae <sub>d</sub> P <sub>e</sub>		Δ			0	
equities			e <sub>s</sub> p <sub>e</sub>				
Σ	0	0	0	0	0	0	

Table 3:

Relative evolution of the rate of accumulation and the debt ratio, measured in new steady-state positions, following a change in one of the parameters

change in	a _	r 1	λο	ρ	x	s f	Υ <sub>o</sub>
dg /dl norm al	-	-	-	+	+	+	-
dg /dl puzzling	+	+	-	+	+	+	-

Table 4 :

Relative evolution of the rate of accumulation and the debt ratio, during the transition, following a change in one of the parameters.

change in	a _	r	λ <sub>o</sub>	ρ	x	s f	Ϋ́o
dg /dl right after shock	-	-	-	-	+	-	-
dg /dl near the steady-state	+	+	+	+	+	+	+

#### REFERENCES

Baker, Dean. 1997. "Conceptual and accounting issues in the analysis of saving, investment, and macroeconom ic activity." In Robert Pollin (ed.), The Macro-Econom ics of Saving, Finance and Investment . Ann Arbor: University of Michigan Press, 35-82.

Bhaduri, Am it, and Marglin, Stephen. 1990. "Unem plym ent and the realwage: the econom ic basis for contesting political ideologies." *Cam bridge Journal of Econom ics* 14,4 (December): 375-390.

Chick, Victoria. 1995. "Is there a case for Post Keynesian econom ics?" Scottish Journal of Political Economy 42,1 (February): 20-36.

Coghlan, Richard. 1978. "A new view of money." Lbyds Bank Review , July, 12-27.

Crotty, James R. 1996. "Is New Keynesian investment theory really Keynesian? Reflections on Fazzari and Variato." *Journal of Post Keynesian Econom ics* 18,3 (Spring): 333-358.

Coutts, Ken, W ynne Godley, and W . Nordhaus. 1978. Industrial Pricing in the United Kingdom . Cam bridge: Cam bridge University Press.

Dalziel, Paul 1999-2000. "A Post Keynesian theory of asset price inflation with endogenous money." Journal of Post Keynesian Economics 22,2 (Winter): 227-246.

Davidson, Paul 1968. "The Dem and and Supply of Securities and Econom ic Growth and its Implications for the Kaldor-Pasinettiversus Samuelson-ModiglianiControversy." American Econom ic Review 58 (May): 252-269.

Davidson, Paul 1972. Money and the RealWorth . London: Macm illan.

Davidson, Paul 1992. "Davidson, Paul (born 1930)." In Philip Arestis and Malcolm Sawyer (eds). A Biographical Dictionary of Dissenting Economists. Aldershot: Edward Elgar, 109-115.

DelliGatti, Domenico, Mauro Gallegati, and Laura Gardini 1990. "Realaccum ulation and financial instability: a model of profit flows, debt comm itm ents and capital asset prices." *StudiEconom ici* 41,92:101-126.

Dum énil, Gérard, and Dom inique Lévy. 1999. "Being Keynesian in the short term and classical in the long term : the traverse to classical long-term equilibrium ." *Manchester School* 67, 6 (December): 684-716.

Dutt, Am itava K. 1990. Growth, Distribution and Uneven Development . Cam bridge: Cam bridge University Press.

Dutt, Am itava K. 1995. "Internal finance and monopoly power in capitalist econom ies: a reform ulation of Steindl's growth model." *Metroeconom ica* 46,1 (February): 16-34.

Fazzari, Steven M., and Tracy L. Mott. 1986-87. "The investment theories of Kaleckiand Keynes: an empirical study of firm data, 1970-1982." *Journal of Post Keynesian Economics* 9, 2 (Winter): 171-187.

Flaschel, Peter, Reine Franke, and WilliSemmler. 1994. Dynamic Macroeconomics: Instability, Fluctuation, and Growth in Monetary Economics. Cambridge, Mass.: MIT Press.

Franke, Reiner, and Willi Semm ler. 1989. "Debt-financing of firms, stability, and cycles in a dynam icalm acroeconom ic growth model." In Willi Semm ler (ed.), *Financial Dynam ics and Business Cycles: New Perspectives*. New York: M.E. Sharpe, 38-64.

Franke, Reiner, and Willi Semm ler. 1991. "A dynam ic macroeconom ic growth model with external financing of firms: a numerical stability analysis." In Nell, Edward J., and Willi Semm ler (eds.), Nicholas Kaldor and Mainstream Econom ics: Confrontation or Convergence? London: Macmillan, 335-359.

Godley, Wynne. 1993. "Time, increasing returns and institutions in macroeconom ics." In Biasco, Salvatore, Alessandro Roncaglia, and Michele Salvati (eds.), *Market and Institutions* in Econom ic Development: Essays in Honour of Paulo Sylos Labini. New York: St. Martin's Press, 59-82.

Godley, W ynne. 1996. "Money, finance and national income determ ination: An integrated approach." W orking Paper No. 167. Annandale-on-Hudson, N.Y.: The Jerom e Levy Econom ics Institute of Bard College.

Godley, W ynne. 1999. "Money and credit in a Keynesian model of income determ ination." Cam bridge Journal of Econom ics 23, 393-411.

Godley, W ynne. 2000. *Monetary Macroeconom ics*. Manuscript. The Jerom e Levy Econom ics Institute of Bard College.

Godley, Wynne, and Cripps, Francis. 1983. Macroeconomics . London: Fontana.

Kaldor, Nicholas. 1966. "Marginal Productivity and the Macro-Economic Theories of Growth and Distribution." Review of Economic Studies 33 (October): 309-319.

Kablor, Nicholas. 1982. The Scourge of Monetarism . Oxford: Oxford University Press.

Kaldor, Nicholas. 1985. Econom ics without Equilibrium . Arm onk, N.Y.: M.E. Sharpe.

Kurz, Heinz D. 1990. "Technical change, growth and distribution: a steady-state approach to unsteady growth." In *Capital, Distribution and Effective Dem and: Studies in the Classical Approach*. Cambridge: Polity Press, 210-239.

Lavoie, Marc. 1984. "The endogenous flow of credit and the Post Keynesian theory of money." Journal of Econom ic Issues 18,3 (September): 771-797.

Lavoie, Marc. 1992. Foundations of Post-Keynesian Econom ic Analysis . Aldershot: Edward Elgar.

Lavoie, Marc. 1996. "Horizontalism, structuralism, liquidity preference and the principle of increasing risk." *Scottish Journal of Political Economy* 43,3 (August): 275-300.

Lavoie, Marc. 1998. "The neo-Pasinettitheorem in Cambridgian and Kaleckian models of growth and distribution." *Eastern Econom ic Journal* 24, 4 (Fall): 417-434.

Lavoie, Marc. 1999. "The credit-led supply of deposits and the dem and form oney: Kaldor's reflux mechanism as previously endorsed by Joan Robinson." *Cam bridge Journal of Econom ics* 23,1 (January):103-114.

Lavoie, Marc, and Mario Seccareccia. 2000 (forthcoming). "Minsky's financial fragility hypothesis: a missing macroeconomic link?" In Bellofiore, Riccardo, and Piero Ferri (eds.), The Economic Legacy of Hym an P. Minsky. Cheltenham: Edward Elgar.

Marglin, Stephen A. 1994. *Growth, Distribution, and Prices*. Cambridge, Mass.: Harvard University Press.

Marris, Robin. 1972. "Why econom ics needs a theory of the firm ." *Econom ic Journal* 82 (March): 321-352.

Minsky, Hyman P. 1975. John Maynard Keynes . New York: Columbia University Press.

Moore, BasilJ. 1973. "Som e m acroeconom ic consequences of corporate equities." *Canadian Journal of Econom ics* 6,4 (Novem ber): 529-544.

Mott, Tracy. 1985-86. "Towards a post-Keynesian form ulation of liquidity preference." Journal of Post Keynesian Econom ics 8,2 (Winter): 222-232.

Moudud, Jam ee K. 1999. "Finance in classical and Harrodian cyclical growth models." Working Paper 290. Annandale-on-Hudson, N.Y.: The Jerom e Levy Economics Institute of Bard College.

Ndikum ana, Leonce. 1999. "Debt service, financing constraints, and fixed investment: evidence from paneldata." *Journal of Post Keynesian Econom ics* 21, 3 (Spring): 455-478.

Palley, Thom as I. 1996. Post Keynesian Econom ics: Debt, Distribution and the Macro Economy. London: Macmillan.

Panico, Carb. 1993. "Two alternative approaches to financialm odelbuilding." Metroeconom ica 44,2 (June): 93-133.

Panico, Carlo. 1997. "Government deficits in Post-Keynesian theories of growth and distribution." *Contributions to PoliticalEconomy* 16:61-86.

Rimmer, R.J. 1993. Income Distribution in a Corporate Economy . Aldershot: Edward Elgar.

Robinson, Joan. 1956. The Accum ulation of Capital . London: Macm illan.

Robinson, Joan. 1971. Econom ic Heresies: Som e Old-Fashioned Questions in Econom ic Theory . London: Macmillan.

Rochon, Louis-Philippe. 2000. Credit, Money and Production: An Alternative Post-Keynesian

Approach . Cheltenham : Edward Elgar.

Row thorn, Robert E. 1981. "Dem and, realwages and econom ic grow th." Tham es Papers in PoliticalEconom y, Autum n, 1-39.

Sargent, Tom J.1970. Macroeconom is Theory . New York: Academ is Press.

Sem m ler, W illi, and Reiner Franke. 1996. "The financial-real interaction and investment in the business cycle: theories and empirical evidence." In Deleplace, Ghislain, and Edward J. Nell (eds.), Money in Motion: The Post Keynesian and Circulation Approaches . London: Macm illan, 606-634.

Shaikh, Anwar. 1989. "Accum ulation, effective dem and and Marx, Keynes and Kalecki" In Semmler, Willi (ed.), *Financial Dynam ics and Business Cycles: New Perspectives*. New York: M.E. Sharpe, 65-86.

Skott, Peter. 1981. "On the Kaldorian Saving Function." Kykbs , 34, 4:563-581.

Skott, Peter. 1988. "Finance, saving and accum ulation." Cam bridge Journal of Econom ics 12 (Septem ber): 339-354.

Skott, Peter. 1989. Conflict and Effective Dem and in Econom ic Growth . Cam bridge: Cam bridge University Press.

Steindl, Joseph. 1952. *Maturity and Stagnation in American Capitalism*. Oxford: Basil Blackwell.

Taybr, Lance, and Stephen A. O 'Connell 1985. "A Minsky crisis." *Quarterly Journal of Econom ics* 100, Supplem ent: 871-885.

Tobin, James. 1969. "A general equilibrium approach to monetary theory." *Journal of Money*, *Credit*, and Banking 1,1 (February): 15-29.

Wolfson, Martin H. 1994. Financial Crises: Understanding the U.S. Postwar Experience . London: M.E. Sharpe, 2<sup>nd</sup> edition.

Wolfson, Martin H. 1996. "A Post Keynesian theory of credit rationing." *Journal of Post Keynesian Econom ics* 18,3 (Spring): 443-470.

Wood, Adrian. 1975. A Theory of Profits . Cam bridge: Cam bridge University Press.



.



















10.0

14 14









1. The rate of profit in a Golden Age equilibrium ... will then be independent of the "personal" savings propensities.... In this way, it is sim ilar to the Pasinetti theorem .... It will hold in any steady growth state, and not only in a "bng-run" Golden Age (Kaldor 1966, p. 318).

2. As in other Kaleckian models, it will be assumed that parameters are such that the rate of capacity utilization does not exceed unity.

3. See Lavoie (1998) for an analysis of Kaldor's 1966 m odelwith endogenous rates of capacity utilization.

4. See Palley (1996) for an analysis of household debt. Of course it would be possible within the present model to suppose that households borrow to speculate on the stock market.

5. For instance, the investment function proposed by Dutt (1995) includes the cash flow ratio, the debt ratio and the rate of utilization.

6. The suggested investment function is also supported by the empirical work of Semmler and Franke (1996).

7. "[The article] Money, Portfolio Balance, Capital Accumulation and Economic Growth', written in 1965 ... presented an alternative approach to money and capital accumulation more in tune with Keynes's *General Theory* and *Treatise on Money*. This alternative to Tobin's 1965 accumulation analysis involved utilizing the forward market price for capitals (that is, the market price of existing real capital relative to the cost of producing real capital) as the relevant 'invisible hand' ratio directing the entrepreneurial determ ination of the rate of investment or disinvestment in real capital. This ratio, is of course, the equivalent of the fam ous q-ratio that Tobin was to discover in 1968". (Davidson 1992, p. 111).

8. "The stock exchange value of a company can fall to say one half of the value of the assets employed in the business. But this does not change the decision as to whether it is worthwhile to undertake some investment or not; the implicit rate of return would only become relevant to the firm 's decisions if the norm almethod of financing investment were to be the issue of ordinary shares for cash — which in fact plays a very small role. Most of the profits come from ploughed back profits, in which case the expected internal rate of return is relevant and not the implicit rate of return". (N. Kaldor, in a letter to one of the authors, Novem ber 9, 1983).

9. It is also what Flaschel, Franke and Semm ler (1997, p. 357) end up doing them selves.

10. Alternative form ulations would have been possible. For instance, Marris (1972) and Skott (1988) assume that the stock of issued securities grows at a constant rate  $g_s$ . That rate could also be assumed to be higher when the valuation ratio exceeds unity.

11. Here, because there is no price inflation, all growth rates are in real term s: the rate of interest is the real rate of interest.

12. Since the propensity to save of households has an in pact on the debt ratio 1, it also has an in pact on the rate of profit, even if there is no change in the rate of growth. Thus, as guessed by Davidson (1968, 1972), introducing money into Kaldor's neo-Pasinettim odeldoes change the main feature which gave it its name!

13. Computing the steady-state value of the debt ratio 1 yields an extraordinarily complicated equation, even in such a simple model.

14. The expression "effective" dem and for bans, to denote the dem and from credit-worthy custom ers, is utilized by both Lavoie (1992, p. 177) and Wolfson (1996, p. 466).

15. This is one of the crucial aspects which distinguish the present model from that of Skott (1988).

16. It should be pointed out, however, that Kaldorwas fully aware that wealthy households could consume without ever having to declare any taxable income. Even if a portion of realized capital gains were to become part of taxable income, these wealthy families could dodge taxation altogether by borrowing their way into consumption, getting bans for consumption purposes, secured on the basis of their large assets, thus slowly depleting their net assets. This is why Kaldorwished to have an expenditure tax replacing the income tax.

17. See Panico (1993, 1997) and Franke and Semm ler (1989, 1991) form odels that purport to integrate Tobin's portfolio adding-up constraint approach with Kaldor's growth models.

18. This assumption can be found in Godley (1996, p. 18): "It is assumed that m istaken expectations about disposable income turn up as differences in holdings of [m oney deposits] compared with what was targeted".

19. Equation (2B) is the result of combining equations (1) and (1), which, given equations (34) and (3A), arise from the following subtractions:

 $(3) V = V_{-1} + Y_{hr} + G_{-C} d (2A) V = M_{d} + e_{d} p_{e}$   $(32) V * = V_{-1} + Y_{hr} * + G * - C_{d} (30) V * = M_{d} * + (e_{d} p_{e}) *$   $(DV - V * = Y^{ss} - Y_{hr} * (D) V V * = M_{d} - M_{d} *$ 

20. By contrast, Godley (1999) shows how, in a world with a more sophisticated banking system, the path of bans and deposits can diverge. But the question of the equality between the dem and for and the supply of money is an entirely different issue.

21. See Rochon (2000, ch.4) for an overview of Robinson's unjustly neglected analysis of endogenous credit money.

22. The reader will see some similarity with Davidson's (1972, p. 335) analysis of growth when the so-called excess flow-dem and for securities is negative. See also Dalziel (1999-2000), for a symmetrical analysis when the excess flow-dem and for securities is positive.

23. Figure 1 (b) shows substantial cyclical fluctuations in the stock market, which are due to the mechanical way in which portfolio decisions are taken. Still the variables do converge to their steady-state values.

24. From the budget constraint of firms, and from the definition of the rate of return on equities, it can be shown that, in the steady-state,  $r = \{r - il + g, (q - 1)\}/(q - 1)$ .

25. In the simulations of the model, the markup p is reduced while the nom inalwage rate w is simultaneously increased, to keep output prices constant at p = 1.

26. In the *puzzling* regime, however, the steady-state rate of profit with higher realwages is much higher than that with low realwages.

27. It turns out that the regime of the modeldoes not matter.

28. Steve Keen tells us that he obtains the same results with his own simulations, based on non-linear models.

29. Other authors, mainly heterodox ones, have made use of balance sheets, to secure appropriate accounting foundations, and of Tobin's adding-up constraint, to achieve portfolio equilibrium, for instance Franke and Semm ler (1989, 1991). But while the stock matrix is given a great deal of attention, the flow matrix is sometimes left out, specially when dealing with the banking sector.